

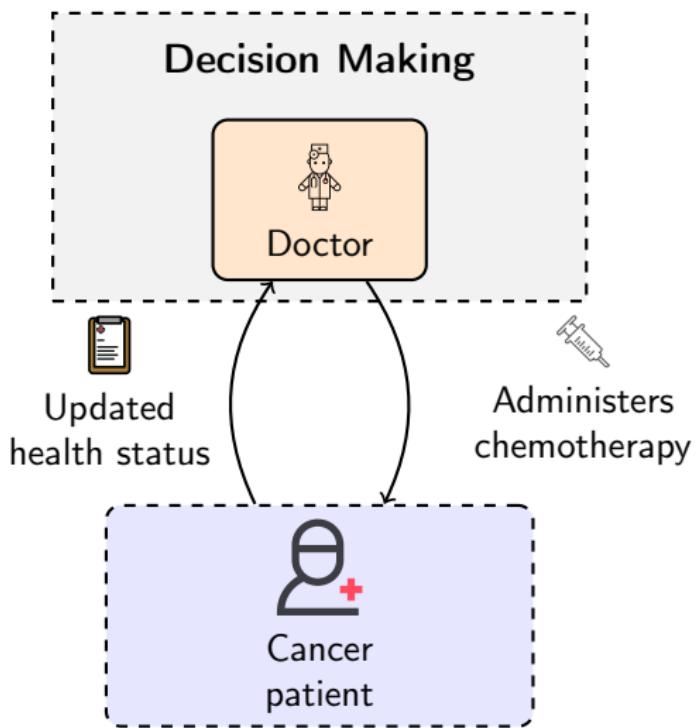
# Interpretability, Decision Trees, and Sequential Decision Making

Hector Kohler

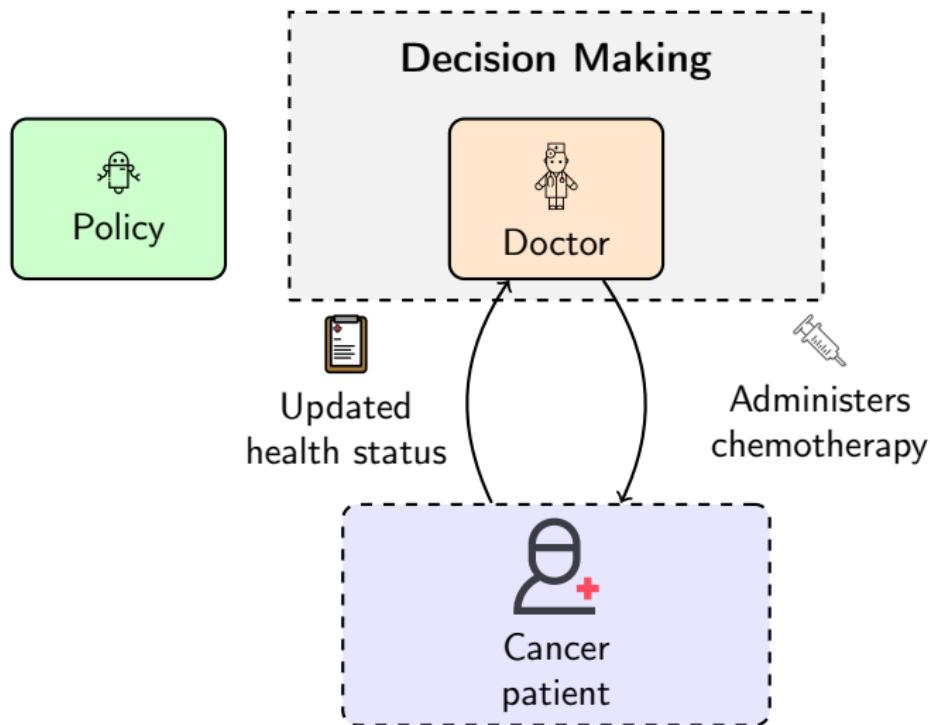
Supervised by Dr. Riad Akrour (HdR) and Prof. Philippe Preux (HdR)  
Université de Lille, CNRS, Inria, UMR CRIStAL 9189, France

November 25, 2025

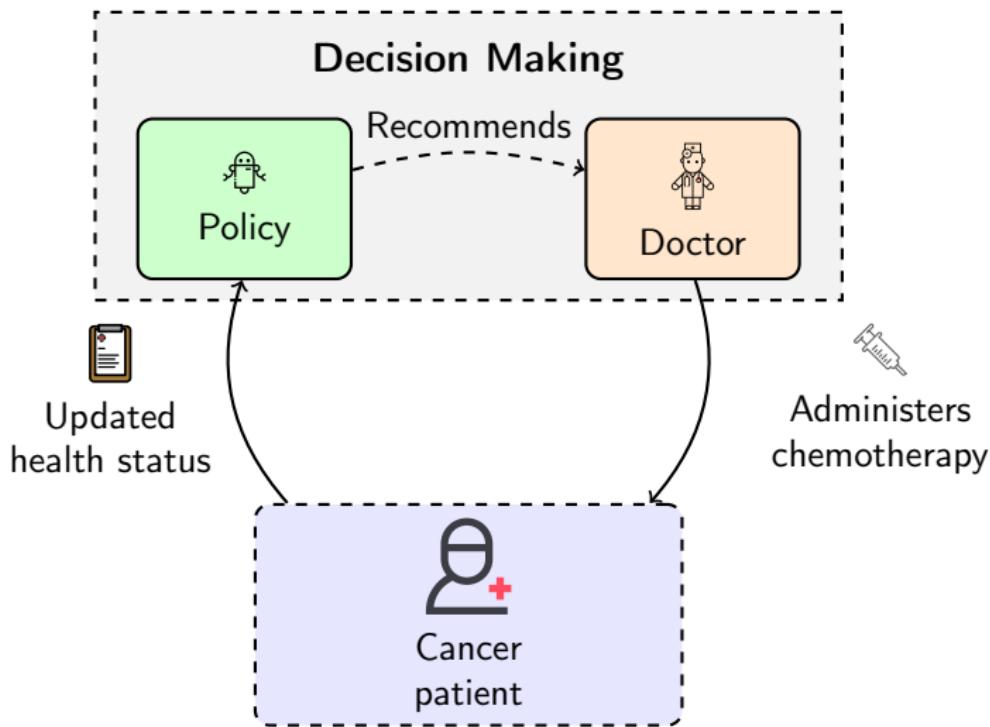
# Sequential decision making (SDM) and machine learning (ML)



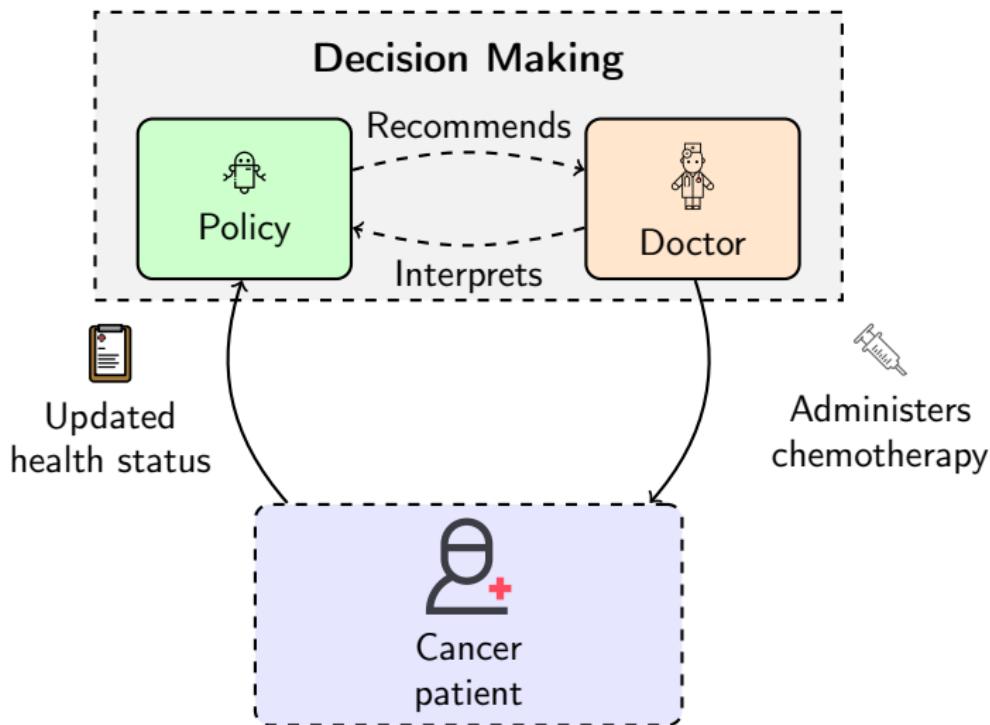
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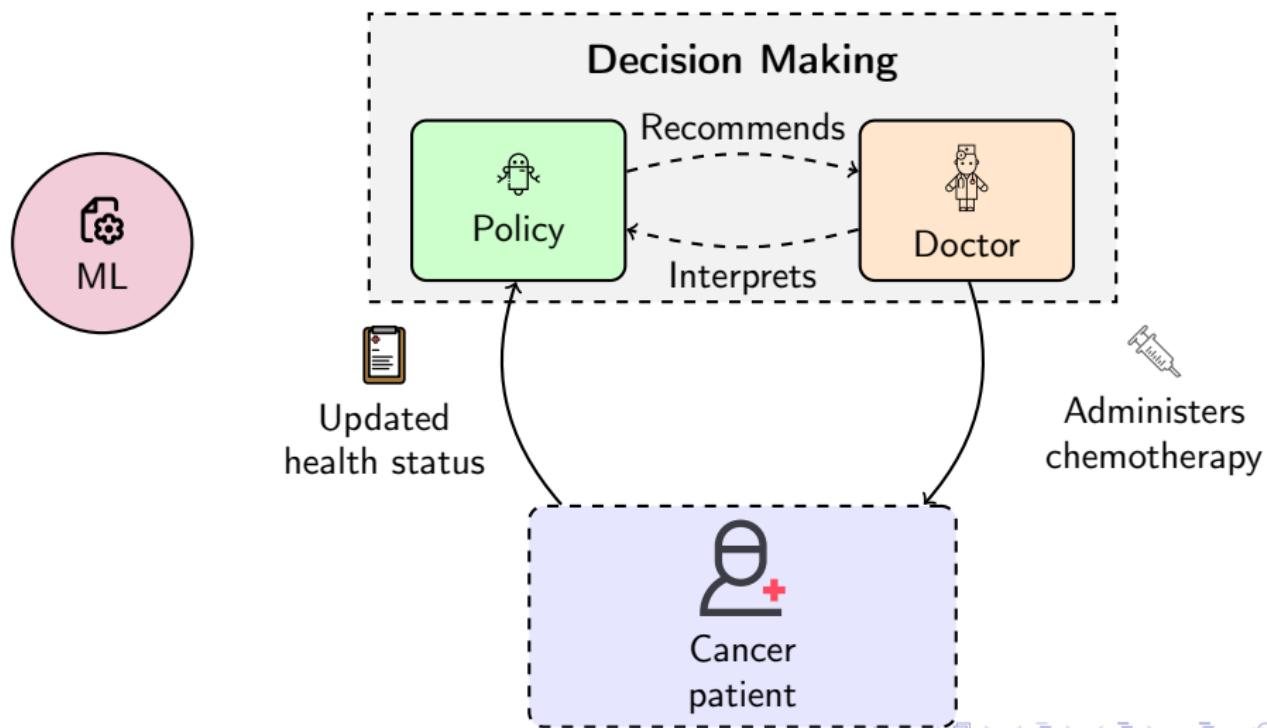
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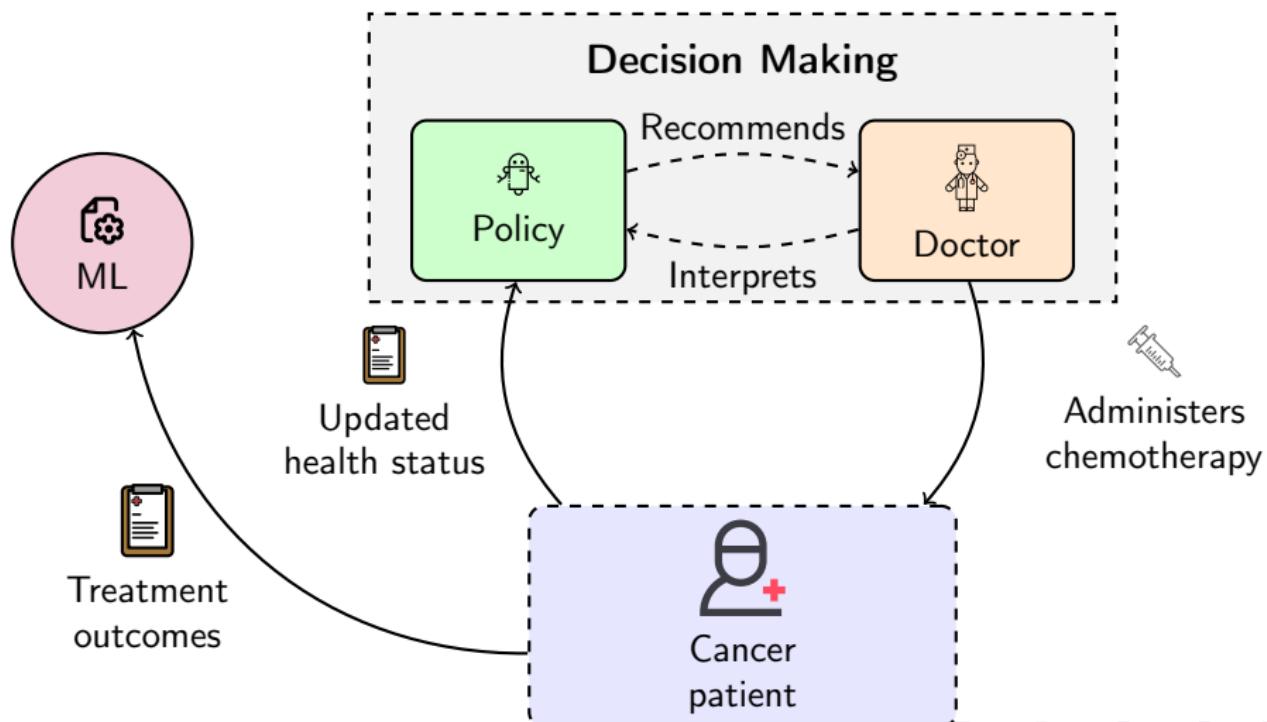
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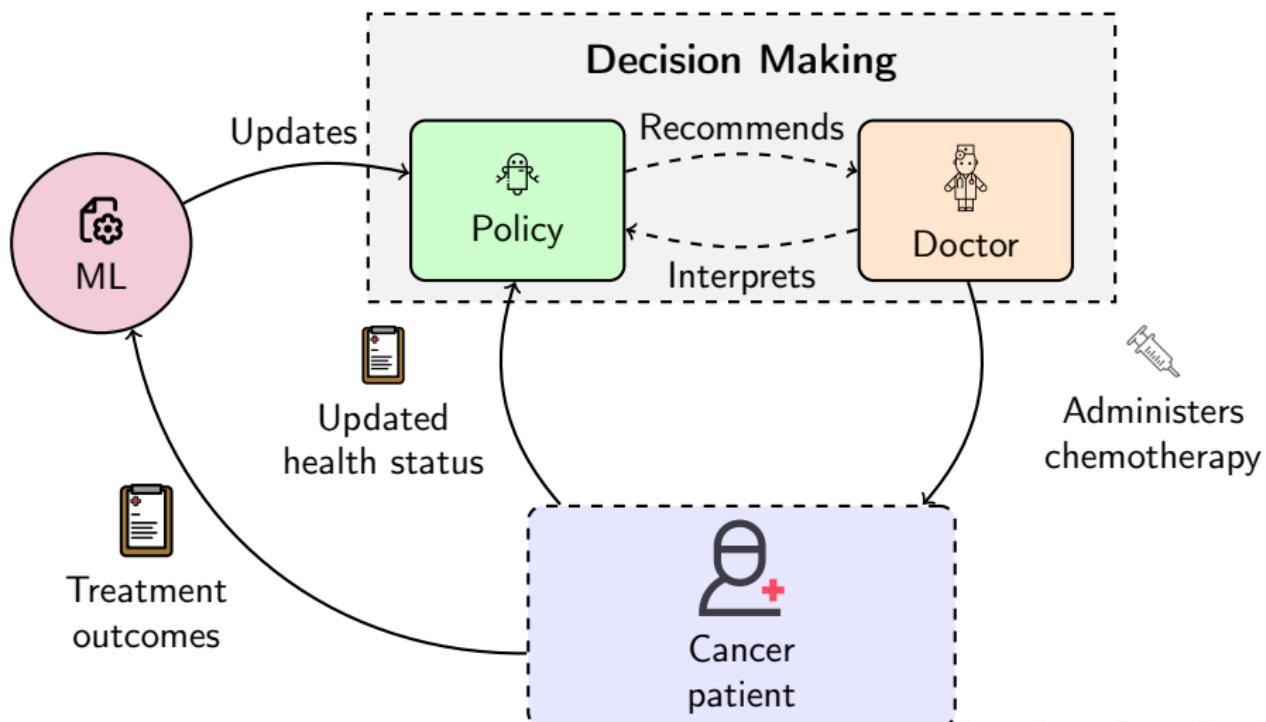
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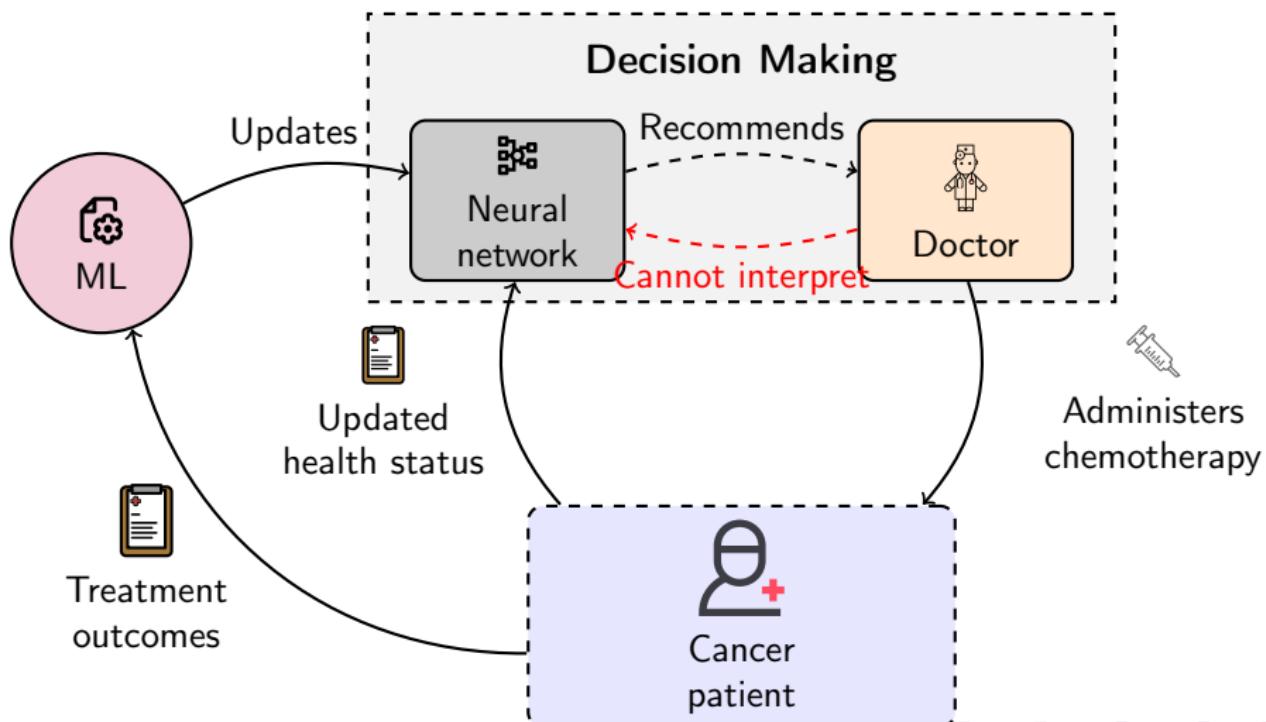
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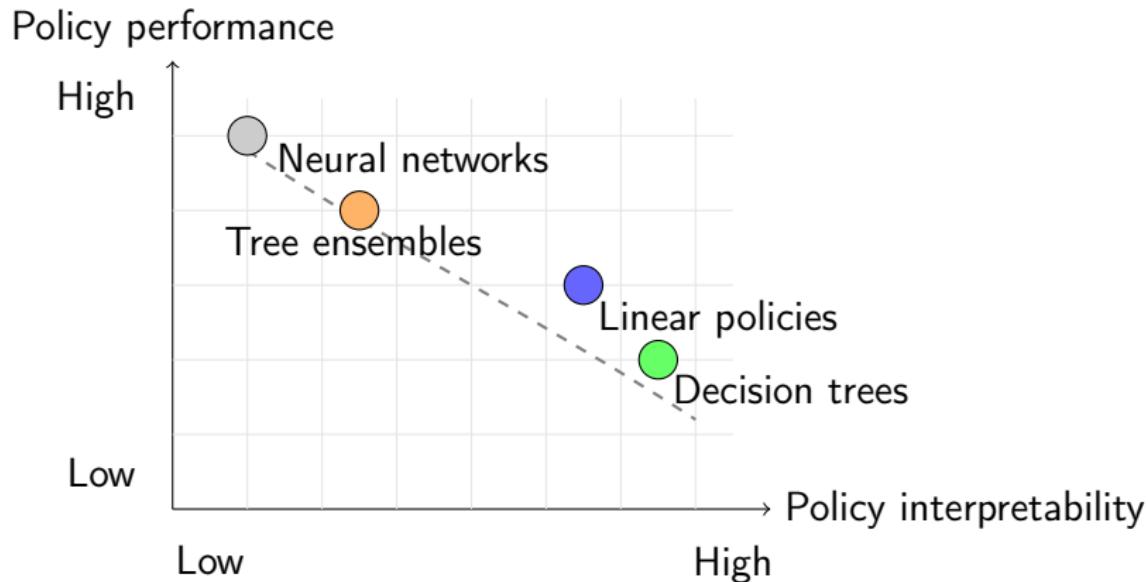
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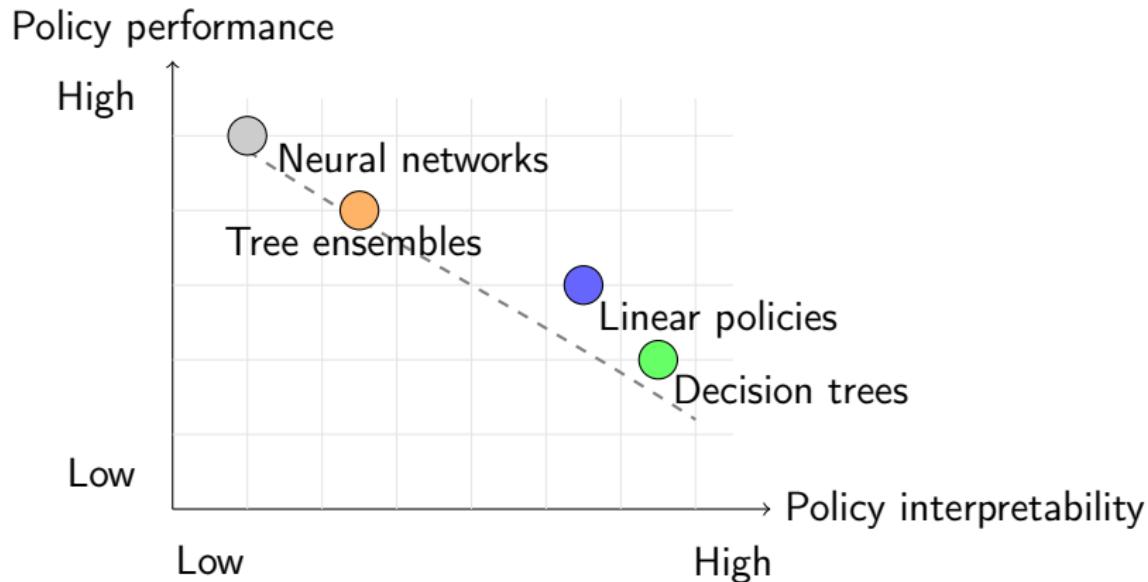


