Interpretability, Decision Trees, and Sequential Decision Making

Hector Kohler

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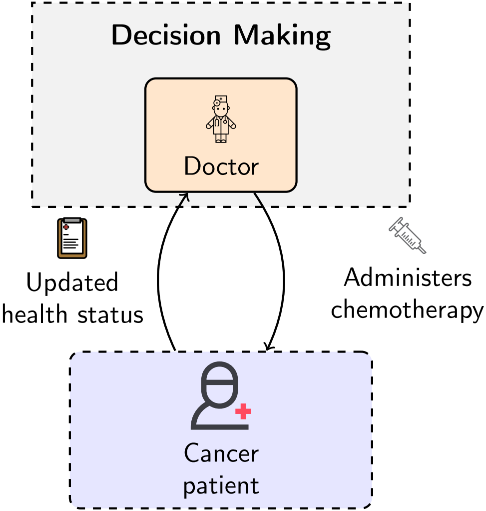
Université de Lille, CNRS, Inria, UMR CRIStAL 9189, France

November 30, 2025

Sequential decision making (SDM)



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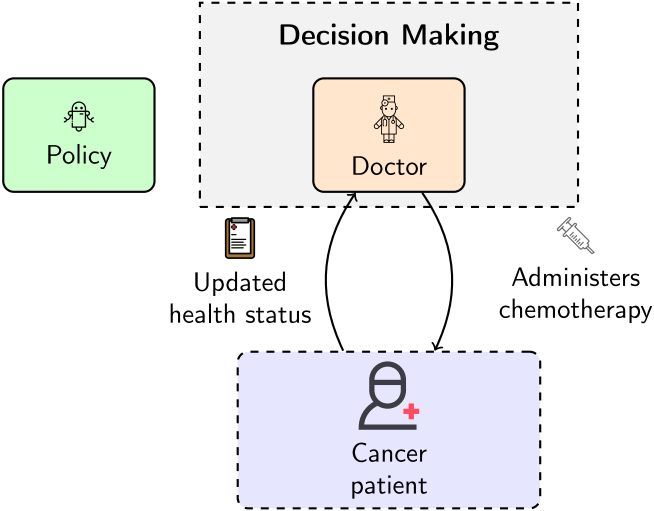

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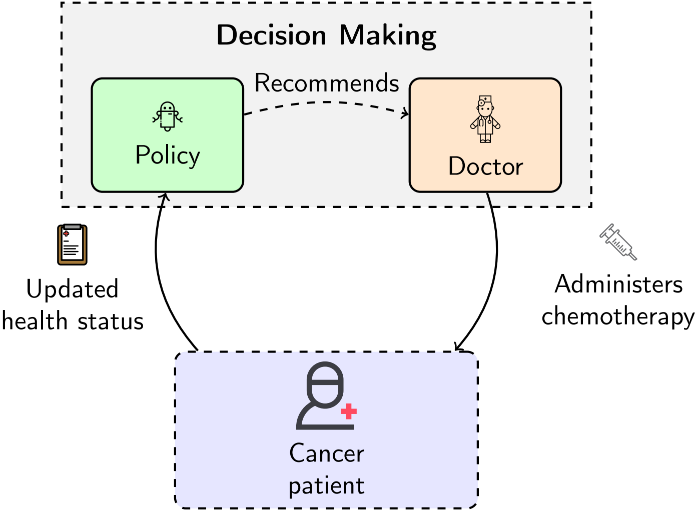
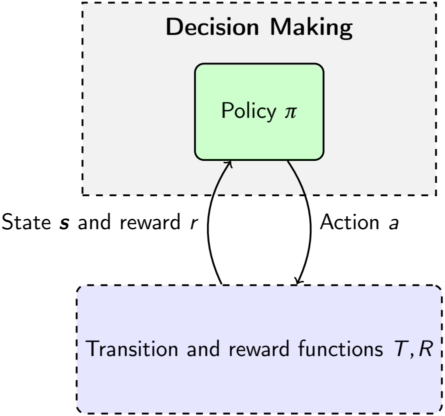


Markov decision processes (MDPs) and reinforcement learning (RL)



Markov decision processes [Put94].

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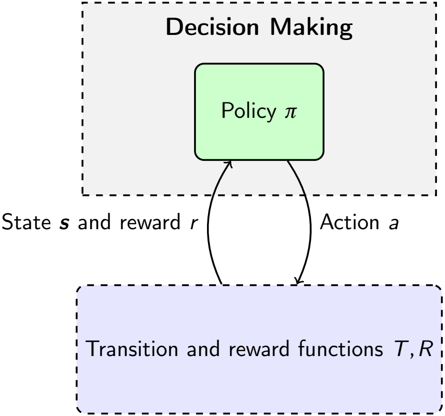

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- RL [SB98] aims to find a policy, $\pi : S \rightarrow A$ that maximizes:

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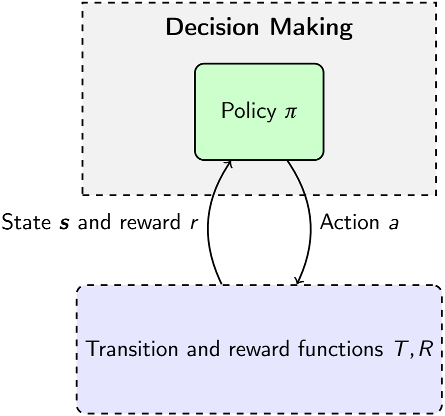

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- Lots of succesful RL algorithms [SB98; Mni+15; Sch+17].

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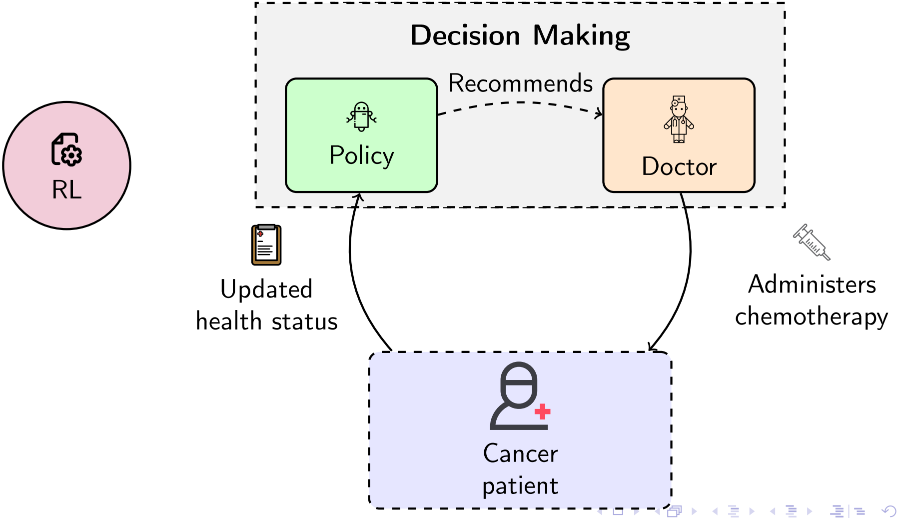

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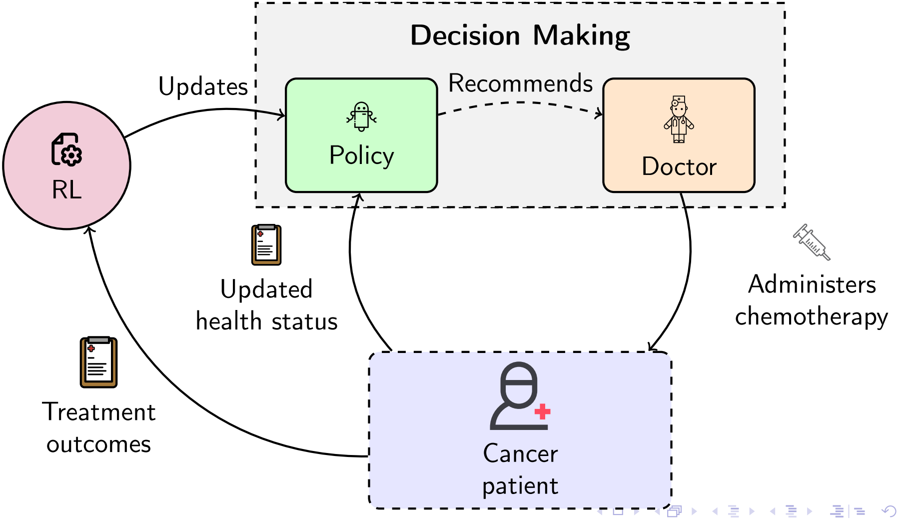

