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UNIVERSITÉ DE LILLE
INRIA

École doctorale **École Graduée MADIS-631**
Unité de recherche **Centre de Recherche en Informatique, Signal et Automatique de Lille**

Thèse présentée par **Hector Kohler**

Soutenue le **1er décembre 2025**

En vue de l'obtention du grade de docteur de l'Université de Lille et de l'Inria

Discipline **Informatique**
Spécialité **Informatique et Applications**

Arbres de Décision pour la Prise de Décision Séquentielle

Thèse dirigée par	Philippe Preux	directeur
	Riad Akrour	co-directeur

Composition du jury

Rapporteurs	René Descartes	professeur à l'IHP	
	Denis Diderot	directeur de recherche au CNRS	
Examinateurs	Victor Hugo	professeur à l'ENS Lyon	président du jury
	Sophie Germain	MCF à l'Université de Paris 13	
	Joseph Fourier	chargé de recherche à l'INRIA	
	Paul Verlaine	chargé de recherche HDR au CNRS	
Invité	George Sand		
Directeurs de thèse	Philippe Preux	professeur à l'Université de Lille	
	Riad Akrour	Inria	

Colophon

Mémoire de thèse intitulé « Arbres de Décision pour la Prise de Décision Séquentielle », écrit par Hector Kohler, achevé le 2 juin 2025, composé au moyen du système de préparation de document LaTeX et de la classe yathesis dédiée aux thèses préparées en France.

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University Department **Centre de Recherche en Informatique, Signal et Automatique de Lille**

Thesis defended by **Hector Kohler**

Defended on **December 1, 2025**

In order to become Doctor from Université de Lille and from Inria

Academic Field **Computer Science**

Speciality **Computer Science and Applications**

Decision Trees for Sequential Decision Making

Thesis supervised by Philippe Preux — Supervisor
Riad Akrour — Co-Supervisor

Committee members

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Guest	George Sand		
Supervisors	Philippe Preux	Professor at Université de Lille	
	Riad Akrour	Inria	

Arbres de Décision pour la Prise de Décision Séquentielle

Résumé

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum. Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

Mots clés : apprentissage par renforcement, arbres de décision, interprétabilité, méthodologie

Centre de Recherche en Informatique, Signal et Automatique de Lille
Université de Lille-Campus scientifique – Bâtiment ESPRIT – Avenue Henri Poincaré – 59655 Villeneuve d'Ascq

Decision Trees for Sequential Decision Making

Abstract

In this Ph.D. thesis, we study algorithms to learn decision trees for classification and sequential decision making. Decision trees are interpretable because humans can read through the decision tree computations from the root to the leaves. This makes decision trees the go-to model when human verification is required like in medicine applications. However, decision trees are non-differentiable making them hard to optimize unlike neural networks that can be trained efficiently with gradient descent. Existing interpretable reinforcement learning approaches usually learn soft trees (non-interpretable as is) or are ad-hoc (train a neural network then fit a tree to it) potentially missing better solutions.

In the first part of this manuscript, we aim to directly learn decision trees for a Markov decision process with reinforcement learning. In practice we show that this amounts to solving a partially observable Markov decision process. Most existing RL algorithms are not suited for POMDPs. This parallel between decision tree learning with RL and POMDPs solving help us understand why in practice it is often easier to obtain a non-interpretable expert policy–a neural network–and then distillate it into a tree rather than learning the decision tree from scratch.

The second contribution from this work arose from the observation that looking for a deicison tree classifier (or regressor) can be seen as sequentially adding nodes to a tree to maximize the accuracy of predictions. We thus formulate decision tree induction as sloving a Markov decision problem and propose a new state-of-the-art algorithm that can be trained with supervised example data and generalizes well to unseen data.

Work from the previous parts rely on the hypothesis that decision trees are indeed an interpretable model that humans can use in sensitive applications. But is it really the case? In the last part of this thesis, we attempt to answer some more general questions about interpretability: can we measure interpretability without humans? And are decision trees really more interpretable than neural networks?

Keywords: reinforcement learning, deicision trees, interpretability, methodology

Sommaire

Preliminary Concepts

What is Sequential Decision Making ?

In this manuscript we are interested in sequential decision making processes. Sequential decision making processes are found in all aspects of life. In medicine, doctors have to decide when to use chimiotherapy next based on the patient's current health in order to heal (cite). In agriculture, agronomists have to decide when to fertilize next based on the current soil and wheather conditions in order to maximize plant growth (cite). In automotive, the auto-pilot system has to decide how to steer the wheel next based on lidar sensors in order to maintain a safe trajectory (cite). In video games, a bot decides what attack to throw next based on the player's and its own state in order to provide the best entertainment (cite). Those sequential decision making processes exhibits key similarities : an agent takes actions based on some current information to achieve some goal. Algorithms presented in this thesis use the formalism of Marko decision processes that we present next to model sequential decision making.

Markov decision processes/problems

Markov decision processes (MDPs) were first introduced in the 1950s by Richard Bellman (cite). The original idea behind MDPs was to fromalize some dynamical systems which states obey the Markov property–the futur only depends on the the present and is independent of the past–and hence can be solved efficiently with dynamic programming (cite). Following Martin L. Puterman's book on MDPS (cite), we formally define an MDP as

Exact solutions

Reinforcement learning

What is Interpretable Sequential Decision Making?

Why do we care about interpretability?

What are existing approaches for interpretable sequential decision making?

What are decision tree policies?

Why are decision tree policies harder to learn than decision tree classifiers?

Outline of the Thesis

Première partie

A difficult problem : Learning Decision Trees for MDP

I have not failed. I've just found
10.000 ways that won't work.

