Dobre Delia
Grupa 131

Tema 10

1) Notam cu $S := \left\{ \begin{pmatrix} \hat{1} & \hat{0} \\ \hat{0} & \hat{1} \end{pmatrix}, \begin{pmatrix} \hat{1} & \hat{1} \\ \hat{0} & \hat{1} \end{pmatrix}, \begin{pmatrix} \hat{1} & \hat{0} \\ \hat{1} & \hat{1} \end{pmatrix} \right\}$

$$S \leq (GL_2(\mathbb{Z}_3), \cdot) \iff \begin{cases} a)\ S \neq \emptyset \\ b)\ s_1, s_2 \in S \Rightarrow s_1 \cdot s_2 \in S \\ c)\ s_1 \in S \Rightarrow s_1^{-1} \in S \end{cases}$$

a) $S \neq \emptyset$ evident

b) $$\underbrace{\begin{pmatrix} \hat{1} & \hat{1} \\ \hat{0} & \hat{1} \end{pmatrix}}_{\in S} \cdot \underbrace{\begin{pmatrix} \hat{1} & \hat{0} \\ \hat{1} & \hat{1} \end{pmatrix}}_{\in S} = \begin{pmatrix} \hat{2} & \hat{1} \\ \hat{1} & \hat{1} \end{pmatrix} \notin S$$

$\Rightarrow \exists\, s_1, s_2 \in S$ a.î. $s_1 \cdot s_2 \notin S$

$\Rightarrow S \not\leq (GL_2(\mathbb{Z}_3), \cdot)$

2) $(\mathbb{Z}_4, +, \cdot)$ inel comutativ cu unitate

În $\mathbb{Z}_4$: $\exists\, x, y \neq 0$ a.î. $x \cdot y = 0$

Exemplu: $x = y = \hat{2}$: $x \cdot y = \hat{2} \cdot \hat{2} = \hat{4} = \hat{0}$

$\Rightarrow \mathbb{Z}_4$ are divizori ai lui zero

$\Rightarrow$ nu e domeniu de integritate

3) $A = \begin{pmatrix} -3 & 1 \\ \alpha & -1 \end{pmatrix}$, $\alpha \in \mathbb{R}$

$\det A = \begin{vmatrix} -3 & 1 \\ \alpha & -1 \end{vmatrix} = 3 - \alpha$

$3 - \alpha = 0 \iff 3 = \alpha$.

Dacă $\alpha = 3 \Rightarrow \det A = 0 \Rightarrow \operatorname{rang} A = 1$.

Dacă $\alpha \neq 3 \Rightarrow \det A \neq 0 \Rightarrow \operatorname{rang} A = 2$.