

Test seminar 2

1) Sătim (\mathbb{Z}_n, \cdot) monoid.

$$e.n. = 1, \quad \hat{0} \cdot \hat{x} = \hat{0} \hat{x} = \hat{0}, \quad \forall \hat{x} \in \mathbb{Z}_n \Rightarrow \hat{0} \text{ nu e inversabil}$$

Deci $(\mathbb{Z}_{23}^*, \cdot)$ monoid.

$$(23, x) = 1, \quad \forall \hat{x} \in \mathbb{Z}_{23}^*$$

$\Rightarrow (\mathbb{Z}_{23}^*, \cdot)$ grup

2) $S = \{ a + b i\sqrt{3} \mid a, b \in \mathbb{Z} \}$

$$\left. \begin{array}{l} 0 = 0 + 0 i\sqrt{3}, \quad 0 \in \mathbb{Z} \Rightarrow 0 \in S \\ 1 = 1 + 0 i\sqrt{3}, \quad 0, 1 \in \mathbb{Z} \Rightarrow 1 \in S \end{array} \right\} \Rightarrow |S| \geq 2$$

$$\text{Fie } x = x_1 + x_2 i\sqrt{3}, \quad x_1, x_2 \in \mathbb{Z}$$

$$y = y_1 + y_2 i\sqrt{3}, \quad y_1, y_2 \in \mathbb{Z}$$

$$x - y = x_1 + x_2 i\sqrt{3} - y_1 - y_2 i\sqrt{3}$$

$$= (x_1 - y_1) + (x_2 - y_2) i\sqrt{3}, \quad x_1 - y_1, x_2 - y_2 \in \mathbb{Z}$$

$$\Rightarrow \forall x, y \in S, \quad x - y \in S$$

$$\text{Fie } y = y_1 + y_2 i\sqrt{3}, \quad y_1, y_2 \in \mathbb{Z}, \quad y \neq 0.$$

$$y^{-1} = \frac{\bar{y}}{y} = \frac{y_1 - y_2 i\sqrt{3}}{y_1^2 + 3y_2^2} = \frac{y_1}{y_1^2 + 3y_2^2} + \frac{-y_2}{y_1^2 + 3y_2^2} i\sqrt{3}$$

$$\text{Alegem } y_1 = 3, \quad y_2 = 0.$$

$$y = 3 + 0 i\sqrt{3}$$

$$y^{-1} = \frac{3}{9} + \frac{-0}{9} i\sqrt{3} = \frac{1}{3} + 0 i\sqrt{3} \notin S$$

$\frac{1}{3} \notin \mathbb{Z}$

$\Rightarrow S$ nu este subcorp.

$$3) f: \mathbb{R}^2 \rightarrow \mathbb{R}^2, f(x_1, x_2) = (-3x_2, x_1^2 + 2)$$

$$\text{Fie } \alpha, \beta \in \mathbb{R}, (x_1, y_1), (x_2, y_2) \in \mathbb{R}^2$$

$$\begin{aligned} f(\alpha(x_1, y_1) + \beta(x_2, y_2)) &= f(\alpha x_1 + \beta x_2, \alpha y_1 + \beta y_2) \\ &= (-3(\alpha y_1 + \beta y_2), (\alpha x_1 + \beta x_2)^2 + 2) \\ &= (-3\alpha y_1, -3\beta y_2, \underbrace{\alpha^2 x_1^2 + 2\alpha\beta x_1 x_2 + \beta^2 x_2^2 + 2}_{\substack{\text{not.} \\ A}}) \end{aligned}$$

$$\begin{aligned} \alpha f(x_1, y_1) + \beta f(x_2, y_2) &= \alpha(-3y_1, x_1^2 + 2) + \beta(-3y_2, x_2^2 + 2) \\ &= (-3\alpha y_1, -3\beta y_2, \underbrace{\alpha x_1^2 + 2\alpha + \beta x_2^2 + 2\beta}_{\substack{\text{not.} \\ B}}) \end{aligned}$$

$A \neq B \Rightarrow f$ nu e transformare liniară.