Dobre Delia
Grupa 131

Tema 5

1) $f:(\mathbb{Z},+)\to(\mathbb{Z},+)$, $f(x)=2x$, $\forall x\in\mathbb{Z}$.

(1) $\forall x,y\in\mathbb{Z}$, $f(x+y)=2(x+y)=2x+2y=f(x)+f(y)$

(2) $f(0)=2\cdot 0=0$, $0=$ e.n.m $(\mathbb{Z},+)$

$\Rightarrow f=$ morfism de la $(\mathbb{Z},+)$ la el însuşi

2) $(\mathbb{Z}_7,+,\cdot)$

Căutam $\hat{x}\in\mathbb{Z}_7$ a.î. $\hat{4}+\hat{x}=\hat{0}\Rightarrow\hat{x}=(-\hat{4})=\hat{3}$

$(4,7)=1\Rightarrow\hat{4}$ este inversabil în $\mathbb{Z}_7$

Căutam $\hat{y}\in\mathbb{Z}_7$ a.î. $\hat{4}\cdot\hat{y}=\hat{1}\Rightarrow\hat{x}=\hat{2}=(\hat{4})^{-1}$

3) $S=\{(x,y,z)\in\mathbb{R}^3 \mid y=x-z\}$ subspaţiu în ${}_{\mathbb{R}}\mathbb{R}^3$

Teorema de caracterizare a subspaţiului:

(i) $S\subseteq\mathbb{R}^3$

(ii) $S\neq\emptyset$

(iii) $\alpha_1,\alpha_2\in\mathbb{R}$, $a,b\in S\Rightarrow\alpha_1 a+\alpha_2 b\in S$

(i) S este formată din $(x,y,z)\in\mathbb{R}^3\Rightarrow S\subseteq\mathbb{R}^3$

(ii) $0=0-0\Rightarrow(0,0,0)\in S\Rightarrow S\neq\emptyset$

(iii) Fie $\alpha_1,\alpha_2\in\mathbb{R}$ şi $a,b\in S$

$$\alpha_1 a+\alpha_2 b=\alpha_1(a_1,a_2,a_3)+\alpha_2(b_1,b_2,b_3)$$
$$=(\alpha_1 a_1+\alpha_2 b_1,\ \alpha_1 a_2+\alpha_2 b_2,\ \alpha_1 a_3+\alpha_2 b_3)\in S\Leftrightarrow$$

$$\alpha_1 a_2+\alpha_2 b_2=\alpha_1 a_1+\alpha_2 b_1-\alpha_1 a_3-\alpha_2 b_3=\alpha_1(a_1-a_3)+\alpha_2(b_1-b_3)$$

$$\Leftrightarrow\begin{cases}a_2=a_1-a_3\\ b_2=b_1-b_3\end{cases}\Leftrightarrow\begin{cases}(a_1,a_2,a_3)\in S\\ (b_1,b_2,b_3)\in S\end{cases}\Leftrightarrow\begin{cases}a\in S\\ b\in S\end{cases}\text{ adevarat}$$

$\Rightarrow\alpha_1 a+\alpha_2 b\in S$

$\Rightarrow S$ subspaţiu în ${}_{\mathbb{R}}\mathbb{R}^3$.