PDEs 2020/21 (winter)

1. Consider $\begin{cases} -\Delta u + cu = f & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \end{cases}$ ($\Omega \subset \mathbb{R}^n$ open, bounded, C^1)
$c > 0$
Define a notion of weak solution similarly to the Lectures.

2. Complete the inequality $\|\mathcal{F}(u)\|_{L^\infty} \leq \frac{1}{(2\pi)^{n/2}} \|u\|_{?}$

3. If $u_0(x) = \sin x$ and $v_0(x) = 0$, use d'Alembert's formula to represent the solution of
$u_{tt} = u_{xx}$ with $\underbrace{u(0,x) = u_0(x) \text{ and } u_t(0,x) = v_0(x)}_{\text{initial conditions}}$.

4. The solution of the transport problem $\begin{cases} u_t = -2u_x \\ u(0,x) = e^{-x^2} \end{cases}$ is ?

5. Give an example of a nonconstant function u which belongs to $H_0^1(\Omega)$, $\Omega = (0,1) \subset \mathbb{R}$.

6. The (~~first~~) Riemann-Green identity is ...

7. Which of the following is not a reaction-diffusion model
a) $u_t = u_{xx}$, b) $u_t = u_{xx} + u(1-u)$, c) $\begin{cases} u_t = u_{xx} + uv \\ v_t = v_{xx} - uv \end{cases}$

8. Give two examples of applications of the maximum principle (either weak or strong).

9. The heat equation with Neumann boundary conditions is ...

10. For $u \in L^1(\mathbb{R}^n) \cap L^2(\mathbb{R}^n)$ the identity $\|u\|_{L^2} = \|\mathcal{F}(u)\|_{L^2}$ is called ..... Theorem (name of the mathematician)