We observe that (P) is given in standard form

$$(P)\begin{cases} \text{Minimize } f(x) = 3x_1 + 2x_2 + x_3 \\ -x_1 - 2x_3 + x_5 = -6 \\ -3x_2 - x_3 + x_4 = -4 \\ x_1, \dots x_3 \geq 0 \end{cases} \quad (S)$$

$n = 5$, $m = 2$

$$c = (c_1, c_2, c_3, c_4, c_5) = (3, 2, 1, 0, 0)$$

$$A = \begin{pmatrix} -1 & 0 & -2 & 0 & 1 \\ 0 & -3 & -1 & 1 & 0 \end{pmatrix} ; \quad b = \begin{pmatrix} -6 \\ -4 \end{pmatrix}$$

$A^1 \quad A^2 \quad A^3 \quad A^4 \quad A^5$

Let $B = (A^5, A^4)$

		0	0		
	1	A^5	A^4	Test d.f.b	
3	A^1	$-1<0$	0	$-3 \leq 0$	$0\cdot(-1)+0\cdot0-3=-3$ $\quad \frac{-3}{-1}$
2	A^2	0	$-3<0$	$-2 \leq 0$	$0\cdot0+0\cdot(-3)-2=-2$
1	A^3	$-2<0$	$-1<0$	$-1 \leq 0$	$-2\cdot0-1\cdot0-1=-1$ $\quad -\frac{1}{2}$ *
	Test r.f.b	$-6<0$	$-4<0$	0	$-6\cdot0-4\cdot0=0$
		*			

If no negative $\Rightarrow$ set empty

Calculate d.f.b or r.f.b

$\Rightarrow$ B is d.f.b $\Rightarrow$ we apply the Dual Simplex Alg

2	A^3	A^4	Ist d.f.b
A^1	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{5}{2}$
A^2	0	-3	-2
A^5	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$
d.f.b	3	-1	3

$\frac{-2}{-3}$

$\frac{-1/2}{-1/2} = 1$

- Divide everything by pivot on same column as pivot
- Divide everything by the opposite of pivot on the same row

3	A^3	A^2	d.f.b
A^1			
A^4			
$A5$			
d.f.b	3	$\frac{1}{3}$	$\frac{11}{3}$

d.f.b, according to DSA

(A^3, A^2) is p.f.b, hence optimal basis

An optimal sol of (P) is

$x^0 = (x_1^0, x_2^0, x_3^0, x_4^0, x_5^0)$

$x_1^0 = 0,\ x_2^0 = \frac{1}{3},\ x_3^0 = 3,\ x_4^0 = 0,\ x_5^0 = 0$

The optimal value (min) of f on S is:

Remark $f(x^0) = 3x_1^0 + 2x_2^0 + x_3^0$
$= 3.0 + 2\frac{1}{3} + 3 = \frac{2}{3} + 3 = \frac{11}{3}$

Ex 2 Solve the foll

$$(P) \begin{cases} \text{Minimise } f(x) = \alpha^2 x_1 + (\alpha - 1)^2 x_2 \\ \left.\begin{array}{l} -x_1 \qquad + x_3 = 0 \\ -2x_1 - x_2 \qquad + x_4 = -5 \\ x_1, \dots, x_5 \geq 0 \end{array}\right] (S) \end{cases}$$

α is a parameter

Solution:

(P) is given in standard form

$n = 4$, $m = 2$

$$c = (\underset{\alpha^2}{\underset{\shortparallel}{c_1}}, \underset{(\alpha-1)^2}{\underset{\shortparallel}{c_2}}, \underset{0}{\underset{\shortparallel}{c_3}}, \underset{0}{\underset{\shortparallel}{c_4}})$$

$$A \begin{pmatrix} -1 & 0 & 1 & 0 \\ -2 & -1 & 0 & 1 \end{pmatrix} \quad b \begin{pmatrix} 0 \\ -5 \end{pmatrix}$$
$$\quad A^1 \quad A^2 \quad A^3 \quad A^4$$

$B = (A^3, A^4)$

		0	0	
	1	A^3	A^4	$\alpha R b$
α^2	A^1	-1	$-2<0$	$-\alpha^2 \leq 0$
$\alpha^2-2\alpha+1$	A^2	0	$-1<0$	$-\alpha^2+2\alpha-1 \leq 0$
	$1Rb$	0	-5	0

B is $\alpha R b \Rightarrow$ $\frac{-\alpha^2}{-2}$ $\Rightarrow$ we can apply $\frac{-\alpha^2+2\alpha-1}{-1}$ O.F.A.