# Policy interpretability



**Heuristic** interpretability-performance trade-offs of different policy classes.

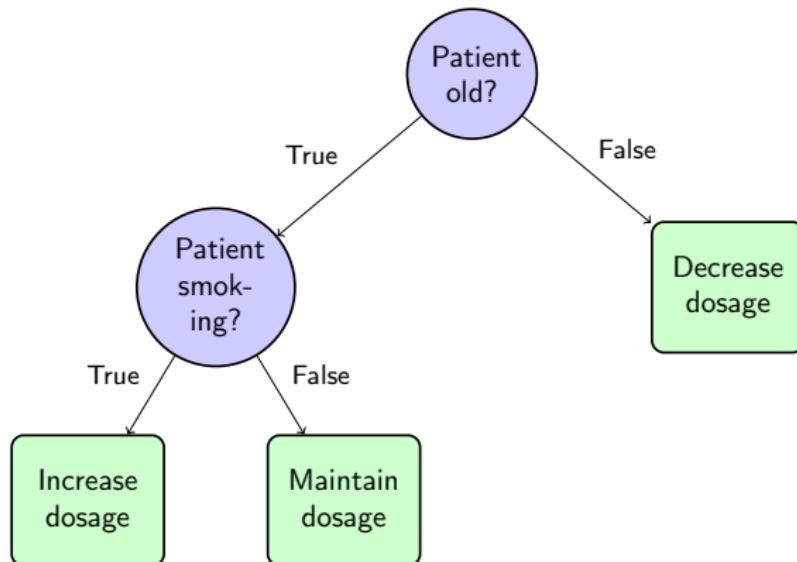
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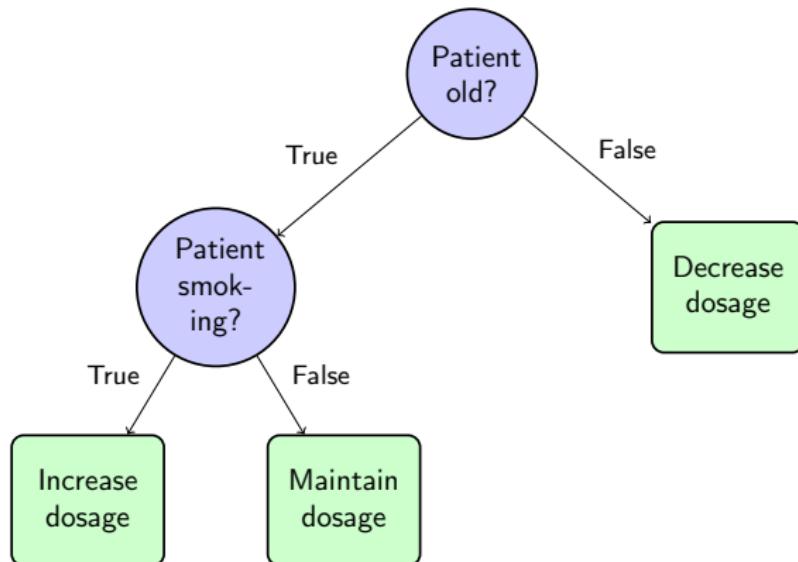
⚠ No definition of interpretability in machine learning models!

# Decision trees



A generic decision tree of depth  $D = 2$ .

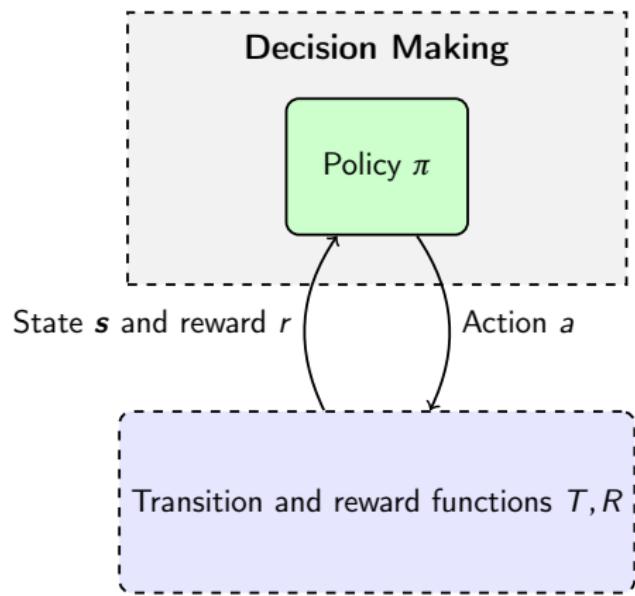
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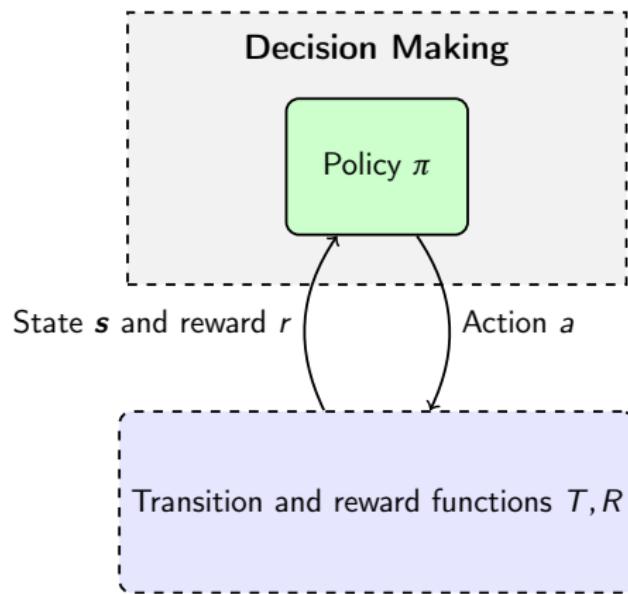
Successful algorithms for non-sequential learning: [Bre+84; BD17; Dem+22;  
VZ19; MMW22] ... What about SDM?

# Markov decision processes (MDPs) and reinforcement learning (RL)



Markov decision processes [Put94].

# Markov decision processes (MDPs) and reinforcement learning (RL)

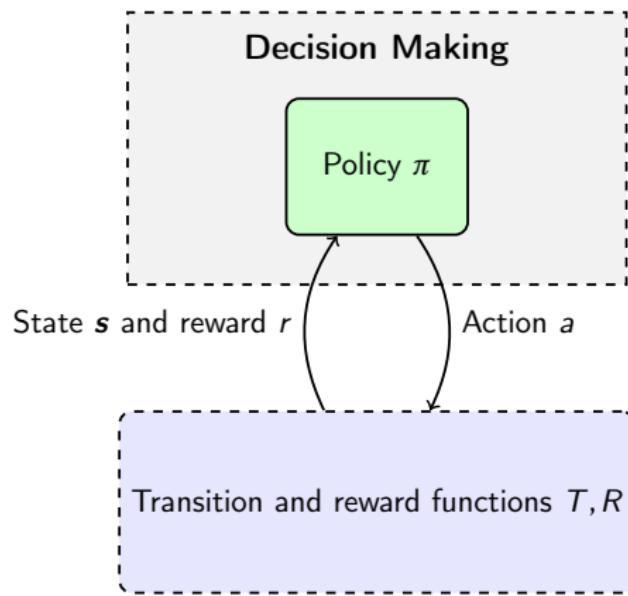


- RL [SB98] aims to find a policy,  $\pi : S \rightarrow A$  that maximizes:

$$\mathbb{E}_{s_t \sim T} \left[ \sum_{t=0}^{\infty} \gamma^t R(s_t, \pi(s_t)) \right]$$

Markov decision processes [Put94].