Thomas A. Edison

Chapitre 1

A Decision Tree Policy for an MDP is a Policy for some Partially Observable MDP

1.1 How to Learn a Decision Tree Policy for an MDP ?

1.1.1 Imitation

1.1.2 Soft Trees

1.1.3 Iterative Bounding MDPs

1.2 How to solve Iterative Bounding MDPs ?

1.2.1 Asymmetric Reinforcement Learning

1.2.2 Learning a decision tree policy is solving a POMDP

1.3 Is it hard to properly learn a Decision Tree Policy for an MDP ?

1.3.1 POMDPs are way harder to solve than MDPs

1.3.2 Memoryless approaches to solve POMDPs seem uneffective

Chapitre 2

An attempt at Learning Decision Tree Policies with Reinforcement Learning

2.1 Grid Worlds

$V(g) = \zeta \sum_{i=0}^{\infty} \gamma^{2i} + \sum_{i=0}^{\infty} \gamma^{2i+1}$
$V(0) = \zeta + \gamma^2 V(g)$
$V(1) = \zeta + \gamma^2 V(0)$
$\frac{1}{4}\gamma \frac{1}{1-\gamma} + \frac{1}{4}\frac{1}{1-\gamma} \leq \frac{1}{4}V(g) + \frac{2}{4}V(0) + \frac{1}{4}V(1)$
$\zeta \cdot \sum_{i=0}^{\infty} \gamma^i \leq \frac{1}{4}V(g) + \frac{2}{4}V(0) + \frac{1}{4}V(1)$
$\frac{1}{4}V(g) + \frac{1}{4}(\zeta + \gamma V(0)) + \frac{1}{4}(\zeta + \gamma V(1)) + \frac{1}{4}V(0) \leq \frac{1}{4}V(g) + \frac{2}{4}V(0) + \frac{1}{4}V(1)$
$\frac{1}{4}V(g) + \frac{1}{4}V(0) + \frac{1}{4}(\zeta + \gamma^2 \zeta \sum_{i=0}^{\infty} \gamma^{2i}) + \frac{1}{4}(\zeta \sum_{i=0}^{\infty} \gamma^{2i}) \leq \frac{1}{4}V(g) + \frac{2}{4}V(0) + \frac{1}{4}V(1)$

2.1.1 Step-by-step derivation of the lower bound on ζ

Step 1 : Simplify the left side of the inequality

$$\frac{1}{4}\gamma \frac{1}{1-\gamma} + \frac{1}{4}\frac{1}{1-\gamma} = \frac{1}{4}\frac{1}{1-\gamma}(\gamma + 1) \tag{2.1}$$

$$= \frac{\gamma + 1}{4(1-\gamma)} \tag{2.2}$$

Step 2 : Express $V(g)$, $V(0)$, and $V(1)$ in simplified forms

$$
\begin{aligned}
V(g) &= \zeta \sum_{i=0}^{\infty} \gamma^{2i} + \sum_{i=0}^{\infty} \gamma^{2i+1} && (2.3)\\
&= \zeta \frac{1}{1-\gamma^2} + \gamma \frac{1}{1-\gamma^2} && (2.4)\\
&= \frac{\zeta + \gamma}{1-\gamma^2} && (2.5)
\end{aligned}
$$

$$
\begin{aligned}
V(0) &= \zeta + \gamma^2 V(g) && (2.6)\\
&= \zeta + \gamma^2 \frac{\zeta + \gamma}{1-\gamma^2} && (2.7)\\
&= \frac{\zeta(1-\gamma^2) + \gamma^2(\zeta + \gamma)}{1-\gamma^2} && (2.8)\\
&= \frac{\zeta + \gamma^3}{1-\gamma^2} && (2.9)
\end{aligned}
$$

$$
\begin{aligned}
V(1) &= \zeta + \gamma^2 V(0) && (2.10)\\
&= \zeta + \gamma^2 \frac{\zeta + \gamma^3}{1-\gamma^2} && (2.11)\\
&= \frac{\zeta(1-\gamma^2) + \gamma^2(\zeta + \gamma^3)}{1-\gamma^2} && (2.12)\\
&= \frac{\zeta + \gamma^5}{1-\gamma^2} && (2.13)
\end{aligned}
$$

Step 3 : Substitute into the right side of the inequality

$$\frac{1}{4}V(g)+\frac{2}{4}V(0)+\frac{1}{4}V(1)=\frac{1}{4}\frac{\zeta+\gamma}{1-\gamma^2}+\frac{1}{2}\frac{\zeta+\gamma^3}{1-\gamma^2}+\frac{1}{4}\frac{\zeta+\gamma^5}{1-\gamma^2} \tag{2.14}$$

$$=\frac{1}{4(1-\gamma^2)}[(\zeta+\gamma)+2(\zeta+\gamma^3)+(\zeta+\gamma^5)] \tag{2.15}$$

$$=\frac{4\zeta+\gamma+2\gamma^3+\gamma^5}{4(1-\gamma^2)} \tag{2.16}$$

Step 4 : Set up the inequality

$$\frac{\gamma+1}{4(1-\gamma)}\leq\frac{4\zeta+\gamma+2\gamma^3+\gamma^5}{4(1-\gamma^2)} \tag{2.17}$$

Step 5 : Use the identity $1-\gamma^2=(1-\gamma)(1+\gamma)$

$$\frac{\gamma+1}{4(1-\gamma)}\leq\frac{4\zeta+\gamma+2\gamma^3+\gamma^5}{4(1-\gamma)(1+\gamma)} \tag{2.18}$$

Step 6 : Multiply both sides by $4(1-\gamma)$

$$\gamma+1\leq\frac{4\zeta+\gamma+2\gamma^3+\gamma^5}{1+\gamma} \tag{2.19}$$

Step 7 : Multiply both sides by $(1+\gamma)$

$$(\gamma+1)(1+\gamma)\leq 4\zeta+\gamma+2\gamma^3+\gamma^5 \tag{2.20}$$

$$(\gamma+1)^2\leq 4\zeta+\gamma+2\gamma^3+\gamma^5 \tag{2.21}$$

Step 8 : Expand and rearrange

$$\gamma^2+2\gamma+1\leq 4\zeta+\gamma+2\gamma^3+\gamma^5 \tag{2.22}$$

$$4\zeta\geq\gamma^2+2\gamma+1-\gamma-2\gamma^3-\gamma^5 \tag{2.23}$$

$$4\zeta\geq\gamma^2+\gamma+1-2\gamma^3-\gamma^5 \tag{2.24}$$

$$\zeta\geq\frac{\gamma^2+\gamma+1-2\gamma^3-\gamma^5}{4} \tag{2.25}$$

Therefore, we obtain a **lower bound** on ζ :

$$\zeta \geq \frac{\gamma^2 + \gamma + 1 - 2\gamma^3 - \gamma^5}{4} \tag{2.26}$$

where $0 < \gamma < 1$.

