1. $_{\mathbb{R}}V$ $\quad B=(v_1,v_2,v_3)$ bază în V

$$u_1 = v_1 + 2v_2 + v_3$$
$$u_2 = v_1 + v_2 + 2v_3$$
$$u_3 = v_1 + v_2$$

$f \in End_{\mathbb{R}}(V)$

1) $B' = (u_1, u_2, u_3)$ bază în $_{\mathbb{R}}V$

2) $[f]_{B} = ?$

$$[f]_{B'} = \begin{array}{c} \begin{pmatrix} 1 & 1 & 3 \\ 0 & 5 & -1 \\ 2 & 7 & -3 \end{pmatrix} \\ \begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ f(u_1) & f(u_2) & f(u_3) \end{array} \end{array}$$

coord lui $f(u_1)$ $f(u_2)$ $f(u_3)$ în baza B'

coef lui u_1, u_2, u_3

$$S = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 1 & 2 & 0 \end{pmatrix}$$

1) $B' = B \cdot [T]_{BB'}$

S-tranziție matrix $\quad \boxed{B' = B \cdot S}$

$$(u_1, u_2, u_3) = (v_1, v_2, v_3) \cdot S$$

B' bază $\Leftrightarrow$ S inversabilă (în $M_3(\mathbb{R})$)

$\Leftrightarrow$ $\det S \neq 0$.

2) $S \cdot \Big| [f]_{B'} = S^{-1} \cdot [f]_B \cdot S \Big| \cdot S^{-1}$

$\Rightarrow$ $[f]_B = S[f]_{B'} \cdot S^{-1}$

$B = B' \cdot S^{-1}$

$$[f]_B = S \cdot [f]_{B'} \cdot S^{-1}$$

2) $(G \ \cdot)$ - grup $\quad H, K \leq G$

a) $H \cap K$ - subgrup în G

b) Un ex în care H K nu e subgrup în G.

· $1 \in H \cap K$

$\forall x, y \in H \cap K \quad xy^{-1} \overset{?}{\in} H \cap K$ (a)

$$\left.\begin{array}{l} x, y \in H \\ x, y \in K \end{array}\right\} \xrightarrow[\text{a subgr.}]{\text{T. carac.}} \left.\begin{array}{l} xy^{-1} \in H \\ xy^{-1} \in K \end{array}\right\} \Rightarrow x \cdot y^{-1} \in H \cap K$$

b) $(\mathbb{Z}, +)$-grup abelian

$2\mathbb{Z} \leq \mathbb{Z}$

$3\mathbb{Z} \leq \mathbb{Z} \qquad 3\mathbb{Z} \cup 2\mathbb{Z} \overset{?}{\leq} \mathbb{Z}$

$$\underset{\substack{\cap \\ 2\mathbb{Z} \\ \cap \\ 2\mathbb{Z}+3\mathbb{Z}}}{2} + \underset{\substack{\cap \\ 3\mathbb{Z} \\ \cap \\ 2\mathbb{Z}+3\mathbb{Z}}}{3} = 5 \notin 2\mathbb{Z} \cup 3\mathbb{Z}$$

nu e multiplu de 2 sau 3.

În general $H \cup K \not\leq G$

* $H \subset K \quad K \subset H$ - e subgrup.

3.5.1.

$(\mathbb{R}_+^*; \cdot)$ – gr. abelian $\quad \mathbb{R}$ v.s. $\quad (\mathbb{R}_+; \cdot)$ ← de aici iau op. între scalari

$$\alpha * x = x^\alpha, \quad \alpha \in \mathbb{R} \quad x \in \mathbb{R}_+^*$$

corp – division ring

${}_{\mathbb{R}}\mathbb{R}_+^*$ - isomorphic to $\mathbb{R}$ - v.s. def. on $(\mathbb{R}+)$ by the usual $+$ and $\cdot$.

$\cdot : \mathbb{R} \times \mathbb{R} \to \mathbb{R} \quad (\alpha, x) \mapsto \alpha x$

Fie $(K; +; \cdot)$ $\underbrace{\text{corp comutativ}}_{\text{field}}$,

$(V; \underline{+})$ - grup abelian și $\underset{\sim}{\cdot} : K \times V \to V$.

Spunem că V este un K v.s. if

1) $\alpha \underset{\sim}{\cdot} (x \underline{+} y) = \alpha x + \alpha y$

2) $(\alpha + \beta) x = \alpha x + \beta x \qquad \forall \alpha, \beta \in K$

3) $(\alpha \cdot \beta) \cdot x = \alpha \cdot (\beta x) \qquad \forall x, y \in V.$

4) $1 \cdot x = x$

• Fie $x, y \in \mathbb{R}_+^* \quad \alpha, \beta \in$

1) $\alpha * (x \cdot y) = (\alpha * x) \cdot (\alpha * y)$

$$\alpha * (x \cdot y) = (xy)^\alpha = x^\alpha \cdot y^\alpha = (\alpha * x) \cdot (\alpha * y)$$

2) $(\alpha + \beta) * x = \alpha * x \cdot \beta * y$

$$(\alpha + \beta) * x = x^{\alpha + \beta} = x^\alpha \cdot x^\beta = (\alpha * x)(\beta * x)$$

3) $(\alpha \cdot \beta) * x \overset{?}{=} \alpha * (\beta * x)$

$$(\alpha \cdot \beta) * x = x^{\alpha \cdot \beta} = (x^\beta)^\alpha = \alpha * (x^\beta) = \alpha * (\beta * x)$$

4) $1 * x$

$f : \mathbb{R} \to \mathbb{R}_+^*$ inj + ~~morfism~~ (transf. liniară) = linear map. } isomor phism

$f(x) = e^x$

$f(x+y) = e^{x+y} = e^x \cdot e^y = f(x) \cdot f(y) \quad \forall\, x, y \in \mathbb{R}$

$f(\alpha x) = e^{\alpha x} = (e^x)^\alpha = \alpha * e^x = \alpha * f(x)$

$\forall\, \alpha \in \mathbb{R} \quad \forall\, x \in \mathbb{R}$