# Markov decision processes (MDPs) and reinforcement learning (RL)



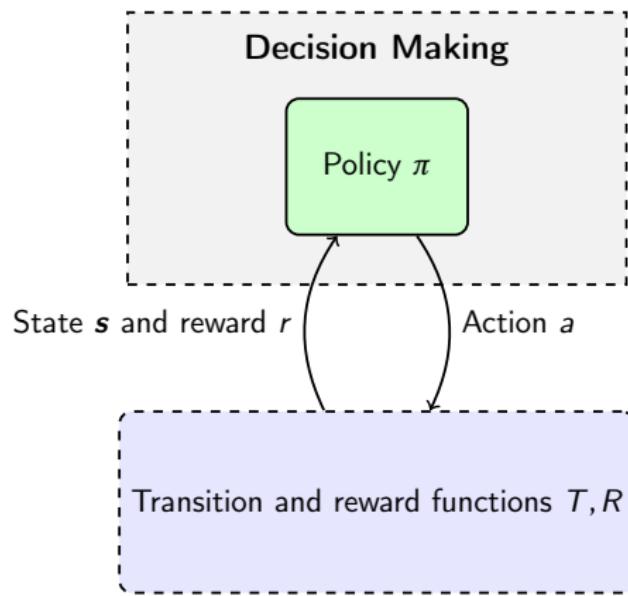
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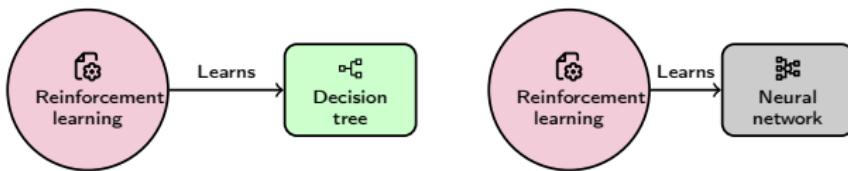
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- Lots of successful RL algorithms [SB98; Mni+15; Sch+17].
- Few interpretability concerns.

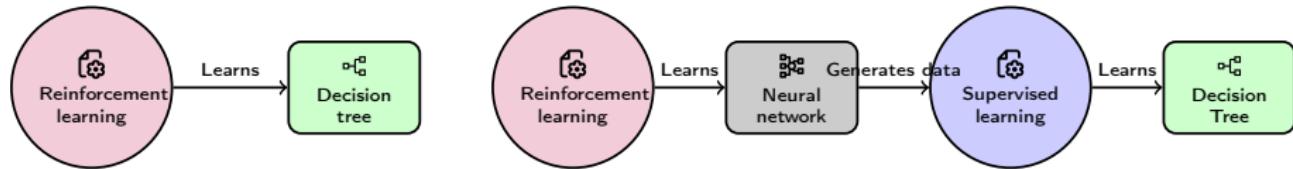
# Two ways to get interpretable policies for SDM



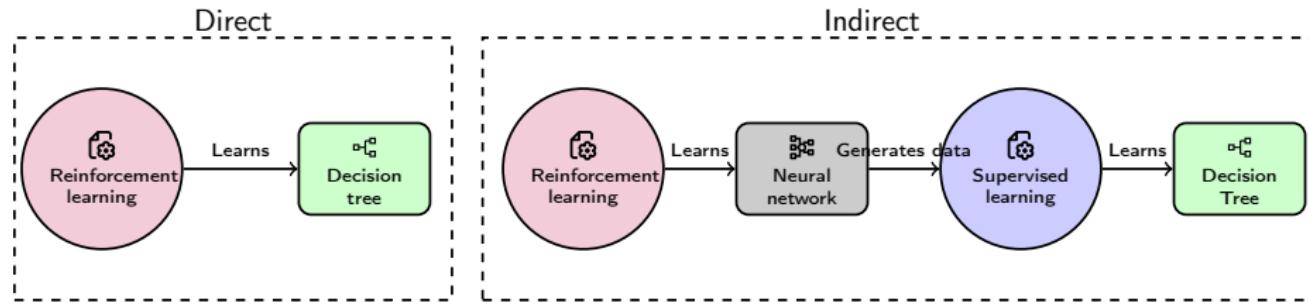
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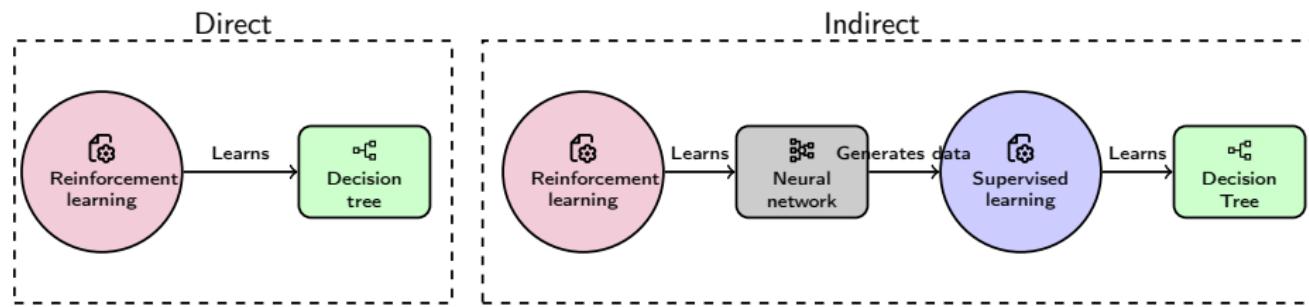
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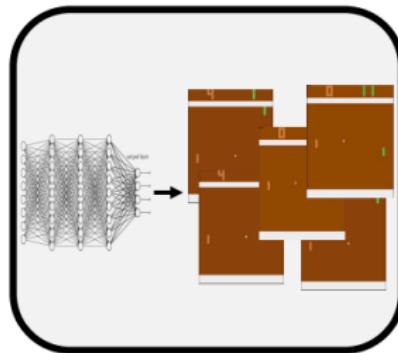


# Two ways to get interpretable policies for SDM

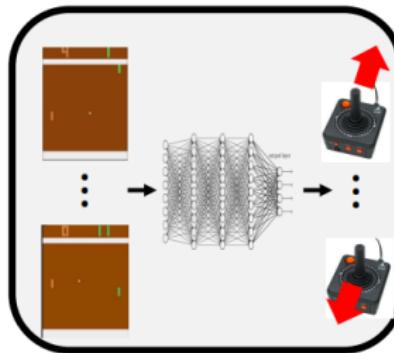


⚠ Policies obtained indirectly optimize a surrogate objective rather than a trade-off between interpretability and cumulative rewards.

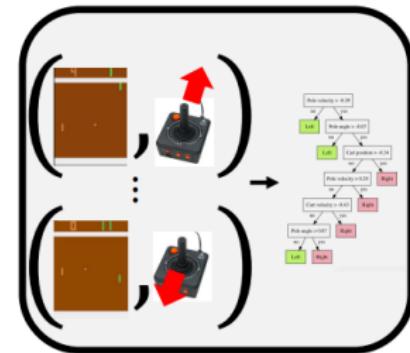
# Indirect approach: imitation learning



**Step 1:** Use NN to generate states

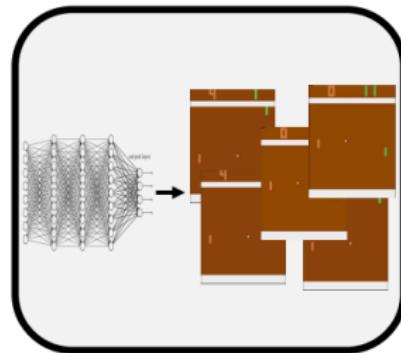


**Step 2:** Use NN to obtain actions

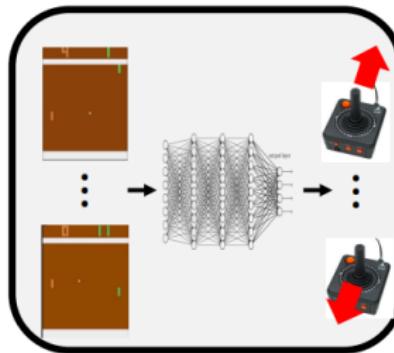


**Step 3:** Use supervised learning  
to train a decision tree

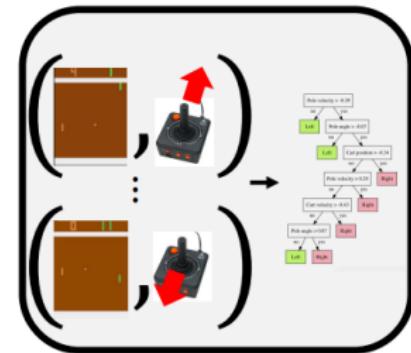
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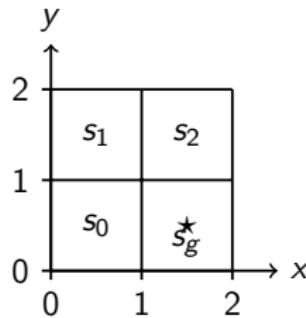
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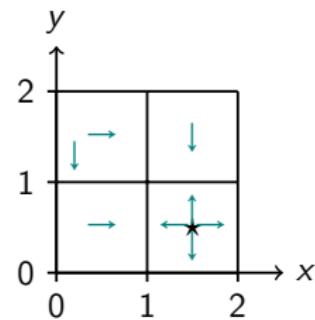
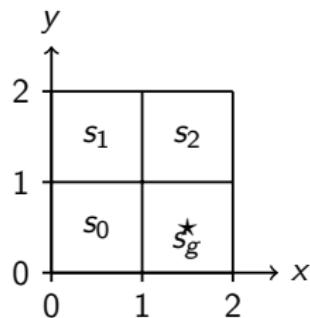
**Step 3:** Use supervised learning  
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Most research focused on indirect learning of interpretable policies [RGB10;  
BPS18; Ver+18; Mil+24].

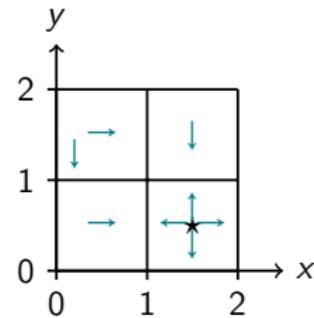
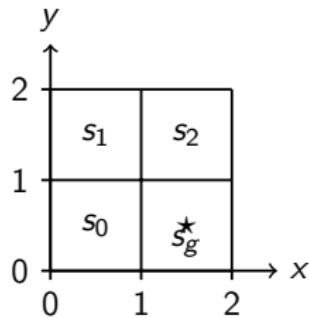
# Grid world MDP and decision tree policies



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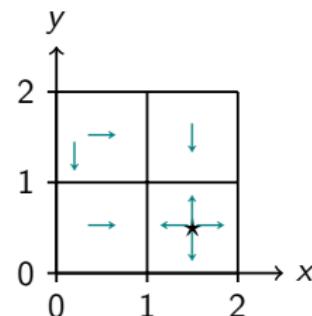
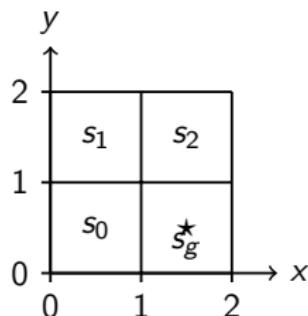


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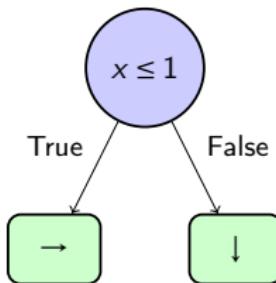


Grid world MDP and optimal actions.

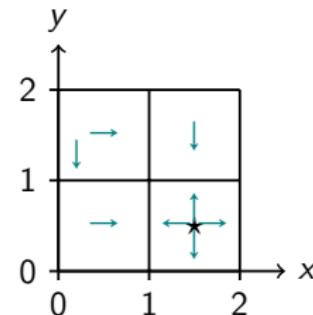
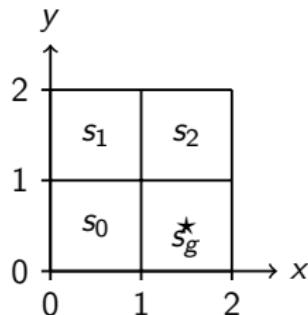
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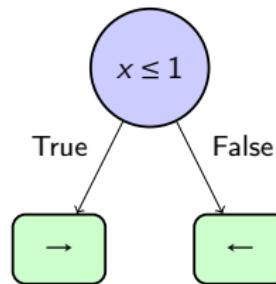
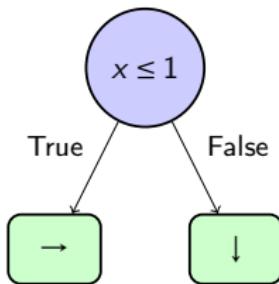
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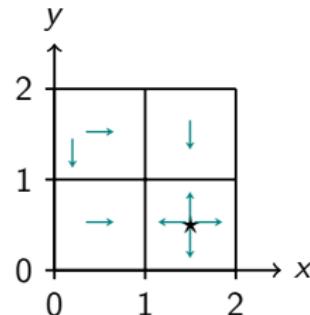
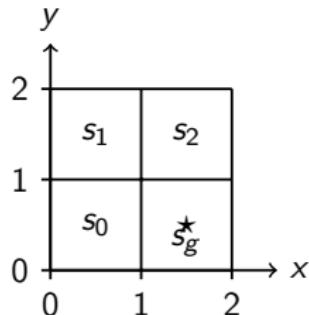
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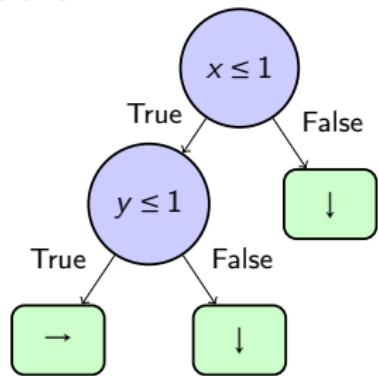
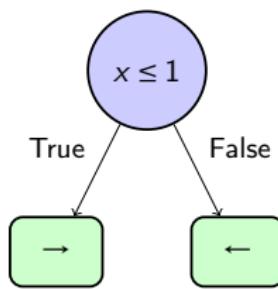
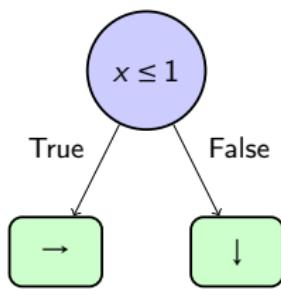
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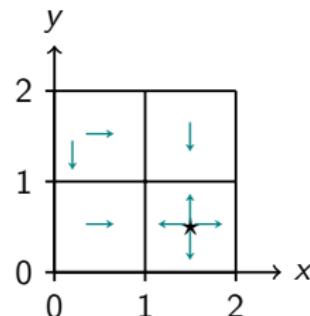
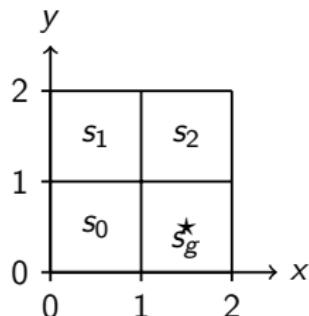
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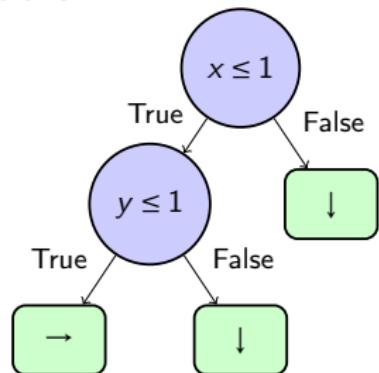
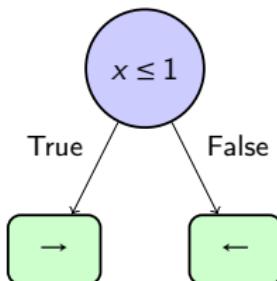
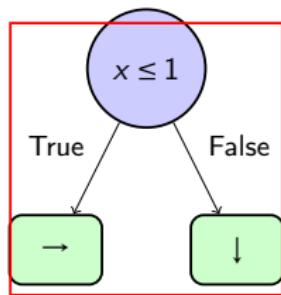
Grid world MDP and optimal actions.



