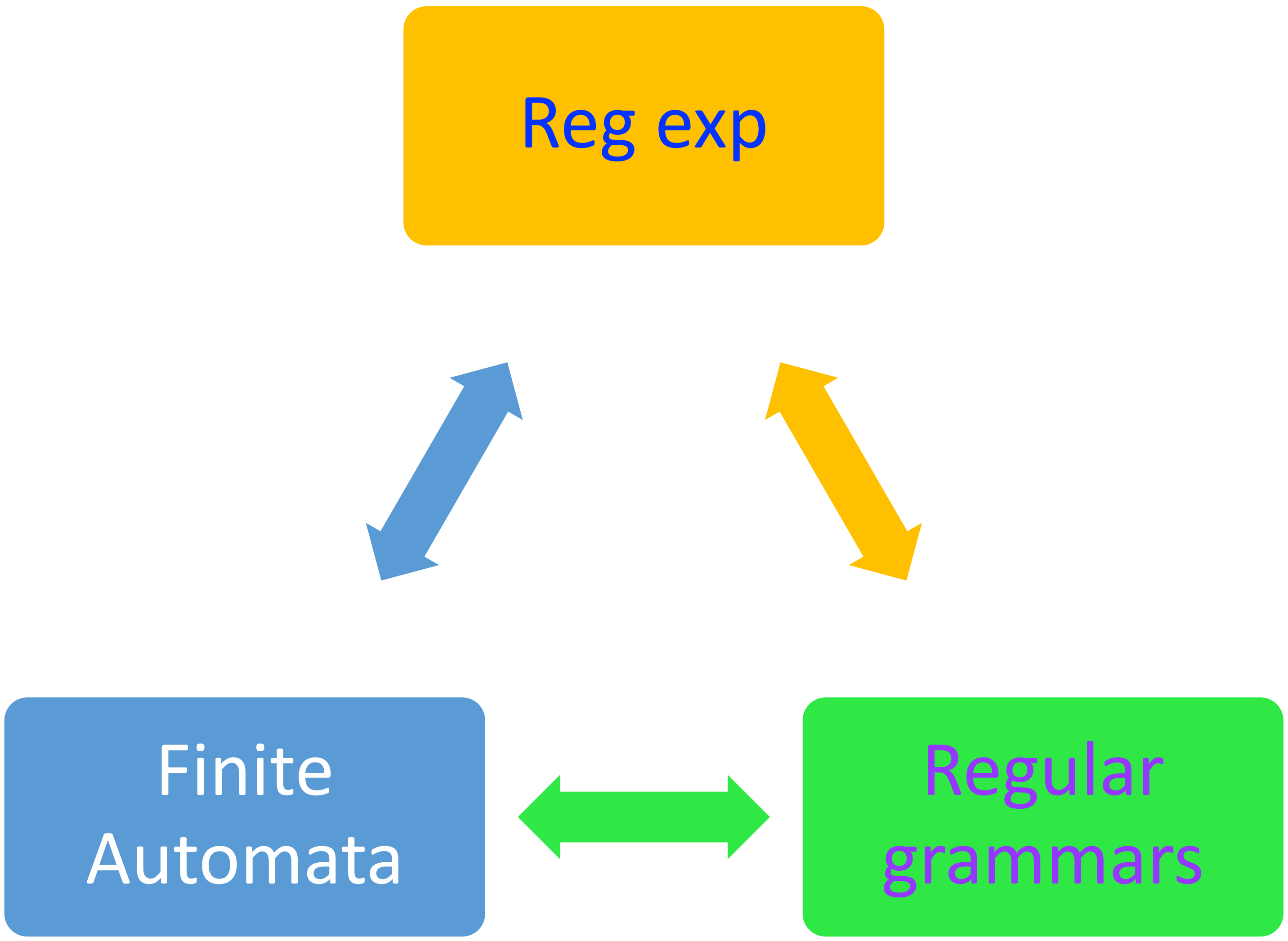
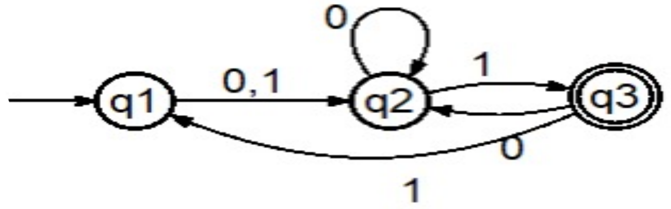
Course 5


