Dobre Della
Grupa 131

Tema 9

1) $(GL_2(\mathbb{R}), \cdot)$ grup necomutativ

Fie $A, B \in GL_2(\mathbb{R})$.

$$A \cdot B = \begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix}\begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix} = \begin{pmatrix} a_1b_1 + a_2b_3 & a_1b_2 + a_2b_4 \\ a_3b_1 + a_4b_3 & a_3b_2 + a_4b_4 \end{pmatrix}$$

$$B \cdot A = \begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix}\cdot\begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} = \begin{pmatrix} b_1a_1 + b_2a_3 & b_1a_2 + b_2a_4 \\ b_3a_1 + b_4a_3 & b_3a_2 + b_4a_4 \end{pmatrix}$$

$\Rightarrow A \cdot B \neq B \cdot A \Rightarrow$ necomutativ

2) $(\mathbb{Z}, +, \cdot)$ inel, $(2\mathbb{Z}, +, \cdot)$ subinel,
$(2\mathbb{Z} \cup 3\mathbb{Z}, +, \cdot)$ nu e subinel.

3) $f: \mathbb{R}^3 \to \mathbb{R}^2$, $f(x_1, x_2, x_3) = (x_2 - 3x_3, 2x_1)$

Fie $x = (x_1, x_2, x_3), y = (y_1, y_2, y_3) \in \mathbb{R}^3$, $\alpha, \beta \in \mathbb{R}$.

$f(\alpha x + \beta y) = f(\alpha x_1 + \beta y_1, \alpha x_2 + \beta y_2, \alpha x_3 + \beta y_3) =$
$= (\alpha x_2 + \beta y_2 - 3\alpha x_3 - 3\beta y_3, 2\alpha x_1 + 2\beta y_1)$
$= (\alpha x_2 - 3\alpha x_3, 2\alpha x_1) + (\beta y_2 - 3\beta y_3, 2\beta y_1)$
$= \alpha(x_2 - 3x_3, 2x_1) + \beta(y_2 - 3y_3, 2y_1)$
$= \alpha f(x) + \beta f(y)$
$\Rightarrow f =$ transformare liniară

$f =$ injectivă $\Leftrightarrow \ker f = \{0_3\}$

$\ker f = \{x \in \mathbb{R}^3 \mid f(x) = 0_2\}$

$(x_2 - 3x_3, 2x_1) = (0, 0) \Leftrightarrow \begin{cases} x_2 - 3x_3 = 0 \\ 2x_1 = 0 \end{cases}$

$\Leftrightarrow \begin{cases} x_2 = 3x_3 \\ x_1 = 0 \end{cases}$

Not. $x_3 = c \in \mathbb{R} \Rightarrow \begin{cases} x_2 = 3c \\ x_3 = c \\ x_1 = 0 \end{cases}$

$\Rightarrow \ker f = \{(0, 3c, c) \mid c \in \mathbb{R}\} \Rightarrow f$ nu e injectivă