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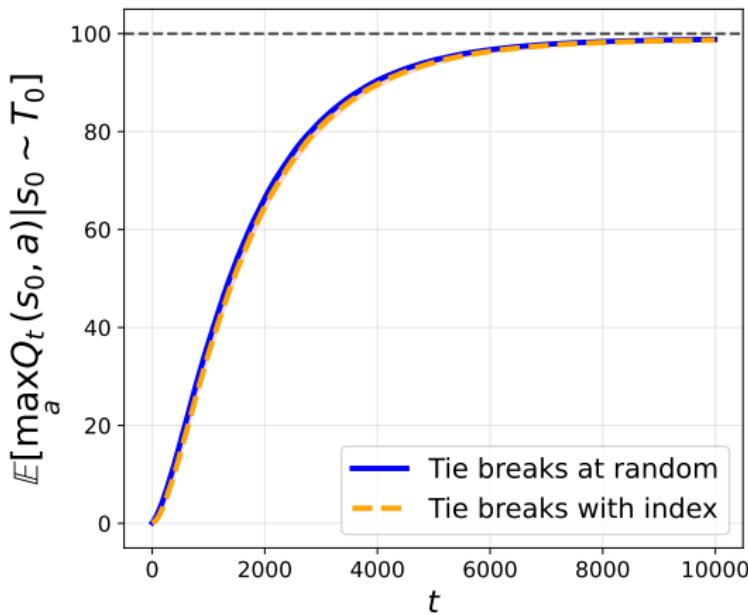


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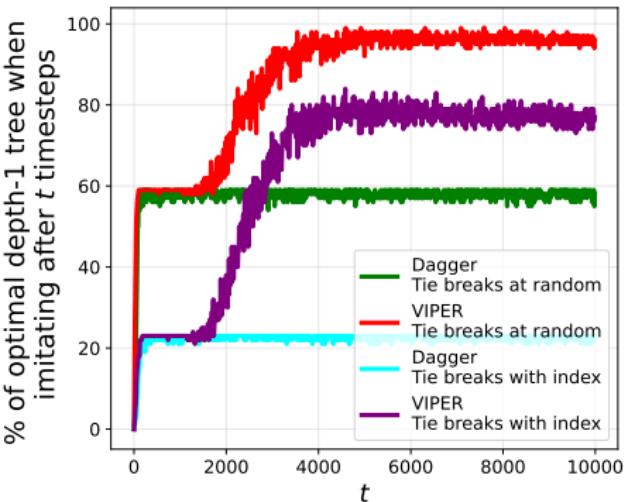
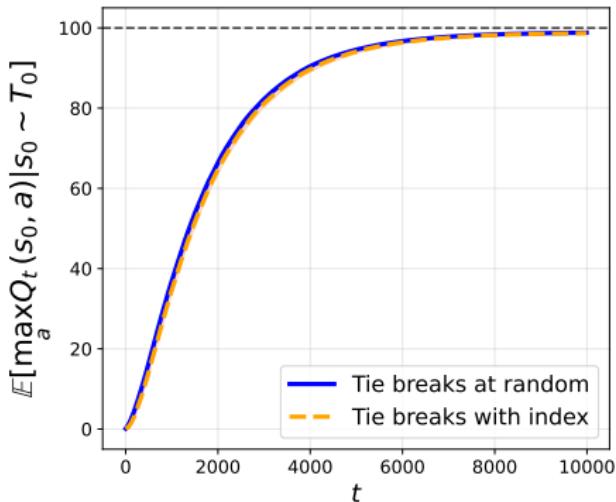
Decision tree policies with different interpretability-performance trade-offs.

# Grid world MDP and decision tree policies



Sample complexity curve of Q-learning over 100 random seeds.

# Grid world MDP and decision tree policies



Sample complexity curve of Q-learning over 100 random seeds and performance of indirect interpretable methods when imitating the greedy policy with a tree at different Q-learning stages.

# Contributions

- ① How difficult is it to directly optimize a trade-off of interpretability and performance in SDM?
- ② How to leverage sequential decision making to learn interpretable classifiers for supervised learning?
- ③ How to measure policy interpretability in sequential decision making?

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# Direct RL of decision tree policies with iterative boudoing Markov decision processes

Given an MDP  $\mathcal{M} \langle S, A, R, T \rangle$ , an associated iterative bounding Markov decision process (IBMDP, [Top+21])  $\mathcal{M}_{IB}$  is a tuple:

$$\langle \overbrace{S \times O}^{\text{Augmented state space}}, \underbrace{A \cup A_{info}}_{\text{Augmented action space}}, \overbrace{(R, \zeta)}^{\text{Augmented rewards}}, \underbrace{(T_{info}, T, T_0)}_{\text{Augmented transitions}} \rangle$$

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## IBMDPs promises

- No need to design new algorithm: we can use deep RL.

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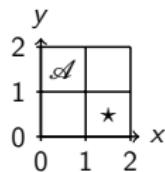
## IBMDPs promises

- No need to design new algorithm: we can use deep RL.
- IBMDP rewards trade-off naturally interpretability and performances.

# Grid world IBMDP example

$t = 0$

$$s_t = (0.5, 1.5)$$

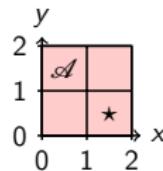


# Grid world IBMDP example

$t = 0$

$$s_t = (0.5, 1.5)$$

$$o_t = (0, 2, 0, 2)$$

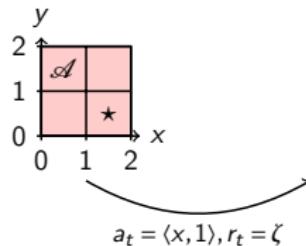


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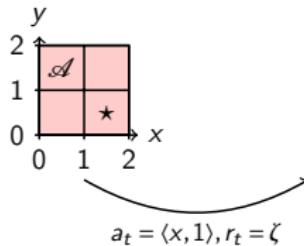
$$a_t = \langle x, 1 \rangle, r_t = \zeta$$

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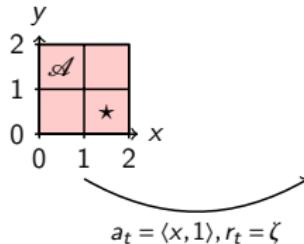
$$x \leq 1$$

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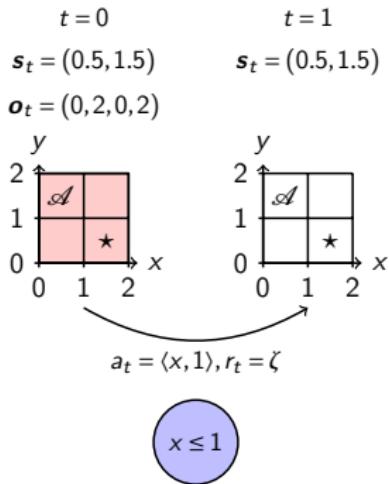
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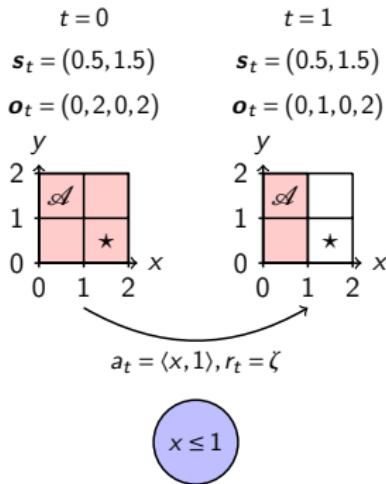


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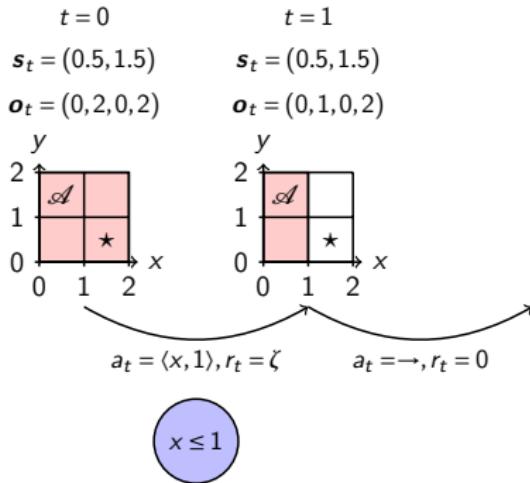
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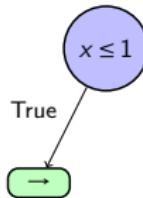
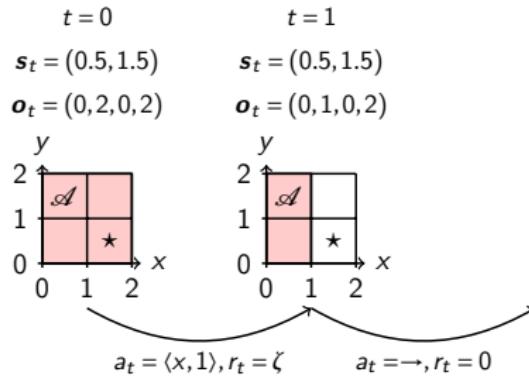
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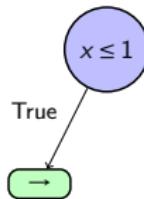
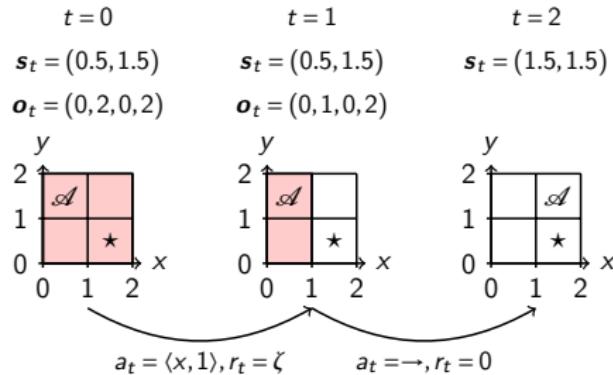
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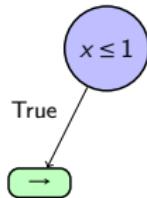
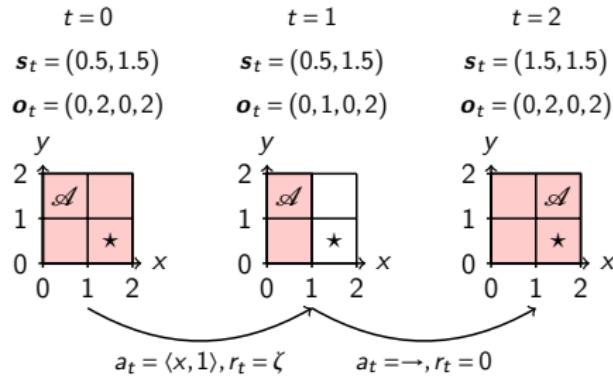
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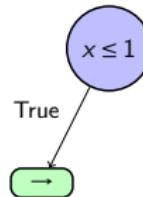
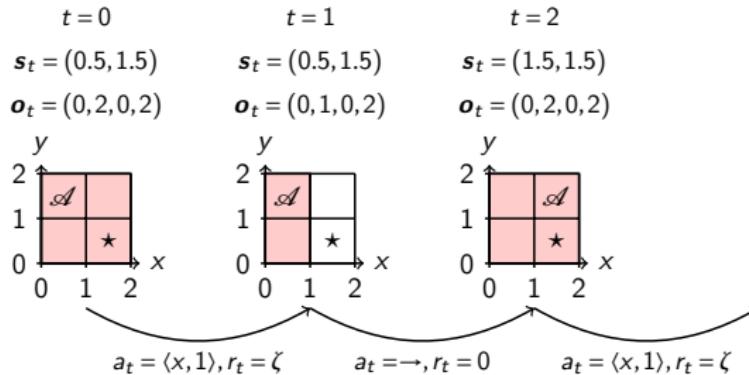
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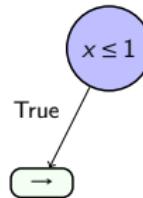
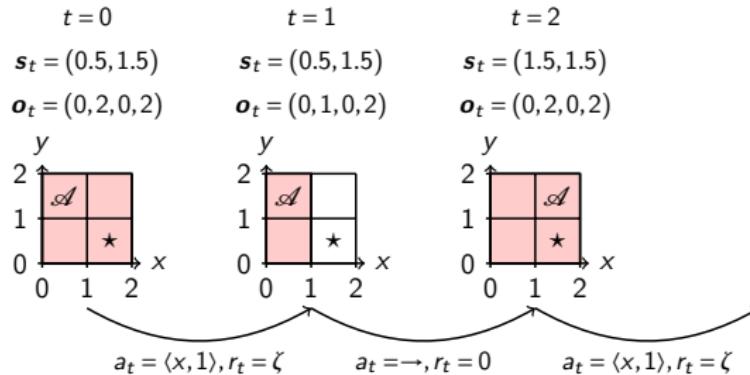
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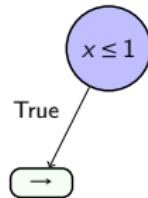
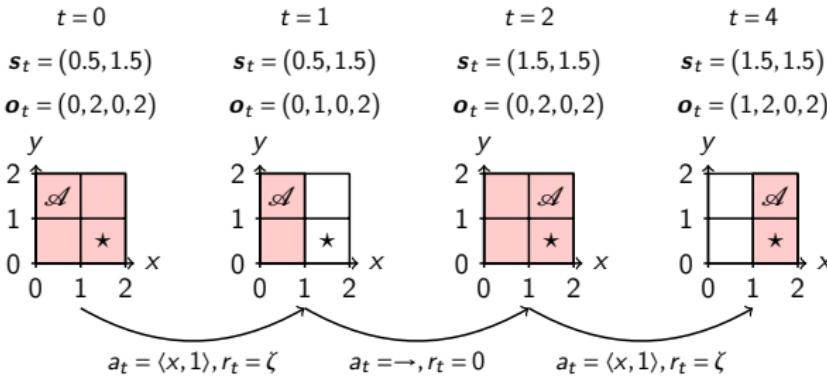
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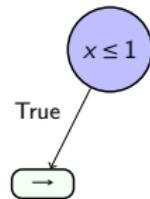
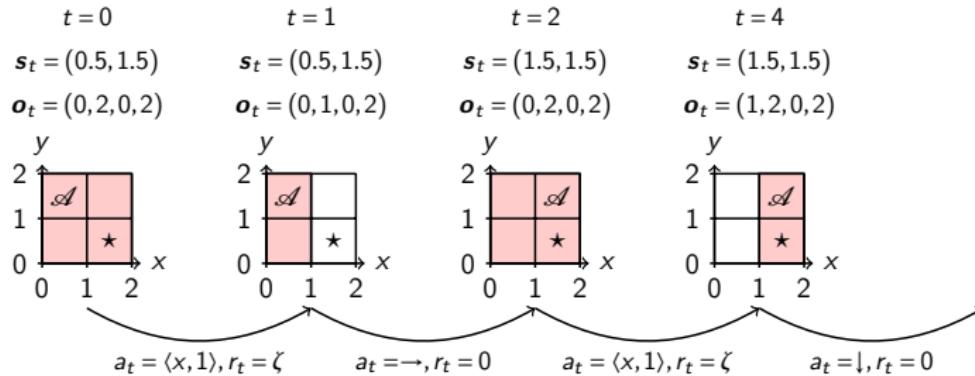
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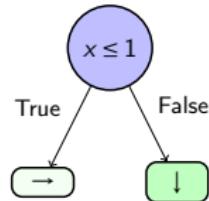
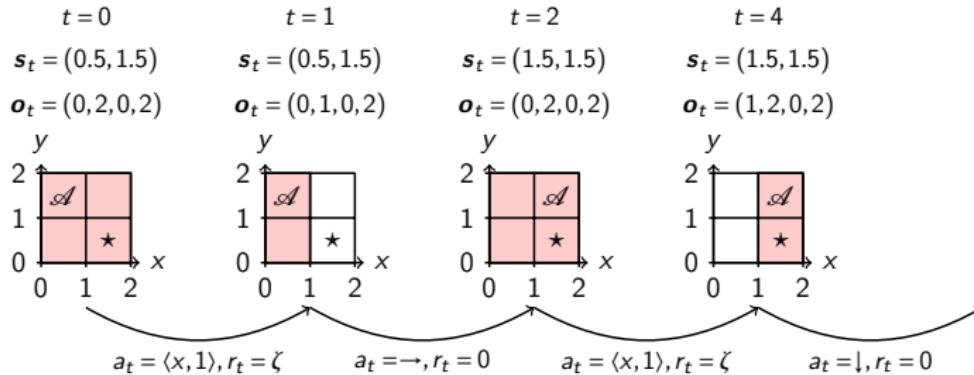
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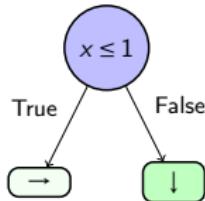
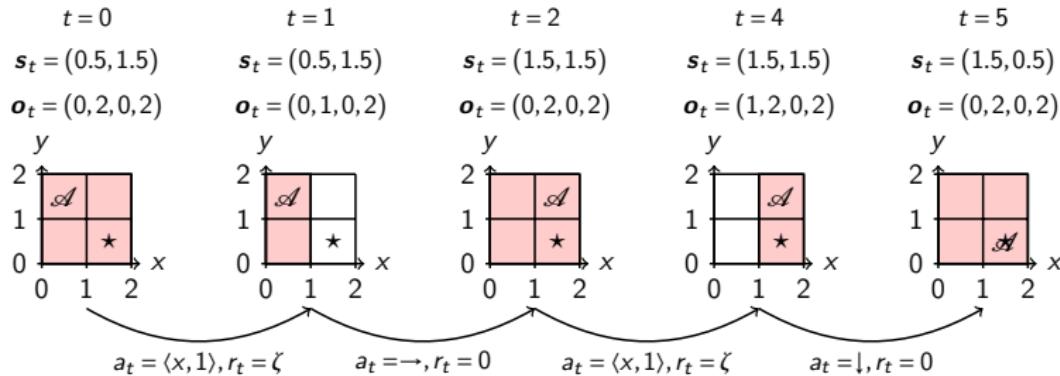
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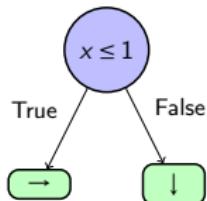
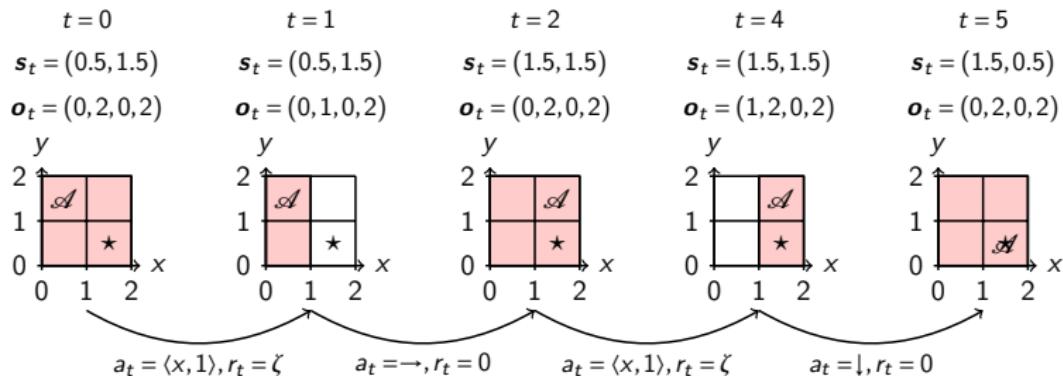
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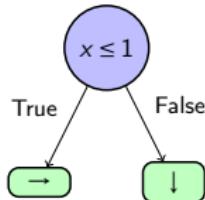
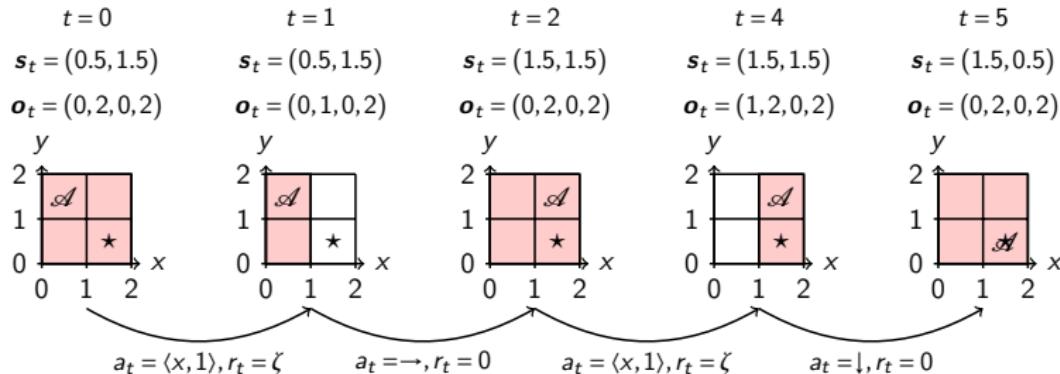
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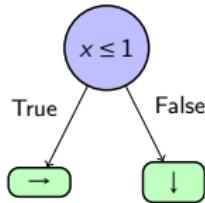
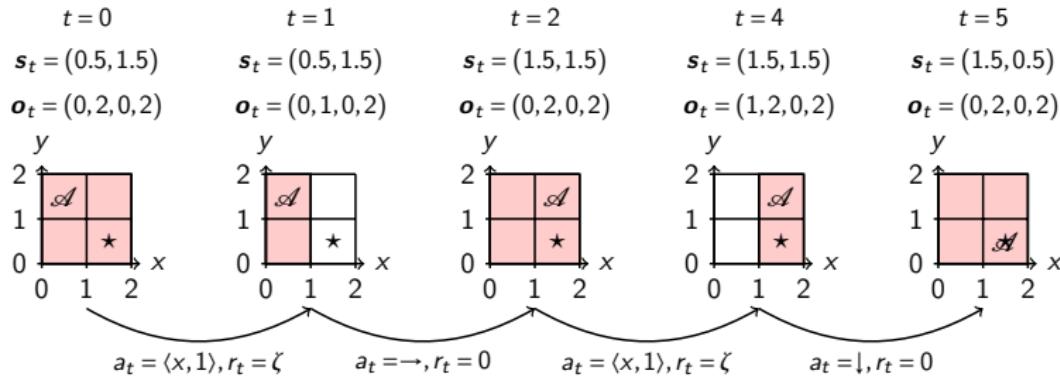