2.1.2 Step-by-step derivation of the upper bound on ζ

Starting from the inequality :

$$\frac{1}{4}V(g) + \frac{1}{4}(\zeta + \gamma V(0)) + \frac{1}{4}(\zeta + \gamma V(1)) + \frac{1}{4}V(0) \leq \frac{1}{4}V(g) + \frac{2}{4}V(0) + \frac{1}{4}V(1)$$

Step 1 : Cancel the $\frac{1}{4}V(g)$ terms from both sides

$$\frac{1}{4}(\zeta + \gamma V(0)) + \frac{1}{4}(\zeta + \gamma V(1)) + \frac{1}{4}V(0) \leq \frac{2}{4}V(0) + \frac{1}{4}V(1) \tag{2.27}$$

Step 2 : Expand the left side

$$\frac{1}{4}\zeta + \frac{1}{4}\gamma V(0) + \frac{1}{4}\zeta + \frac{1}{4}\gamma V(1) + \frac{1}{4}V(0) \leq \frac{2}{4}V(0) + \frac{1}{4}V(1) \tag{2.28}$$

Step 3 : Combine like terms

$$\frac{1}{2}\zeta + \frac{1}{4}\gamma V(0) + \frac{1}{4}\gamma V(1) \leq \frac{2}{4}V(0) - \frac{1}{4}V(0) + \frac{1}{4}V(1) \tag{2.29}$$

$$\frac{1}{2}\zeta + \frac{1}{4}\gamma V(0) + \frac{1}{4}\gamma V(1) \leq \frac{1}{4}V(0) + \frac{1}{4}V(1) \tag{2.30}$$

Step 4 : Factor out common terms

$$\frac{1}{2}\zeta \leq \frac{1}{4}V(0) + \frac{1}{4}V(1) - \frac{1}{4}\gamma V(0) - \frac{1}{4}\gamma V(1) \tag{2.31}$$

$$\frac{1}{2}\zeta \leq \frac{1}{4}V(0)(1-\gamma) + \frac{1}{4}V(1)(1-\gamma) \tag{2.32}$$

$$\frac{1}{2}\zeta \leq \frac{1-\gamma}{4}(V(0) + V(1)) \tag{2.33}$$

$$\zeta \leq \frac{1-\gamma}{2}(V(0) + V(1)) \tag{2.34}$$

Step 5 : Substitute the expressions for $V(0)$ and $V(1)$

$$V(0) + V(1) = \frac{\zeta + \gamma^3}{1 - \gamma^2} + \frac{\zeta + \gamma^5}{1 - \gamma^2} \tag{2.35}$$

$$= \frac{2\zeta + \gamma^3 + \gamma^5}{1 - \gamma^2} \tag{2.36}$$

Step 6 : Substitute back into the inequality

$$\zeta \leq \frac{1 - \gamma}{2} \cdot \frac{2\zeta + \gamma^3 + \gamma^5}{1 - \gamma^2} \tag{2.37}$$

$$= \frac{(1 - \gamma)(2\zeta + \gamma^3 + \gamma^5)}{2(1 - \gamma^2)} \tag{2.38}$$

Step 7 : Use the identity $1 - \gamma^2 = (1 - \gamma)(1 + \gamma)$

$$\zeta \leq \frac{(1 - \gamma)(2\zeta + \gamma^3 + \gamma^5)}{2(1 - \gamma)(1 + \gamma)} \tag{2.39}$$

$$= \frac{2\zeta + \gamma^3 + \gamma^5}{2(1 + \gamma)} \tag{2.40}$$

Step 8 : Multiply both sides by $2(1 + \gamma)$

$$2(1 + \gamma)\zeta \leq 2\zeta + \gamma^3 + \gamma^5 \tag{2.41}$$

$$2\zeta + 2\gamma\zeta \leq 2\zeta + \gamma^3 + \gamma^5 \tag{2.42}$$

$$2\gamma\zeta \leq \gamma^3 + \gamma^5 \tag{2.43}$$

$$\zeta \leq \frac{\gamma^3 + \gamma^5}{2\gamma} \tag{2.44}$$

$$\zeta \leq \frac{\gamma^2 + \gamma^4}{2} \tag{2.45}$$

Therefore, we obtain an **upper bound** on ζ :

$$\zeta \leq \frac{\gamma^2 + \gamma^4}{2} \tag{2.46}$$

Combined bounds :

$$\frac{\gamma^2+\gamma+1-2\gamma^3-\gamma^5}{4} \leq \zeta \leq \frac{\gamma^2+\gamma^4}{2} \tag{2.47}$$

where $0 < \gamma < 1$.

2.1.3 Step-by-step derivation for the third inequality

Starting from the inequality :

$$\zeta \cdot \sum_{i=0}^{\infty} \gamma^i \leq \frac{1}{4}V(g) + \frac{2}{4}V(0) + \frac{1}{4}V(1)$$

Step 1 : Simplify the left side using the geometric series

$$\zeta \cdot \sum_{i=0}^{\infty} \gamma^i = \zeta \cdot \frac{1}{1-\gamma} \tag{2.48}$$

$$= \frac{\zeta}{1-\gamma} \tag{2.49}$$

Step 2 : Use the previously derived expression for the right side From our earlier calculation :

$$\frac{1}{4}V(g) + \frac{2}{4}V(0) + \frac{1}{4}V(1) = \frac{4\zeta+\gamma+2\gamma^3+\gamma^5}{4(1-\gamma^2)} \tag{2.50}$$

Step 3 : Set up the inequality

$$\frac{\zeta}{1-\gamma} \leq \frac{4\zeta+\gamma+2\gamma^3+\gamma^5}{4(1-\gamma^2)} \tag{2.51}$$

Step 4 : Use the identity $1-\gamma^2 = (1-\gamma)(1+\gamma)$

$$\frac{\zeta}{1-\gamma} \leq \frac{4\zeta+\gamma+2\gamma^3+\gamma^5}{4(1-\gamma)(1+\gamma)} \tag{2.52}$$

Step 5 : Multiply both sides by $(1-\gamma)$

$$\zeta \le \frac{4\zeta + \gamma + 2\gamma^3 + \gamma^5}{4(1+\gamma)} \tag{2.53}$$

Step 6 : Multiply both sides by $4(1+\gamma)$

$$4(1+\gamma)\zeta \le 4\zeta + \gamma + 2\gamma^3 + \gamma^5 \tag{2.54}$$

$$4\zeta + 4\gamma\zeta \le 4\zeta + \gamma + 2\gamma^3 + \gamma^5 \tag{2.55}$$

Step 7 : Subtract 4ζ **from both sides**

$$4\gamma\zeta \le \gamma + 2\gamma^3 + \gamma^5 \tag{2.56}$$

$$\zeta \le \frac{\gamma + 2\gamma^3 + \gamma^5}{4\gamma} \tag{2.57}$$

$$\zeta \le \frac{1 + 2\gamma^2 + \gamma^4}{4} \tag{2.58}$$

Therefore, we obtain another **upper bound** on ζ :

$$\zeta \le \frac{1 + 2\gamma^2 + \gamma^4}{4} \tag{2.59}$$

Final combined bounds from all three inequalities :

$$\frac{\gamma^2 + \gamma + 1 - 2\gamma^3 - \gamma^5}{4} \le \zeta \le \min\left\{\frac{\gamma^2 + \gamma^4}{2}, \frac{1 + 2\gamma^2 + \gamma^4}{4}\right\} \tag{2.60}$$

where $0 < \gamma < 1$.

