

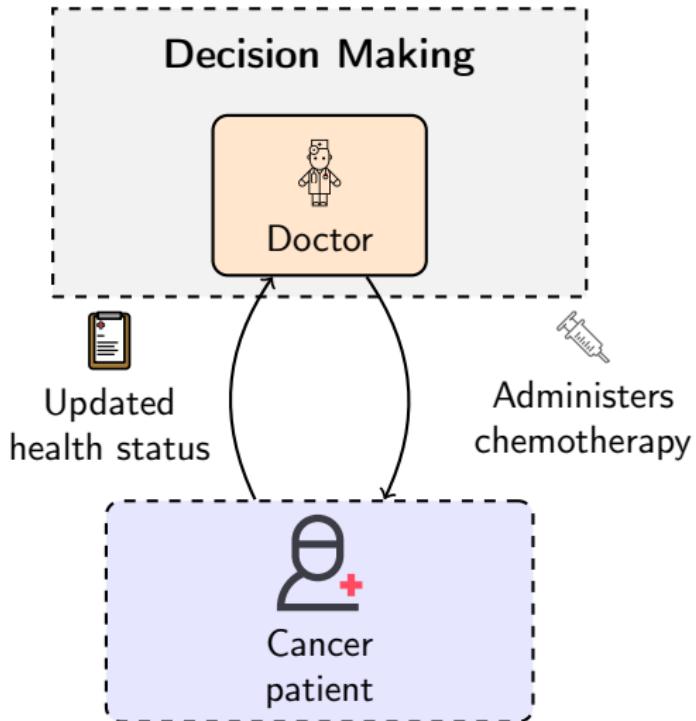
Interpretability, Decision Trees, and Sequential Decision Making

Hector Kohler

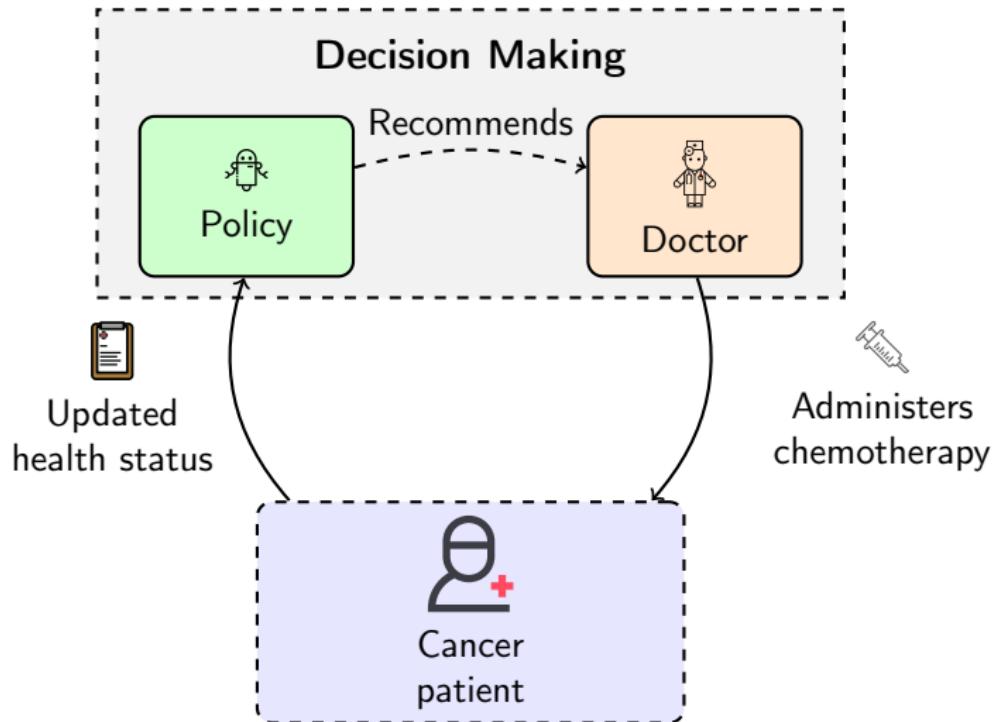
Supervised by Dr. Riad Akrour (HdR) and Prof. Philippe Preux (HdR)
Université de Lille, CNRS, Inria, UMR CRIStAL 9189, France

December 5, 2025

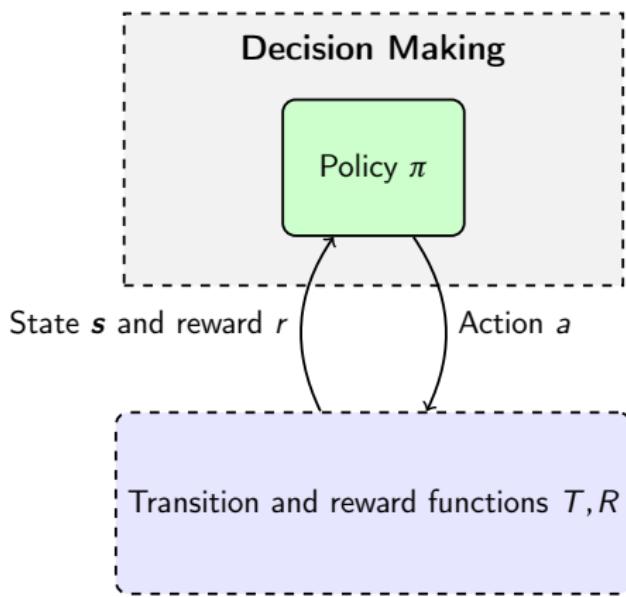
Sequential decision making (SDM)



Sequential decision making (SDM)

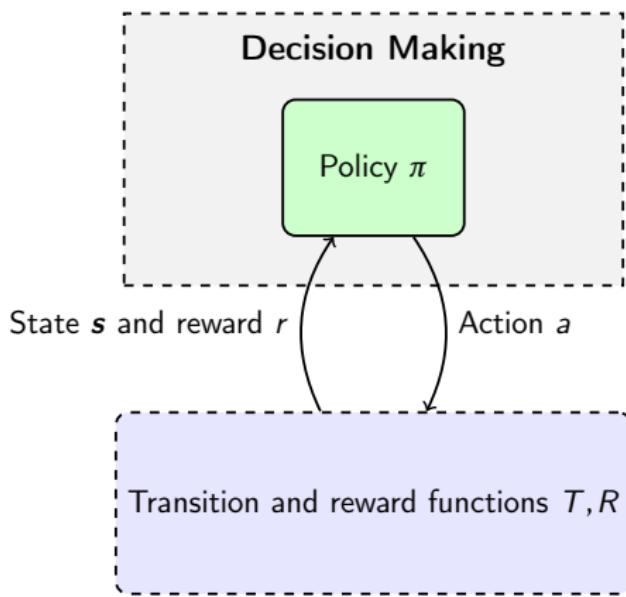


Markov decision processes (MDPs) and reinforcement learning (RL)



Markov decision processes ([Puterman 1994](#)).

Markov decision processes (MDPs) and reinforcement learning (RL)

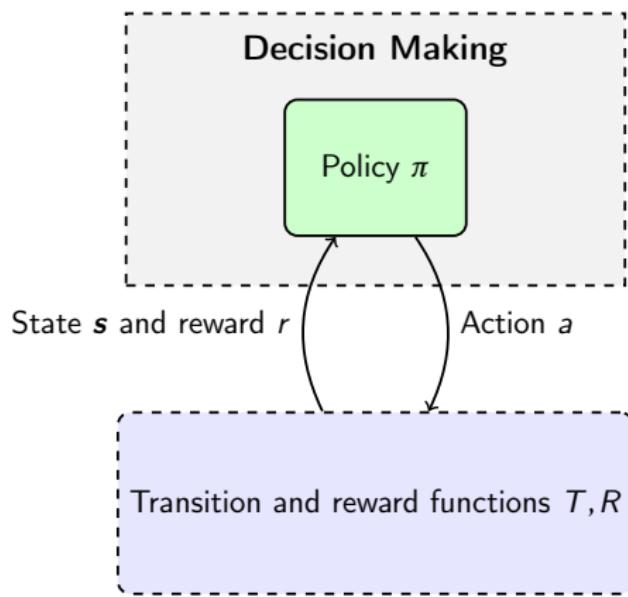


- RL (Sutton and Barto 1998) aims to find a policy, $\pi : S \rightarrow A$ that maximizes:

$$J(\pi) = \mathbb{E}_{s_t \sim T} \left[\sum_{t=0}^{\infty} \gamma^t R(s_t, \pi(s_t)) \right]$$

Markov decision processes (Puterman 1994).

Markov decision processes (MDPs) and reinforcement learning (RL)



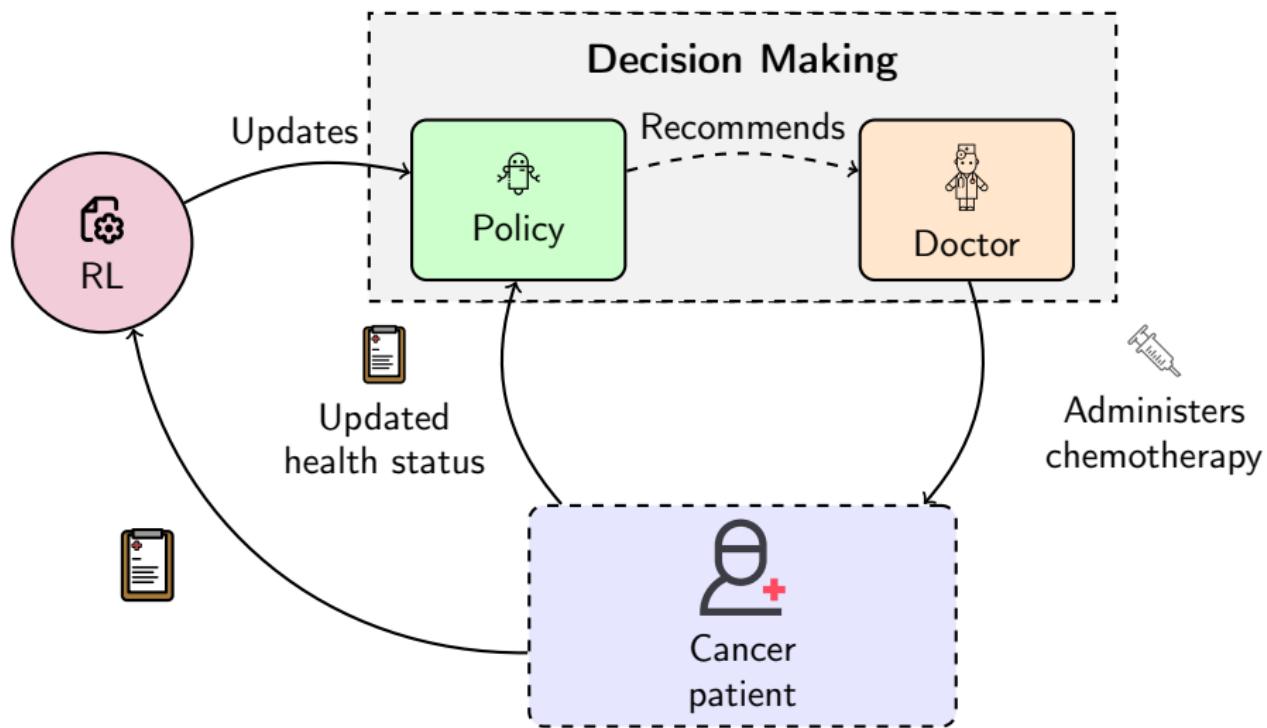
- RL (Sutton and Barto 1998) aims to find a policy,
 $\pi : S \rightarrow A$ that maximizes:

$$J(\pi) = \mathbb{E}_{s_t \sim T} \left[\sum_{t=0}^{\infty} \gamma^t R(s_t, \pi(s_t)) \right]$$