Reg exp
Finite Automata
Regular grammars

Theorem: *A language is a regular set if and only if is accepted by a FA*



Proof:

=> Apply lemma 1' and lemma 2' (to follow, similar to RG)

<= construct a system of regular exp equations where:

- Indeterminants – states
- Coefficients – terminals
- Equation for A: all the possibilities that put the FA in state A
- Equation of the form: X=Xa+b => solution **X=ba***

$$\begin{cases} q_1 = q_3 0 + \boldsymbol{\varepsilon} \\ q_2 = q_1 0 + q_1 1 + q_2 0 + q_3 0 \\ q_3 = q_2 1 \end{cases}$$

Regular exp = union of solutions corresponding to final states

Lemma 1': $\boldsymbol{\Phi}$,{$\boldsymbol{\varepsilon}$}, {a},$\forall a \in \boldsymbol{\Sigma}$ are accepted by FA

Reg exp	FA
$\boldsymbol{\Phi}$	$M = (Q, \boldsymbol{\Sigma}, \delta, q_0, \boldsymbol{\Phi})$
$\boldsymbol{\varepsilon}$	$M = (Q, \boldsymbol{\Sigma}, \boldsymbol{\Phi}, q_0, \{q_0\})$
a,$\forall a \in \boldsymbol{\Sigma}$	$M = (\{q_0,q_1\}, \boldsymbol{\Sigma}, \{\delta(q_0,a) = q_1\}, q_0, \{q_1\})$

Lemma 2': If L_1 and L_2 are accepted by a FA then: $L_1 \cup L_2$, L_1L_2 and L_1^* are accepted by FA

Proof:

$M_1 = (Q_1, \boldsymbol{\Sigma}_1, \delta_1, q_{01}, F_1)$ such that $L_1 = L(M_1)$

$M_2 = (Q_2, \boldsymbol{\Sigma}_2, \delta_2, q_{02}, F_2)$ such that $L_2 = L(M_2)$

$M_3 = (Q_3, \boldsymbol{\Sigma}_{1U}, \delta_3, q_{03}, F_3)$

$Q_3 = Q_1 \cup Q_2 \cup \{q_{03}\}$; $\Sigma_3 = \Sigma_1 \cup \Sigma_2$

$F_3 = F_1 \cup F_2 \cup \{q_{03} \mid \text{if } q_{01} \in F_1 \text{ or } q_{02} \in F_2\}$

$\delta_3 = \delta_1 \cup \delta_2 \cup \{\delta_3(q_{03},a) = p \mid \exists \delta_1(q_{01},a) = p\} \cup$
$\{\delta_3(q_{03},a) = p \mid \exists \delta_2(q_{02},a) = p\}$

$L(M_3) = L(M_1) \cup L(M_2)$

PROOF!!! Homework

$M_4 = (Q_4, \boldsymbol{\Sigma}_4, \delta_4, q_{04}, F_4)$

$Q_4 = Q_1 \cup Q_2$; $q_{04} = q_{01}$;

$F_4 = F_2 \cup \{q \in F_1 \mid \text{if } q_{02} \in F_2\}$

$$\delta_4(q,a) = \begin{cases} \delta_1(q,a), \text{ if } q \in Q_1\text{-}F_1 \\ \delta_1(q,a) \cup \delta_2(q_{02},a) \text{ if } q \in F_1 \\ \delta_2(q,a), \text{ if } q \in Q_2 \end{cases}$$

$L(M_3) = L(M_1)L(M_2)$

PROOF!!! Homework

$M_5 = (Q_5, \boldsymbol{\Sigma}_1, \delta_5, q_{05}, F_5)$

$Q_5 = Q_1;$ $\quad q_{05} = q_{01}$

$F_5 = F_1 \cup \{q_{01}\}$

$$\delta_5(q,a) = \begin{cases} \delta_1(q,a), \text{ if } q \in Q_1 - F_1 \\ \delta_1(q,a) \cup \delta_1(q_{01},a) \text{ if } q \in F_1 \end{cases}$$

$L(M_3) = L(M_1)^*$

PROOF!!! Homework

Pumping Lemma

- Not all languages are regular
- How to decide if a language is regular or not?