2.1.4 Step-by-step derivation for the fourth inequality

Starting from the inequality :

$$\frac{1}{4}V(g) + \frac{1}{4}V(0) + \frac{1}{4}(\zeta + \gamma^2\zeta\sum_{i=0}^{\infty}\gamma^{2i}) + \frac{1}{4}(\zeta\sum_{i=0}^{\infty}\gamma^{2i}) \le \frac{1}{4}V(g) + \frac{2}{4}V(0) + \frac{1}{4}V(1)$$

Step 1 : Cancel the $\frac{1}{4}V(g)$ terms from both sides

$$\frac{1}{4}V(0)+\frac{1}{4}(\zeta+\gamma^2\zeta\sum_{i=0}^{\infty}\gamma^{2i})+\frac{1}{4}(\zeta\sum_{i=0}^{\infty}\gamma^{2i})\leq\frac{2}{4}V(0)+\frac{1}{4}V(1) \tag{2.61}$$

Step 2 : Simplify using the geometric series $\sum_{i=0}^{\infty}\gamma^{2i}=\frac{1}{1-\gamma^2}$

$$\frac{1}{4}V(0)+\frac{1}{4}\left(\zeta+\gamma^2\zeta\frac{1}{1-\gamma^2}\right)+\frac{1}{4}\left(\zeta\frac{1}{1-\gamma^2}\right)\leq\frac{2}{4}V(0)+\frac{1}{4}V(1) \tag{2.62}$$

Step 3 : Factor out common terms

$$\frac{1}{4}V(0)+\frac{1}{4}\zeta\left(1+\frac{\gamma^2}{1-\gamma^2}\right)+\frac{1}{4}\zeta\frac{1}{1-\gamma^2}\leq\frac{2}{4}V(0)+\frac{1}{4}V(1) \tag{2.63}$$

Step 4 : Simplify the coefficient of ζ

$$1+\frac{\gamma^2}{1-\gamma^2}+\frac{1}{1-\gamma^2}=\frac{1-\gamma^2+\gamma^2+1}{1-\gamma^2}=\frac{2}{1-\gamma^2} \tag{2.64}$$

So the inequality becomes :

$$\frac{1}{4}V(0)+\frac{1}{4}\zeta\frac{2}{1-\gamma^2}\leq\frac{2}{4}V(0)+\frac{1}{4}V(1) \tag{2.65}$$

$$\frac{1}{4}V(0)+\frac{\zeta}{2(1-\gamma^2)}\leq\frac{1}{2}V(0)+\frac{1}{4}V(1) \tag{2.66}$$

Step 5 : Rearrange to isolate the ζ term

$$\frac{\zeta}{2(1-\gamma^2)}\leq\frac{1}{2}V(0)-\frac{1}{4}V(0)+\frac{1}{4}V(1) \tag{2.67}$$

$$\frac{\zeta}{2(1-\gamma^2)}\leq\frac{1}{4}V(0)+\frac{1}{4}V(1) \tag{2.68}$$

$$\zeta\leq\frac{(1-\gamma^2)}{2}(V(0)+V(1)) \tag{2.69}$$

Step 6 : Substitute the expressions for $V(0)$ and $V(1)$

$$V(0)+V(1)=\frac{\zeta+\gamma^3}{1-\gamma^2}+\frac{\zeta+\gamma^5}{1-\gamma^2} \tag{2.70}$$

$$=\frac{2\zeta+\gamma^3+\gamma^5}{1-\gamma^2} \tag{2.71}$$

Step 7 : Substitute back into the inequality

$$\zeta \leq \frac{(1-\gamma^2)}{2}\cdot\frac{2\zeta+\gamma^3+\gamma^5}{1-\gamma^2} \tag{2.72}$$

$$=\frac{2\zeta+\gamma^3+\gamma^5}{2} \tag{2.73}$$

Step 8 : Multiply both sides by 2

$$2\zeta \leq 2\zeta+\gamma^3+\gamma^5 \tag{2.74}$$

$$0 \leq \gamma^3+\gamma^5 \tag{2.75}$$

$$0 \leq \gamma^3(1+\gamma^2) \tag{2.76}$$

Since $0 < \gamma < 1$, we have $\gamma^3 > 0$ and $(1+\gamma^2) > 0$, so this inequality is always satisfied. This means the fourth inequality does not provide an additional constraint on ζ.

Updated final bounds from all four inequalities :

$$\frac{\gamma^2+\gamma+1-2\gamma^3-\gamma^5}{4} \leq \zeta \leq \min\left\{\frac{\gamma^2+\gamma^4}{2}, \frac{1+2\gamma^2+\gamma^4}{4}\right\} \tag{2.77}$$

where $0 < \gamma < 1$. The fourth inequality is automatically satisfied and does not further constrain the bounds.

2.2 Q-Learning

2.3 Preferences over Decision Tree Policies

2.4 Results

Chapitre 3

Conclusion

3.1 What happens when the MDP's transitions are independent of the current state ?

Deuxième partie

An easier problem : Learning Decision Trees for MDPs that are Classification tasks

Chapitre 4

DPDT-intro

Chapitre 5

DPDT-paper

Chapitre 6

Conclusion

Troisième partie

Beyond Decision Trees : what can be done with other Interpretable Policies ?

Conclusion générale

Annexe A

Documents juridiques

Cette partie regroupe les documents juridiques officiels.

A.1 Licence sous laquelle est publié notre travail

Sed feugiat. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Ut pellentesque augue sed urna. Vestibulum diam eros, fringilla et, consectetuer eu, nonummy id, sapien. Nullam at lectus. In sagittis ultrices mauris. Curabitur malesuada erat sit amet massa. Fusce blandit. Aliquam erat volutpat. Aliquam euismod. Aenean vel lectus. Nunc imperdiet justo nec dolor.

Etiam euismod. Fusce facilisis lacinia dui. Suspendisse potenti. In mi erat, cursus id, nonummy sed, ullamcorper eget, sapien. Praesent pretium, magna in eleifend egestas, pede pede pretium lorem, quis consectetuer tortor sapien facilisis magna. Mauris quis magna varius nulla scelerisque imperdiet. Aliquam non quam. Aliquam porttitor quam a lacus. Praesent vel arcu ut tortor cursus volutpat. In vitae pede quis diam bibendum placerat. Fusce elementum convallis neque. Sed dolor orci, scelerisque ac, dapibus nec, ultricies ut, mi. Duis nec dui quis leo sagittis commodo.