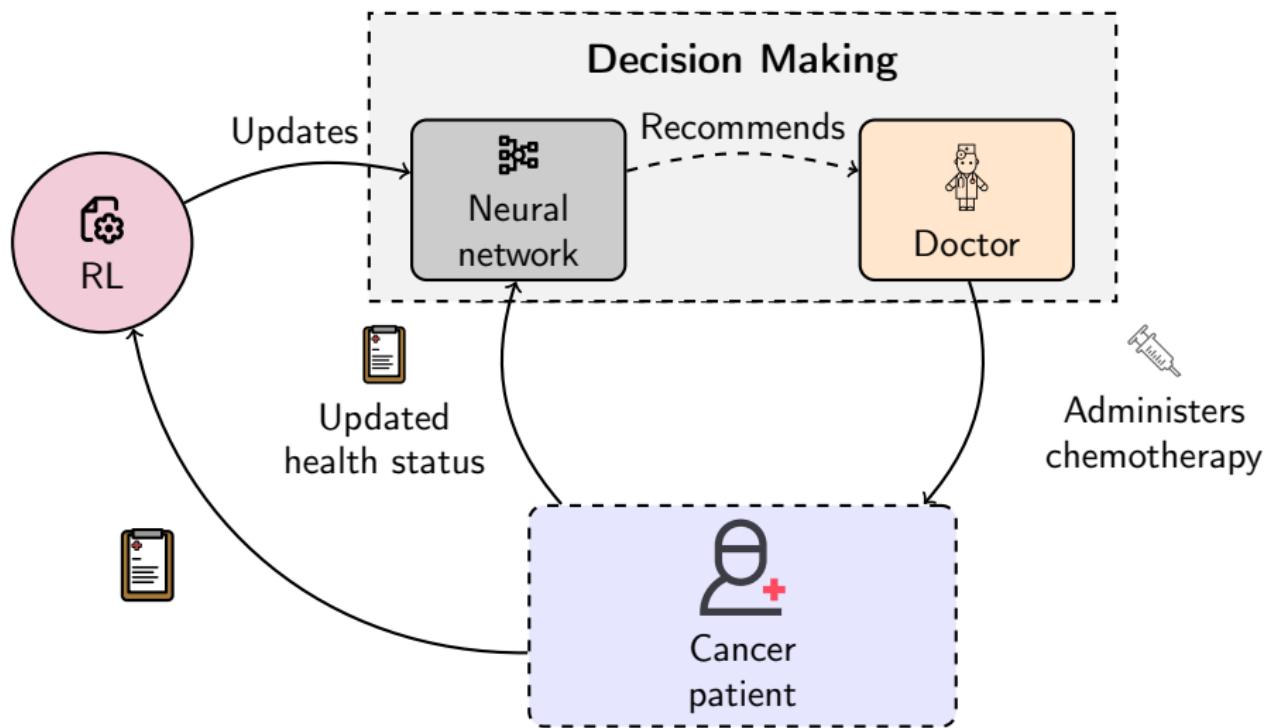
- Lots of successful (deep) RL algorithms (Schulman et al. 2017).

Markov decision processes (Puterman 1994).

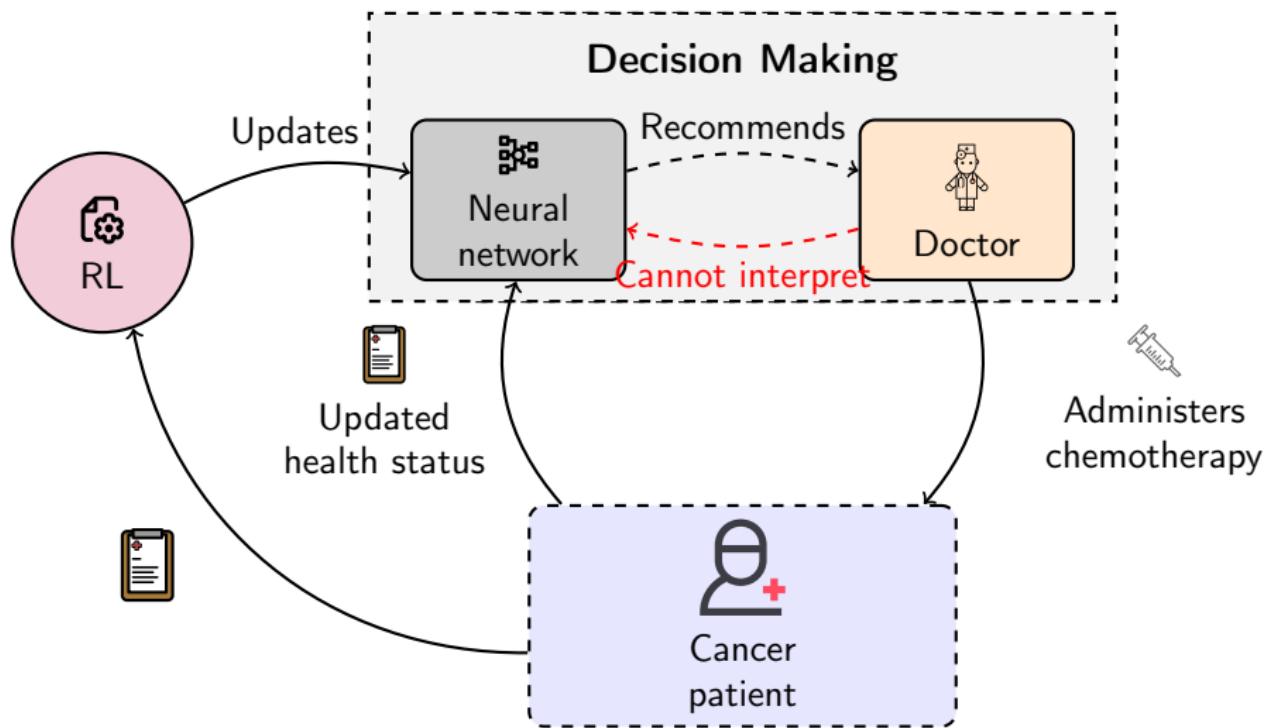
Sequential decision making and reinforcement learning



Sequential decision making and reinforcement learning



Sequential decision making and reinforcement learning



What is interpretability?

⚠ No consensus on what interpretability is.

What is interpretability?

⚠️ No consensus on what interpretability is.

- Evaluation dependent (e.g. with or without humans?) ([Doshi-Velez and Kim 2017](#)).

What is interpretability?

⚠️ No consensus on what interpretability is.

- Evaluation dependent (e.g. with or without humans?) (Doshi-Velez and Kim 2017).
- Set of properties (e.g. transparency) (Lipton 2018).

What is interpretability?

⚠️ No consensus on what interpretability is.

- Evaluation dependent (e.g. with or without humans?) (Doshi-Velez and Kim 2017).
- Set of properties (e.g. transparency) (Lipton 2018).
- Different aspects of SDM (e.g. MDP, RL algorithms, policy class . . .) (Glanois et al. 2024; Milani et al. 2024).

Explainability vs. interpretability

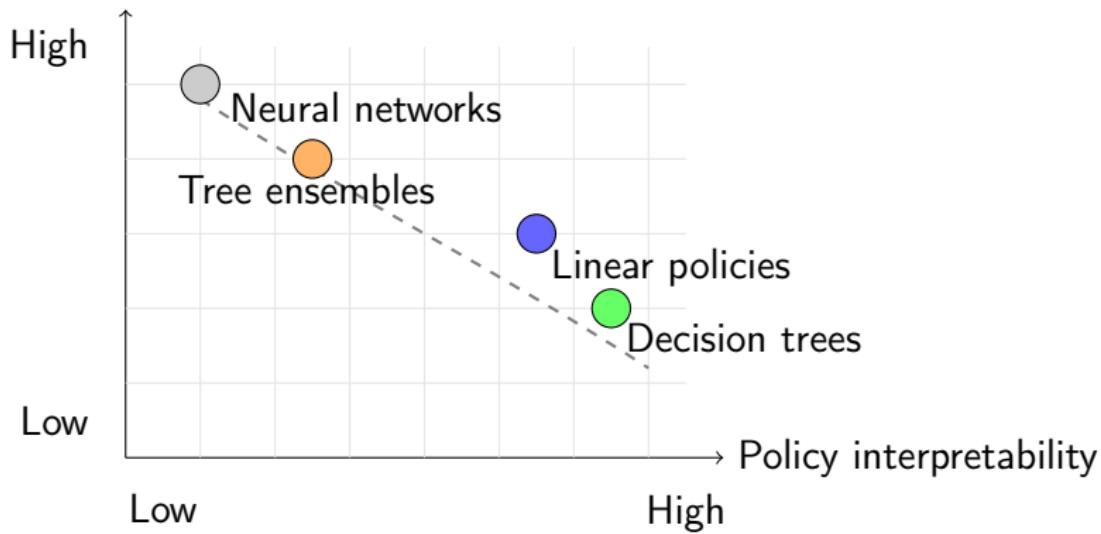
Explainability vs. interpretability



Explainability of different decisions with post-hoc saliency maps (Greydanus et al. 2018).

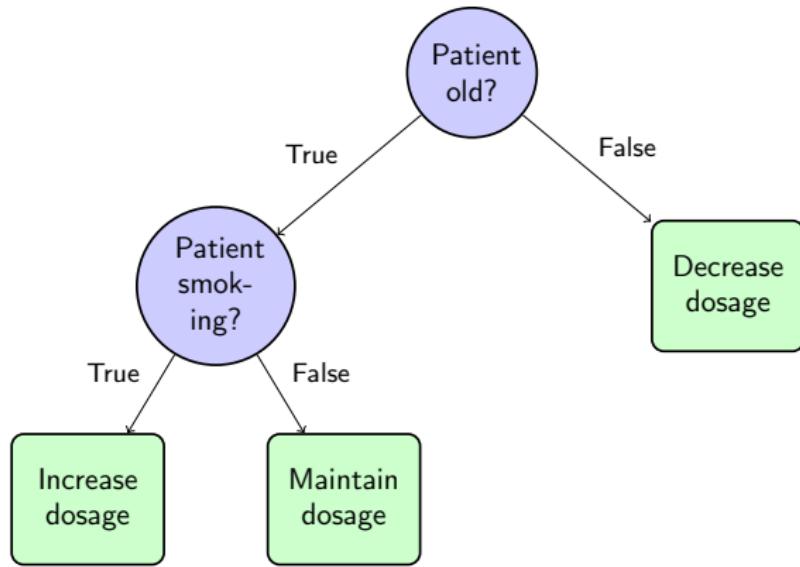
Explainability vs. interpretability

Policy performance



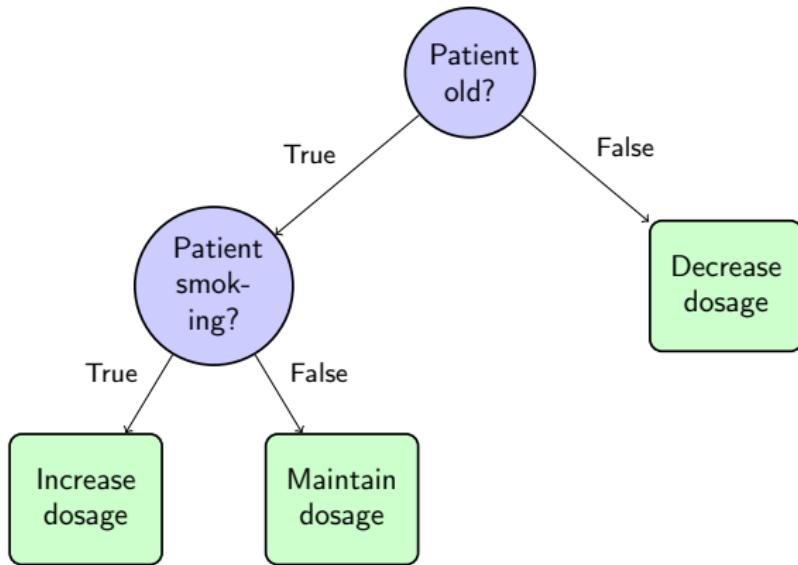
We aim for *interpretability as a property* of policies.