# Grid world IBMDP example



- Deterministic and partially observable policies  $\Rightarrow$  decision trees.

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- **Deterministic and partially observable** policies  $\Rightarrow$  decision trees.
- $\triangleleft$  Finding the best **deterministic and partially observable** policy is NP-hard [Lit94]!

# Re-formulation

*Q: Can we use reinforcement learning to directly optimize trade-offs of performance and interpretability in SDM?*

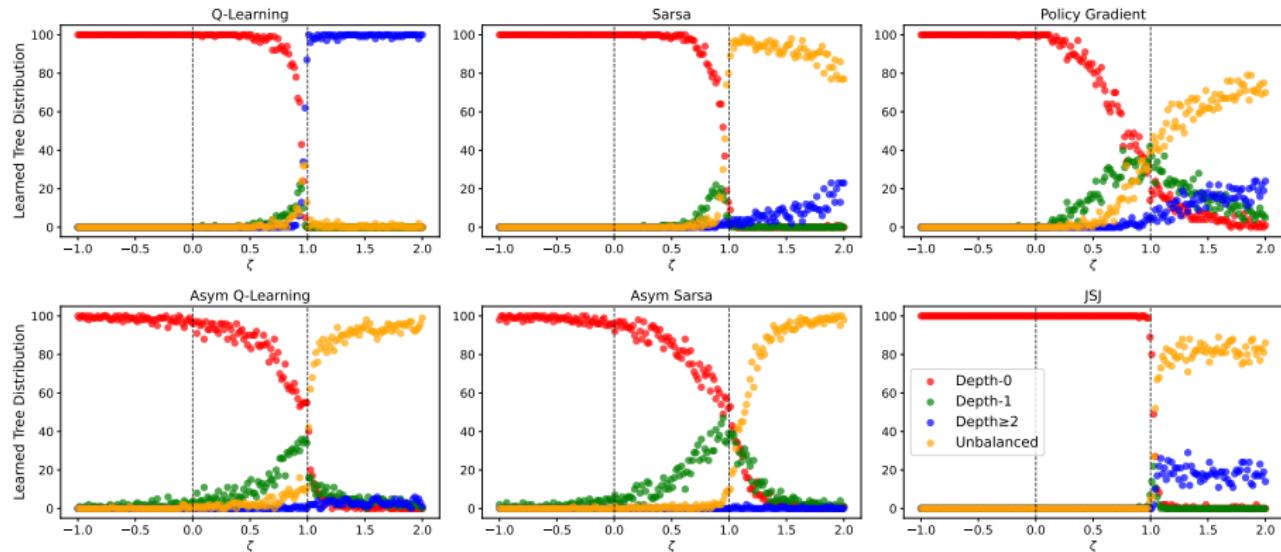
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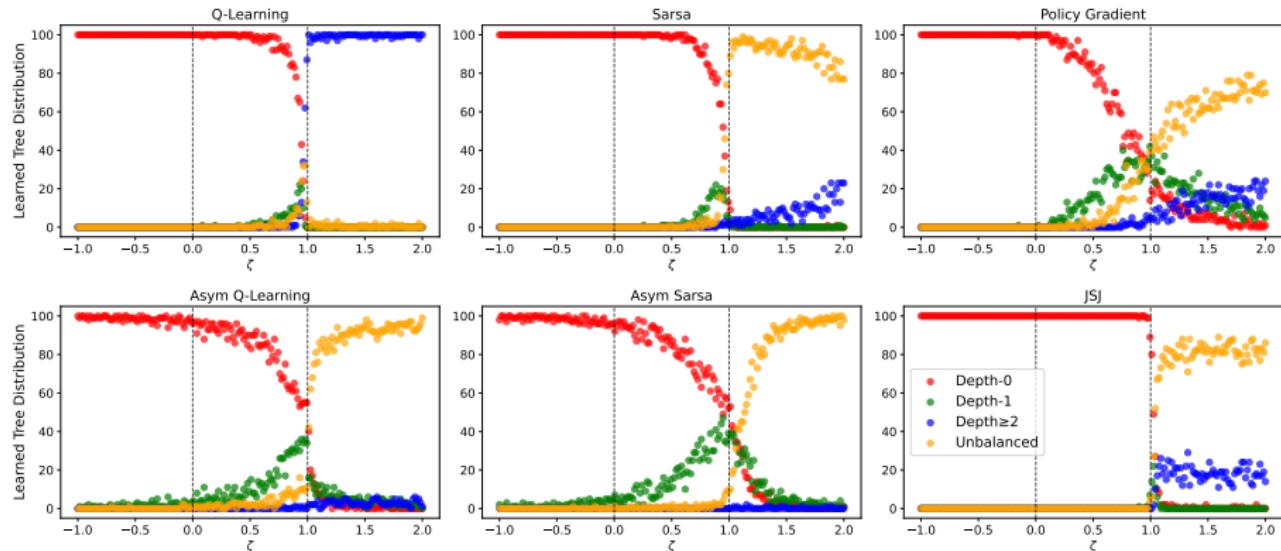
$\approx$

*Q: How does RL perform for optimizing **deterministic** and **partially observable** policies in IBMDPs?*

# Result: RL cannot retrieve optimal depth-1 trees for the grid world MDP

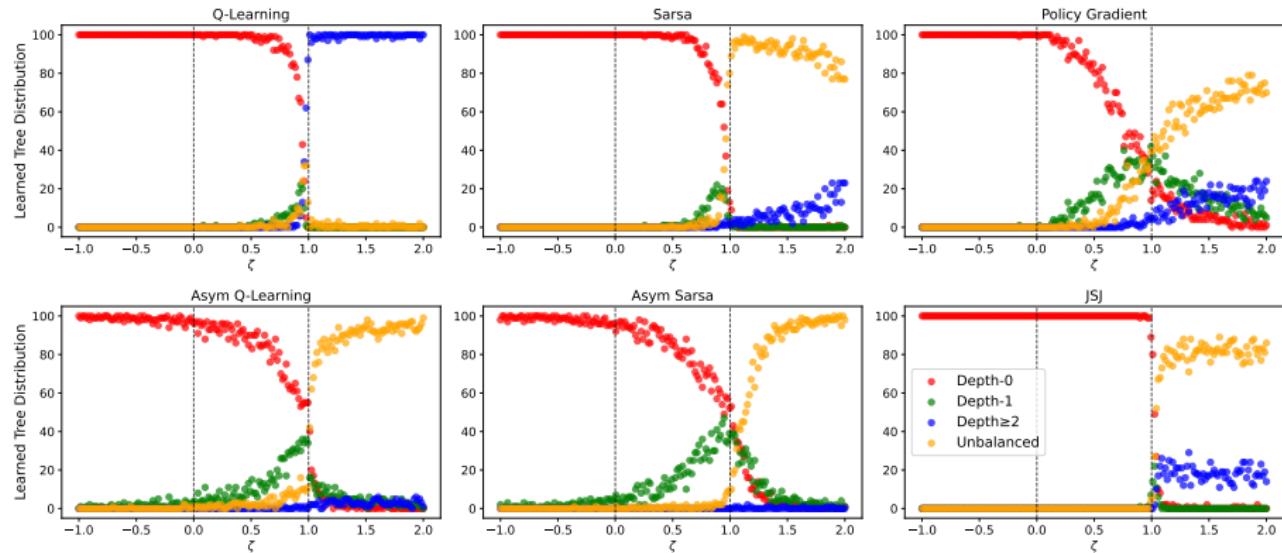


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Distributions of tree policies learned with (asymmetric) RL algorithms [SB98; SJ94; LS98; BA22; BDA22] as a function of the interpretability reward  $\zeta$ .