Aliquam lectus. Vivamus leo. Quisque ornare tellus ullamcorper nulla. Mauris porttitor pharetra tortor. Sed fringilla justo sed mauris. Mauris tellus. Sed non leo. Nullam elementum, magna in cursus sodales, augue est scelerisque sapien, venenatis congue nulla arcu et pede. Ut suscipit enim vel sapien. Donec congue. Maecenas urna mi, suscipit in, placerat ut, vestibulum ut, massa. Fusce ultrices nulla et nisl.

Etiam ac leo a risus tristique nonummy. Donec dignissim tincidunt nulla. Vestibulum rhoncus molestie odio. Sed lobortis, justo et pretium lobortis, mauris turpis condimentum augue, nec ultricies nibh arcu pretium enim. Nunc purus neque, placerat id, imperdiet sed, pellentesque nec, nisl. Vestibulum imperdiet neque non sem accumsan laoreet. In hac habitasse platea dictumst. Etiam condimentum facilisis libero. Suspendisse in elit quis nisl aliquam dapibus. Pellentesque auctor sapien. Sed egestas sapien nec lectus. Pellentesque vel dui vel neque bibendum viverra. Aliquam porttitor nisl nec pede. Proin mattis libero vel turpis. Donec rutrum mauris et libero. Proin euismod porta felis. Nam lobortis, metus quis elementum commodo, nunc lectus elementum mauris, eget vulputate ligula tellus eu neque. Vivamus eu dolor.

Nulla in ipsum. Praesent eros nulla, congue vitae, euismod ut, commodo a, wisi. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Aenean nonummy magna non leo. Sed felis erat, ullamcorper in, dictum non, ultricies ut, lectus. Proin vel arcu a odio lobortis euismod. Vestibulum ante ipsum primis in faucibus orci luctus et ultrices posuere cubilia Curae; Proin ut est. Aliquam odio. Pellentesque massa turpis, cursus eu, euismod nec, tempor congue, nulla. Duis viverra gravida mauris. Cras tincidunt. Curabitur eros ligula, varius ut, pulvinar in, cursus faucibus, augue.

Nulla mattis luctus nulla. Duis commodo velit at leo. Aliquam vulputate magna et leo. Nam vestibulum ullamcorper leo. Vestibulum condimentum rutrum mauris. Donec id mauris. Morbi molestie justo et pede. Vivamus eget turpis sed nisl cursus tempor. Curabitur mollis sapien condimentum nunc. In wisi nisl, malesuada at, dignissim sit amet, lobortis in, odio. Aenean consequat arcu a ante. Pellentesque porta elit sit amet orci. Etiam at turpis nec elit ultricies imperdiet. Nulla facilisi. In hac habitasse platea dictumst. Suspendisse viverra aliquam risus. Nullam pede justo, molestie nonummy, scelerisque eu, facilisis vel, arcu.

Curabitur tellus magna, porttitor a, commodo a, commodo in, tortor. Donec interdum. Praesent scelerisque. Maecenas posuere sodales odio. Vivamus metus lacus, varius quis, imperdiet quis, rhoncus a, turpis. Etiam ligula arcu, elementum a, venenatis quis, sollicitudin sed, metus. Donec nunc pede, tincidunt in, venenatis vitae, faucibus vel, nibh. Pellentesque wisi. Nullam malesuada. Morbi ut tellus ut pede tincidunt porta. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Etiam congue neque id dolor.

Donec et nisl at wisi luctus bibendum. Nam interdum tellus ac libero. Sed sem justo, laoreet vitae, fringilla at, adipiscing ut, nibh. Maecenas non sem quis tortor eleifend fermentum. Etiam id tortor ac mauris porta vulputate. Integer porta neque vitae massa. Maecenas tempus libero a libero posuere dictum. Vestibulum ante ipsum primis in faucibus orci luctus et ultrices posuere cubilia Curae; Aenean quis mauris sed elit commodo placerat. Class aptent taciti sociosqu ad litora torquent per conubia nostra, per inceptos hymenaeos. Vivamus rhoncus tincidunt libero. Etiam elementum pretium justo. Vivamus est. Morbi a tellus eget pede tristique commodo. Nulla nisl. Vestibulum sed nisl eu sapien cursus rutrum.

Nulla non mauris vitae wisi posuere convallis. Sed eu nulla nec eros scelerisque pharetra. Nullam varius. Etiam dignissim elementum metus. Vestibulum faucibus, metus sit amet mattis rhoncus, sapien dui laoreet odio, nec ultricies nibh augue a enim. Fusce in ligula. Quisque at magna et nulla commodo consequat. Proin accumsan imperdiet sem. Nunc porta. Donec feugiat mi at justo. Phasellus facilisis ipsum quis ante. In ac elit eget ipsum pharetra faucibus. Maecenas viverra nulla in massa.

Nulla ac nisl. Nullam urna nulla, ullamcorper in, interdum sit amet, gravida ut, risus. Aenean ac enim. In luctus. Phasellus eu quam vitae turpis viverra pellentesque. Duis feugiat felis ut enim. Phasellus pharetra, sem id porttitor sodales, magna nunc aliquet nibh, nec blandit nisl mauris at pede. Suspendisse risus risus, lobortis eget, semper at, imperdiet sit amet, quam. Quisque scelerisque dapibus nibh. Nam enim. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Nunc ut metus. Ut metus justo, auctor at, ultrices eu, sagittis ut, purus. Aliquam aliquam.

Etiam pede massa, dapibus vitae, rhoncus in, placerat posuere, odio. Vestibulum luctus commodo lacus. Morbi lacus dui, tempor sed, euismod eget, condimentum at, tortor. Phasellus aliquet odio ac lacus tempor faucibus. Praesent sed sem. Praesent iaculis. Cras rhoncus tellus sed justo ullamcorper sagittis. Donec quis orci. Sed ut tortor quis tellus euismod tincidunt. Suspendisse congue nisl eu elit. Aliquam tortor diam, tempus id, tristique eget, sodales vel, nulla. Praesent tellus mi, condimentum sed, viverra at, consectetuer quis, lectus. In auctor vehicula orci. Sed pede sapien, euismod in, suscipit in, pharetra placerat, metus. Vivamus commodo dui non odio. Donec et felis.