Decision trees



A generic decision tree of depth $D = 2$.

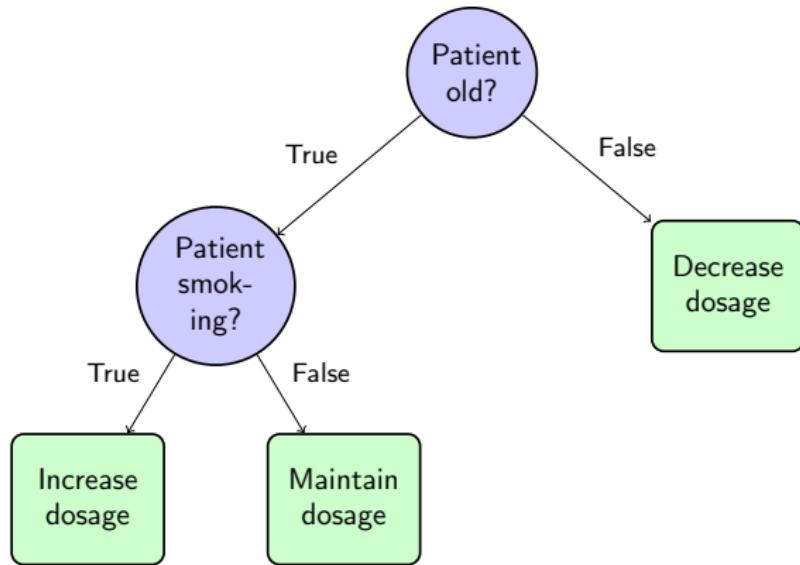
Decision trees



A generic decision tree of depth $D = 2$.

Successful algorithms for classification/regression (Breiman et al. 1984).

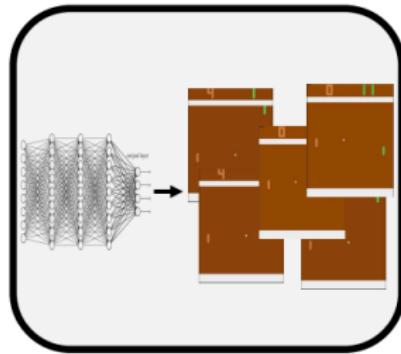
Decision trees



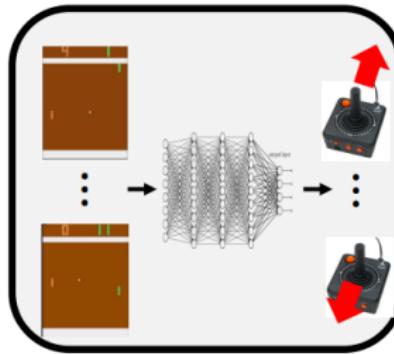
A generic decision tree of depth $D = 2$.

Successful algorithms for classification/regression (Breiman et al. 1984).
What about SDM?

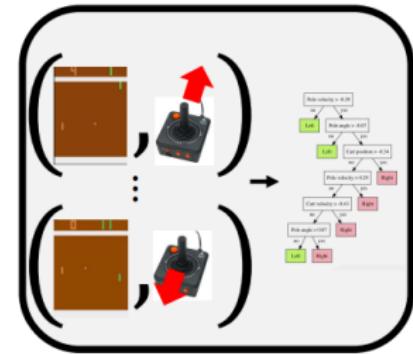
Imitation learning



Step 1: Use NN to generate states

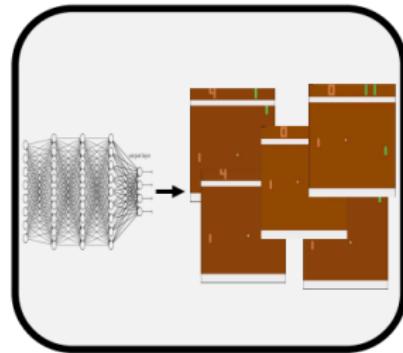


Step 2: Use NN to obtain actions

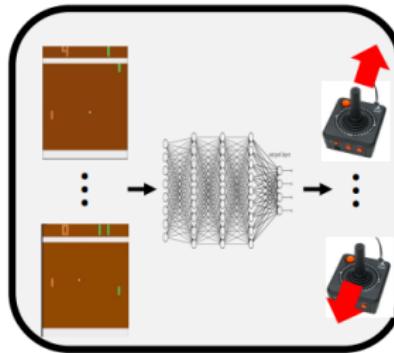


Step 3: Use supervised learning
to train a decision tree

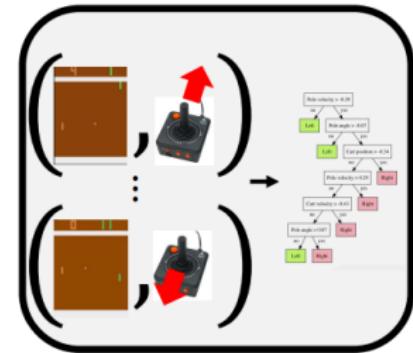
Imitation learning



Step 1: Use NN to generate states



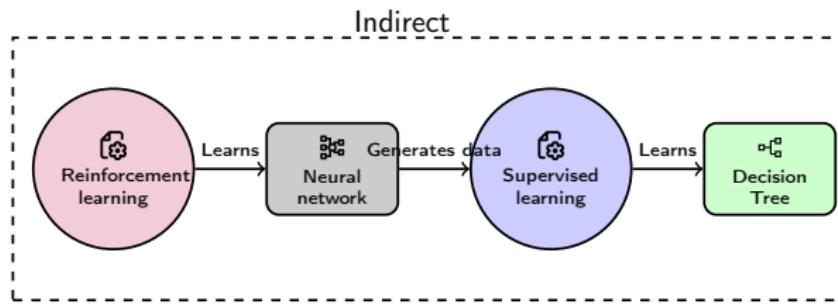
Step 2: Use NN to obtain actions



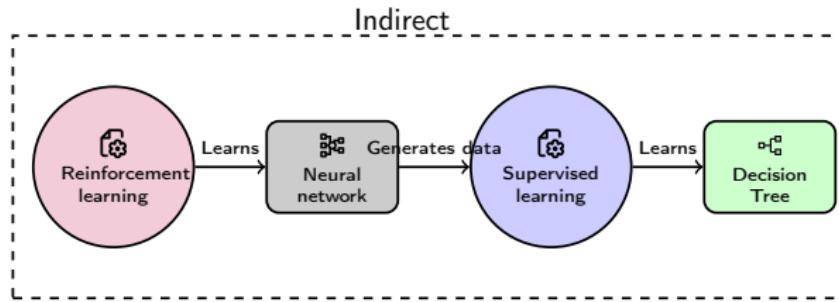
Step 3: Use supervised learning
to train a decision tree

Most research focuses on imitation learning of interpretable policies
(Bastani, Pu, and Solar-Lezama 2018; Milani et al. 2024).

Two ways to get interpretable policies for SDM (Glanois et al. 2024)

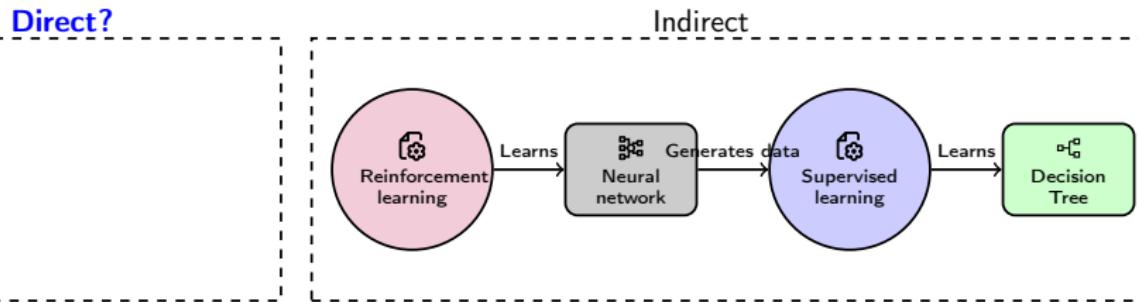


Two ways to get interpretable policies for SDM (Glanois et al. 2024)



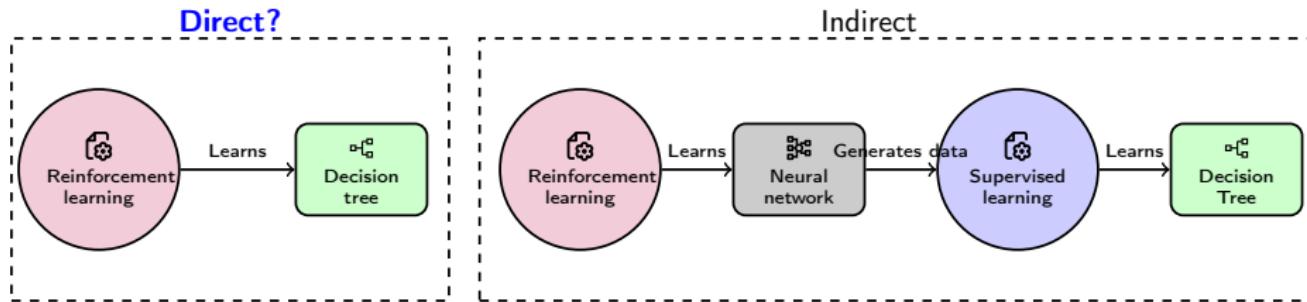
⚠ Policies obtained indirectly optimize a surrogate objective rather than an MDP cumulative rewards.

Two ways to get interpretable policies for SDM (Glanois et al. 2024)



⚠ Policies obtained indirectly optimize a surrogate objective rather than an MDP cumulative rewards.

Two ways to get interpretable policies for SDM (Glanois et al. 2024)



⚠ Policies obtained indirectly optimize a surrogate objective rather than an MDP cumulative rewards.

Contributions

- ① Can we directly train decision tree policies that trade off interpretability and performances for SDM?
- ② Can we leverage SDM to learn decision trees for classification/regression?
- ③ How to measure policy interpretability in SDM?

Contributions

- ① Can we directly train decision tree policies that trade off interpretability and performances for SDM?
- ② Can we leverage SDM to learn decision trees for classification/regression?
- ③ How to measure policy interpretability in SDM?

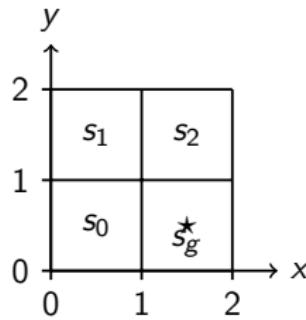
Contributions

- ① Can we directly train decision tree policies that trade off interpretability and performances for SDM?
- ② Can we leverage SDM to learn decision trees for classification/regression?
- ③ How to measure policy interpretability in SDM?

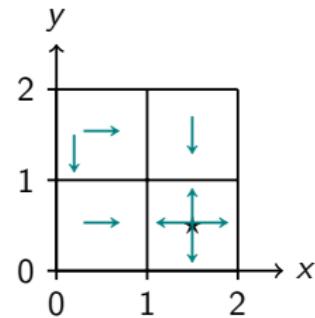
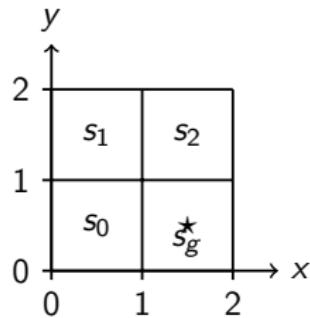
Contributions

- ① Can we directly train decision tree policies that trade off interpretability and performances for SDM?
- ② Can we leverage SDM to learn decision trees for classification/regression?
- ③ How to measure policy interpretability in SDM?