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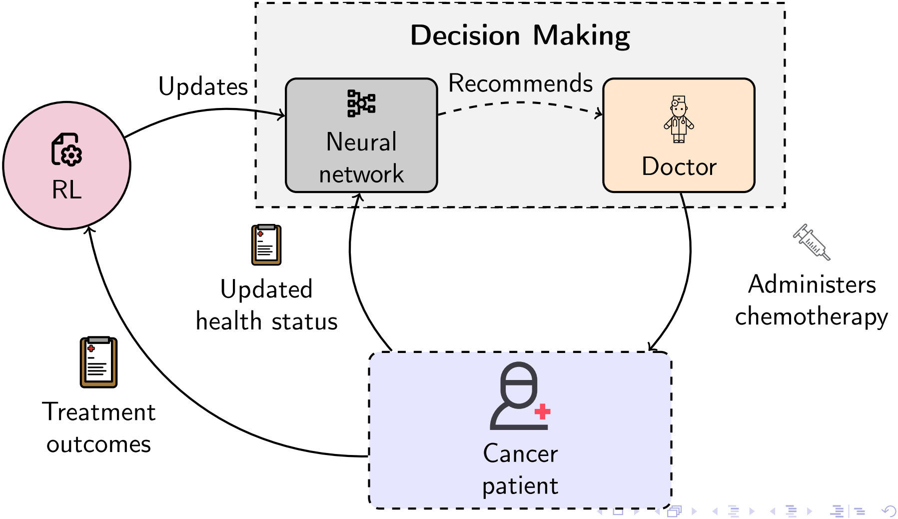

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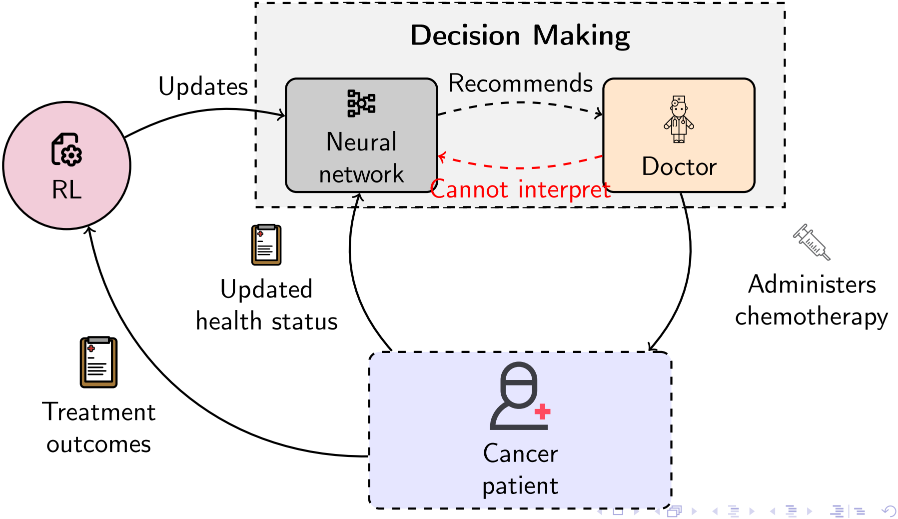

Interpretability?

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Local vs. global interpretability [Mil+24; Gla+24].

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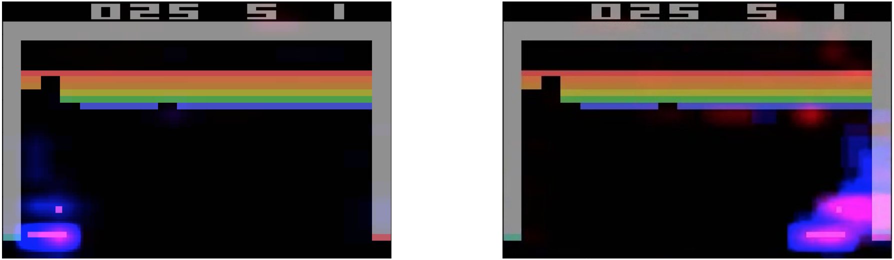

Saliency maps [Gre+18].

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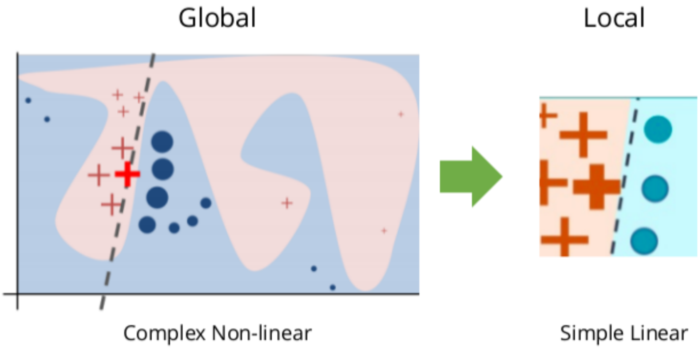

Local interpretable model-agnostic explanations (LIME) [RSG16].

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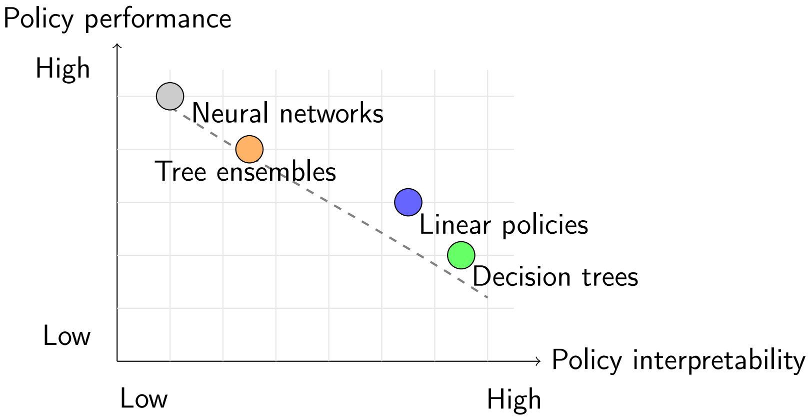

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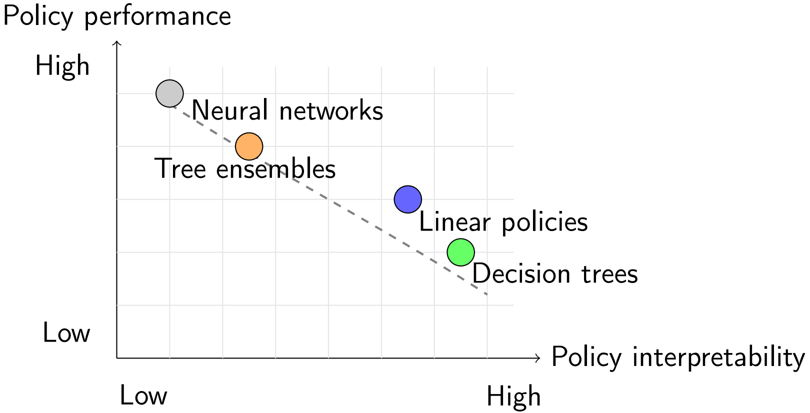

Global interpretability → works for all states.

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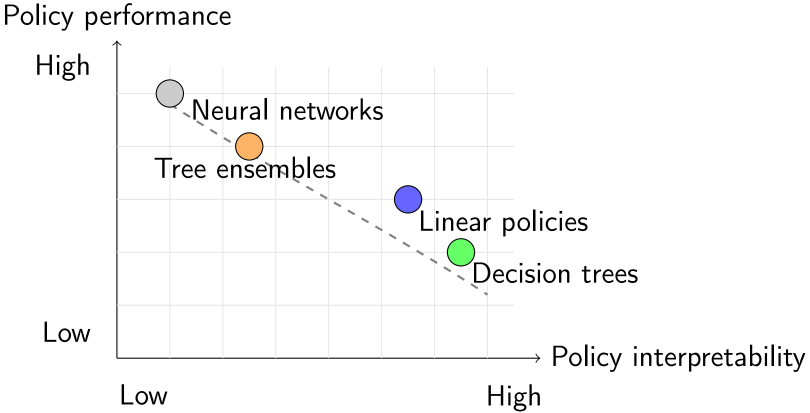
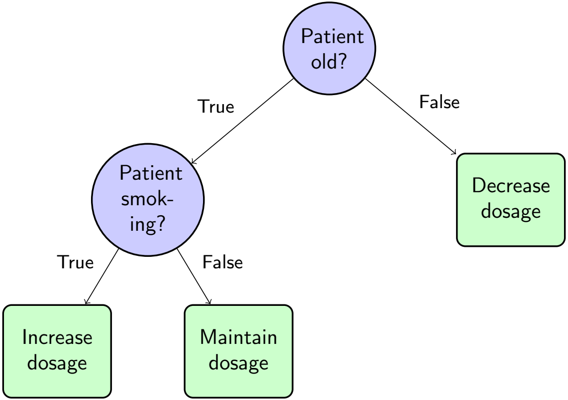


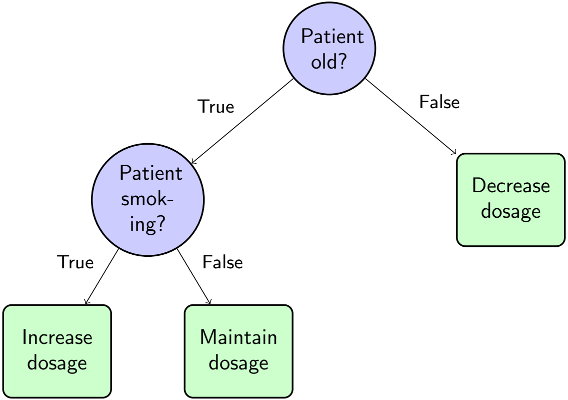
⚠**Multiple definitions [Lip18; DK17; Bar+20]**.