Etiam suscipit aliquam arcu. Aliquam sit amet est ac purus bibendum congue. Sed in eros. Morbi non orci. Pellentesque mattis lacinia elit. Fusce molestie velit in ligula. Nullam et orci vitae nibh vulputate auctor. Aliquam eget purus. Nulla auctor wisi sed ipsum. Morbi porttitor tellus ac enim. Fusce ornare. Proin ipsum enim, tincidunt in, ornare venenatis, molestie a, augue. Donec vel pede in lacus sagittis porta. Sed hendrerit ipsum quis nisl. Suspendisse quis massa ac nibh pretium cursus. Sed sodales. Nam eu neque quis pede dignissim ornare. Maecenas eu purus ac urna tincidunt congue.

Donec et nisl id sapien blandit mattis. Aenean dictum odio sit amet risus. Morbi purus. Nulla a est sit amet purus venenatis iaculis. Vivamus viverra purus vel magna. Donec in justo sed odio malesuada dapibus. Nunc ultrices aliquam nunc. Vivamus facilisis pellentesque velit. Nulla nunc velit, vulputate dapibus, vulputate id, mattis ac, justo. Nam mattis elit dapibus purus. Quisque enim risus, congue non, elementum ut, mattis quis, sem. Quisque elit.

Maecenas non massa. Vestibulum pharetra nulla at lorem. Duis quis quam id lacus dapibus interdum. Nulla lorem. Donec ut ante quis dolor bibendum condimentum. Etiam egestas tortor vitae lacus. Praesent cursus. Mauris bibendum pede at elit. Morbi et felis a lectus interdum facilisis. Sed suscipit gravida turpis. Nulla at lectus. Vestibulum ante ipsum primis in faucibus orci luctus et ultrices posuere cubilia Curae; Praesent nonummy luctus nibh. Proin turpis nunc, congue eu, egestas ut, fringilla at, tellus. In hac habitasse platea dictumst.

Vivamus eu tellus sed tellus consequat suscipit. Nam orci orci, malesuada id, gravida nec, ultricies vitae, erat. Donec risus turpis, luctus sit amet, interdum quis, porta sed, ipsum. Suspendisse condimentum, tortor at egestas posuere, neque metus tempor orci, et tincidunt urna nunc a purus. Sed facilisis blandit tellus. Nunc risus sem, suscipit nec, eleifend quis, cursus quis, libero. Curabitur et dolor. Sed vitae sem. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Maecenas ante. Duis ullamcorper enim. Donec tristique enim eu leo. Nullam molestie elit eu dolor. Nullam bibendum, turpis vitae tristique gravida, quam sapien tempor lectus, quis pretium tellus purus ac quam. Nulla facilisi.

Duis aliquet dui in est. Donec eget est. Nunc lectus odio, varius at, fermentum in, accumsan non, enim. Aliquam erat volutpat. Proin sit amet nulla ut eros consectetuer cursus. Phasellus dapibus aliquam justo. Nunc laoreet. Donec consequat placerat magna. Duis pretium tincidunt justo. Sed sollicitudin vestibulum quam. Nam quis ligula. Vivamus at metus. Etiam imperdiet imperdiet pede. Aenean turpis. Fusce augue velit, scelerisque sollicitudin, dictum vitae, tempor et, pede. Donec wisi sapien, feugiat in, fermentum ut, sollicitudin adipiscing, metus.

Donec vel nibh ut felis consectetuer laoreet. Donec pede. Sed id quam id wisi laoreet suscipit. Nulla lectus dolor, aliquam ac, fringilla eget, mollis ut, orci. In pellentesque justo in ligula. Maecenas turpis. Donec eleifend leo at felis tincidunt consequat. Aenean turpis metus, malesuada sed, condimentum sit amet, auctor a, wisi. Pellentesque sapien elit, bibendum ac, posuere et, congue eu, felis. Vestibulum mattis libero quis metus scelerisque ultrices. Sed purus.

Donec molestie, magna ut luctus ultrices, tellus arcu nonummy velit, sit amet pulvinar elit justo et mauris. In pede. Maecenas euismod elit eu erat. Aliquam augue wisi, facilisis congue, suscipit in, adipiscing et, ante. In justo. Cras lobortis neque ac ipsum. Nunc fermentum massa at ante. Donec orci tortor, egestas sit amet, ultrices eget, venenatis eget, mi. Maecenas vehicula leo semper est. Mauris vel metus. Aliquam erat volutpat. In rhoncus sapien ac tellus. Pellentesque ligula.

Cras dapibus, augue quis scelerisque ultricies, felis dolor placerat sem, id porta velit odio eu elit. Aenean interdum nibh sed wisi. Praesent sollicitudin vulputate dui. Praesent iaculis viverra augue. Quisque in libero. Aenean gravida lorem vitae sem ullamcorper cursus. Nunc adipiscing rutrum ante. Nunc ipsum massa, faucibus sit amet, viverra vel, elementum semper, orci. Cras eros sem, vulputate et, tincidunt id, ultrices eget, magna. Nulla varius ornare odio. Donec accumsan mauris sit amet augue. Sed ligula lacus, laoreet non, aliquam sit amet, iaculis tempor, lorem. Suspendisse eros. Nam porta, leo sed congue tempor, felis est ultrices eros, id mattis velit felis non metus. Curabitur vitae elit non mauris varius pretium. Aenean lacus sem, tincidunt ut, consequat quis, porta vitae, turpis. Nullam laoreet fermentum urna. Proin iaculis lectus.

Sed mattis, erat sit amet gravida malesuada, elit augue egestas diam, tempus scelerisque nunc nisl vitae libero. Sed consequat feugiat massa. Nunc porta, eros in eleifend varius, erat leo rutrum dui, non convallis lectus orci ut nibh. Sed lorem massa, nonummy quis, egestas id, condimentum at, nisl. Maecenas at nibh. Aliquam et augue at nunc pellentesque ullamcorper. Duis nisl nibh, laoreet suscipit, convallis ut, rutrum id, enim. Phasellus odio. Nulla nulla elit, molestie non, scelerisque at, vestibulum eu, nulla. Ut odio nisl, facilisis id, mollis et, scelerisque nec, enim. Aenean sem leo, pellentesque sit amet, scelerisque sit amet, vehicula pellentesque, sapien.

A.2 Transposition de la licence précédente en droit français

Sed consequat tellus et tortor. Ut tempor laoreet quam. Nullam id wisi a libero tristique semper. Nullam nisl massa, rutrum ut, egestas semper, mollis id, leo. Nulla ac massa eu risus blandit mattis. Mauris ut nunc. In hac habitasse platea dictumst. Aliquam eget tortor. Quisque dapibus pede in erat. Nunc enim. In dui nulla, commodo at, consectetuer nec, malesuada nec, elit. Aliquam ornare tellus eu urna. Sed nec metus. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas.