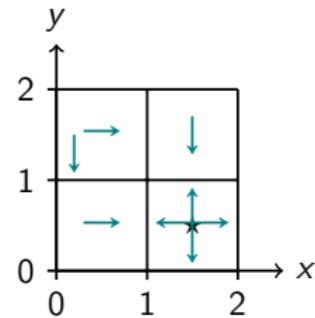
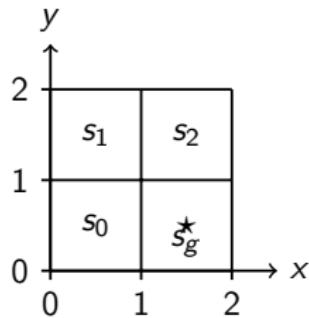
Grid world MDP and decision tree policies



Grid world MDP and decision tree policies

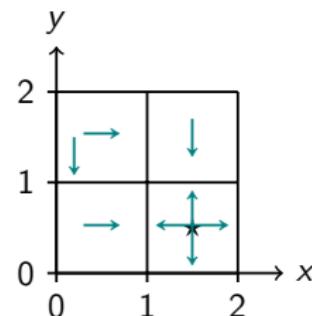
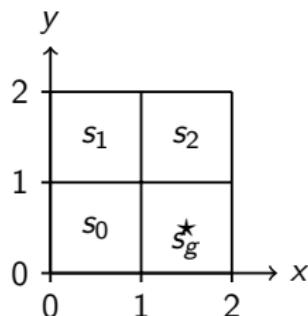


Grid world MDP and decision tree policies

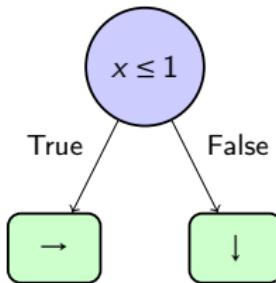


Grid world MDP and optimal actions.

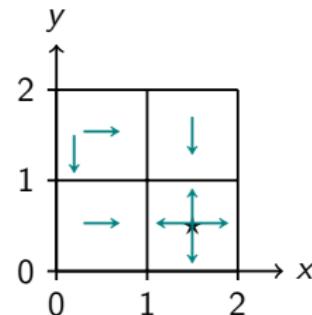
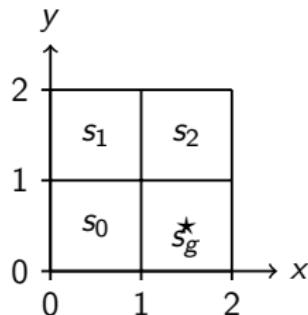
Grid world MDP and decision tree policies



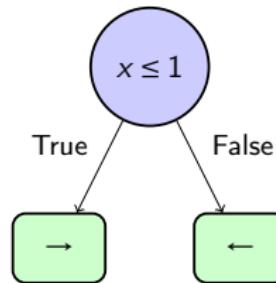
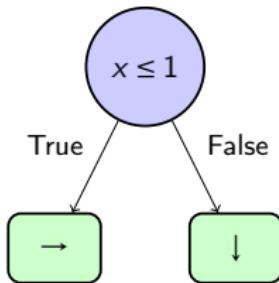
Grid world MDP and optimal actions.



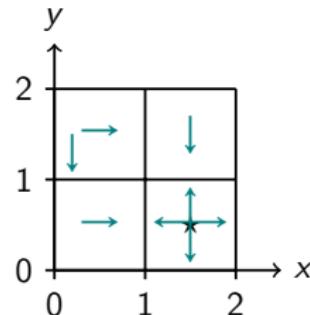
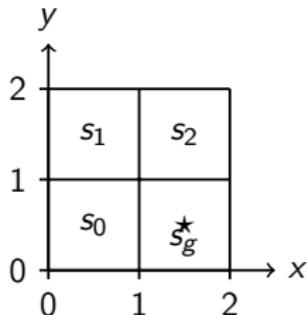
Grid world MDP and decision tree policies



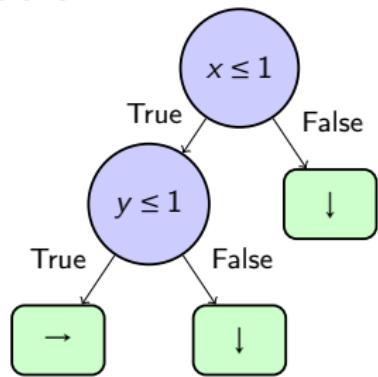
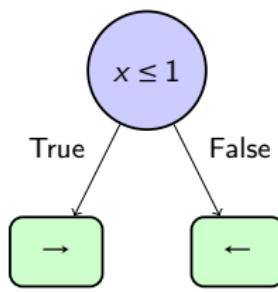
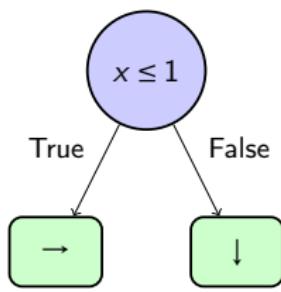
Grid world MDP and optimal actions.



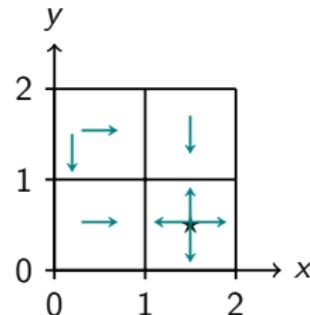
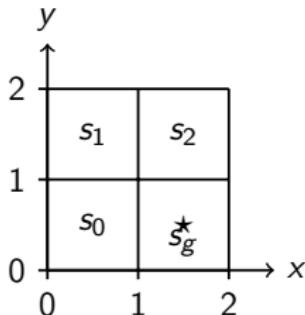
Grid world MDP and decision tree policies



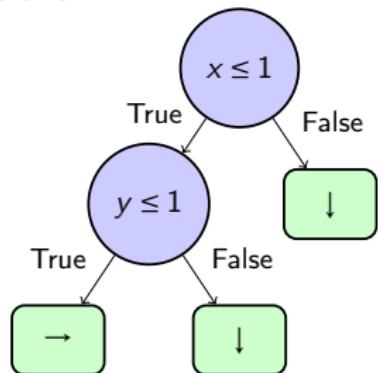
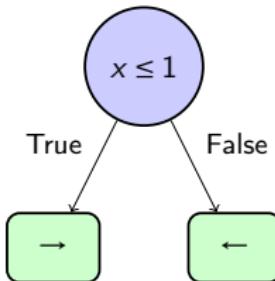
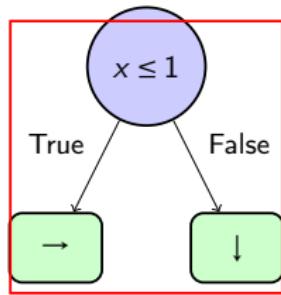
Grid world MDP and optimal actions.



Grid world MDP and decision tree policies

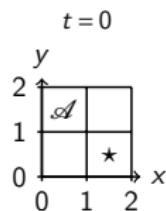


Grid world MDP and optimal actions.

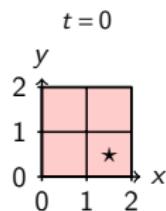


Decision tree policies with different interpretability-performance trade-offs.

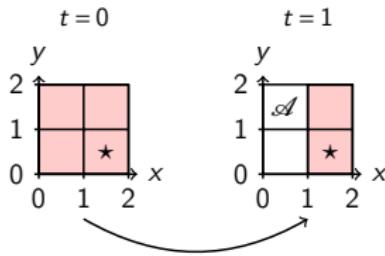
Iterative bounding Markov decision processes (Topin et al. 2021)



Iterative bounding Markov decision processes (Topin et al. 2021)

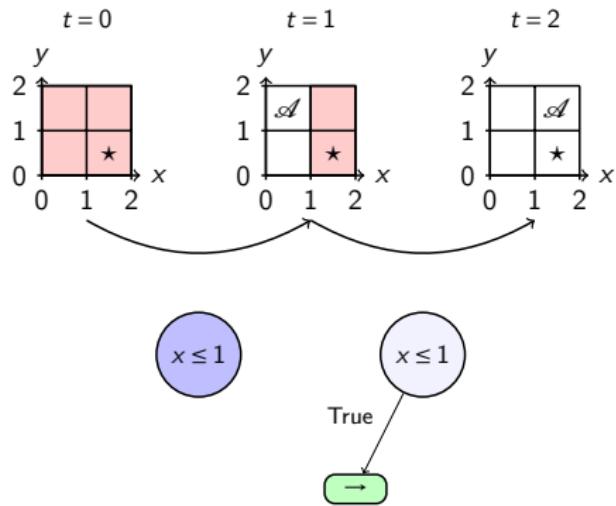


Iterative bounding Markov decision processes (Topin et al. 2021)

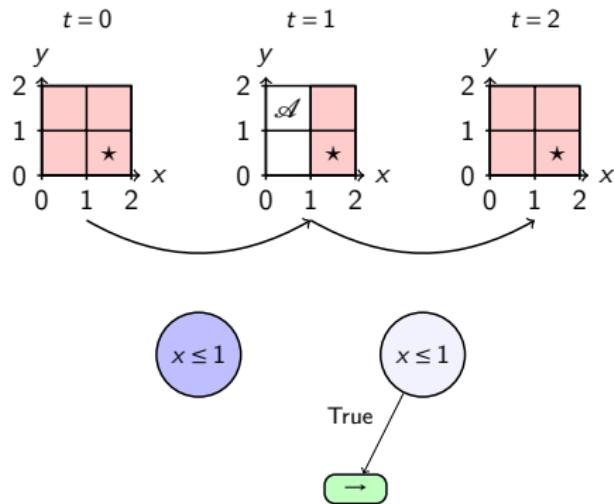


A purple circle contains the mathematical expression $x \leq 1$.

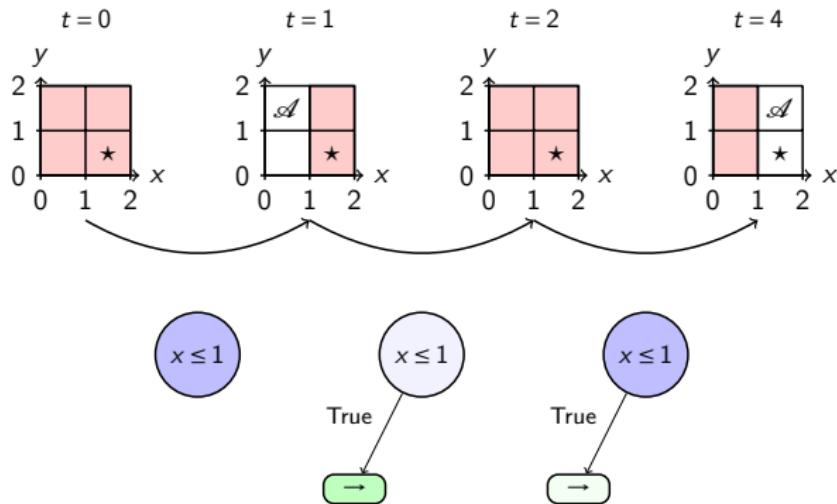
Iterative bounding Markov decision processes (Topin et al. 2021)



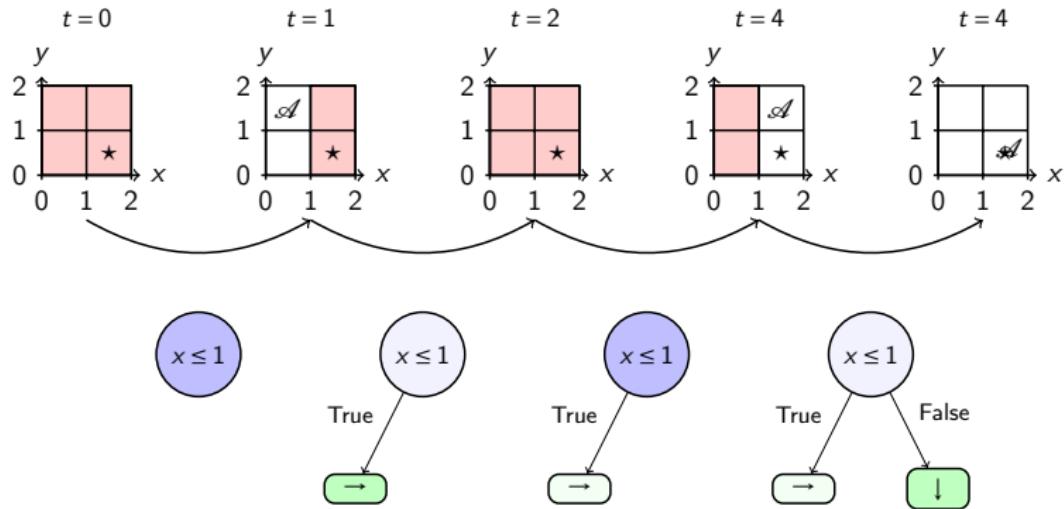
Iterative bounding Markov decision processes (Topin et al. 2021)



