Find a grammar that generates the language

$L = \{x^{2n} \mid n \in N\}$

Prove that the languages are equal.

$n=0 \Rightarrow \varepsilon$

$n=1 \Rightarrow x^2 \Leftrightarrow xx$

$n=2 \Rightarrow x^4 = xxxx$

$n=3 \Rightarrow x^6 \Rightarrow xxxxxx$

$L(G) = \{\underbrace{x^{2n} \mid n \in N\}}_{L}$

$G = (N, \Sigma, P, S)$

$N = \{S\}$

$\Sigma = \{x\}$

$P: S \to xxS \mid \varepsilon$

<u>Proof</u> $L(G) = L$

$L \subseteq L(G)$ and $L(G) \subseteq L$

I $P(n)$ $\forall n \in N$ $x^{2n} \in L(G)$

$n=0 \Rightarrow x^{2\cdot 0} = \varepsilon \in L(G)$, $S \to \varepsilon$

Suppose $P(k)$ true $\Rightarrow x^{2k} \in L(G)$

Prove $P(k+1)$ true $\Rightarrow x^{2(k+1)} \in L(G)$

$S \Rightarrow x x S \; (x^2 S) \Rightarrow x x \, x^2 S \; (x^4 S) \Rightarrow$

$\Rightarrow x x (x^4 S) \, x^6 S \Rightarrow x^{2(k-1)} S \Rightarrow$

$\Rightarrow x^{2k} \quad (1)$

If $S \Rightarrow x^{2k} S \Rightarrow x^{2(k+1)} S \Rightarrow x^{2(k+1)} \quad (2)$

$(1), (2) \Rightarrow L \subseteq L(G)$

II $L(G) \subseteq L$

$S \Rightarrow x^2 S \Rightarrow x^4 S \Rightarrow \dots x^{2(n-1)} S \Rightarrow$

$\cancel{\Rightarrow x^{2n} S \Rightarrow x^{2n}}$

$\Rightarrow x^{2(n-1)} x x S \Rightarrow x^{2n} S \Rightarrow x^{2n} \in L$