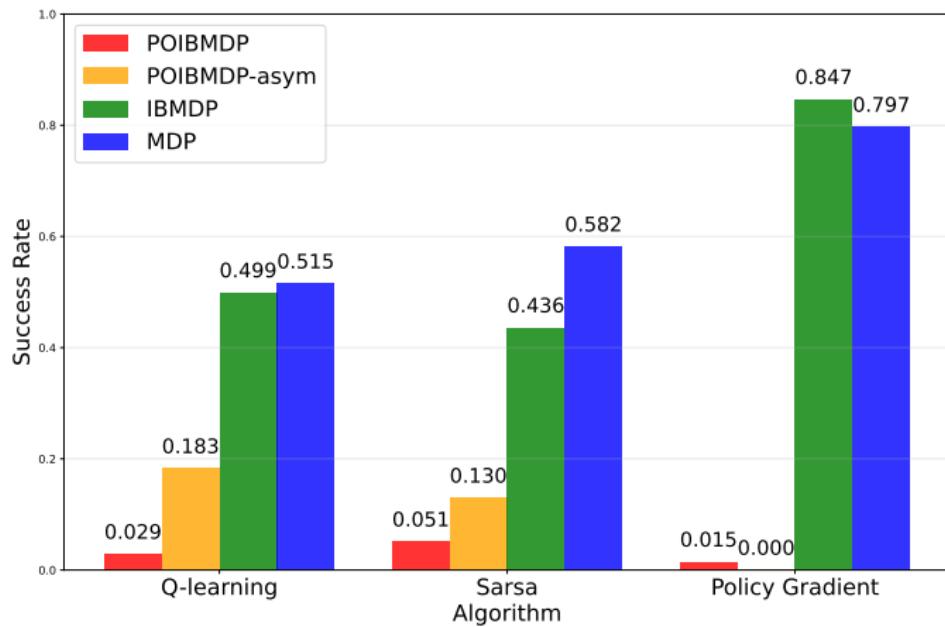
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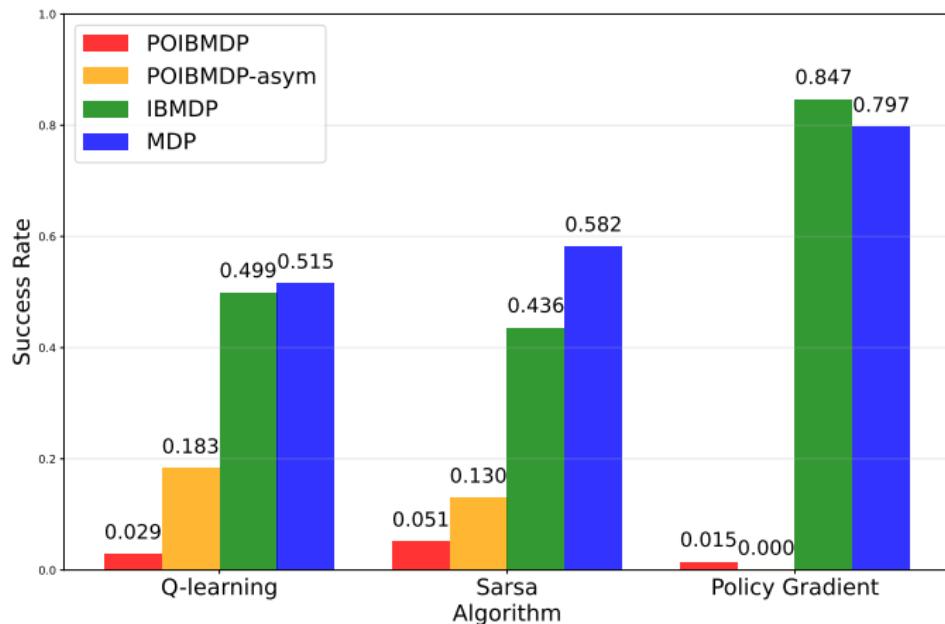
Are the poor performances due to partial observability?

Result: for similar problems, RL struggles more when there is partial observability



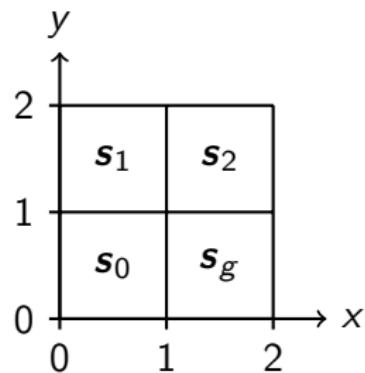
Success rates when learning deterministic partially observable or Markovian policies in the same IBMDP.

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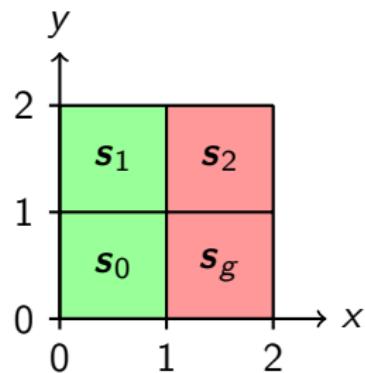


Success rates when learning deterministic partially observable or Markovian policies in the same IBMDP. **Is it all for nothing?**

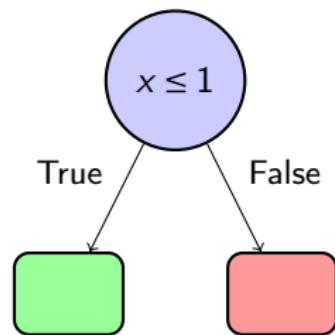
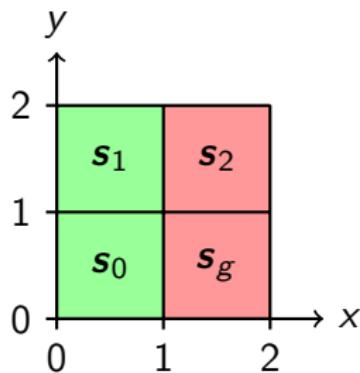
# Interesting sub-class of MDPs: classification MDPs



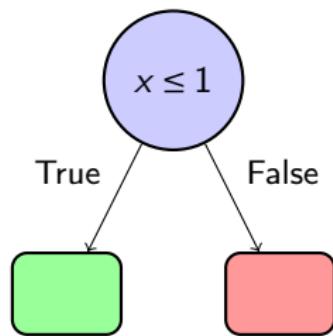
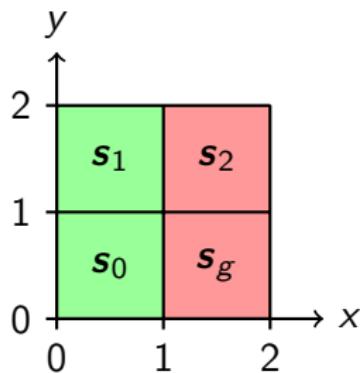
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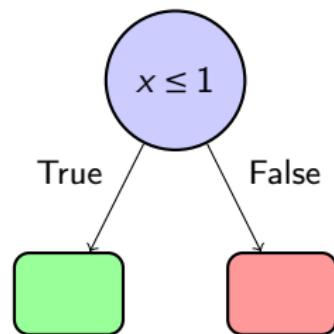
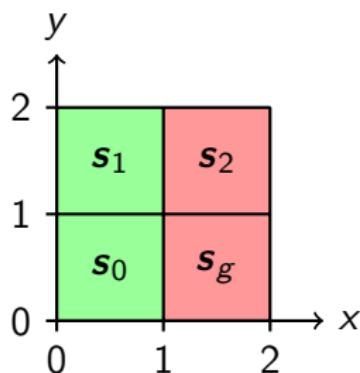


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Classification MDP and the unique optimal depth-1 tree.

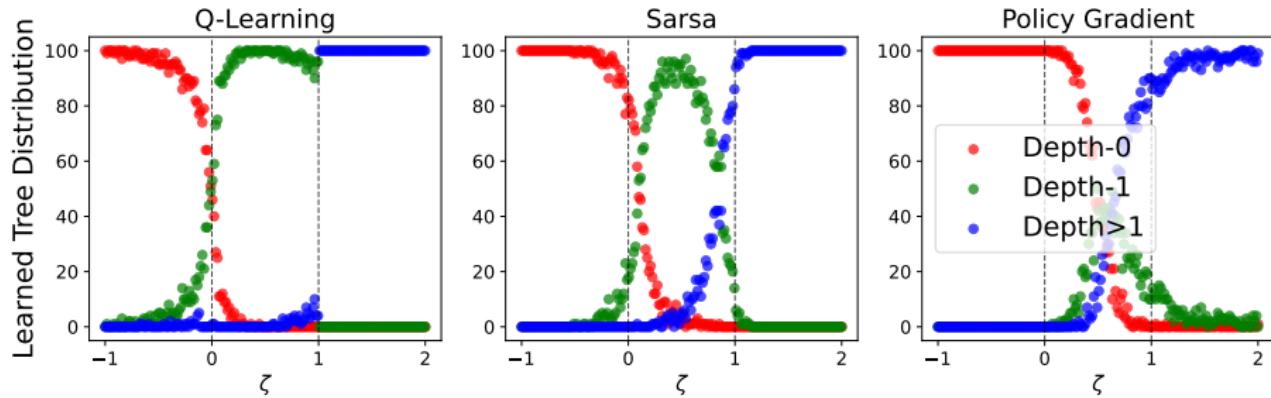
## Interesting sub-class of MDPs: classification MDPs



Classification MDP and the unique optimal depth-1 tree.

**⚠ Deterministic partially observable policies for classification IBMDPs ( $\Leftrightarrow$  decision tree policies) are in fact Markovian.**

Result: RL can retrieve optimal depth-1 trees for the toy classification MDPs



Distributions of tree policies learned with various RL algorithms.

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*Q: Can we leverage SDM to design new decision tree induction algorithms for the supervised learning setting? A: Yes!*

# Decision trees in supervised learning

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- In between optimal and greedy?

# Decision tree induction as solving MDPs

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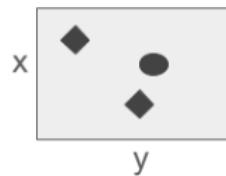
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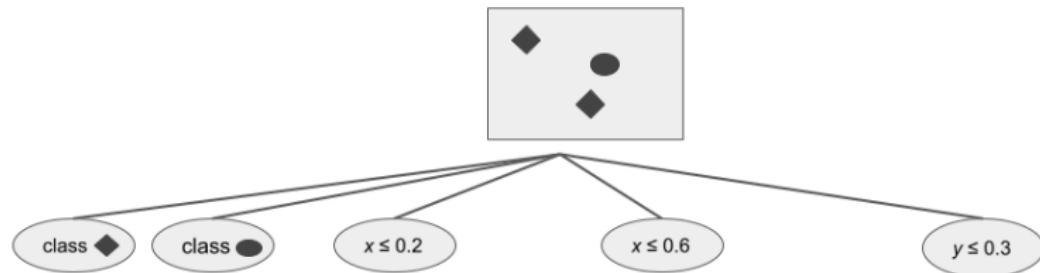
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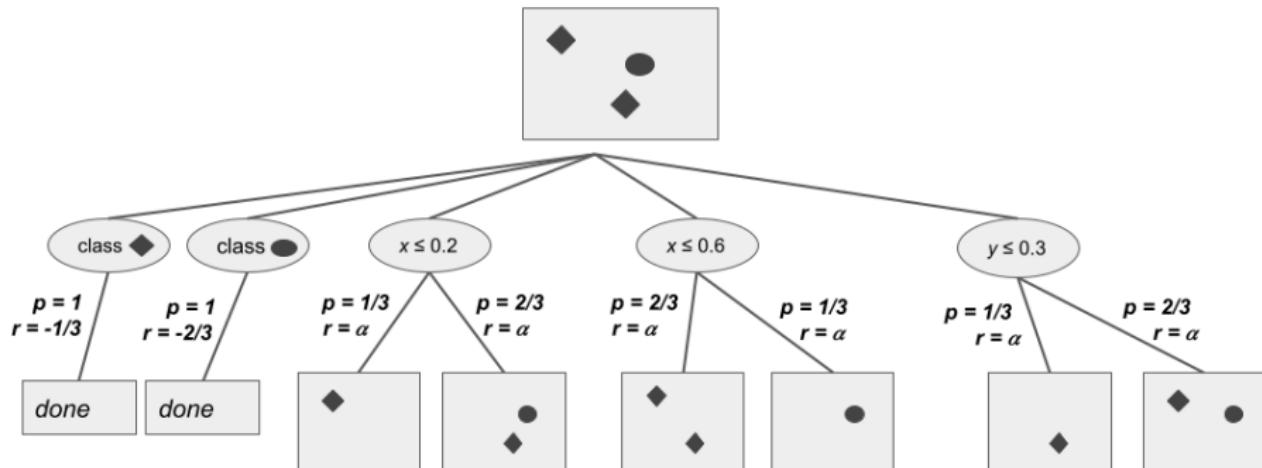
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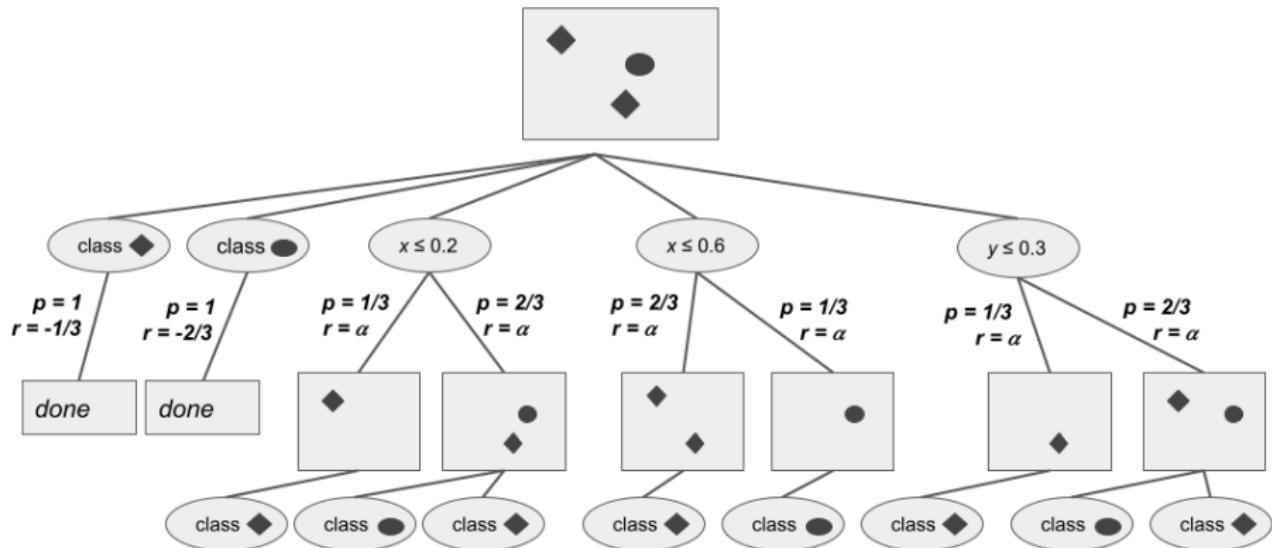
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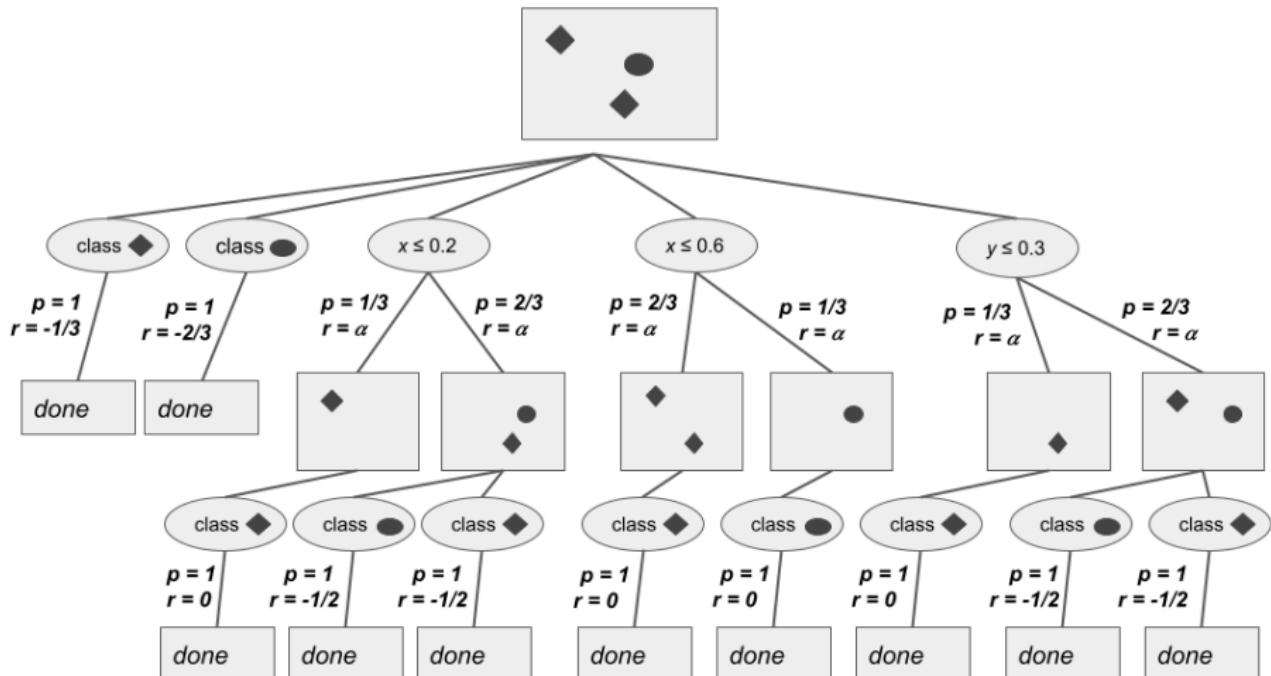
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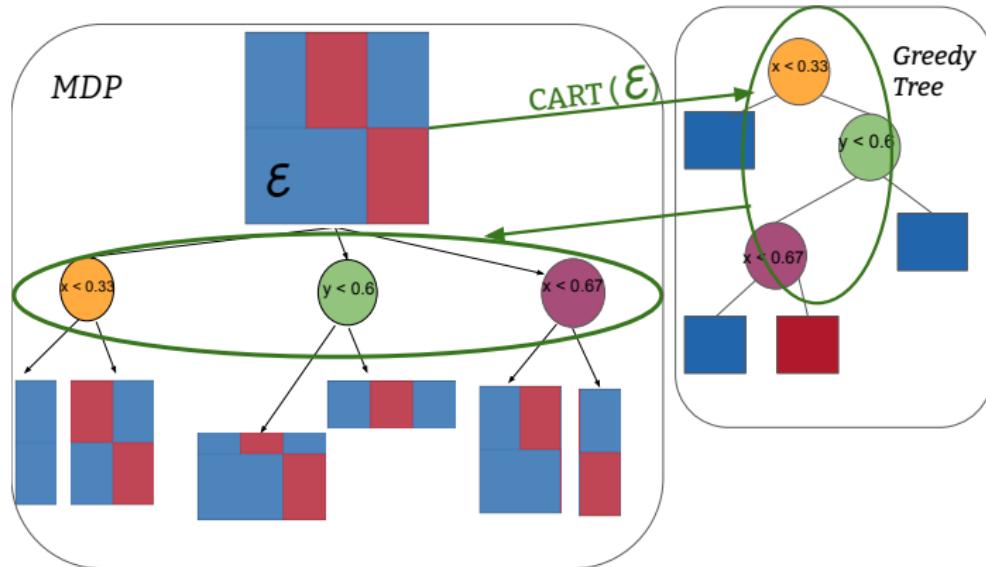
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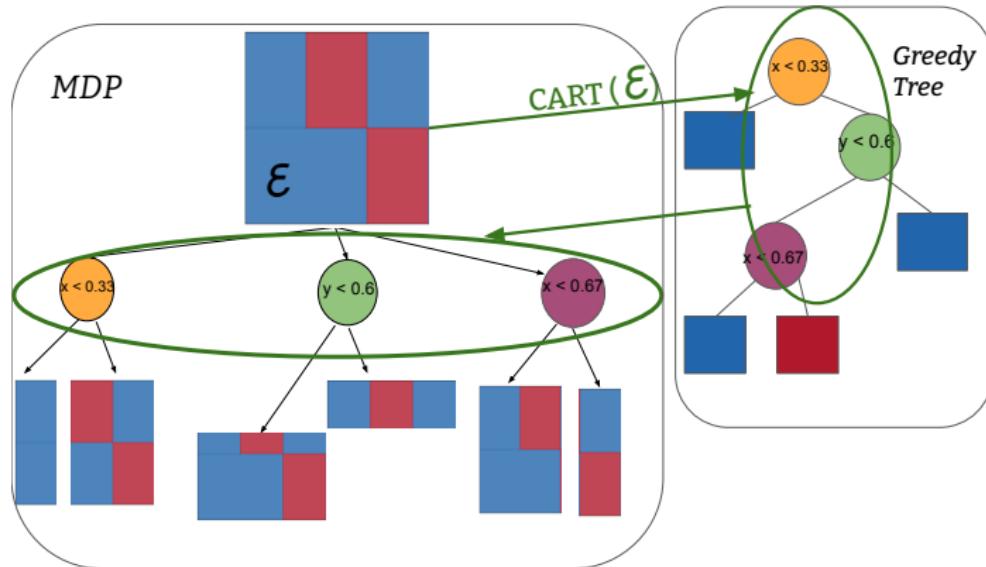
How to choose the  $B$  candidate actions/splits?