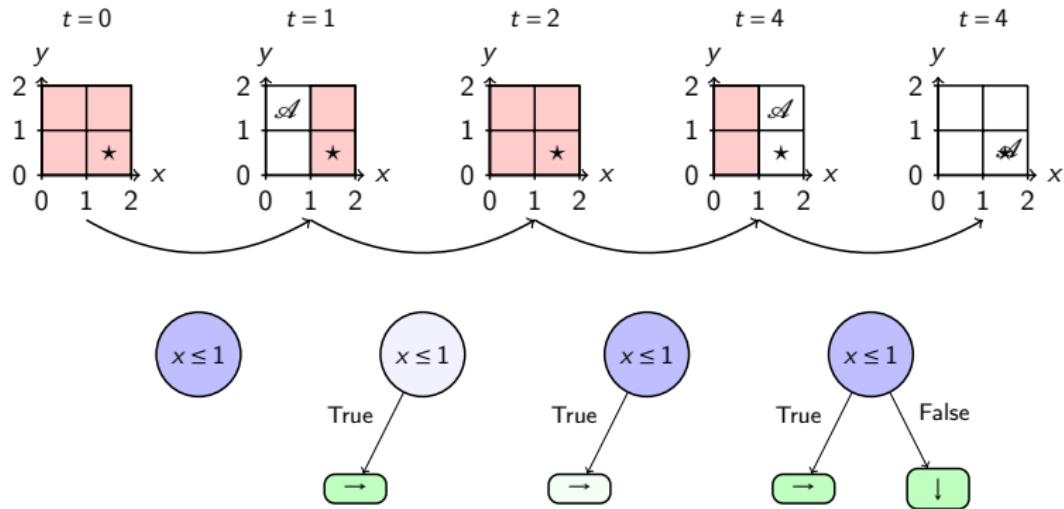
Iterative bounding Markov decision processes (Topin et al. 2021)



Iterative bounding Markov decision processes (Topin et al. 2021)



Iterative bounding Markov decision processes (Topin et al. 2021)



Iterative bounding Markov decision processes (Topin et al. 2021)

Given an MDP $\mathcal{M} \langle S, A, R, T \rangle$

Iterative bounding Markov decision processes (Topin et al. 2021)

Given an MDP $\mathcal{M} \langle S, A, R, T \rangle$, an IBMDP for \mathcal{M} is an MDP

$$\langle S \times O, A \cup A_{info}, (R, \zeta), (T_{info}, T) \rangle$$

where:

Iterative bounding Markov decision processes (Topin et al. 2021)

Given an MDP $\mathcal{M} \langle S, A, R, T \rangle$, an IBMDP for \mathcal{M} is an MDP

$$\langle S \times O, A \cup A_{info}, (R, \zeta), (T_{info}, T) \rangle$$

where:

- O : observations of some state features.

Iterative bounding Markov decision processes (Topin et al. 2021)

Given an MDP $\mathcal{M} \langle S, A, R, T \rangle$, an IBMDP for \mathcal{M} is an MDP

$$\langle S \times O, A \cup A_{info}, (R, \zeta), (T_{info}, T) \rangle$$

where:

- O : observations of some state features.
- A_{info} : actions that gather information about some state features.

Iterative bounding Markov decision processes (Topin et al. 2021)

Given an MDP $\mathcal{M} \langle S, A, R, T \rangle$, an IBMDP for \mathcal{M} is an MDP

$$\langle S \times O, A \cup A_{info}, (R, \zeta), (T_{info}, T) \rangle$$

where:

- O : observations of some state features.
- A_{info} : actions that gather information about some state features.
- ζ : reward for taking $a \in A_{info}$.

Iterative bounding Markov decision processes (Topin et al. 2021)

Given an MDP $\mathcal{M} \langle S, A, R, T \rangle$, an IBMDP for \mathcal{M} is an MDP

$$\langle S \times O, A \cup A_{info}, (R, \zeta), (T_{info}, T) \rangle$$

where:

- O : observations of some state features.
- A_{info} : actions that gather information about some state features.
- ζ : reward for taking $a \in A_{info}$.
- T_{info} : transitions following $a \in A_{info}$ that update the partial observation with gathered info.

Iterative bounding Markov decision processes (Topin et al. 2021)

Given an MDP $\mathcal{M} \langle S, A, R, T \rangle$, an IBMDP for \mathcal{M} is an MDP

$$\langle S \times O, A \cup A_{info}, (R, \zeta), (T_{info}, T) \rangle$$

where:

- O : observations of some state features.
- A_{info} : actions that gather information about some state features.
- ζ : reward for taking $a \in A_{info}$.
- T_{info} : transitions following $a \in A_{info}$ that update the partial observation with gathered info.
- **⚠ Topin et al. 2021: deterministic memoryless policies**
 $\pi_{po} : O \rightarrow A \cup A_{info}$ in an IBMDP for \mathcal{M} are decision trees for \mathcal{M} .

RL for memoryless policies in POMDPs

RL for memoryless policies

- Finding the best deterministic and memoryless policy in a POMDP is NP-hard (Littman 1994)!
- The best memoryless policy can be stochastic (Singh, Jaakkola, and Jordan 1994).
- Value-based RL converges to sub-optimal solutions.

Asymmetric RL (Pinto et al. 2017)

- Value-based → learns $Q(o, a)$ with TD targets $U(s, a)$ (Baisero, Daley, and Amato 2022).
- Actor-critic → policy gradient on $\pi(o, a)$ using a critic $V(s)$ (Baisero and Amato 2022).
- Topin et al. 2021 uses asymmetric RL (not knowing it) for their experiments.

RL for memoryless policies

- Finding the best deterministic and memoryless policy in a POMDP is NP-hard (Littman 1994)!
- The best memoryless policy can be stochastic (Singh, Jaakkola, and Jordan 1994).
- Value-based RL converges to sub-optimal solutions.

Asymmetric RL (Pinto et al. 2017)

- Value-based → learns $Q(o, a)$ with TD targets $U(s, a)$ (Baisero, Daley, and Amato 2022).
- Actor-critic → policy gradient on $\pi(o, a)$ using a critic $V(s)$ (Baisero and Amato 2022).
- Topin et al. 2021 uses asymmetric RL (not knowing it) for their experiments.

RL for memoryless policies

- Finding the best deterministic and memoryless policy in a POMDP is NP-hard (Littman 1994)!
- The best memoryless policy can be stochastic (Singh, Jaakkola, and Jordan 1994).
- Value-based RL converges to sub-optimal solutions.

Asymmetric RL (Pinto et al. 2017)

- Value-based → learns $Q(o, a)$ with TD targets $U(s, a)$ (Baisero, Daley, and Amato 2022).
- Actor-critic → policy gradient on $\pi(o, a)$ using a critic $V(s)$ (Baisero and Amato 2022).
- Topin et al. 2021 uses asymmetric RL (not knowing it) for their experiments.

RL for memoryless policies

- Finding the best deterministic and memoryless policy in a POMDP is NP-hard (Littman 1994)!
- The best memoryless policy can be stochastic (Singh, Jaakkola, and Jordan 1994).
- Value-based RL converges to sub-optimal solutions.

Asymmetric RL (Pinto et al. 2017)

- Value-based → learns $Q(o, a)$ with TD targets $U(s, a)$ (Baisero, Daley, and Amato 2022).
- Actor-critic → policy gradient on $\pi(o, a)$ using a critic $V(s)$ (Baisero and Amato 2022).
- Topin et al. 2021 uses asymmetric RL (not knowing it) for their experiments.

RL for memoryless policies

- Finding the best deterministic and memoryless policy in a POMDP is NP-hard (Littman 1994)!
- The best memoryless policy can be stochastic (Singh, Jaakkola, and Jordan 1994).
- Value-based RL converges to sub-optimal solutions.

Asymmetric RL (Pinto et al. 2017)

- Value-based → learns $Q(o, a)$ with TD targets $U(s, a)$ (Baisero, Daley, and Amato 2022).
- Actor-critic → policy gradient on $\pi(o, a)$ using a critic $V(s)$ (Baisero and Amato 2022).
- Topin et al. 2021 uses asymmetric RL (not knowing it) for their experiments.

RL for memoryless policies

- Finding the best deterministic and memoryless policy in a POMDP is NP-hard (Littman 1994)!
- The best memoryless policy can be stochastic (Singh, Jaakkola, and Jordan 1994).
- Value-based RL converges to sub-optimal solutions.

Asymmetric RL (Pinto et al. 2017)

- Value-based → learns $Q(o, a)$ with TD targets $U(s, a)$ (Baisero, Daley, and Amato 2022).
- Actor-critic → policy gradient on $\pi(o, a)$ using a critic $V(s)$ (Baisero and Amato 2022).
- Topin et al. 2021 uses asymmetric RL (not knowing it) for their experiments.

RL for memoryless policies

- Finding the best deterministic and memoryless policy in a POMDP is NP-hard (Littman 1994)!
- The best memoryless policy can be stochastic (Singh, Jaakkola, and Jordan 1994).
- Value-based RL converges to sub-optimal solutions.

Asymmetric RL (Pinto et al. 2017)

- Value-based → learns $Q(o, a)$ with TD targets $U(s, a)$ (Baisero, Daley, and Amato 2022).
- Actor-critic → policy gradient on $\pi(o, a)$ using a critic $V(s)$ (Baisero and Amato 2022).
- Topin et al. 2021 uses asymmetric RL (not knowing it) for their experiments.

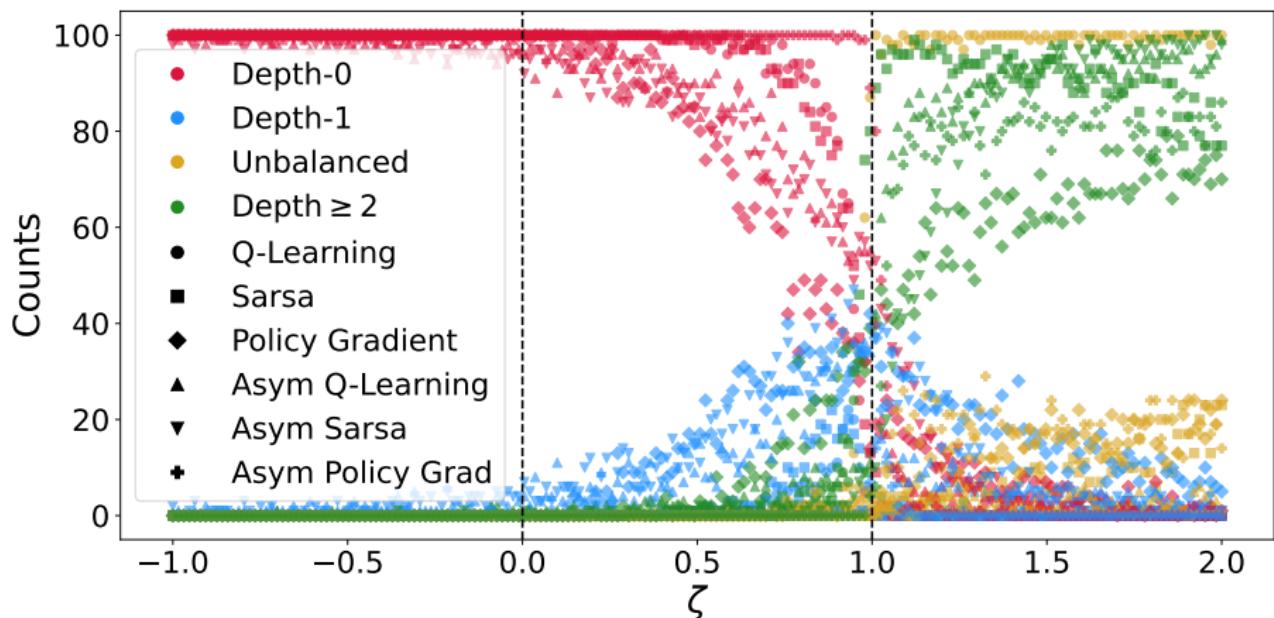
RL for memoryless policies

- Finding the best deterministic and memoryless policy in a POMDP is NP-hard (Littman 1994)!
- The best memoryless policy can be stochastic (Singh, Jaakkola, and Jordan 1994).
- Value-based RL converges to sub-optimal solutions.

