

- 1) (2 points)** Let X_1, X_2, \dots, X_n be a random sample drawn from a discrete distribution with pdf $f(x; \theta) = C_5^x \theta^x (1 - \theta)^{5-x}$, $x = 0, 1, \dots, 5$, with $\theta \in (0, 1)$ unknown. Let $S = \sum_{i=1}^n X_i$. Is S a sufficient statistic for the estimation of θ ? Explain.
- 2)** Let X_1, X_2, \dots, X_n be a random sample drawn from a distribution with pdf $f(x; \theta) = \frac{1}{\theta} e^{-\frac{x}{\theta}}$, for $x > 0$, $E(X) = \theta$, $V(X) = \theta^2$, with $\theta > 0$, unknown.
- a) (1.5 points)** Find the maximum likelihood estimator, $\bar{\theta}$, for θ .
 - b) (1 point)** Is it an absolutely correct estimator? Explain.
 - c) (1.5 points)** Find the efficiency of $\bar{\theta}$, $e(\bar{\theta})$.
 - d) (1 point)** At the 1% significance level, find a most powerful test for testing $H_0 : \theta = 1$ against $H_1 : \theta = 2$.
- 3) (2 points)** Let X be a discrete random variable with probability distribution $P(X = 1) = \lambda$, $P(X = 5) = 1 - \lambda$, with $\lambda \in (0, 1)$ unknown. The sample 1, 1, 5, 5, 5, 5 is collected from this distribution. Find the method of moments estimator, $\hat{\lambda}$, for λ .