Decision trees



A generic decision tree of depth $D = 2$.

Decision trees



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Successful algorithms for classification/regression: [Bre+84; BD17; Dem+22; VZ19; MMW22] . . .

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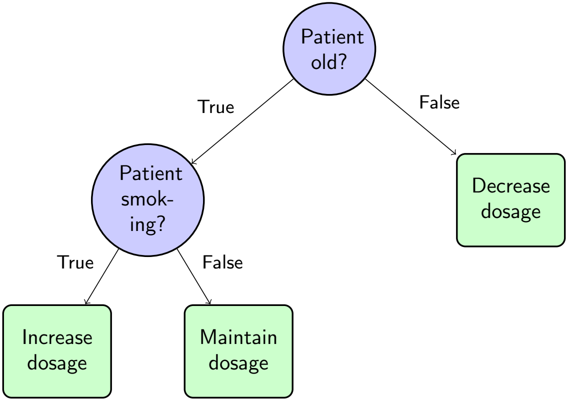

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What about SDM?

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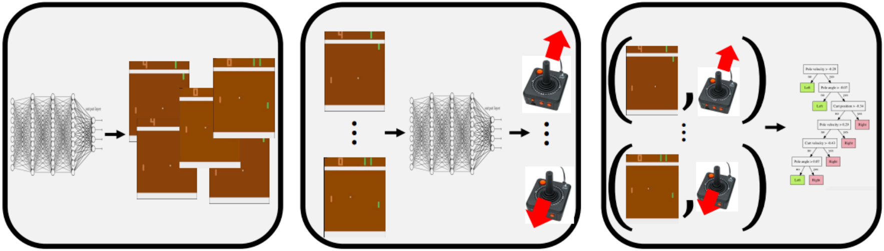

Step 1: Use NN to generate states

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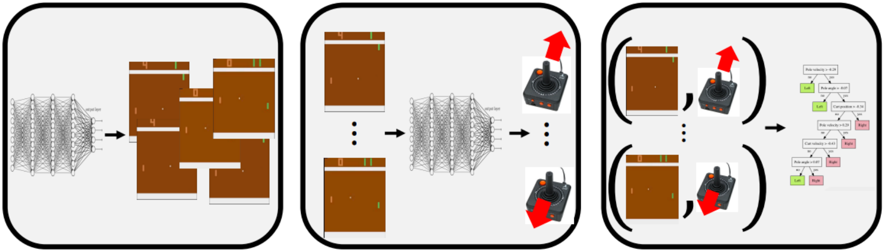

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Most research focused on indirect learning of interpretable policies [RGB10; BPS18; Ver+18; Mil+24].

Two ways to get interpretable policies for SDM

Indirect

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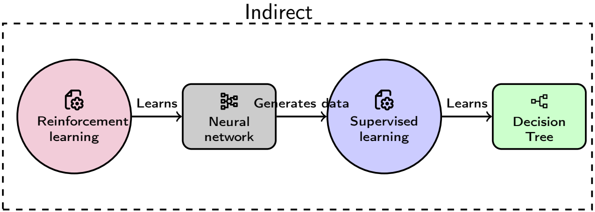

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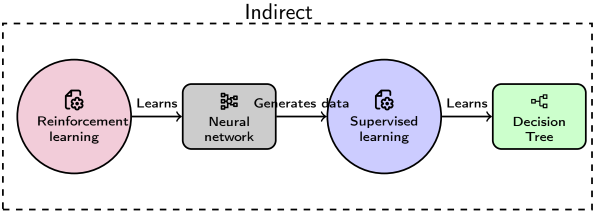

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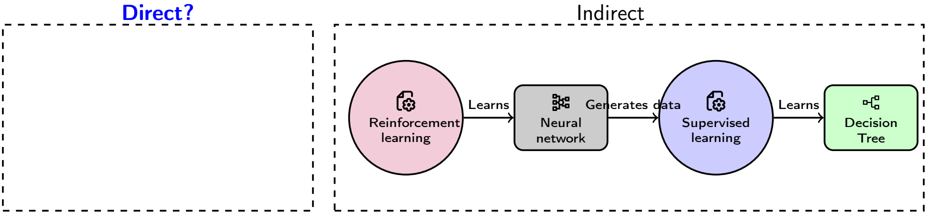

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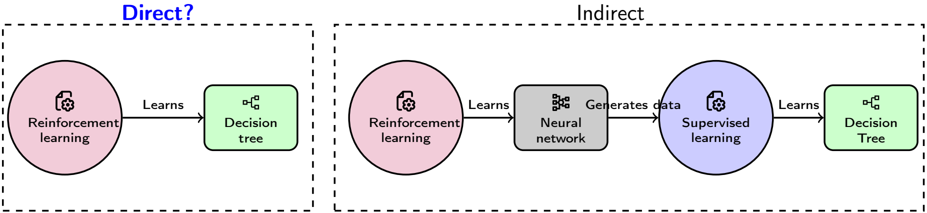

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Contributions

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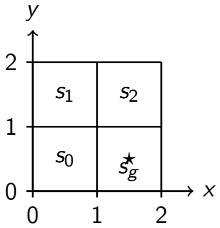

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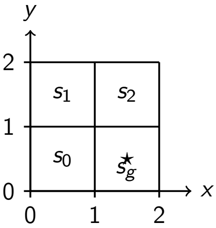

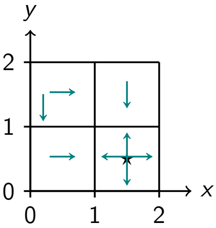

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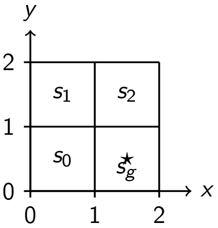

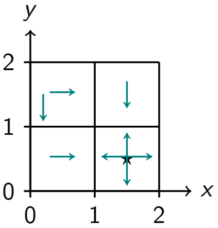

Grid world MDP and optimal actions.

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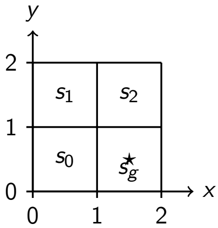

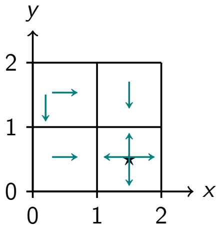

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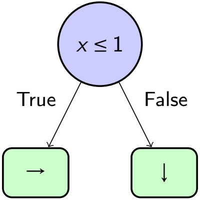

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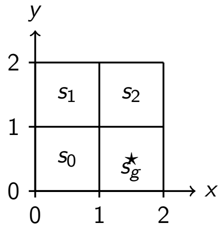

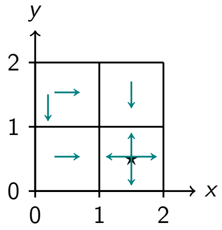

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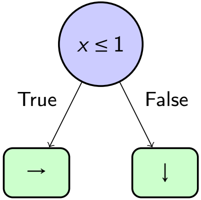

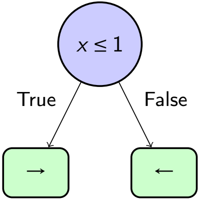

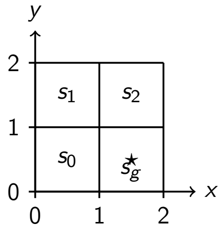

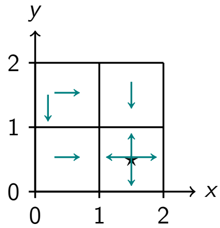

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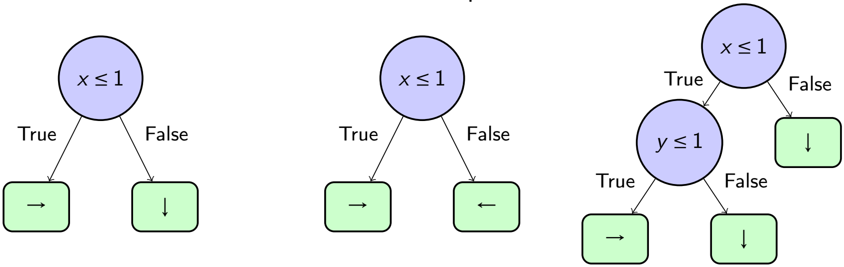

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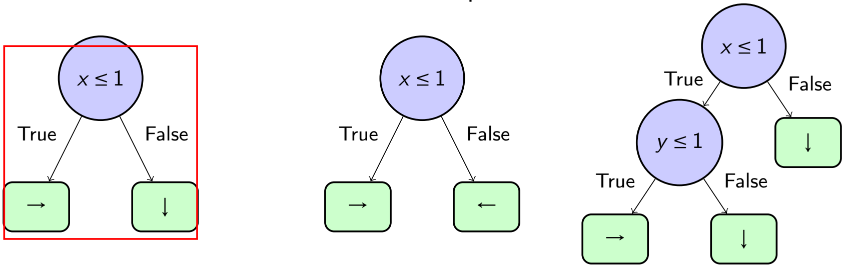
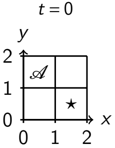


Decision tree policies with different interpretability-performance trade-offs.