Simplexul primal nu merge pt că $-5<0$

$-\alpha^2+2-1 = -(-\alpha-1)^2 \leq 0$

Alegem cel mai mic nr din sectorul orizontal

$$\frac{-\alpha^2}{-2} = \frac{\alpha^2}{2}$$

$$\frac{-\alpha^2+2\alpha-1}{-1} = \alpha^2-2\alpha+1$$

$$\frac{\alpha^2}{2} \leq \alpha^2-2\alpha+1$$

$$\Leftrightarrow \alpha^2 \leq 2\alpha^2-4\alpha+2 \Rightarrow \alpha^2-4\alpha+2 \geq 0$$

$$\Delta = \cancel{\quad} (-4)^2-4\cdot 2 = 16-8 = 8 > 0$$

$$\alpha_{1,2} = \frac{4 \pm \sqrt{8}}{2} = 2 \pm \sqrt{2}$$

α	$-\infty$		$2-\sqrt{2}$		$2+\sqrt{2}$		$+\infty$
		$+++$	0	$--$	0	$+++$	

Case 1) $\alpha \in (-\infty, 2-\sqrt{2}) \cup (2+\sqrt{2}, \infty)$

1	A^3	A^4	dfb
A^1	-1	-2	$-\alpha^2$
A^2	0	-1	$-\alpha^2+\alpha-1$
rfb	0	-5	0

$\frac{\alpha^2}{2}$ * (in Case 1)

$\alpha^2-2\alpha+1$

...

2	A^3	A^1	dfb
A^4			
A^2			
rfb	$\frac{5}{2}$	$\frac{5}{2}$	
	≥ 0	≥ 0	

$\Rightarrow (A^3, A^1)$ is rfb, hence an optimal basis

An optimal sol of (P) in Case 1 is:

$$x^0 = (x_1^0, x_2^0, x_3^0, x_4^0) = \left(\frac{5}{2}, 0, \frac{5}{2}, 0\right)$$

and the optimal value (min) of f on S is

$$\alpha_{00} = \frac{5\alpha^2}{2}$$

Case 2:

1	A^3	A^4	dfb
A^1	-1	-2	$-\alpha^2$
A^2	0	-1	$\alpha^2+2\alpha-1$
nfb	0	-5	0

* Case 2

2	A^3	A^2	dfb
A^1			
A^4			
pbb	0	5	~~[illegible]~~ $-5\alpha^2+10\alpha-5$

$\Downarrow$

(A^3, A^2) is pbb, hence optimal basis

$\Rightarrow$ An optimal sol of (P) in Case 2 is

$$x^0 = (x_1^0, x_2^0, x_3^0, x_4^0) = (0, 5, 0, 0)$$

The optimal value of f is $\alpha_{00} = -5\alpha^2+10\alpha-5$

Ex 3 — Apply the dual simplex algorithm to the foll pb.

$$(P)\begin{cases} \text{Minimize } f(x) = -x_2 - x_3 + 3x_5 - 3x_6 \\ 2x_1 + x_2 + x_3 + x_5 + 3x_6 = 0 \\ x_1 + x_3 + x_4 - x_5 = -2 \\ x_1, \dots x_6 \geq 0 \end{cases} \quad (S)$$

(P) is in std form

$n=6, m=2$

$$c = (c_1, c_2, c_3, c_4, c_5, c_6) = (0, -1, -1, 0, 3, -3)$$

$$A = \begin{pmatrix} 2 & 1 & 1 & 0 & 1 & 3 \\ 1 & 0 & 1 & 1 & -1 & 0 \end{pmatrix}, \quad b = \begin{pmatrix} 0 \\ -2 \end{pmatrix}$$

$$\begin{array}{cccccc} A^1 & A^2 & A^3 & A^4 & A^5 & A^6 \end{array}$$

	1	A^2 (−1)	A^4 (0)	dfb
0	A^1	2	1	$-2 \le 0$
−1	A^3	1	1	$0 \le 0$
3	A^5	1	$-1 < 0$	$-4 \le 0$
−3	A^6	3	0	$0 \le 0$
	fb	0	$-2 < 0$	0
			*	

=> B- is DFB

We apply D.FA

$\frac{-4}{-1}$ *

2	A^2	A^5	dfb
A^1	$3 \not< 0$		
A^3	$2 \not< 0$		
A^4	$1 \not< 0$		
A^6	$3 \not< 0$		
fb	$-2 < 0$		

=> (P) has no feasible sol => $S = \emptyset$