Phasellus id magna. Duis malesuada interdum arcu. Integer metus. Morbi pulvinar pellentesque mi. Suspendisse sed est eu magna molestie egestas. Quisque mi lorem, pulvinar eget, egestas quis, luctus at, ante. Proin auctor vehicula purus. Fusce ac nisl aliquam ante hendrerit pellentesque. Class aptent taciti sociosqu ad litora torquent per conubia nostra, per inceptos hymenaeos. Morbi wisi. Etiam arcu mauris, facilisis sed, eleifend non, nonummy ut, pede. Cras ut lacus tempor metus mollis placerat. Vivamus eu tortor vel metus interdum malesuada.

Sed eleifend, eros sit amet faucibus elementum, urna sapien consectetuer mauris, quis egestas leo justo non risus. Morbi non felis ac libero vulputate fringilla. Mauris libero eros, lacinia non, sodales quis, dapibus porttitor, pede. Class aptent taciti sociosqu ad litora torquent per conubia nostra, per inceptos hymenaeos. Morbi dapibus mauris condimentum nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Etiam sit amet erat. Nulla varius. Etiam tincidunt dui vitae turpis. Donec leo. Morbi vulputate convallis est. Integer aliquet. Pellentesque aliquet sodales urna.

Nullam eleifend justo in nisl. In hac habitasse platea dictumst. Morbi nonummy. Aliquam ut felis. In velit leo, dictum vitae, posuere id, vulputate nec, ante. Maecenas vitae pede nec dui dignissim suscipit. Morbi magna. Vestibulum id purus eget velit laoreet laoreet. Praesent sed leo vel nibh convallis blandit. Ut rutrum. Donec nibh. Donec interdum. Fusce sed pede sit amet elit rhoncus ultrices. Nullam at enim vitae pede vehicula iaculis.

Class aptent taciti sociosqu ad litora torquent per conubia nostra, per inceptos hymenaeos. Aenean nonummy turpis id odio. Integer euismod imperdiet turpis. Ut nec leo nec diam imperdiet lacinia. Etiam eget lacus eget mi ultricies posuere. In placerat tristique tortor. Sed porta vestibulum metus. Nulla iaculis sollicitudin pede. Fusce luctus tellus in dolor. Curabitur auctor velit a sem. Morbi sapien. Class aptent taciti sociosqu ad litora torquent per conubia nostra, per inceptos hymenaeos. Donec adipiscing urna vehicula nunc. Sed ornare leo in leo. In rhoncus leo ut dui. Aenean dolor quam, volutpat nec, fringilla id, consectetuer vel, pede.

Nulla malesuada risus ut urna. Aenean pretium velit sit amet metus. Duis iaculis. In hac habitasse platea dictumst. Nullam molestie turpis eget nisl. Duis a massa id pede dapibus ultricies. Sed eu leo. In at mauris sit amet tortor bibendum varius. Phasellus justo risus, posuere in, sagittis ac, varius vel, tortor. Quisque id enim. Phasellus consequat, libero pretium nonummy fringilla, tortor lacus vestibulum nunc, ut rhoncus ligula neque id justo. Nullam accumsan euismod nunc. Proin vitae ipsum ac metus dictum tempus. Nam ut wisi. Quisque tortor felis, interdum ac, sodales a, semper a, sem. Curabitur in velit sit amet dui tristique sodales. Vivamus mauris pede, lacinia eget, pellentesque quis, scelerisque eu, est. Aliquam risus. Quisque bibendum pede eu dolor.

Donec tempus neque vitae est. Aenean egestas odio sed risus ullamcorper ullamcorper. Sed in nulla a tortor tincidunt egestas. Nam sapien tortor, elementum sit amet, aliquam in, porttitor faucibus, enim. Nullam congue suscipit nibh. Quisque convallis. Praesent arcu nibh, vehicula eget, accumsan eu, tincidunt a, nibh. Suspendisse vulputate, tortor quis adipiscing viverra, lacus nibh dignissim tellus, eu suscipit risus ante fringilla diam. Quisque a libero vel pede imperdiet aliquet. Pellentesque nunc nibh, eleifend a, consequat consequat, hendrerit nec, diam. Sed urna. Maecenas laoreet eleifend neque. Vivamus purus odio, eleifend non, iaculis a, ultrices sit amet, urna. Mauris faucibus odio vitae risus. In nisl. Praesent purus. Integer iaculis, sem eu egestas lacinia, lacus pede scelerisque augue, in ullamcorper dolor eros ac lacus. Nunc in libero.

Fusce suscipit cursus sem. Vivamus risus mi, egestas ac, imperdiet varius, faucibus quis, leo. Aenean tincidunt. Donec suscipit. Cras id justo quis nibh scelerisque dignissim. Aliquam sagittis elementum dolor. Aenean consectetuer justo in pede. Curabitur ullamcorper ligula nec orci. Aliquam purus turpis, aliquam id, ornare vitae, porttitor non, wisi. Maecenas luctus porta lorem. Donec vitae ligula eu ante pretium varius. Proin tortor metus, convallis et, hendrerit non, scelerisque in, urna. Cras quis libero eu ligula bibendum tempor. Vivamus tellus quam, malesuada eu, tempus sed, tempor sed, velit. Donec lacinia auctor libero.

Praesent sed neque id pede mollis rutrum. Vestibulum iaculis risus. Pellentesque lacus. Ut quis nunc sed odio malesuada egestas. Duis a magna sit amet ligula tristique pretium. Ut pharetra. Vestibulum imperdiet magna nec wisi. Mauris convallis. Sed accumsan sollicitudin massa. Sed id enim. Nunc pede enim, lacinia ut, pulvinar quis, suscipit semper, elit. Cras accumsan erat vitae enim. Cras sollicitudin. Vestibulum rutrum blandit massa.

Sed gravida lectus ut purus. Morbi laoreet magna. Pellentesque eu wisi. Proin turpis. Integer sollicitudin augue nec dui. Fusce lectus. Vivamus faucibus nulla nec lacus. Integer diam. Pellentesque sodales, enim feugiat cursus volutpat, sem mauris dignissim mauris, quis consequat sem est fermentum ligula. Nullam justo lectus, condimentum sit amet, posuere a, fringilla mollis, felis. Morbi nulla nibh, pellentesque at, nonummy eu, sollicitudin nec, ipsum. Cras neque. Nunc augue. Nullam vitae quam id quam pulvinar blandit. Nunc sit amet orci. Aliquam erat elit, pharetra nec, aliquet a, gravida in, mi. Quisque urna enim, viverra quis, suscipit quis, tincidunt ut, sapien. Cras placerat consequat sem. Curabitur ac diam. Curabitur diam tortor, mollis et, viverra ac, tempus vel, metus.