# Dynamic Programming Decision Trees (DPDT)<sup>1</sup>



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DPDT was presented at the 31st ACM SIGKDD conference.

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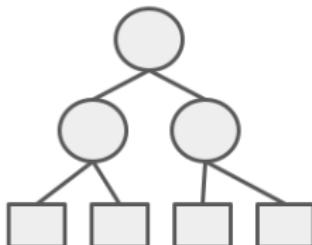
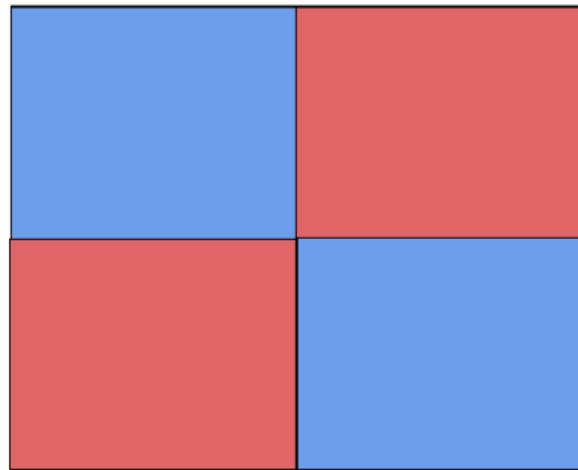
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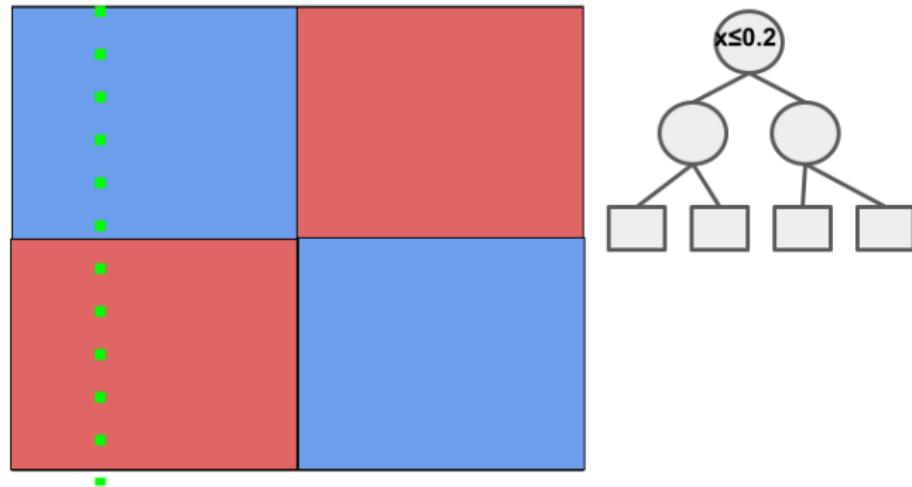
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*There exist a depth budget  $D$  and a dataset for which DPDT trees are strictly better than greedy trees.*

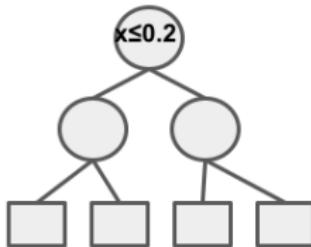
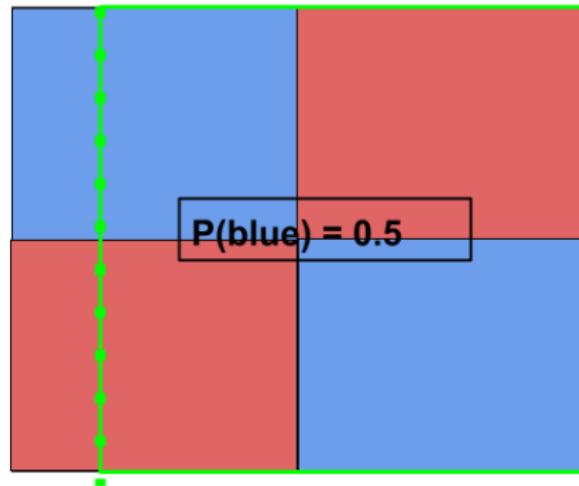
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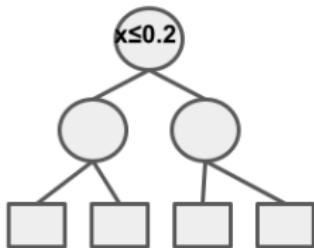
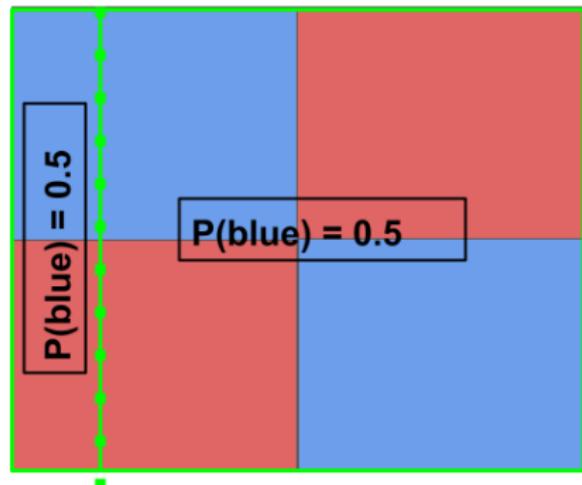
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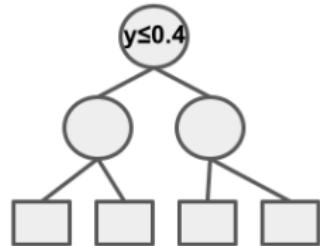
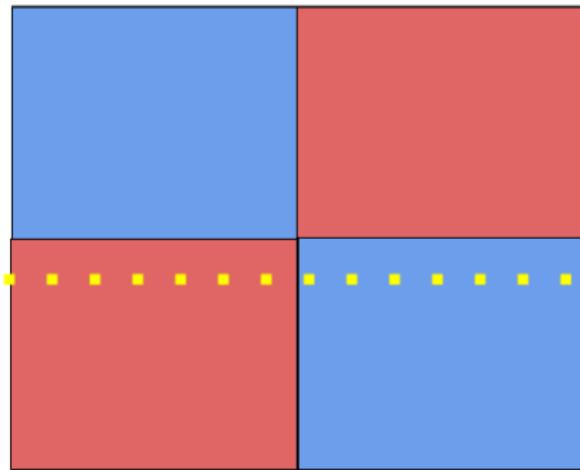
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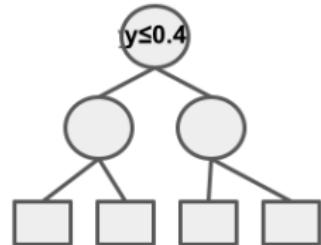
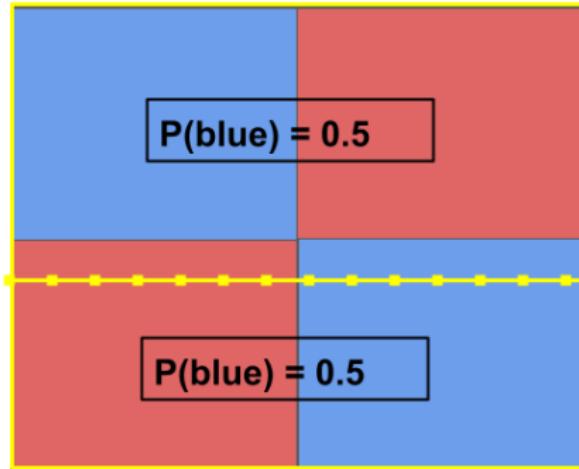
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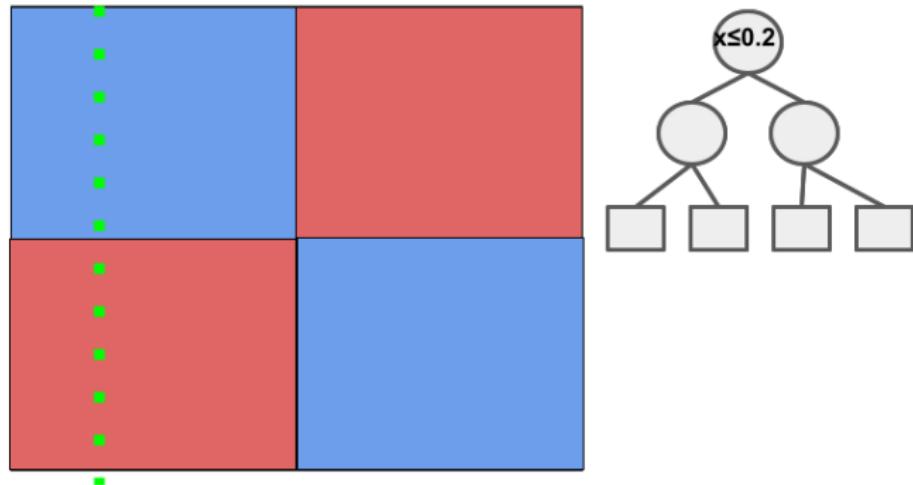
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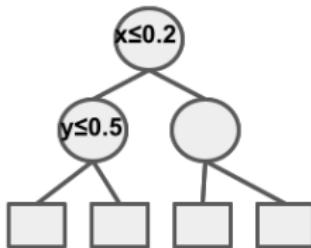
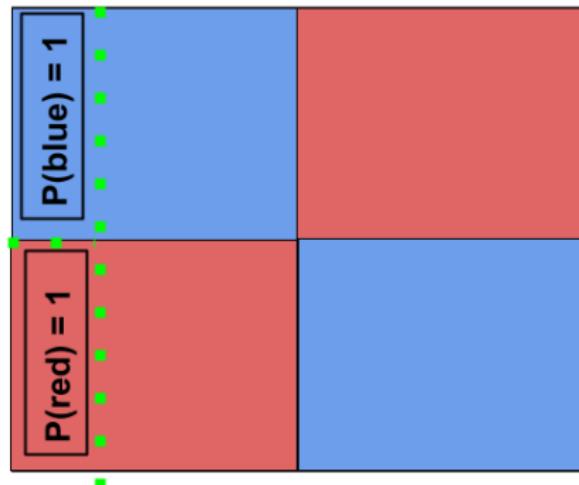
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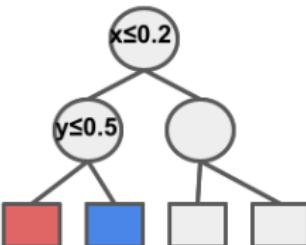
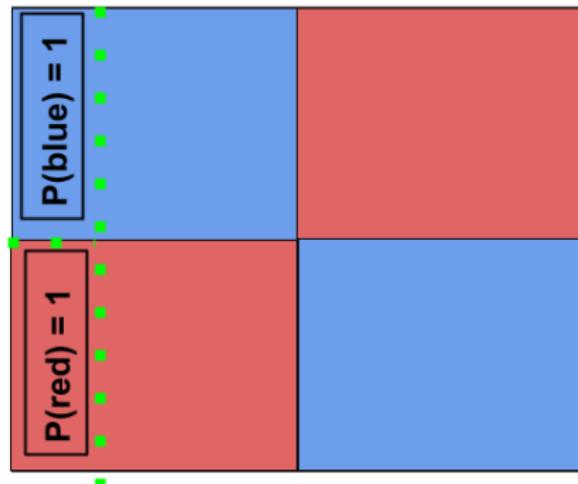
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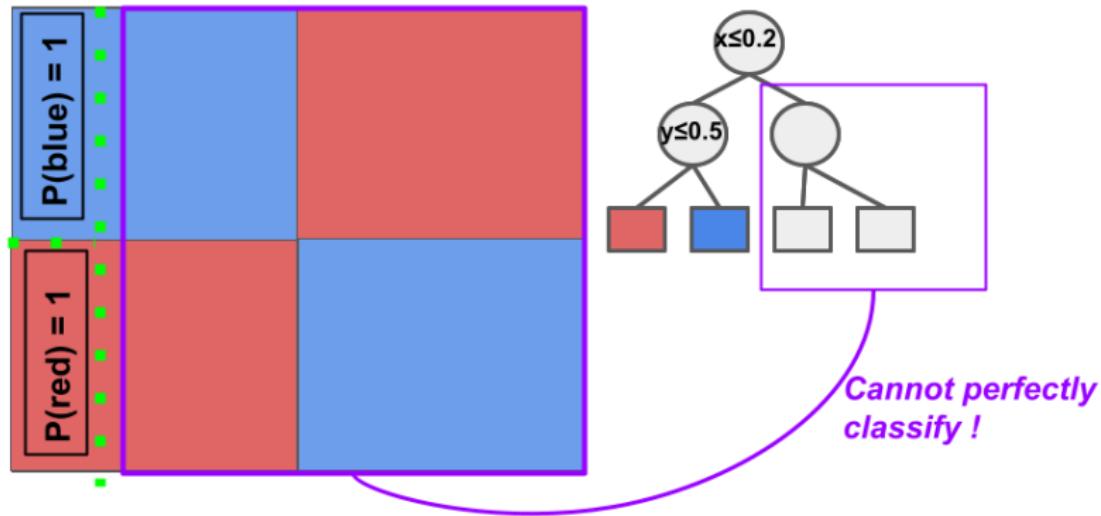
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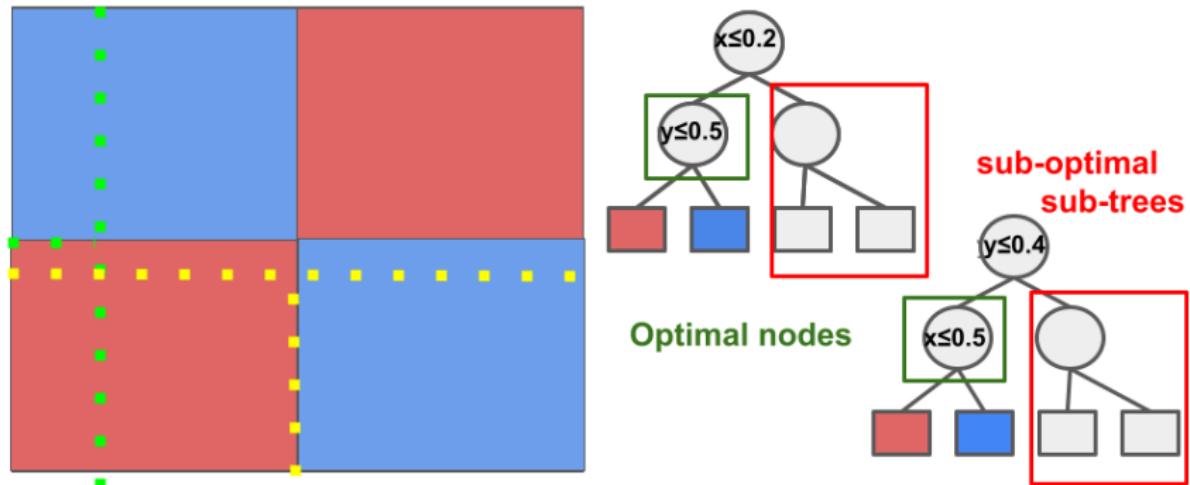
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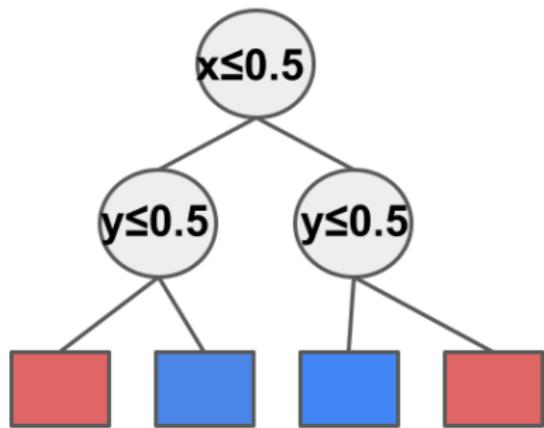
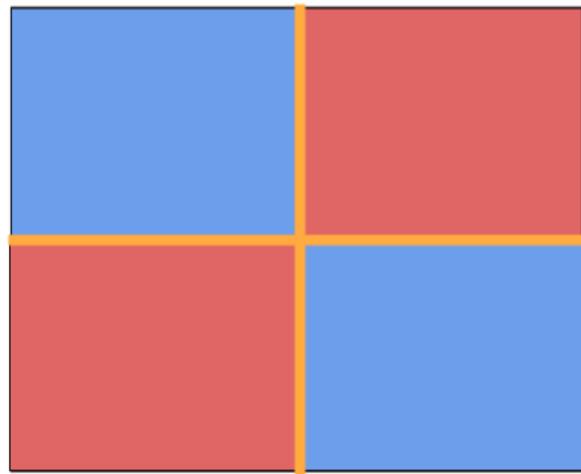
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# Fast like greedy trees, accurate like optimal trees