Asymmetric RL (Pinto et al. 2017)

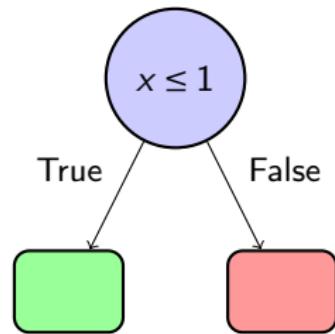
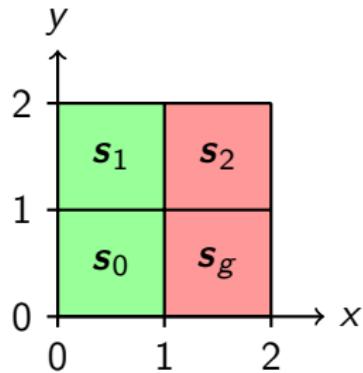
- Value-based → learns $Q(o, a)$ with TD targets $U(s, a)$ (Baisero, Daley, and Amato 2022).
- Actor-critic → policy gradient on $\pi(o, a)$ using a critic $V(s)$ (Baisero and Amato 2022).
- Topin et al. 2021 uses asymmetric RL (not knowing it) for their experiments.

Result: RL cannot retrieve optimal depth-1 trees for the grid world MDP



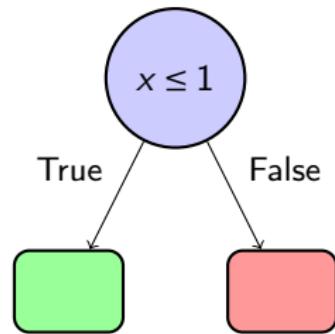
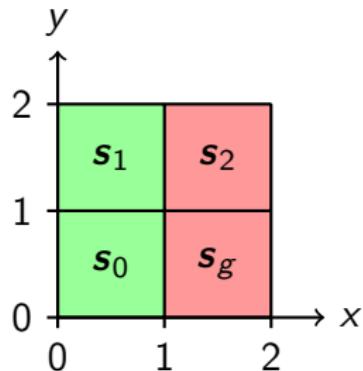
Distributions of tree policies learned with (asymmetric) RL algorithms as a function of the interpretability reward ζ .

Direct RL of decision trees for classification tasks does not involve partial observability



Classification MDP and the unique optimal depth-1 tree.

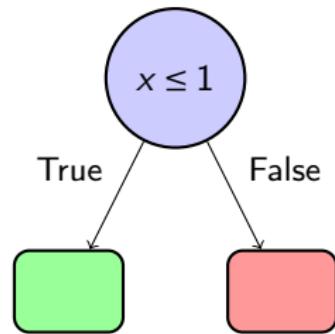
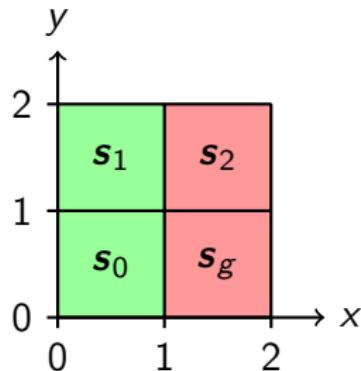
Direct RL of decision trees for classification tasks does not involve partial observability



Classification MDP and the unique optimal depth-1 tree.

Partial observations are sufficient statistics of histories in IBMDPs
for classification tasks → easy to solve (Sigaud and Buffet 2013).

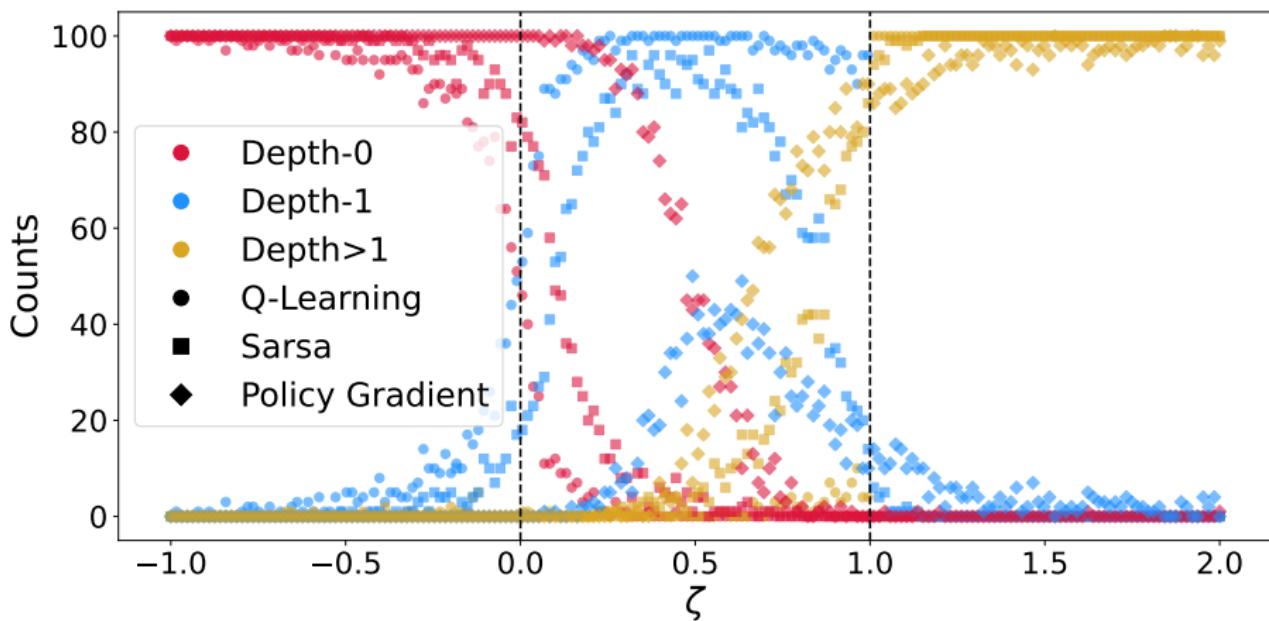
Direct RL of decision trees for classification tasks does not involve partial observability



Classification MDP and the unique optimal depth-1 tree.

Partial observations are sufficient statistics of histories in IBMDPs for classification tasks → easy to solve (**Sigaud and Buffet 2013**).

Result: RL can retrieve optimal depth-1 trees for the toy classification MDP



Distributions of tree policies learned with various RL algorithms.

Perspectives for direct RL of decision tree policies.

- It seems that directly learning decision tree trading off interpretability and performances in MDPs can be difficult to achieve because of **partial observability**.
- Should we focus on indirect approaches? Hybrid approaches ([Wu et al. 2020](#))?
- Fixing the policy tree structure a priori (parametric trees, ([Marton et al. 2025](#)))?

RL can train good decision trees for classification MDPs

Q: Can we leverage SDM to design new decision tree induction algorithms for classification/regression?

Perspectives for direct RL of decision tree policies.

- It seems that directly learning decision tree trading off interpretability and performances in MDPs can be difficult to achieve because of **partial observability**.
- Should we focus on indirect approaches? Hybrid approaches ([Wu et al. 2020](#))?
- Fixing the policy tree structure a priori (parametric trees, ([Marton et al. 2025](#)))?

RL can train good decision trees for classification MDPs

Q: Can we leverage SDM to design new decision tree induction algorithms for classification/regression?

Perspectives for direct RL of decision tree policies.

- It seems that directly learning decision tree trading off interpretability and performances in MDPs can be difficult to achieve because of **partial observability**.
- Should we focus on indirect approaches? Hybrid approaches ([Wu et al. 2020](#))?
- Fixing the policy tree structure a priori (parametric trees, ([Marton et al. 2025](#)))?

RL can train good decision trees for classification MDPs

Q: Can we leverage SDM to design new decision tree induction algorithms for classification/regression?

Perspectives for direct RL of decision tree policies.

- It seems that directly learning decision tree trading off interpretability and performances in MDPs can be difficult to achieve because of **partial observability**.
- Should we focus on indirect approaches? Hybrid approaches ([Wu et al. 2020](#))?
- Fixing the policy tree structure a priori (parametric trees, ([Marton et al. 2025](#)))?

RL can train good decision trees for classification MDPs

Q: Can we leverage SDM to design new decision tree induction algorithms for classification/regression?

Perspectives for direct RL of decision tree policies.

- It seems that directly learning decision tree trading off interpretability and performances in MDPs can be difficult to achieve because of **partial observability**.
- Should we focus on indirect approaches? Hybrid approaches ([Wu et al. 2020](#))?
- Fixing the policy tree structure a priori (parametric trees, ([Marton et al. 2025](#)))?

RL can train good decision trees for classification MDPs

Q: Can we leverage SDM to design new decision tree induction algorithms for classification/regression?

Perspectives for direct RL of decision tree policies.

- It seems that directly learning decision tree trading off interpretability and performances in MDPs can be difficult to achieve because of **partial observability**.
- Should we focus on indirect approaches? Hybrid approaches ([Wu et al. 2020](#))?
- Fixing the policy tree structure a priori (parametric trees, ([Marton et al. 2025](#)))?

RL can train good decision trees for classification MDPs

Q: Can we leverage SDM to design new decision tree induction algorithms for classification/regression?

Decision trees in supervised learning

- N data points $\{x_i, y_i\}$. Each x_i is described by p features and has a label $y_i \in \mathcal{Y}$. We want to find a tree of depth at most D , $T \in \mathcal{T}_D$ that minimizes:

$$\mathcal{L}_\alpha(T) = \frac{1}{N} \sum_{i=1}^N \ell(y_i, T(x_i)) + \alpha C(T)$$

- Trees **interpretable and competitive with neural nets** (Grinsztajn, Oyallon, and Varoquaux 2022).
- Greedy algorithms **sub-optimal accuracy**, but $O(2^D)$ operations (Breiman et al. 1984) .
- Optimal algorithms, **optimal accuracy**, but $O((2Np)^D)$ operations (NP-hard) (Bertsimas and Dunn 2017).
- In between optimal and greedy?

Decision trees in supervised learning

- N data points $\{x_i, y_i\}$. Each x_i is described by p features and has a label $y_i \in \mathcal{Y}$. We want to find a tree of depth at most D , $T \in \mathcal{T}_D$ that minimizes:

$$\mathcal{L}_\alpha(T) = \frac{1}{N} \sum_{i=1}^N \ell(y_i, T(x_i)) + \alpha C(T)$$

- Trees interpretable and competitive with neural nets (Grinsztajn, Oyallon, and Varoquaux 2022).
- Greedy algorithms sub-optimal accuracy, but $O(2^D)$ operations (Breiman et al. 1984).
- Optimal algorithms, optimal accuracy, but $O((2Np)^D)$ operations (NP-hard) (Bertsimas and Dunn 2017).
- In between optimal and greedy?

