

Tema 3

1) $(\mathbb{Z}_6, +)$

	$\hat{0}$	$\hat{1}$	$\hat{2}$	$\hat{3}$	$\hat{4}$	$\hat{5}$
$\hat{0}$	$\hat{0}$	$\hat{1}$	$\hat{2}$	$\hat{3}$	$\hat{4}$	$\hat{5}$
$\hat{1}$	$\hat{1}$	$\hat{2}$	$\hat{3}$	$\hat{4}$	$\hat{5}$	$\hat{0}$
$\hat{2}$	$\hat{2}$	$\hat{3}$	$\hat{4}$	$\hat{5}$	$\hat{0}$	$\hat{1}$
$\hat{3}$	$\hat{3}$	$\hat{4}$	$\hat{5}$	$\hat{0}$	$\hat{1}$	$\hat{2}$
$\hat{4}$	$\hat{4}$	$\hat{5}$	$\hat{0}$	$\hat{1}$	$\hat{2}$	$\hat{3}$
$\hat{5}$	$\hat{5}$	$\hat{0}$	$\hat{1}$	$\hat{2}$	$\hat{3}$	$\hat{4}$

Căutăm o submulțime cu 3 elemente pentru care "+" să fie parte stabilă.

Din tabla lui $(\mathbb{Z}_6, +)$, ne rezultă că $\{\hat{0}, \hat{2}, \hat{4}\}$ ar fi un exemplu.

	$\hat{0}$	$\hat{2}$	$\hat{4}$
$\hat{0}$	$\hat{0}$	$\hat{2}$	$\hat{4}$
$\hat{2}$	$\hat{2}$	$\hat{4}$	$\hat{0}$
$\hat{4}$	$\hat{4}$	$\hat{0}$	$\hat{2}$

Fică $H \subseteq \mathbb{Z}_6$, $|H| = 4$.

$H \subseteq \mathbb{Z}_6 \xleftrightarrow{\text{T. Lagrange}} |H| \mid |\mathbb{Z}_6| \Leftrightarrow 4 \mid 6$ fals \Rightarrow

\mathbb{Z}_6 nu are subgrupuri cu 4 elemente.

2) Fie $S = \{a\sqrt{2} \mid a \in \mathbb{R}\} \subseteq \mathbb{R}$

$S \neq \emptyset$

$\forall s_1 = a\sqrt{2}, s_2 = b\sqrt{2} \in S \Rightarrow s_1 - s_2 = (a-b)\sqrt{2} \in S$
 $a, b \in \mathbb{R}$

$\forall s_1 = a\sqrt{2}, s_2 = b\sqrt{2} \in S \Rightarrow s_1 \cdot s_2 = 2ab \in S$
 $a, b \in \mathbb{R}$

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S subinel al lui $(\mathbb{R}, +, \cdot)$

3) $f: \mathbb{R}^2 \rightarrow \mathbb{R}^2$, $f(x_1, x_2) = (-3x_1 + 4x_2, 2x_1 - x_2)$

Fie $\alpha, \beta \in \mathbb{R}$, $(x_1, x_2), (y_1, y_2) \in \mathbb{R}^2$

$f(\alpha(x_1, x_2) + \beta(y_1, y_2)) = f(\alpha x_1 + \beta y_1, \alpha x_2 + \beta y_2)$

$= (-3\alpha x_1 - 3\beta y_1 + 4\alpha x_2 + 4\beta y_2, 2\alpha x_1 + 2\beta y_1 - \alpha x_2 - \beta y_2)$

$$\begin{aligned}
 &= (-3\alpha x_1 + 4\alpha x_2, 2\alpha x_1 - \alpha x_2) + (-3\beta y_1 + 4\beta y_2, 2\beta y_1 - \beta y_2) \\
 &= \alpha(-3x_1 + 4x_2, 2x_1 - x_2) + \beta(-3y_1 + 4y_2, 2y_1 - y_2) \\
 &= \underline{\alpha f(x_1, x_2) + \beta f(y_1, y_2)}
 \end{aligned}$$

$\Rightarrow f = \text{transformare liniară de } \mathbb{R}\text{-spațiu vectoriale}$

$$\begin{aligned}
 f(e) = f(1, 0) &= (-3, 2) \\
 f(e) = f(0, 1) &= (4, -1)
 \end{aligned}
 \Rightarrow [f]_e = \begin{pmatrix} -3 & 4 \\ 2 & -1 \end{pmatrix}$$