Seminar 13 04.04.2022

• Exercise 1 : The data set are independent:

40 computers with old battery $\overline{X_1} = 3.9$ = the average duration of old battery

$S_1 = 0,1$ = standard deviation
" how shattered the data is

30 computers with new battery $\overline{X_2} = 4$
$S_2 = 0.2$

Old battery life is X_1, $E(x_1) = \mu_1$
New battery life is X_2, $E(x_2) = \mu_2$

characteristic - we don't know to much (0 or a little)
sample statistic - we know about it

Accept or reject "the new batteries lasts longer than the ~~[illegible]~~ old ones"

$\underbrace{\text{the new batteries lasts longer than the old ones}}_{\text{hyphothesis}}$

$H_0 = \mu_1 - \mu_2 = 0$ (they lasts the same)

$H_1 = \mu_1 - \mu_2 < 0$

left-tailed test

a) $\sigma_1 = \sigma_2$
b) $\sigma_1 \neq \sigma_2$ } the probab } unknown (see table $\mu_1 - \mu_2$ second)

$$Z = \frac{\overline{X_1} - \overline{X_2} - (\mu_1 - \mu_2)}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \in T(n_1 + n_2 - 2)$$

$$s_p = \sqrt{\frac{(n_1 - 1) \cdot S_1^2 + (n_2 - 1) \cdot S_2^2}{n_1 + n_2 - 2}}$$

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$n_1 = 40$ $\Rightarrow S_p = \sqrt{\dfrac{39 \cdot \frac{1}{100} + 29 \cdot \frac{4}{100}}{68}} = \sqrt{\dfrac{39+116}{68}} \cdot \dfrac{1}{10}$

$n_2 = 30$

$= \dfrac{1}{10} \cdot \sqrt{\dfrac{155}{68}}$

$$Z_0 = \frac{3.9 - 4 - \overbrace{(\mu_1 - \mu_2)}^{=0}}{\underbrace{\frac{1}{10} \cdot \sqrt{\frac{155}{68}}}_{S} \cdot \sqrt{\frac{1}{40} + \frac{1}{30}}}$$

$$Z_0 = \frac{-1}{\sqrt{\frac{155}{68}} \sqrt{\frac{7}{120}}} \in T(n_1 + n_2 - 2) = -2.7424$$

RR = rejection region $= \begin{cases} (-\infty, z_\alpha], & \text{left-tailed} \\ [z_{1-\alpha}, \infty), & \text{right-tailed} \\ (-\infty, z_{\frac{\alpha}{2}}] \cup [z_{1-\frac{\alpha}{2}}, \infty), & \text{two-tailed} \end{cases}$

$z_\alpha = z_{0.05} =$ tinv $(0.05, 68) = -1,66$

$z_0 = -2,7424 \in (-\infty, -1,66) = RR$

$\in RR$

$\Rightarrow$ We reject H_0 and accept H_1 $\Rightarrow$ do the same ↑ for H_0

case b

$$\boxed{\begin{gathered} \frac{1}{n} = \frac{c^2}{n_1 - 1} + \frac{(1-c)^2}{n_2 - 1} \\ c = \frac{\frac{S_1^2}{n_1}}{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}} \end{gathered}}$$

$H_0: \mu_1 - \mu_2 = 0$

$H_1: \mu_1 - \mu_2 < 0$ left tailed

$$z = \frac{\overline{X_1} - \overline{X_2} - (\mu_1 - \mu_2)}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} \in T(n)$$

$n_1 = 40$

$n_2 = 30$

$$z_0 = \frac{3.9 - 4 - \overbrace{(\mu_1 - \mu_2)}^{=0}}{\sqrt{\frac{0.1}{40} + \frac{0.2}{30}}} = \frac{-1}{\sqrt{\frac{1}{40} + \frac{2}{30}}} = \frac{-1}{\sqrt{\frac{19}{120}}} = -2.51$$

$RR = (-\infty, z_\alpha]$

$z_\alpha = -1,6840$ ← tinv(α, n) ...

$$c = \frac{\frac{S_1^2}{n_1}}{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}} = \frac{\frac{(0.1)^2}{40}}{\frac{(0.1)^2}{40} + \frac{(0.2)^2}{39}} = \frac{\frac{\frac{1}{100}}{40}}{\frac{\frac{1}{100}}{40} + \frac{\frac{4}{100}}{39}} \overset{(2)}{=} \frac{\frac{1}{4}}{\frac{1}{4} + \frac{16}{3}} = \frac{3}{19}$$

$$\frac{1}{n} = \frac{\left(\frac{3}{19}\right)^2}{39} + \frac{\left(\frac{16}{19}\right)^2}{29} \iff n = 39.845$$

$\Rightarrow z \in T(39.8)$

$\Rightarrow -2.51 \in (-\infty, -1,6840) \Rightarrow$ we reject H_0 and accept H_1

• <u>Exercise 2</u>:

The data set is <u>dependent</u>

Same people before / after taking this drug.

b: before: 145 147 152 145 156 141 151 148 144 15.

a: after: 140 149 142 144 152 143 145 140 142 150

$n = 10$ patients

$$Z = \frac{\bar{X}_d - (\mu_1 - \mu_2)}{\frac{S_d}{\sqrt{m}}} \sim T(m-1)$$

$D = b - a =$ the vector of differences

$D = 5 \; -2 + 10 + 1 + 4 - 2 \; 6 + 8 + 2 + 1$

$\bar{X}_d = \text{mean}(D) = \frac{D}{10} = 3.30$

$S_d = \text{std}(D) =$ standard deviation $= 4.029$

If we want to use significance testing

$H_0 : \mu_1 - \mu_2 = 0$

$H_1 : \mu_1 - \mu_2 > 0$

$$p = 1 - F_z(z_0)$$

cdf

$$z_0 = \frac{3.30}{\frac{4{,}029}{\sqrt{10}}} = 2{,}5901$$

$$p = 1 - \underbrace{\text{tcdf}(2.5901, 9)}_{\text{Octave}} = 1 - 0.98539$$

$= 0{,}01461$

$0.01461 < \alpha = 0.05 \Rightarrow$ We reject $H_0 \Rightarrow$ accept H_1 hypothesis

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