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Grupa 131

# Tema 7

1) $f: \mathbb{Z}_{12} \to \mathbb{Z}_{12}, \quad f(x) = \hat{2}x$

$\ker f = \{x \in \mathbb{Z}_{12} \mid f(x) = \hat{0}\} = \{x \in \mathbb{Z}_{12} \mid \hat{2}x = \hat{0}\}$
$= \{\hat{0}, \hat{6}\}$

2) $(\mathbb{Z}_{12}, +, \cdot)$

$U(\mathbb{Z}_{12}) = \{\hat{a} \in \mathbb{Z}_{12} \mid (a, 12) = 1\} = \{\hat{1}, \hat{5}, \hat{7}, \hat{11}\}$

$\hat{1}$ = e.n. în $U(\mathbb{Z}_{12})$

$\hat{5} \cdot \hat{7} = \widehat{35} = \widehat{11}$, $\hat{5} \cdot \widehat{11} = \widehat{55} = \hat{7}$, $\hat{7} \cdot \widehat{11} = \widehat{77} = \hat{5}$

$\Rightarrow$ parte stabilă

Toate elementele sunt inversabile.

$\Rightarrow (U(\mathbb{Z}_{12}), \cdot)$ grup.

3) $P_1[\mathbb{R}] = \{f = a_0 + a_1 X \mid a_0, a_1 \in \mathbb{R}\}$

$P_1 \leq_{\mathbb{R}} \mathbb{R}[X] \Leftrightarrow \begin{cases} a) P_1[\mathbb{R}] \neq \emptyset \ \checkmark \\ b) f_1, f_2 \in P_1[\mathbb{R}] \Rightarrow f_1 - f_2 \in P_1[\mathbb{R}] \\ c) \alpha \in \mathbb{R}, f \in P_1[\mathbb{R}] \Rightarrow \alpha f \in P_1[\mathbb{R}] \end{cases}$

Fie $\alpha = 0 \in \mathbb{R} \Rightarrow \alpha f = 0 \cdot f = 0 \notin P_1[\mathbb{R}] \Rightarrow$

$\Rightarrow P_1[\mathbb{R}] \not\leq_{\mathbb{R}} \mathbb{R}[X]$

# Tema 8

1) $f:(\mathbb{Z},+) \to (\mathbb{Z},+)$, $f(x)=3x$, $\forall x \in \mathbb{Z}$.

(1) $\forall x,y \in \mathbb{Z}$ : $f(x+y) = 3(x+y) = 3x+3y = f(x)+f(y)$

(2) $f(0) = 3\cdot 0 = 0$

$\Rightarrow$ f morfism de la $(\mathbb{Z},+)$ la el însuşi.

2) $(\mathbb{Z}_n,+,\cdot)$, $n \geq 2$, inel comutativ cu unitate.

În $\mathbb{Z}_n$ : $\exists x,y \neq 0$ cu $x\cdot y = 0 \Rightarrow$

$\Rightarrow \mathbb{Z}_n$ are divizori ai lui zero

$\Rightarrow \mathbb{Z}_n$ nu e domeniu de integritate.

3) $f:\mathbb{R}^3 \to \mathbb{R}^2$ transformare liniară

$\dim \ker f = 1$.

$\dim \mathbb{R}^3 = \dim \ker f + \dim \operatorname{Im} f$.

$3 = 1 + \dim \operatorname{Im} f$.

$\left.\begin{array}{l} \dim \operatorname{Im} f = 3-1 = 2 \\ \dim \mathbb{R}^2 = 2 \end{array}\right\} \Rightarrow$

$\Rightarrow \dim \operatorname{Im} f = \dim \mathbb{R}^2$

$\Rightarrow$ f e surjectivă.