Iterative bounding Markov decision processes [Top+21]



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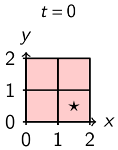

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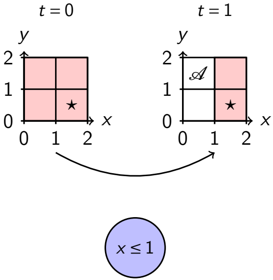

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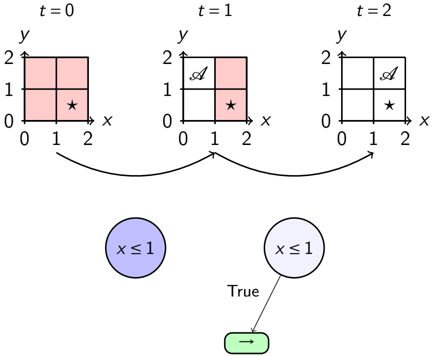

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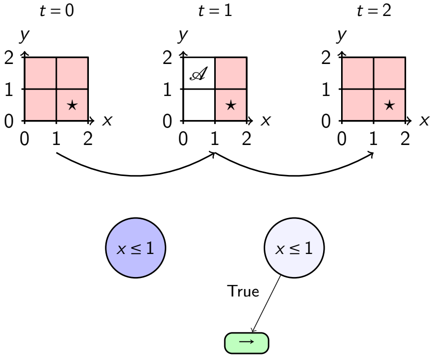

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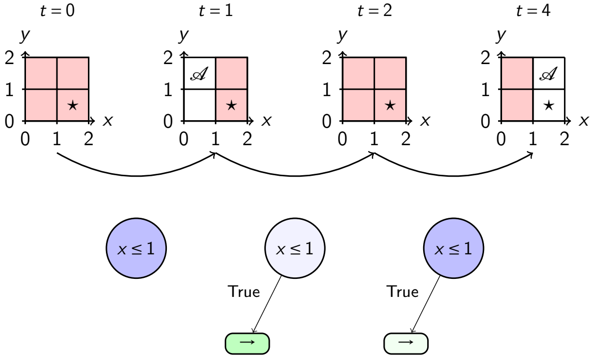

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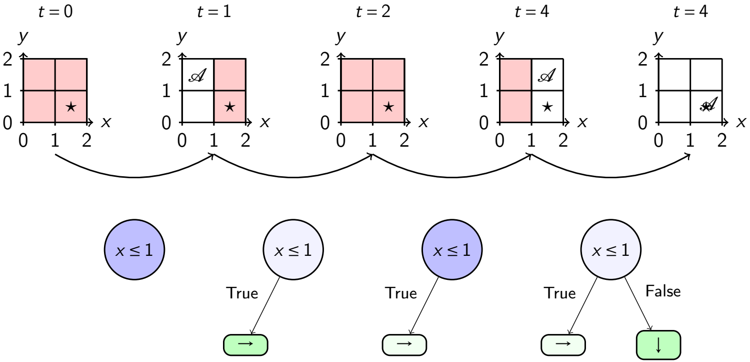

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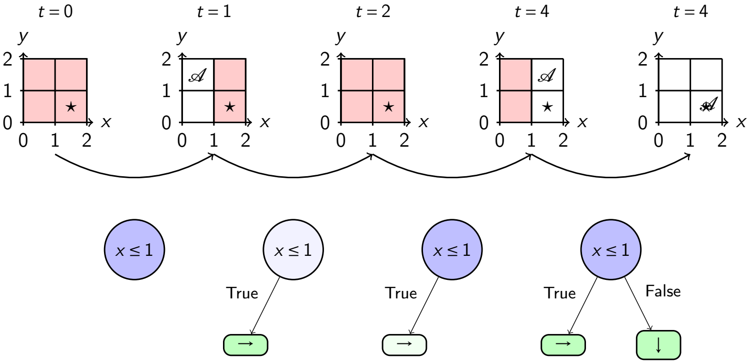

Given an MDP $\mathscr{M}$ $\langle S, A, R, T\rangle$

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Given an MDP $\mathscr{M}$ $\langle S, A, R, T \rangle$, an IBMDP is an MDP

$$\langle \overbrace{S \times O}^{\text{State space}}, \underbrace{A \cup A_{info}}_{\text{Action space}}, \overbrace{(R, \zeta)}^{\text{Reward function}}, \underbrace{(T_{info}, T)}_{\text{Transitions}} \rangle$$

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- ⚠**IBMDP policies $\pi_{po} : O \to A \cup A_{info}$ are decision tree policies for $\mathscr{M}$.**

RL for memoryless policies in POMDPs

RL for memoryless policies

- Finding the best **deterministic** and memoryless policy in a POMDP is NP-hard [Lit94]!
- The best memoryless policy can be stochastic [SJJ94].
- Value-based RL converges to sub-optimal solutions [SJJ94; LS98].

Asymmetric RL

- Access to hidden states during training but not at execution [Pin+17].
- Value-based → learns $Q(o, a)$ with TD targets $U(s, a)$ [BDA22].
- Actor-critic[a] → policy gradient on $\pi(o, a)$ using a critic $V(s)$ [BA22].
- Supposed to work better for our problem [LEM25].

[a] Although those return stochastic policies, we can be greedy.

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- Finding the best **deterministic** and memoryless policy in a POMDP is NP-hard [Lit94]!
- The best memoryless policy can be stochastic [SJJ94].
- Value-based RL converges to sub-optimal solutions [SJJ94; LS98].

Asymmetric RL

- Access to hidden states during training but not at execution [Pin+17].
- Value-based → learns $Q(o, a)$ with TD targets $U(s, a)$ [BDA22].
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Success rates over thousands of RL runs with varying hyperparameters when learning different policies in the same IBMDP[1].

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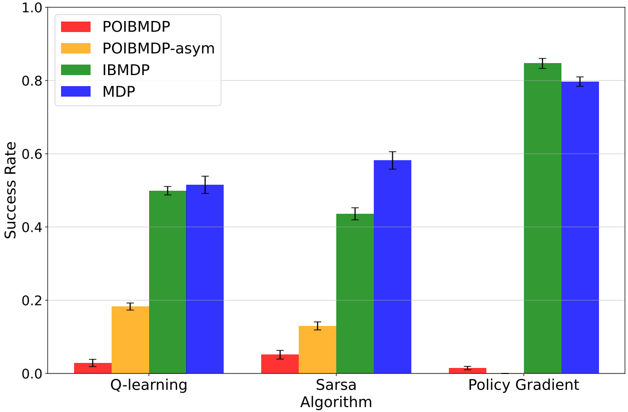

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Q: Can we leverage SDM to design new decision tree induction algorithms for classification/regression?

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Decision trees in supervised learning

- N data points $\{\boldsymbol{x}_i, y_i\}$. Each $\boldsymbol{x}_i$ is described by p features and has a label $y_i \in \mathcal{Y}$. We want to find a tree of depth at most D $T \in \mathcal{T}_D$ that minimizes:

$$\mathcal{L}_\alpha(T) = \frac{1}{N}\sum_{i=1}^{N} \ell(y_i, T(x_i)) + \alpha C(T)$$

- Trees **interpretable** and **competitive with neural nets** [GOV22].
- Greedy algorithms **sub-optimal accuracy**, but $\boldsymbol{O(2^D)}$ operations [Bre+84; Qui86; Qui93] .
- Optimal algorithms, optimal accuracy, but $\boldsymbol{O((2Np)^D)}$ operations (NP-hard) [BD17; Dem+22; LWD23; CRB24; HR76].
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Decision tree induction as solving MDPs

Intuition

The induction of a decision tree is made of a sequence of decisions: at each node, we must decide whether it is better to split (a subset of) the training data, or to create a leaf node.

- S: data subsets.
- A: test or leaf nodes that can be added to the tree.
- R: interpretability term $-\alpha$ and accuracies.
- T: node traversals.

Proposition (Objective Equivalence)

Let π be a deterministic policy of the MDP. Then $J_\alpha(\pi) = -\mathscr{L}_\alpha(E(\pi, s_0))$ where E is an algorithm that extracts a decision tree from π [Top+21].