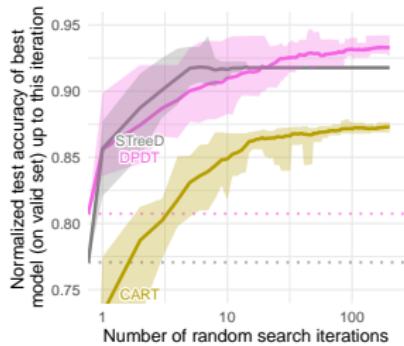
Comparison of greedy, optimal, and DPDT depth-2 trees on the checkersboard dataset.

# Comparing tree accuracy to complexity

Train accuracy and operation count when learning depth-3 decision trees.

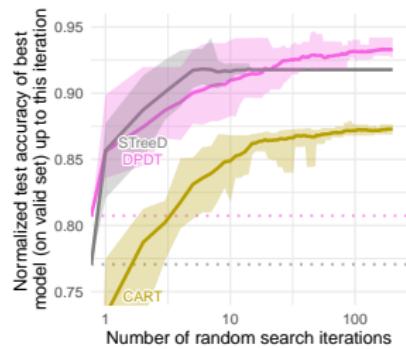
Dataset	N	p	Accuracy				Operations			
			Opt Quant-BnB	Greedy CART	DPDT light	DPDT full	Opt Quant-BnB	Greedy CART	DPDT light	DPDT full
room	8103	16	0.992	0.968	0.991	0.992	$10^6$	15	286	16100
bean	10888	16	0.871	0.777	0.812	0.853	$5 \cdot 10^6$	15	295	25900
eeg	11984	14	0.708	0.666	0.689	0.706	$2 \cdot 10^6$	13	289	26000
avila	10430	10	0.585	0.532	0.574	0.585	$3 \cdot 10^7$	9	268	24700
magic	15216	10	0.831	0.801	0.822	0.828	$6 \cdot 10^6$	15	298	28000
htru	14318	8	0.981	0.979	0.979	0.980	$6 \cdot 10^7$	15	295	25300
occup.	8143	5	0.994	0.989	0.991	0.994	$7 \cdot 10^5$	13	280	16300
skin	196045	3	0.969	0.966	0.966	0.966	$7 \cdot 10^4$	15	301	23300
fault	1552	27	0.682	0.553	0.672	0.674	$9 \cdot 10^8$	13	295	24200
segment	1848	18	0.887	0.574	0.812	0.879	$2 \cdot 10^6$	7	220	16300
page	4378	10	0.971	0.964	0.970	0.970	$10^7$	15	298	22400
bidding	5056	9	0.993	0.981	0.985	0.993	$3 \cdot 10^5$	13	256	9360
raisin	720	7	0.894	0.869	0.879	0.886	$4 \cdot 10^6$	15	295	20900
rice	3048	7	0.938	0.933	0.934	0.937	$2 \cdot 10^7$	15	298	25500
wilt	4339	5	0.996	0.993	0.994	0.995	$3 \cdot 10^5$	13	274	11300
bank	1097	4	0.983	0.933	0.971	0.980	$6 \cdot 10^4$	13	271	7990

# DPDT trees generalization

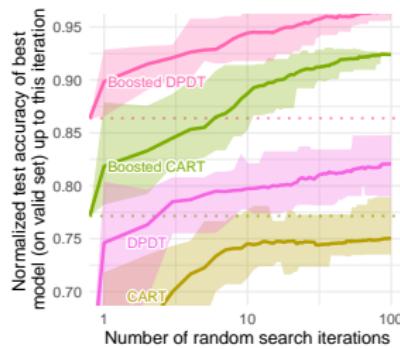


DPDT depth-5 trees vs.  
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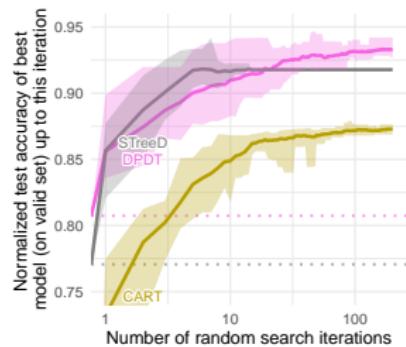


DPDT depth-5 trees vs.  
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Boosted DPDT vs. Boosted  
CART

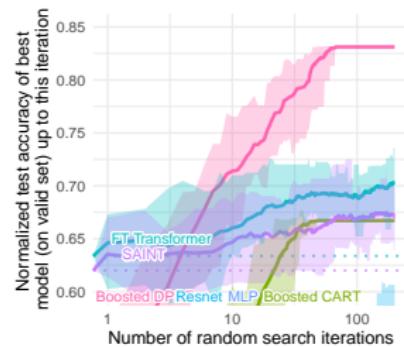
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Boosted DPDT vs. other  
classifiers

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**A: It depends.**

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- Time to formally verify a policy decreases with interpretability [Bar+20].

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- Different implementations (matrix operations vs fully sequentially) ...

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```
# Decision tree for Mountain Car
def play(x):
    if x[1] <= -0.2597:
        if x[1] <= -0.6378:
            return 0
        else:
            if x[0] <= -1.0021:
                return 2
            else:
                return 0
    else:
        if x[1] <= -0.0508:
            if x[0] <= 0.2979:
                if x[0] <= 0.0453:
                    return 2
                else:
                    if x[1] <=
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                    else:
                        return 2
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                    else:
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```
# Small ReLU MLP for Pendulum
def play(x):
    h_layer_0_0 = 1.238*x[0]+0.971*x
    [1]
                           +0.430*x[2]+0.933
    h_layer_0_0 = max(0, h_layer_0_0
    )
    h_layer_0_1 = -1.221*x[0]+1.001
                           *x[1]-0.423*x[2]
                           +0.475
    h_layer_0_1 = max(0, h_layer_0_1
    )
    h_layer_1_0 = -0.109*h_layer_0_0
                           -0.377*h_layer_0_1
                           +1.694
    h_layer_1_0 = max(0, h_layer_1_0
    )
    h_layer_1_1 = -3.024*h_layer_0_0
                           -1.421*h_layer_0_1
                           +1.530
    h_layer_1_1 = max(0, h_layer_1_1
    )
    h_layer_2_0 = -1.790*h_layer_1_0
                           +2.840*h_layer_1_1
                           +0.658
    y_0 = h_layer_2_0
    return [y_0]
```

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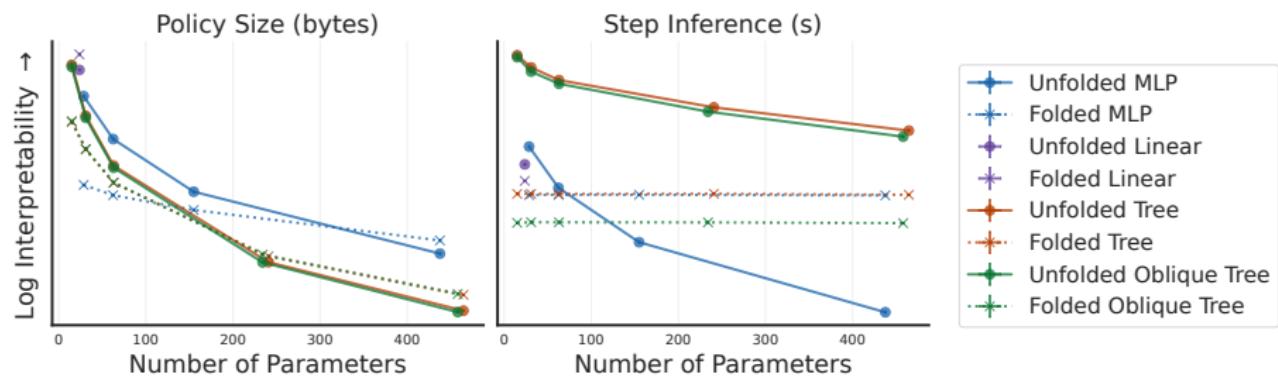
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## Setup

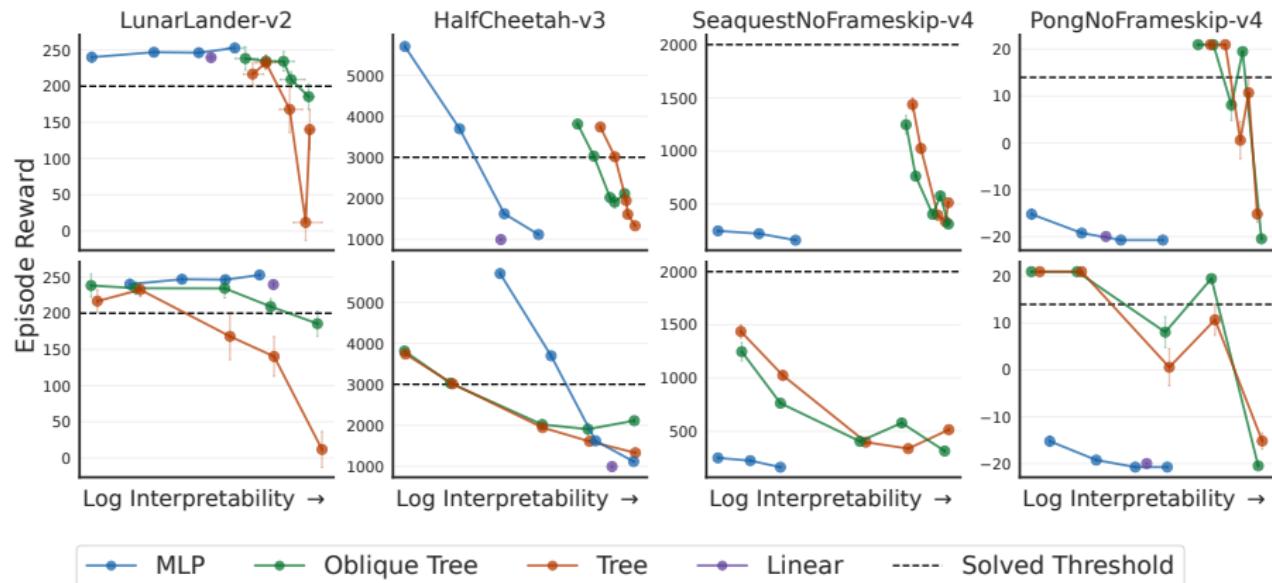
We imitate ~40000 expert policies from stable-baselines3 using various policy classes/nb parameters on various environments.

# Result: unfolding policies is necessary to respect consensus



Aggregated policies interpretability on classic control environments

Result: there is no dominating policy class for all environments



Interpretability-Performance trade-offs. Top row, interpretability is measured with step inference times. Bottom row, the interpretability is measured with policy size.

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- Tree-like policy classes can have good inductive bias (e.g. Atari).
- What about (very) big models?
- Can we use our policy programs as low level skills (hierarchical RL)?

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## My hope

Motivate interpretability by finding a real-world problem where interpretability is *really* necessary [Nag+24].

# Broader perspectives

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- **Combinatorial optimization:** Can we formulate other combinatorial/NP-hard problems as MDPs and design other DPDT-like algorithms?
- **Human-computer interaction:** Can we do large scale human study of the ~40K programs interpretability?

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