- Idea: pump symbols

Example: $L = \{0^n1^n \mid n >= 0\}$

Theorem: (Pumping lemma, Bar-Hillel)

Let L be a regular language. $\exists p \in N$, such that if $w \in L$ with $|w|>p$, then

$w = xyz$, where $0<|y|<=p$

and

$xy^iz \in L, \forall i \geq 0$

Proof

L regular => $\exists$ M = (Q, $\boldsymbol{\Sigma}$, $\boldsymbol{\delta}$, q_0, F) such that L= L(M)

Let |Q| = p

$$\left.\begin{array}{l} \text{If } w \in L(M)\text{: } (q_0,w) \vdash^{*} (q_f , \boldsymbol{\varepsilon}) , q_f \in F \\ \qquad\quad \text{and} \\ |w|>p \end{array}\right\} \begin{array}{l} \text{process at least } p+1 \text{ symbols} \\ \\ p \text{ states} \end{array}$$

$\Rightarrow \exists$ q_1 that appear in at least 2 configurations

$(q_0,xyz) \vdash^{*} (q1,yz) \vdash^{+} (q1,z) \vdash^{*} (q_f , \boldsymbol{\varepsilon}) , q_f \in F \Rightarrow 0 <= |y| <= p$

Proof (cont)

$(q_0, xy^iz) \overset{*}{\vdash} (q_1, y^iz)$

$\overset{*}{\vdash} (q_1, y^{i-1}z)$

$\overset{*}{\vdash} \ldots$

$\overset{*}{\vdash} (q_1, yz)$

$\overset{*}{\vdash} (q_1, z)$

$\overset{*}{\vdash} (q_f, \boldsymbol{\varepsilon}), q_f \in F$

So, if $w=xyz \in L$ then $xy^iz \in L$, for all $i>0$

If $i=0$: $(q_0, xz) \overset{*}{\vdash} (q_1, z) \overset{*}{\vdash} (q_f, \boldsymbol{\varepsilon}), q_f \in F$

Example: $L = \{0^n1^n \mid n >= 0\}$

Suppose L is regular => $w = xyz = 0^n1^n$

Consider all possible decomposition =>

Case 1. $y = 0^k$

$xyz = 0^{n-k}0^k1^n$; $xy^iz = 0^{n-k}0^{ik}1^n \notin L$

Case 2. $y = 1^k$

$xyz = 0^n1^k1^{n-k}$; $xy^iz = 0^n1^{ik}1^{n-k} \notin L$

Case 3. $y = 0^k 1^l$

$xyz = 0^{n-k}0^k1^l1^{n-l}$; $xy^iz = 0^{n-k}(0^k1^l)^i1^{n-l} \notin L$

Case 4. $y = 0^k 1^K$

$xyz = 0^{n-k}0^k1^k1^{n-k}$; $xy^iz = 0^{n-k}0^k1^k0^k1^k\ldots1^{n-l} \notin L$

=> L is not regular

Context free grammar (cfg)

- Produtions of the form: A $\rightarrow\alpha$, A∈N, α∈(N∪$\boldsymbol{\Sigma}$)*
- More powerful
- Can model programming language:

  G = (N, $\boldsymbol{\Sigma}$,P,S) s.t. L(G) = programming language

Syntax tree

Definition: A syntax tree corresponding to a cfg G = (N, Σ,P,S) is a tree obtained in the following way:

1. Root is the starting symbol S
2. Nodes ∈ N∪Σ:
    1. Internal nodes ∈N
    2. Leaves ∈Σ
3. For a node A the descendants in order from left to right are $X_1, X_2, ..., X_n$ only if $A \rightarrow X_1X_2... X_n \in P$

Remarks:

a) Parse tree = syntax tree – result of parsing (syntatic analysis)

b) Derivation tree – condition 2.2 not satisfied

c) Abstract syntax tree (AST) ≠ syntax tree (semantic analysis)

Syntax tree (cont)

Property: In a cfg $G = (N, \boldsymbol{\Sigma}, P, S)$, $w \in L(G)$ <u>if and only if</u> there exists a syntax tree with frontier w.

Proof: HW

Definition: A cfg G = (N, $\boldsymbol{\Sigma}$,P,S) is ambigous if for a w ∈ L(G) there exists 2 distinct syntax tree with frontier w.

Example:

Parsing (syntax analysis) modeled with cfg:

cfg G = (N, $\boldsymbol{\Sigma}$,P,S):

- N – nonterminal: syntactical constructions: declaration, statement, expression, a.s.o.
- $\boldsymbol{\Sigma}$ – terminals; elements of the language: identifiers, constants, reserved words, operators, separators
- P – syntactical rules – expressed in BNF – simple transformation
- S – syntactical construct corresponding to program

THEN

Program syntactical correct <=> w ∊ L(G)

Back to compiler construction