Curabitur ac lorem. Vivamus non justo in dui mattis posuere. Etiam accumsan ligula id pede. Maecenas tincidunt diam nec velit. Praesent convallis sapien ac est. Aliquam ullamcorper euismod nulla. Integer mollis enim vel tortor. Nulla sodales placerat nunc. Sed tempus rutrum wisi. Duis accumsan gravida purus. Nunc nunc. Etiam facilisis dui eu sem. Vestibulum semper. Praesent eu eros. Vestibulum tellus nisl, dapibus id, vestibulum sit amet, placerat ac, mauris. Maecenas et elit ut erat placerat dictum. Nam feugiat, turpis et sodales volutpat, wisi quam rhoncus neque, vitae aliquam ipsum sapien vel enim. Maecenas suscipit cursus mi.

Quisque consectetuer. In suscipit mauris a dolor pellentesque consectetuer. Mauris convallis neque non erat. In lacinia. Pellentesque leo eros, sagittis quis, fermentum quis, tincidunt ut, sapien. Maecenas sem. Curabitur eros odio, interdum eu, feugiat eu, porta ac, nisl. Curabitur nunc. Etiam fermentum convallis velit. Pellentesque laoreet lacus. Quisque sed elit. Nam quis tellus. Aliquam tellus arcu, adipiscing non, tincidunt eleifend, adipiscing quis, augue. Vivamus elementum placerat enim. Suspendisse ut tortor. Integer faucibus adipiscing felis. Aenean consectetuer mattis lectus. Morbi malesuada faucibus dolor. Nam lacus. Etiam arcu libero, malesuada vitae, aliquam vitae, blandit tristique, nisl.

Maecenas accumsan dapibus sapien. Duis pretium iaculis arcu. Curabitur ut lacus. Aliquam vulputate. Suspendisse ut purus sed sem tempor rhoncus. Ut quam dui, fringilla at, dictum eget, ultricies quis, quam. Etiam sem est, pharetra non, vulputate in, pretium at, ipsum. Nunc semper sagittis orci. Sed scelerisque suscipit diam. Ut volutpat, dolor at ullamcorper tristique, eros purus mollis quam, sit amet ornare ante nunc et enim.

Phasellus fringilla, metus id feugiat consectetuer, lacus wisi ultrices tellus, quis lobortis nibh lorem quis tortor. Donec egestas ornare nulla. Mauris mi tellus, porta faucibus, dictum vel, nonummy in, est. Aliquam erat volutpat. In tellus magna, porttitor lacinia, molestie vitae, pellentesque eu, justo. Class aptent taciti sociosqu ad litora torquent per conubia nostra, per inceptos hymenaeos. Sed orci nibh, scelerisque sit amet, suscipit sed, placerat vel, diam. Vestibulum nonummy vulputate orci. Donec et velit ac arcu interdum semper. Morbi pede orci, cursus ac, elementum non, vehicula ut, lacus. Cras volutpat. Nam vel wisi quis libero venenatis placerat. Aenean sed odio. Quisque posuere purus ac orci. Vivamus odio. Vivamus varius, nulla sit amet semper viverra, odio mauris consequat lacus, at vestibulum neque arcu eu tortor. Donec iaculis tincidunt tellus. Aliquam erat volutpat. Curabitur magna lorem, dignissim volutpat, viverra et, adipiscing nec, dolor. Praesent lacus mauris, dapibus vitae, sollicitudin sit amet, nonummy eget, ligula.

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Annexe B

Programmes informatiques

Les listings suivants sont au cœur de notre travail.

Listing B.1 – Il est l'heure

```
#include <stdio.h>
int heures, minutes, secondes;

/*****************************************************/
/*                                                   */
/*             print_heure                           */
/*                                                   */
/*   But:                                            */
/*      Imprime l'heure␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣*/
/*␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣*/
/*␣␣␣Interface:␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣*/
/*␣␣␣␣␣␣Utilise␣les␣variables␣globales␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣*/
/*␣␣␣␣␣␣␣␣␣␣␣heures,␣minutes,␣secondes␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣*/
/*␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣*/
/*****************************************************/

void␣print_heure(void)
{
␣␣printf("Il␣est␣%d␣heure",heures);
␣␣if␣(heures␣>␣1)␣printf("s");
␣␣printf("␣%d␣minute",minutes);
␣␣if␣(minutes␣>␣1)␣printf("s");
␣␣printf("␣%d␣seconde",secondes);
␣␣if␣(secondes␣>␣1)␣printf("s");
```

```
␣␣printf("\n");
}
```

Listing B.2 – Factorielle

```
int factorielle(int n)
{
  if (n > 2) return n * factorielle(n - 1);
  return n;
}
```

Table des matières

Arbres de Décision pour la Prise de Décision Séquentielle

Résumé

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Mots clés : apprentissage par renforcement, arbres de décision, interprétabilité, méthodologie

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Decision Trees for Sequential Decision Making

Abstract

In this Ph.D. thesis, we study algorithms to learn decision trees for classification and sequential decision making. Decision trees are interpretable because humans can read through the decision tree computations from the root to the leaves. This makes decision trees the go-to model when human verification is required like in medicine applications. However, decision trees are non-differentiable making them hard to optimize unlike neural networks that can be trained efficiently with gradient descent. Existing interpretable reinforcement learning approaches usually learn soft trees (non-interpretable as is) or are ad-hoc (train a neural network then fit a tree to it) potentially missing better solutions.

In the first part of this manuscript, we aim to directly learn decision trees for a Markov decision process with reinforcement learning. In practice we show that this amounts to solving a partially observable Markov decision process. Most existing RL algorithms are not suited for POMDPs. This parallel between decision tree learning with RL and POMDPs solving help us understand why in practice it is often easier to obtain a non-interpretable expert policy–a neural network–and then distillate it into a tree rather than learning the decision tree from scratch.

The second contribution from this work arose from the observation that looking for a deicison tree classifier (or regressor) can be seen as sequentially adding nodes to a tree to maximize the accuracy of predictions. We thus formulate decision tree induction as sloving a Markov decision problem and propose a new state-of-the-art algorithm that can be trained with supervised example data and generalizes well to unseen data.

Work from the previous parts rely on the hypothesis that decision trees are indeed an interpretable model that humans can use in sensitive applications. But is it really the case? In the last part of this thesis, we attempt to answer some more general questions about interpretability: can we measure interpretability without humans? And are decision trees really more interpretable than neural networks?

Keywords: reinforcement learning, deicision trees, interpretability, methodology