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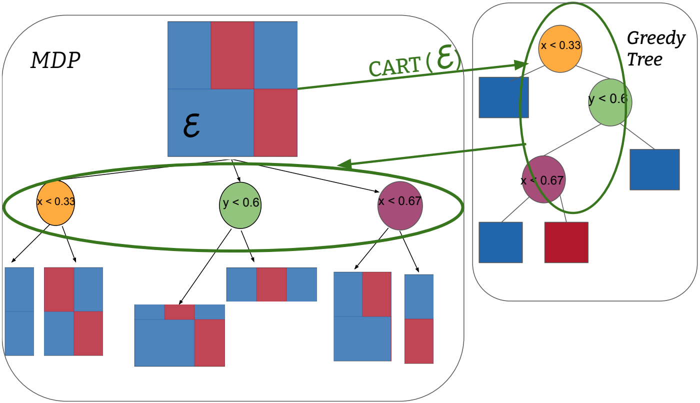

We can use greedy trees nodes as candidate actions.

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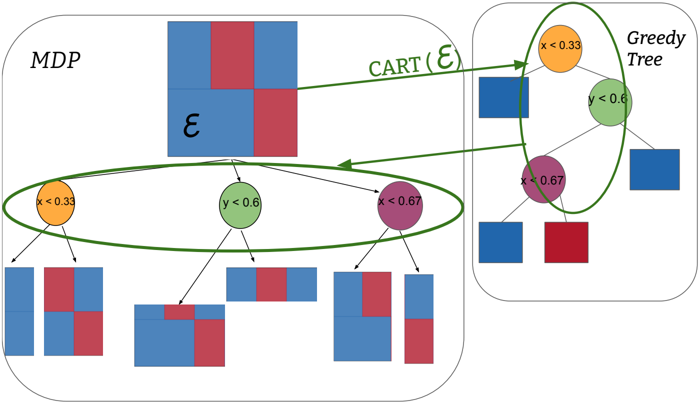

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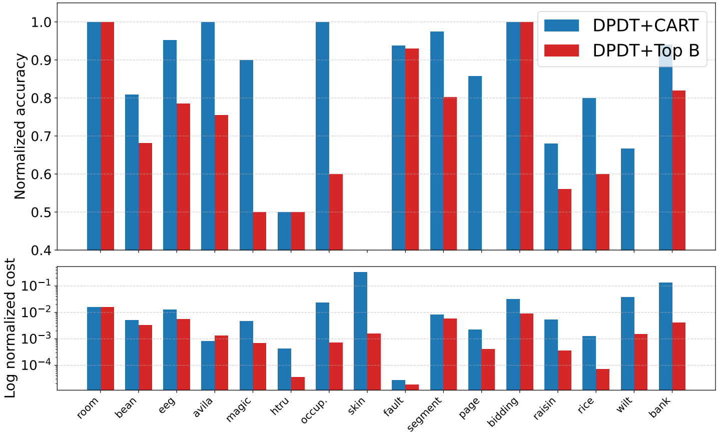
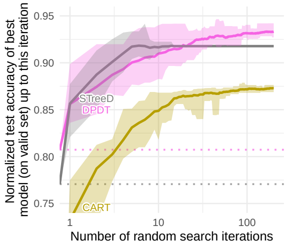
Fast like greedy trees, accurate like optimal trees



Train accuracies against cost for detph-3 trees.

Large scale evaluation of DPDT trees generalization [GOV22]



DPDT depth-5 trees vs. other detph-5 trees

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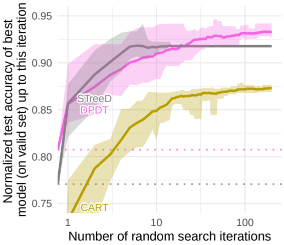

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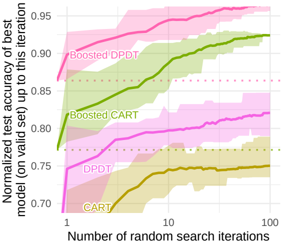

Boosted DPDT vs. Boosted CART

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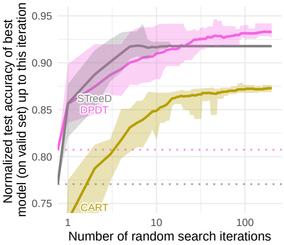

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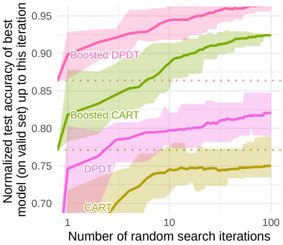

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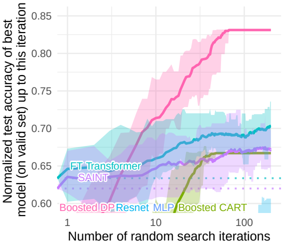
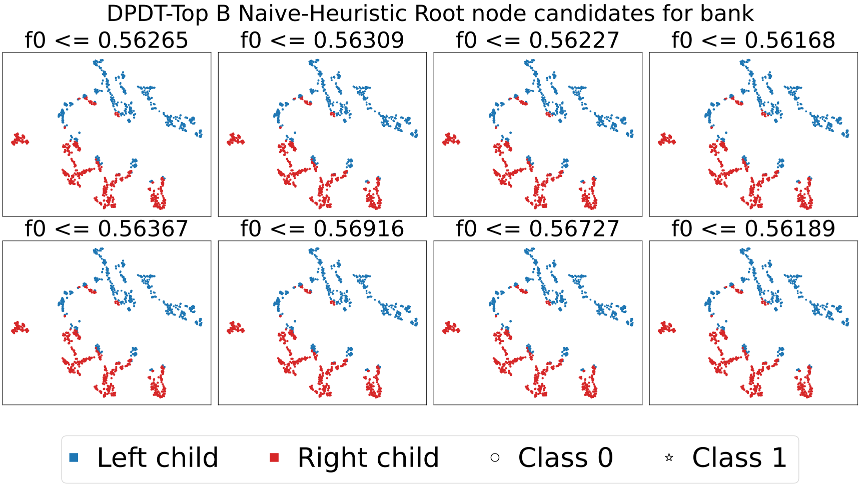


Boosted DPDT vs. other classifiers

CART generates more diverse splits than Top B



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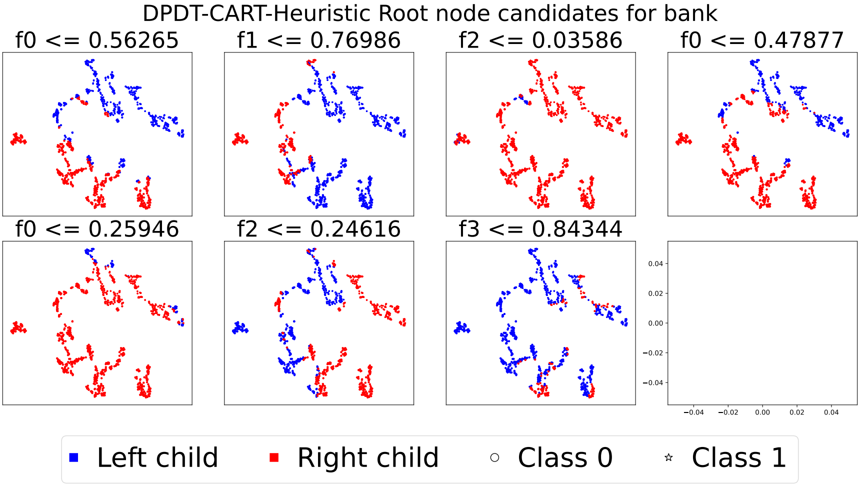

Perspectives

- New SOTA decision tree induction with dynamic programming in MDPs.
- What about using DPDT for indirect decision tree policy learning for SDM?
- What performances could we reach with an industry-grade implementation of XGboost+DPDT?

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How to measure policy interpretability?

Challenges [Gla+24; Lip18; DK17]

- No definition of interpretability.
- Measuring might require humans.
- Different hardwares (CPUs vs GPUs).
- Different implementations (matrix operations vs fully sequentially) [Luo+24]

The notion of *simulatability* [Lip18]

- Less parameters mean more interpretability [Fre14; Lav99].
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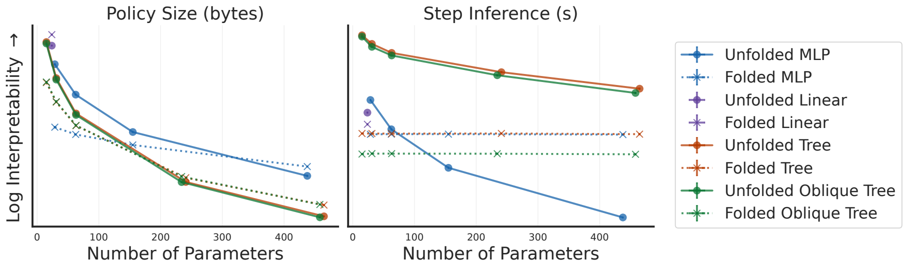