Decision trees in supervised learning

- N data points $\{x_i, y_i\}$. Each x_i is described by p features and has a label $y_i \in \mathcal{Y}$. We want to find a tree of depth at most D , $T \in \mathcal{T}_D$ that minimizes:

$$\mathcal{L}_\alpha(T) = \frac{1}{N} \sum_{i=1}^N \ell(y_i, T(x_i)) + \alpha C(T)$$

- Trees **interpretable** and **competitive with neural nets** (Grinsztajn, Oyallon, and Varoquaux 2022).
- Greedy algorithms **sub-optimal accuracy**, but $O(2^D)$ operations (Breiman et al. 1984).
- Optimal algorithms, **optimal accuracy**, but $O((2Np)^D)$ operations (NP-hard) (Bertsimas and Dunn 2017).
- In between optimal and greedy?

Decision trees in supervised learning

- N data points $\{x_i, y_i\}$. Each x_i is described by p features and has a label $y_i \in \mathcal{Y}$. We want to find a tree of depth at most D , $T \in \mathcal{T}_D$ that minimizes:

$$\mathcal{L}_\alpha(T) = \frac{1}{N} \sum_{i=1}^N \ell(y_i, T(x_i)) + \alpha C(T)$$

- Trees **interpretable** and **competitive with neural nets** (Grinsztajn, Oyallon, and Varoquaux 2022).
- Greedy algorithms **sub-optimal accuracy**, but $O(2^D)$ operations (Breiman et al. 1984).
- Optimal algorithms, **optimal accuracy**, but $O((2Np)^D)$ operations (NP-hard) (Bertsimas and Dunn 2017).
- In between optimal and greedy?

Decision trees in supervised learning

- N data points $\{x_i, y_i\}$. Each x_i is described by p features and has a label $y_i \in \mathcal{Y}$. We want to find a tree of depth at most D , $T \in \mathcal{T}_D$ that minimizes:

$$\mathcal{L}_\alpha(T) = \frac{1}{N} \sum_{i=1}^N \ell(y_i, T(x_i)) + \alpha C(T)$$

- Trees **interpretable** and **competitive with neural nets** (Grinsztajn, Oyallon, and Varoquaux 2022).
- Greedy algorithms **sub-optimal accuracy**, but $O(2^D)$ operations (Breiman et al. 1984).
- Optimal algorithms, **optimal accuracy**, but $O((2Np)^D)$ operations (NP-hard) (Bertsimas and Dunn 2017).
- In between optimal and greedy?

Decision trees in supervised learning

- N data points $\{x_i, y_i\}$. Each x_i is described by p features and has a label $y_i \in \mathcal{Y}$. We want to find a tree of depth at most D , $T \in \mathcal{T}_D$ that minimizes:

$$\mathcal{L}_\alpha(T) = \frac{1}{N} \sum_{i=1}^N \ell(y_i, T(x_i)) + \alpha C(T)$$

- Trees **interpretable** and **competitive with neural nets** (Grinsztajn, Oyallon, and Varoquaux 2022).
- Greedy algorithms **sub-optimal accuracy**, but $O(2^D)$ operations (Breiman et al. 1984).
- Optimal algorithms, **optimal accuracy**, but $O((2Np)^D)$ operations (NP-hard) (Bertsimas and Dunn 2017).
- In between optimal and greedy?

Decision tree induction as solving MDPs

Intuition

The induction of a decision tree is made of a sequence of decisions: at each node, we must decide whether it is better to split (a subset of) the training data, or to create a leaf node.

- S: data subsets.
- A: test or leaf nodes that can be added to the tree.
- R: interpretability term $-\alpha$ and misclassifications.
- T: node traversals.

Proposition (Objective Equivalence)

Let π be a deterministic policy of the MDP. Then $J_\alpha(\pi) = -\mathcal{L}_\alpha(E(\pi, s_0))$ where E is an algorithm that extracts a decision tree from π (Topin et al. 2021).

Decision tree induction as solving MDPs

Intuition

The induction of a decision tree is made of a sequence of decisions: at each node, we must decide whether it is better to split (a subset of) the training data, or to create a leaf node.

- S: data subsets.
- A: test or leaf nodes that can be added to the tree.
- R: interpretability term $-\alpha$ and misclassifications.
- T: node traversals.

Proposition (Objective Equivalence)

Let π be a deterministic policy of the MDP. Then $J_\alpha(\pi) = -\mathcal{L}_\alpha(E(\pi, s_0))$ where E is an algorithm that extracts a decision tree from π (Topin et al. 2021).

Decision tree induction as solving MDPs

Intuition

The induction of a decision tree is made of a sequence of decisions: at each node, we must decide whether it is better to split (a subset of) the training data, or to create a leaf node.

- S: data subsets.
- A: test or leaf nodes that can be added to the tree.
- R: interpretability term $-\alpha$ and misclassifications.
- T: node traversals.

Proposition (Objective Equivalence)

Let π be a deterministic policy of the MDP. Then $J_\alpha(\pi) = -\mathcal{L}_\alpha(E(\pi, s_0))$ where E is an algorithm that extracts a decision tree from π (Topin et al. 2021).

Decision tree induction as solving MDPs

Intuition

The induction of a decision tree is made of a sequence of decisions: at each node, we must decide whether it is better to split (a subset of) the training data, or to create a leaf node.

- S: data subsets.
- A: test or leaf nodes that can be added to the tree.
- R: interpretability term $-\alpha$ and misclassifications.
- T: node traversals.

Proposition (Objective Equivalence)

Let π be a deterministic policy of the MDP. Then $J_\alpha(\pi) = -\mathcal{L}_\alpha(E(\pi, s_0))$ where E is an algorithm that extracts a decision tree from π (Topin et al. 2021).

Decision tree induction as solving MDPs

Intuition

The induction of a decision tree is made of a sequence of decisions: at each node, we must decide whether it is better to split (a subset of) the training data, or to create a leaf node.

- S: data subsets.
- A: test or leaf nodes that can be added to the tree.
- R: interpretability term $-\alpha$ and misclassifications.
- T: node traversals.

Proposition (Objective Equivalence)

Let π be a deterministic policy of the MDP. Then $J_\alpha(\pi) = -\mathcal{L}_\alpha(E(\pi, s_0))$ where E is an algorithm that extracts a decision tree from π (Topin et al. 2021).

Decision tree induction as solving MDPs

Intuition

The induction of a decision tree is made of a sequence of decisions: at each node, we must decide whether it is better to split (a subset of) the training data, or to create a leaf node.

- S: data subsets.
- A: test or leaf nodes that can be added to the tree.
- R: interpretability term $-\alpha$ and misclassifications.
- T: node traversals.

Proposition (Objective Equivalence)

Let π be a deterministic policy of the MDP. Then $J_\alpha(\pi) = -\mathcal{L}_\alpha(E(\pi, s_0))$ where E is an algorithm that extracts a decision tree from π (Topin et al. 2021).

Decision tree induction as solving MDPs

Intuition

The induction of a decision tree is made of a sequence of decisions: at each node, we must decide whether it is better to split (a subset of) the training data, or to create a leaf node.

- S: data subsets.
- A: test or leaf nodes that can be added to the tree.
- R: interpretability term $-\alpha$ and misclassifications.
- T: node traversals.

Proposition (Objective Equivalence)

Let π be a deterministic policy of the MDP. Then $J_\alpha(\pi) = -\mathcal{L}_\alpha(E(\pi, s_0))$ where E is an algorithm that extracts a decision tree from π (*Topin et al. 2021*).

Controlling the time complexity of decision tree induction

- Greedy algorithms consider only one candidate action in each state which is the test that minimizes some impurity criterion
→ MDP state space size is $O(2^D)$.
- Optimal algorithms consider all possible actions in each state
→ MDP state space size is $O((2Np)^D)$.
- Dynamic Programming Decision Trees (DPDT):
→ for each MDP state consider B actions: state space size is $O((2B)^D)$.
→ solve the MDP exactly with DP.

How to choose the B candidate actions/splits?

Top-B greedy splits (Blanc et al. 2023), quantiles, random...

- Greedy algorithms consider only one candidate action in each state which is the test that minimizes some impurity criterion
→ MDP state space size is $O(2^D)$.
- Optimal algorithms consider all possible actions in each state
→ MDP state space size is $O((2Np)^D)$.
- Dynamic Programming Decision Trees (DPDT):
→ for each MDP state consider B actions: state space size is $O((2B)^D)$.
→ solve the MDP exactly with DP.

How to choose the B candidate actions/splits?

Top-B greedy splits (Blanc et al. 2023), quantiles, random...

Controlling the time complexity of decision tree induction

- Greedy algorithms consider only one candidate action in each state which is the test that minimizes some impurity criterion
→ MDP state space size is $O(2^D)$.
- Optimal algorithms consider all possible actions in each state
→ MDP state space size is $O((2Np)^D)$.
- Dynamic Programming Decision Trees (DPDT):
→ for each MDP state consider B actions: state space size is $O((2B)^D)$.
→ solve the MDP exactly with DP.

How to choose the B candidate actions/splits?

Top-B greedy splits (Blanc et al. 2023), quantiles, random...

Controlling the time complexity of decision tree induction

- Greedy algorithms consider only one candidate action in each state which is the test that minimizes some impurity criterion
→ MDP state space size is $O(2^D)$.
- Optimal algorithms consider all possible actions in each state
→ MDP state space size is $O((2Np)^D)$.
- Dynamic Programming Decision Trees (DPDT):
→ for each MDP state consider B actions: state space size is $O((2B)^D)$.
→ solve the MDP exactly with DP.

How to choose the B candidate actions/splits?

Top-B greedy splits (Blanc et al. 2023), quantiles, random...

Controlling the time complexity of decision tree induction

- Greedy algorithms consider only one candidate action in each state which is the test that minimizes some impurity criterion
→ MDP state space size is $O(2^D)$.
- Optimal algorithms consider all possible actions in each state
→ MDP state space size is $O((2Np)^D)$.
- Dynamic Programming Decision Trees (DPDT):
→ for each MDP state consider B actions: state space size is $O((2B)^D)$.
→ solve the MDP exactly with DP.

How to choose the B candidate actions/splits?

Top-B greedy splits (Blanc et al. 2023), quantiles, random...

Controlling the time complexity of decision tree induction

- Greedy algorithms consider only one candidate action in each state which is the test that minimizes some impurity criterion
→ MDP state space size is $O(2^D)$.
- Optimal algorithms consider all possible actions in each state
→ MDP state space size is $O((2Np)^D)$.
- **Dynamic Programming Decision Trees (DPDT):**
 - for each MDP state consider B actions: state space size is $O((2B)^D)$.
 - solve the MDP exactly with DP.

How to choose the B candidate actions/splits?

Top-B greedy splits (Blanc et al. 2023), quantiles, random...

Controlling the time complexity of decision tree induction

- Greedy algorithms consider only one candidate action in each state which is the test that minimizes some impurity criterion
→ MDP state space size is $O(2^D)$.
- Optimal algorithms consider all possible actions in each state
→ MDP state space size is $O((2Np)^D)$.
- Dynamic Programming Decision Trees (DPDT):
→ for each MDP state consider B actions: state space size is $O((2B)^D)$.
→ solve the MDP exactly with DP.

How to choose the B candidate actions/splits?

Top-B greedy splits (Blanc et al. 2023), quantiles, random...

Controlling the time complexity of decision tree induction

- Greedy algorithms consider only one candidate action in each state which is the test that minimizes some impurity criterion
→ MDP state space size is $O(2^D)$.
- Optimal algorithms consider all possible actions in each state
→ MDP state space size is $O((2Np)^D)$.
- Dynamic Programming Decision Trees (DPDT):
→ for each MDP state consider B actions: state space size is $O((2B)^D)$.
→ solve the MDP exactly with DP.

How to choose the B candidate actions/splits?

Top-B greedy splits (Blanc et al. 2023), quantiles, random...

