Seminar 4   Primal simplex algorithm (PSA)

Ex 1 Solve the following problem by using (PSA)

$$(P)\quad \begin{cases} \text{Minimize } f(x) = 2x_1 - x_2 + x_3 \\ \text{s.t.} \\ \left.\begin{array}{l} -x_1 + 2x_3 + x_4 = 0 \\ 3x_1 + x_2 - x_3 = 2 \\ x_1, \dots, x_4 \geq 0 \end{array}\right] \quad (S) \end{cases}$$

Solution: Problem (P) is given in standard form
(see Lecture 7)

$$(P)\quad \begin{cases} \text{Minimize } f(x) = \langle c, x \rangle = c_1 x_1 + \dots + c_n x_n \\ Ax = b \\ x \geq 0_n \text{ (componentwise)}, \quad x = (x_1, \dots, x_n) \end{cases}$$

$$0_n = (0, \dots, 0)$$

We have

$n = 4$

$$c = (\underset{\substack{\parallel \\ 2}}{c_1}, \underset{\substack{\parallel \\ -1}}{c_2}, \underset{\substack{\parallel \\ 1}}{c_3}, \underset{\substack{\parallel \\ 0}}{c_4})$$

$$A = \begin{pmatrix} -1 & 0 & 2 & 1 \\ 3 & 1 & -1 & 0 \end{pmatrix}; \quad b = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\quad A^1 \quad A^2 \quad A^3 \quad A^4$$

$$\operatorname{rank} A = \underset{\substack{\parallel \\ 2}}{m} < \underset{\substack{\parallel \\ 4}}{n}$$

Consider the basis

$B = (A^4, A^2)$

Then the det $[A^4, A^2] = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 \neq 0$

Hence rank $A = 2$

The simplex tableau associated to B is:

B is a dual feasible basis (d.f.b)

$A^1 = (-1)A^4 + 3A^2$

		0	-1	
	1	A^4	A^2	Test d.f.b
2	A^1	-1	3	-5
1	A^3	2	-1	0
	b	$0 \geq 0$	$2 \geq 0$	-2

$[(-1)\cdot 0 + 3\cdot(-1)] - 2$

$[2\cdot 0 - 1(-1)] - 1 = 0$

Test p.f.b = primal feasibility of the basis

B is a primal feasible basis (p.f.b) (bază primal admisibilă)

$\Rightarrow$ we can use P.S.A

$$d_{i,0} = \sum_{j \in B_B} \alpha_{ij} c_j - c_i$$

$$d_{0,b} = \sum_{j \in B_B} \alpha_{0,j} c_j$$

Since b is both p.f.b and d.f.b, it follows that B is an optimal basis

An optimal solution of problem (P) is

$x^0 = x^B = (x_1^0, x_2^0, x_3^0, x_4^0)$

$= (0, 2, 0, 0)$

if L7

The optimal value (min) of f on S is:

$\alpha_{0,0} = -2$

of L7

Checking: $f(x^0) = 2x_1^0 - x_2^0 + x_3^0 = 2\cdot 0 - 2 + 0 = -2$

$f(x) = 2x_1 - x_2 + x_3$

$x^0 = (0, 2, 0, 0)$

Solve the Resource allocation Problem with the following data (see Seminar 1) by means of P.S.A.

Resources \ Products	P1	P2	Available amount of resources ~~the requested amount of nutrients~~
R1	1	2	6
R2	0	1	4
R3	3	0	9
Unit cost	1	3	

$$\begin{cases} \text{Maximize } f(x) = 1x_1 + 3x_2 \\ \text{s.t.} \\ 1x_1 + 2x_2 \geq 6 \\ 0x_1 + 1x_2 \geq 4 \\ 3x_1 + 0x_2 \geq 9 \\ x_1, x_2 \geq 0 \end{cases}$$

This problem has no standard form (because of "$\leq$" type constraints)

In order to apply the P.S.A. we will transform it into an equivalent problem whose constraints are in standard form

$$
(P)\quad \begin{cases} \text{Minimize } g(x) = -f(x) = -x_1 - 3x_2 \\ \left.\begin{aligned} x_1 + 2x_2 + x_3 &= 6 \\ x_2 \qquad + x_4 &= 4 \\ 3x_1 \qquad + x_5 &= 9 \\ x_1, \dots, x_5 &\geq 0 \end{aligned}\right] (S) \end{cases}
$$

$n = 5 \qquad m = 3,\ c = (c_1, c_2, c_3, c_4, c_5) = (-1, -3, 0, 0, 0)$

$$
A = \begin{pmatrix} 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 3 & 0 & 0 & 0 & 1 \end{pmatrix} ; \quad b = \begin{pmatrix} 6 \\ 4 \\ 9 \end{pmatrix}
$$

$\quad A^1 \quad A^2 \quad A^3 \quad A^4 \quad A^5$

Let $B = (A^3, A^4, A^5)$

		0	0	0	
	1	A^3	A^4	A^5	dfb
-1	A^1	$1 > 0$	0	$3 > 0$	$1 > 0$
-3	A^2	$2 > 0$	$1 > 0$	0	$3 > 0$ *
	rfb	$6 \geq 0$	$4 \geq 0$	$9 \geq 0$	0

$\frac{6}{2}$ * $\quad \frac{4}{2}$ $\longrightarrow$ B is a r.f.b, hence we apply P.S.A.

2	A^2	A^4	A^5	dfb
A^1	$\frac{1}{2}$			$-\frac{1}{2}$
A^3	$\frac{1}{2}$	$-\frac{1}{2}$	0	$-\frac{3}{2}$
ffl	3	1	9	-9

$-\frac{1}{2} \leq 0$, $-\frac{3}{2} \leq 0$ $\Rightarrow$ (A^2, A^4, A^5) is d.f.b. Hence an optimal basis

$\frac{1\cdot 2 - 1\cdot 3}{2}$ Gauss Jordan

Dederbarn - muzen

An optimal sol of (P) is x^0 standard $= (x_1^0, x_2^0, x_3^0, x_4^0, x_5^0) = (0, 3, 0, 1, 9)$

$\min g = \alpha_{00} = 9 \Rightarrow \max f = 9$

The sol of R.A.P is

$x^0 = (x_1^0, x_2^0) = (0, 3)$