Aggregated policies interpretability on classic control environments

Perspectives

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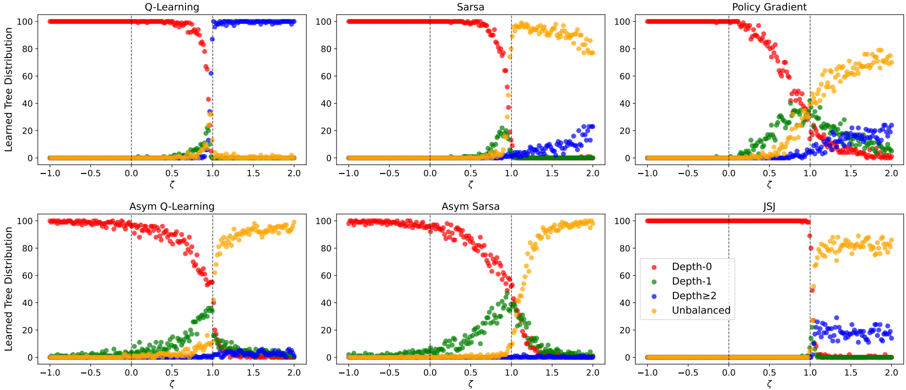

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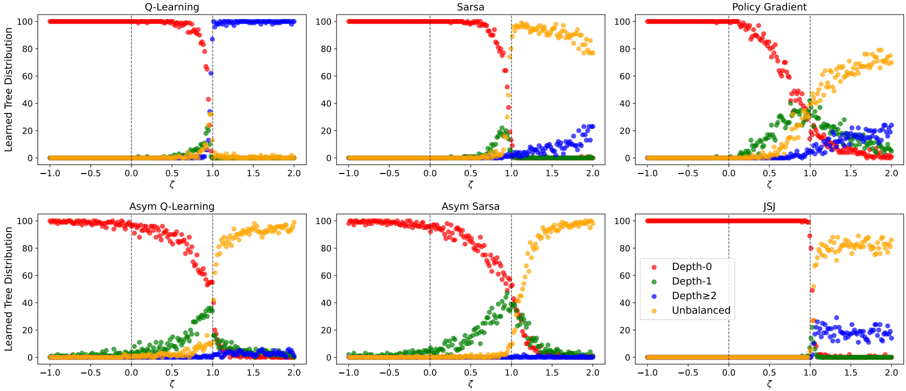

Distributions of tree policies learned with (asymmetric) RL algorithms [SB98; SJJ94; LS98; BA22; BDA22] as a function of the interpretability reward ζ.

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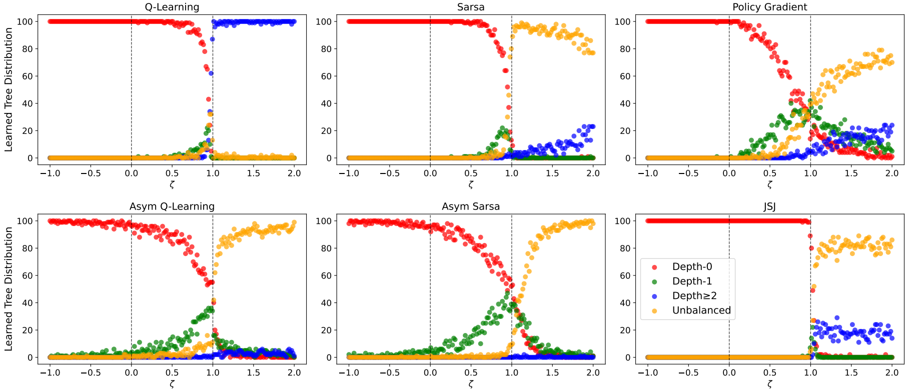

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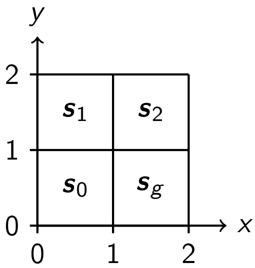
Are the poor performances due to partial observability?

Result: decision tree policies for classification MDPs are standard Markovian policies in IBMDPs



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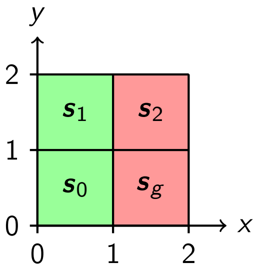

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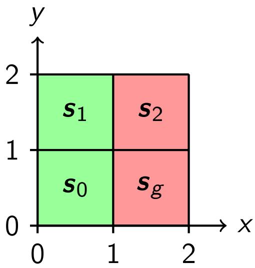

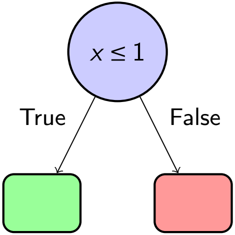

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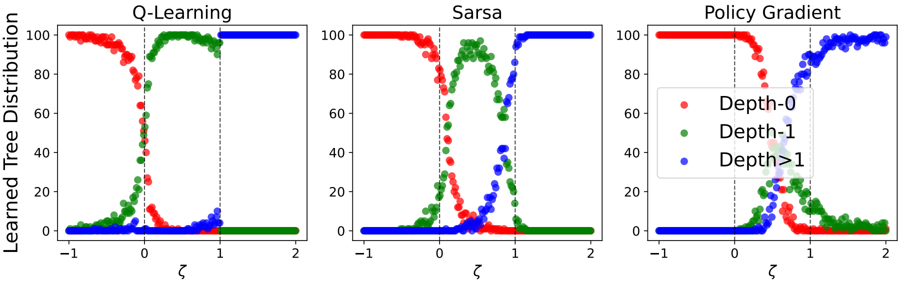
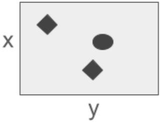
Deterministic memoryless policies for classification IBMDPs ($\Leftrightarrow$ decision tree policies) are in fact Markovian.

Result: RL can retrieve optimal depth-1 trees for the toy classification MDP



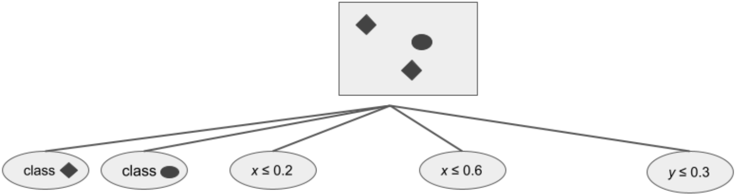
Distributions of tree policies learned with various RL algorithms.

Decision tree induction as solving MDPs



Example of decision tree induction as an MDP.

Decision tree induction as solving MDPs



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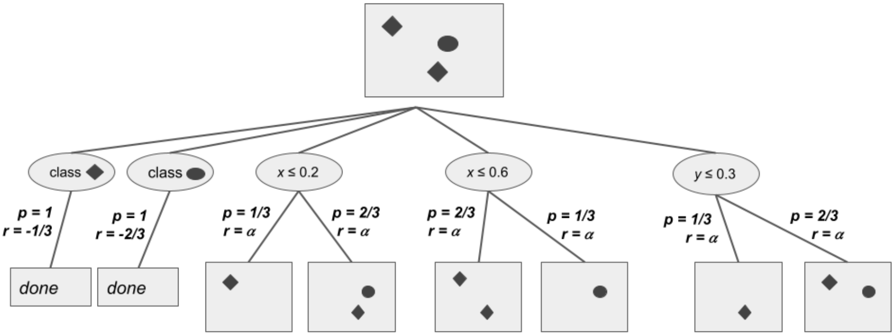

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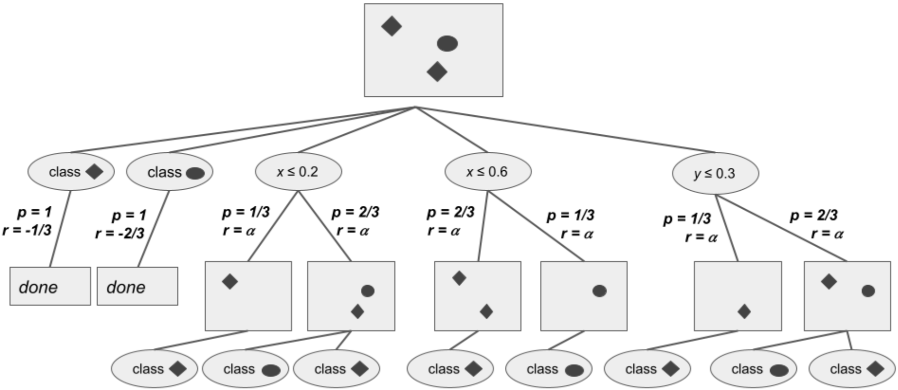

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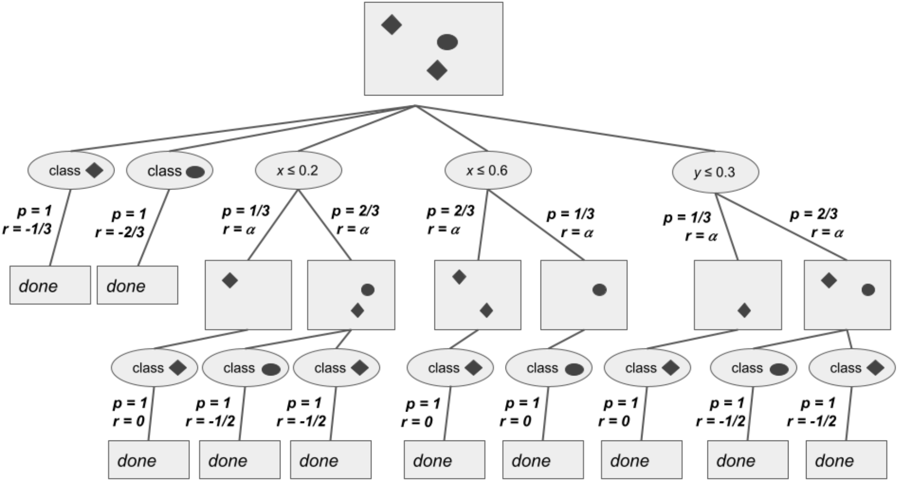

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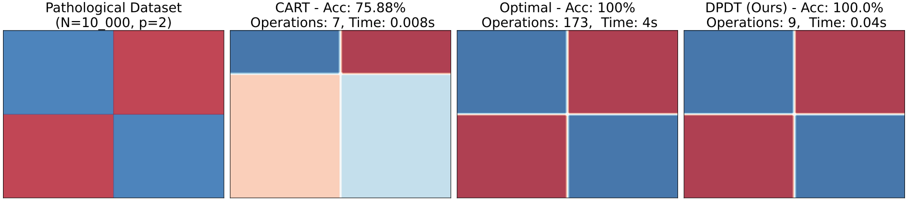
Fast like greedy trees, accurate like optimal trees



Comparison of greedy, optimal, and DPDT depth-2 trees on the checkersboard dataset.

Fast like greedy trees, accurate like optimal trees

Comparison of accuracies and operations for depth-3 trees.

	Accuracy						Operations					
Dataset	Opt	Greedy	DPDT				Opt	Greedy	DPDT			
			CART$^-$	CART$^+$	TopB$^-$	TopB$^+$			CART$^-$	CART$^+$	TopB$^-$	TopB$^+$
room	**0.992**	0.968	0.991	**0.992**	0.990	**0.992**	10^6	15	286	16100	111	16100
bean	**0.871**	0.777	0.812	0.853	0.804	0.841	$5 \cdot 10^6$	15	295	25900	112	16800
eeg	**0.708**	0.666	0.689	0.706	0.684	0.699	$2 \cdot 10^6$	13	289	26000	95	11000
avila	**0.585**	0.532	0.574	**0.585**	0.563	0.572	$3 \cdot 10^7$	9	268	24700	60	38900
magic	**0.831**	0.801	0.822	0.828	0.807	0.816	$6 \cdot 10^6$	15	298	28000	70	4190
htru	**0.981**	0.979	0.979	0.980	0.979	0.980	$6 \cdot 10^7$	15	295	25300	55	2180
occup.	**0.994**	0.989	0.991	**0.994**	0.990	0.992	$7 \cdot 10^5$	13	280	16300	33	510
skin	**0.969**	0.966	0.966	0.966	0.966	0.966	$7 \cdot 10^4$	15	301	23300	20	126
fault	**0.682**	0.553	0.672	0.674	0.672	0.673	$9 \cdot 10^8$	13	295	24200	111	16800
segment	**0.887**	0.574	0.812	0.879	0.786	0.825	$2 \cdot 10^6$	7	220	16300	68	11400
page	**0.971**	0.964	0.970	0.970	0.964	0.965	10^7	15	298	22400	701	4050
bidding	**0.993**	0.981	0.985	**0.993**	0.985	**0.993**	$3 \cdot 10^5$	13	256	9360	58	2700
raisin	**0.894**	0.869	0.879	0.886	0.875	0.883	$4 \cdot 10^6$	15	295	20900	48	1440
rice	**0.938**	0.933	0.934	0.937	0.933	0.936	$2 \cdot 10^7$	15	298	25500	49	1470
wilt	**0.996**	0.993	0.994	0.995	0.994	0.994	$3 \cdot 10^5$	13	274	11300	33	465
bank	**0.983**	0.933	0.971	0.980	0.951	0.974	$6 \cdot 10^4$	13	271	7990	26	256

Why generating candidate splits with CART?

Theorem (DPDT trees are not worse than greedy trees)

The greedy tree is always a solution of the MDPs we solve. Because we solve the MDPs exactly with DP, if the greedy tree is the best solution, DPDT will find it.

Theorem (DPDT trees can be strictly better than greedy trees)

There exist a depth budget D and a dataset for which DPDT trees are strictly better than greedy trees.[a]

[a] cf. checkersboard dataset.

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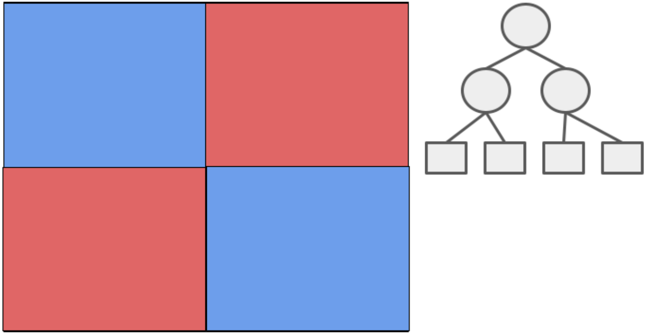
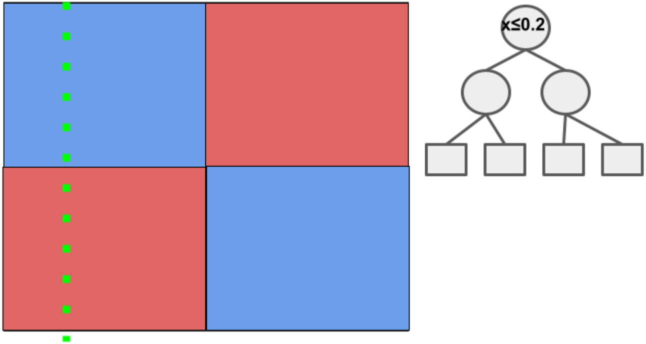
DPDT trees can be strictly better than greedy trees



DPDT trees can be strictly better than greedy trees



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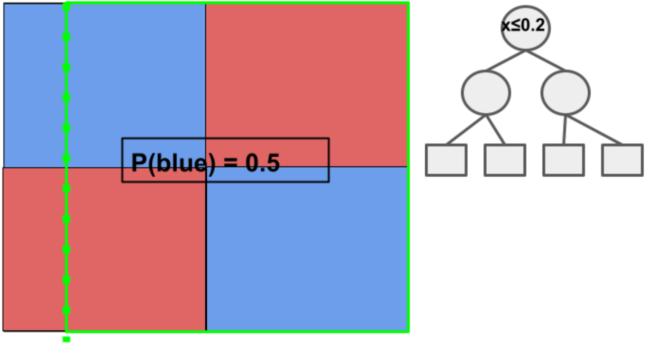

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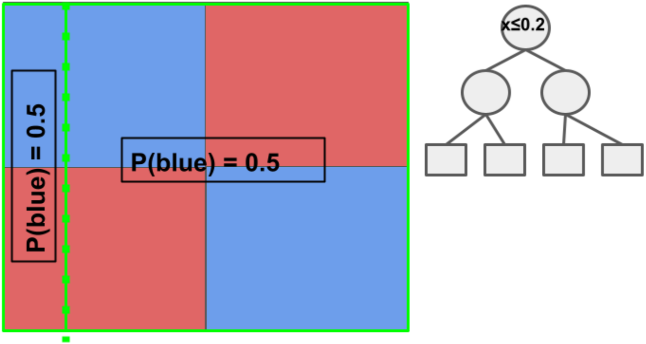

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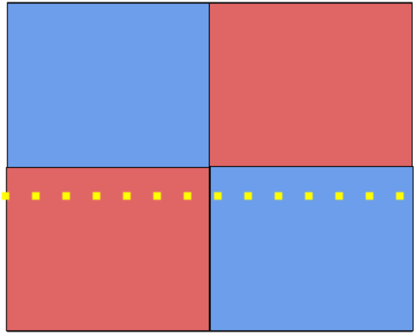
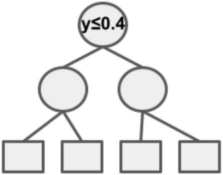
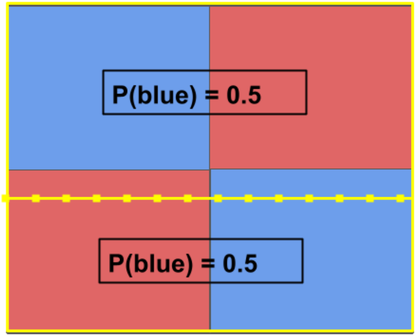
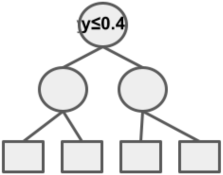


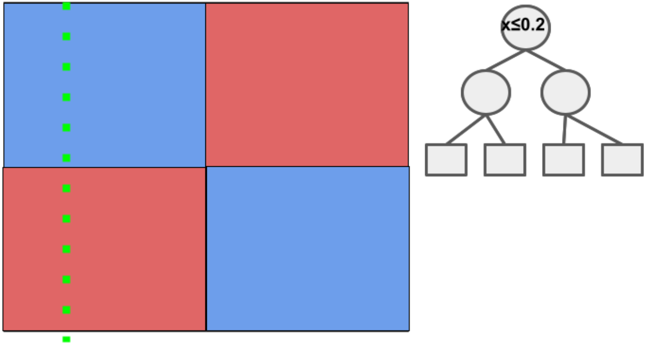


DPDT trees can be strictly better than greedy trees

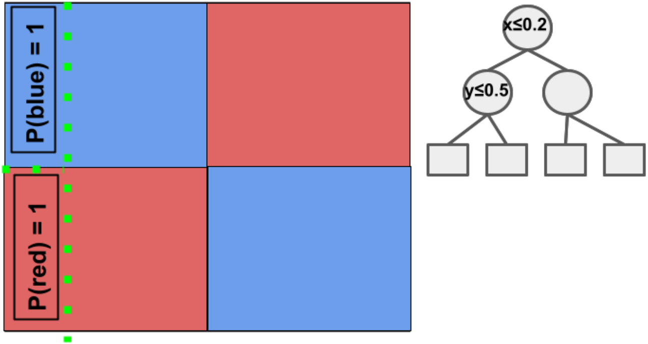




DPDT trees can be strictly better than greedy trees



DPDT trees can be strictly better than greedy trees



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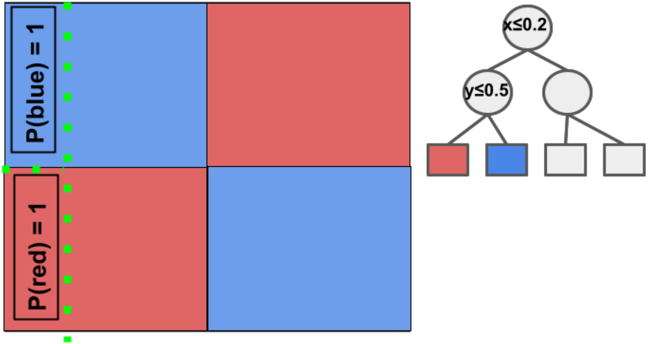
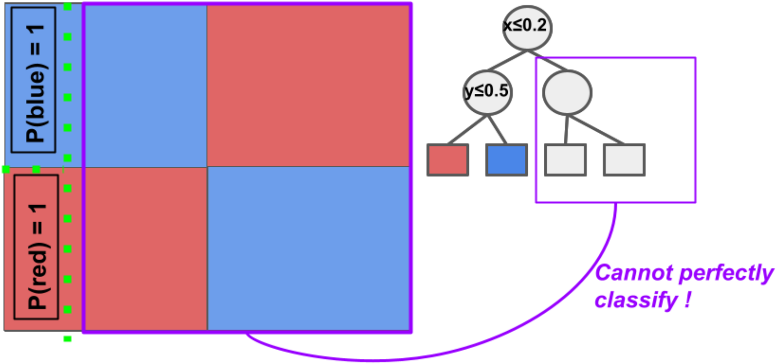
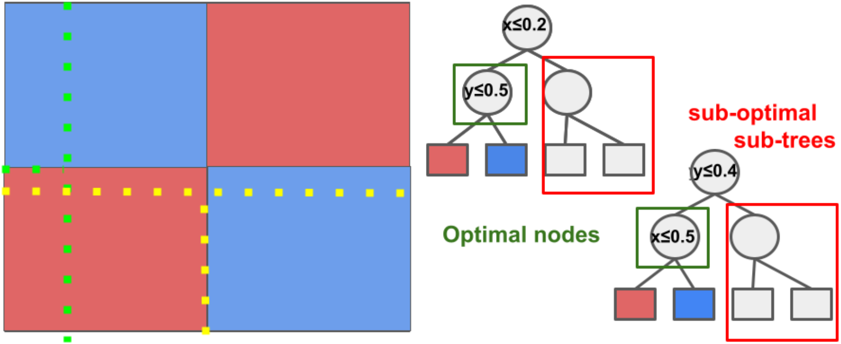



P(blue) = 1
P(red) = 1
x≤0.2
y≤0.5
Cannot perfectly classify !

DPDT trees can be strictly better than greedy trees



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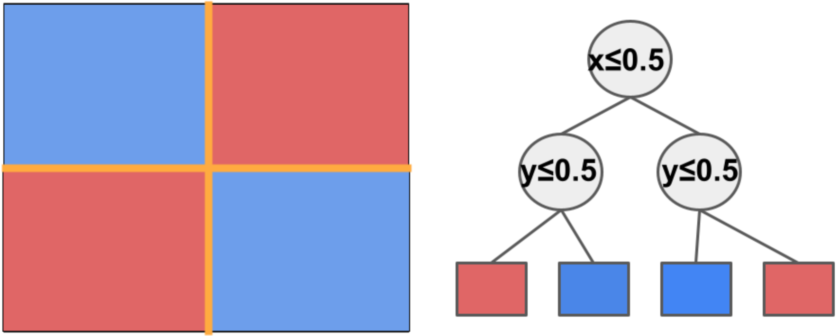
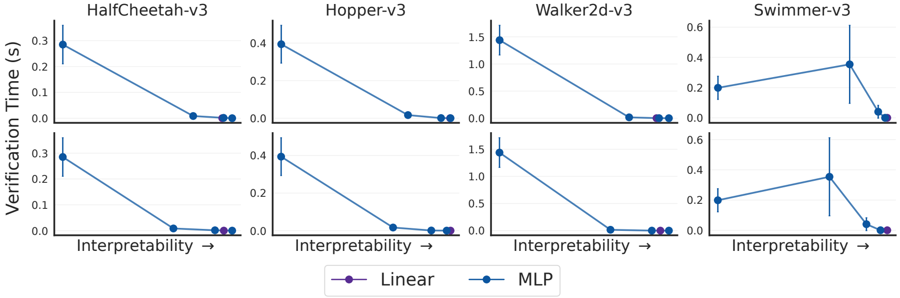
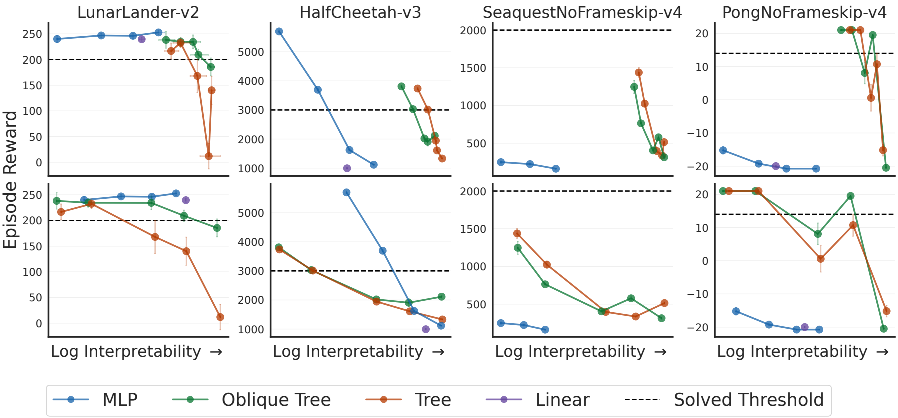


Result: verification time does scale with step inference time



Verification time as a function of policy interpretability. Top row, interpretability is measured with step inference times. Bottom row, the interpretability is measured with policy size.

Result: there is no dominating policy class for all environments



Interpretability-Performance trade-offs. Top row, interpretability is measured with step inference times. Bottom row, the interpretability is measured with policy size.

We propose policy unfolding

```
# Decision tree for Mountain Car
def play(x):
    if x[1] <= -0.2597:
        if x[1] <= -0.6378:
            return 0
        else:
            if x[0] <= -1.0021:
                return 2
            else:
                return 0
    else:
        if x[1] <= -0.0508:
            if x[0] <= 0.2979:
                if x[0] <= 0.0453:
                    return 2
                else:
                    if x[1] <=
  -0.2156:
                        return 0
                    else:
                        return 2
            else:
                return 0
        else:
            return 2
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                    else:
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            return 2
```

```python
# Small ReLU MLP for Pendulum
def play(x):
    h_layer_0_0 = 1.238*x[0]+0.971*x
     [1]
                  +0.430*x[2]+0.933
    h_layer_0_0 = max(0, h_layer_0_0
     )
    h_layer_0_1 = -1.221*x[0]+1.001
                  *x[1]-0.423*x[2]
                  +0.475
    h_layer_0_1 = max(0, h_layer_0_1
     )
    h_layer_1_0 = -0.109*h_layer_0_0
                  -0.377*h_layer_0_1
                  +1.694
    h_layer_1_0 = max(0, h_layer_1_0
     )
    h_layer_1_1 = -3.024*h_layer_0_0
                  -1.421*h_layer_0_1
                  +1.530
    h_layer_1_1 = max(0, h_layer_1_1
     )

    h_layer_2_0 = -1.790*h_layer_1_0
                  +2.840*h_layer_1_1
                  +0.658
    y_0 = h_layer_2_0
    return [y_0]
```