- Greedy algorithms consider only one candidate action in each state which is the test that minimizes some impurity criterion
→ MDP state space size is $O(2^D)$.
- Optimal algorithms consider all possible actions in each state
→ MDP state space size is $O((2Np)^D)$.
- Dynamic Programming Decision Trees (DPDT):
→ for each MDP state consider B actions: state space size is $O((2B)^D)$.
→ solve the MDP exactly with DP.

How to choose the B candidate actions/splits?

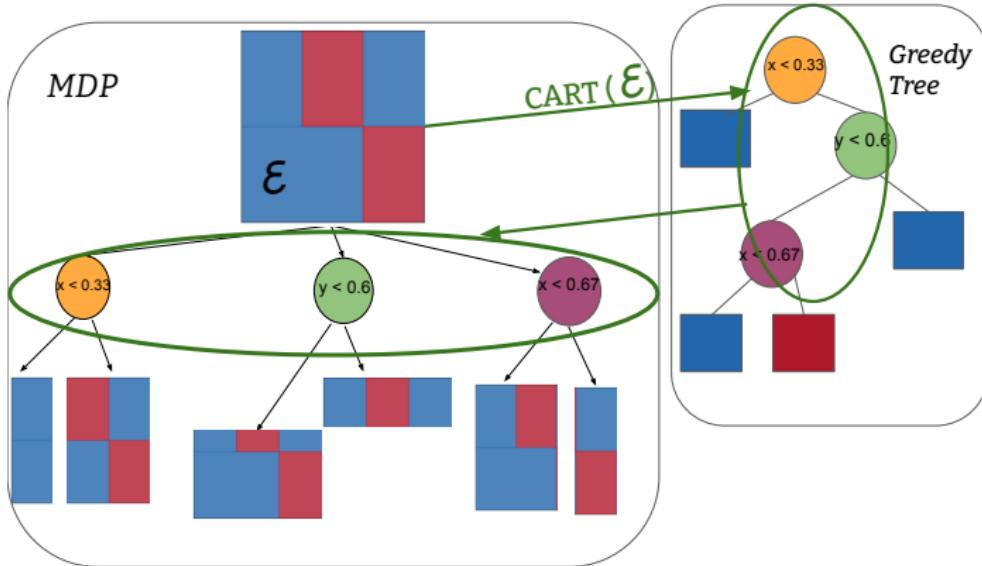
Top-B greedy splits (Blanc et al. 2023), quantiles, random...

- Greedy algorithms consider only one candidate action in each state which is the test that minimizes some impurity criterion
→ MDP state space size is $O(2^D)$.
- Optimal algorithms consider all possible actions in each state
→ MDP state space size is $O((2Np)^D)$.
- Dynamic Programming Decision Trees (DPDT):
→ for each MDP state consider B actions: state space size is $O((2B)^D)$.
→ solve the MDP exactly with DP.

How to choose the B candidate actions/splits?

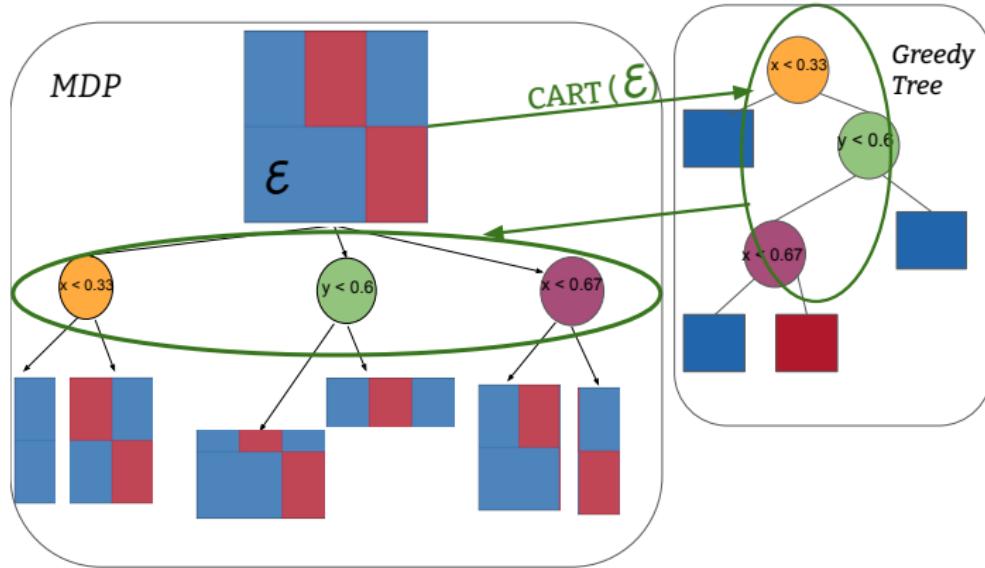
Top-B greedy splits (Blanc et al. 2023), quantiles, random...

Practical implementaion of DPDT



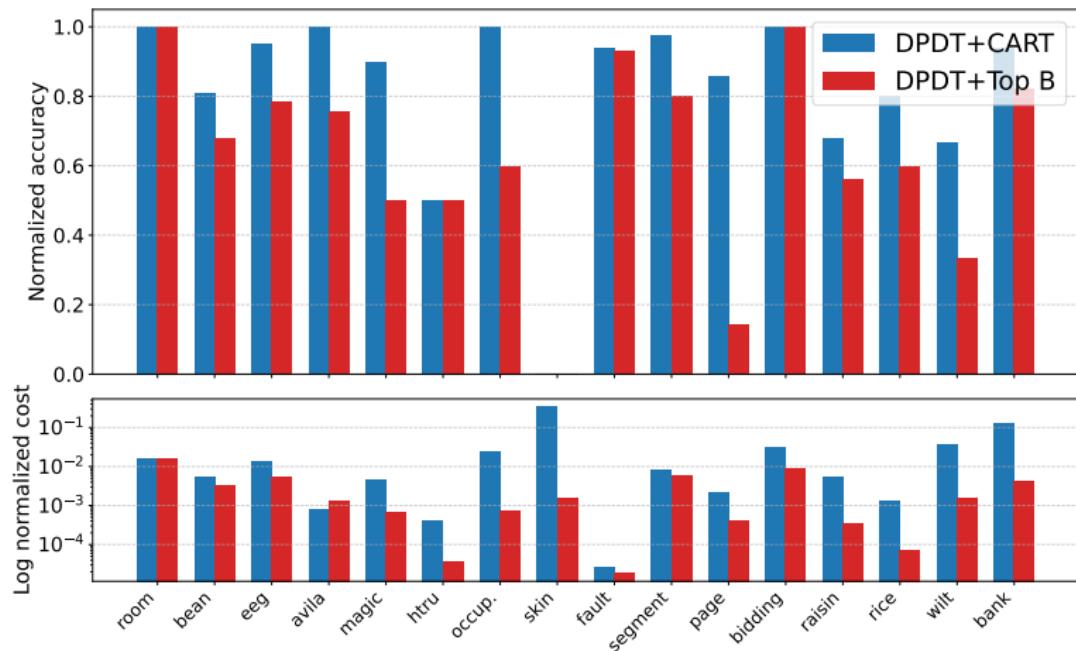
We can use greedy trees nodes as candidate actions.

Practical implementaion of DPDT



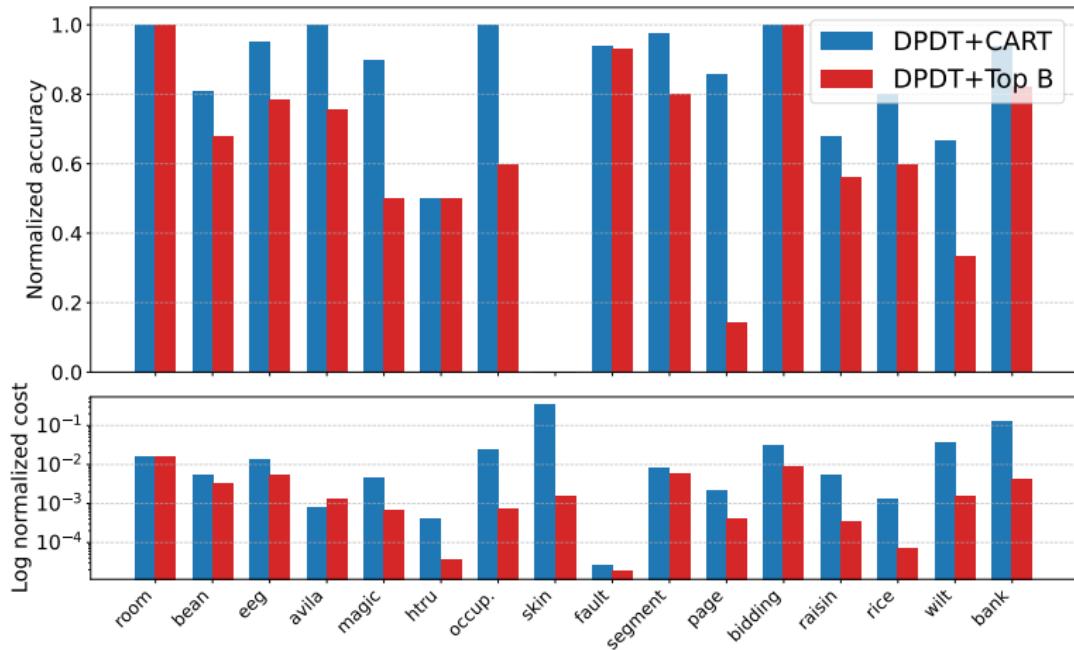
We can use greedy trees nodes as candidate actions.

Fast like greedy trees, accurate like optimal trees



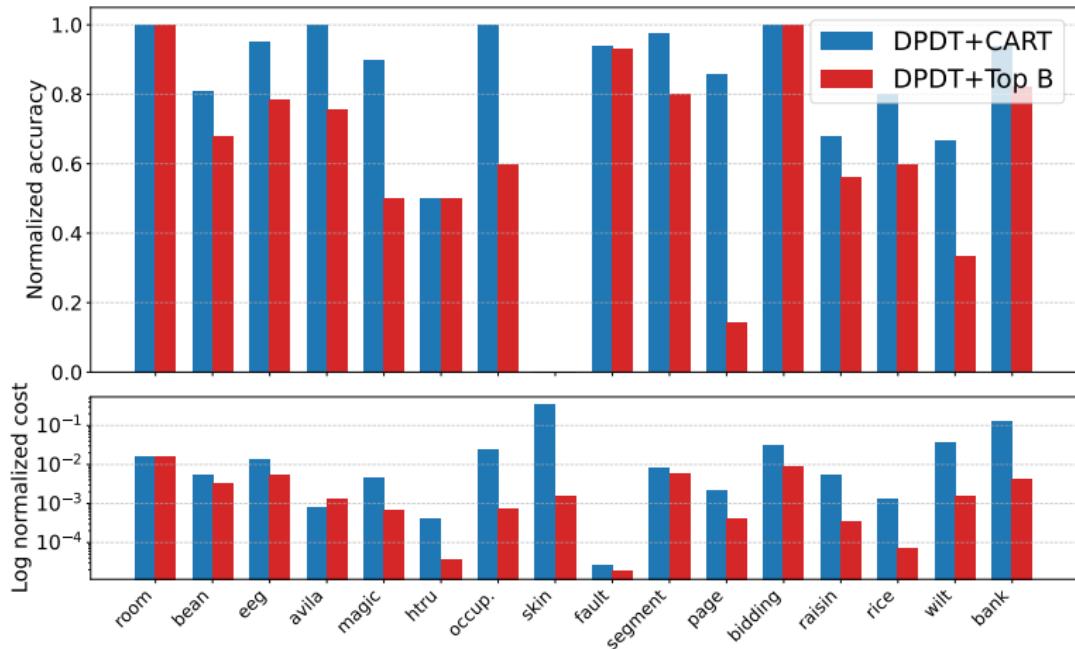
Train accuracies against cost for depth-3 trees.

Fast like greedy trees, accurate like optimal trees



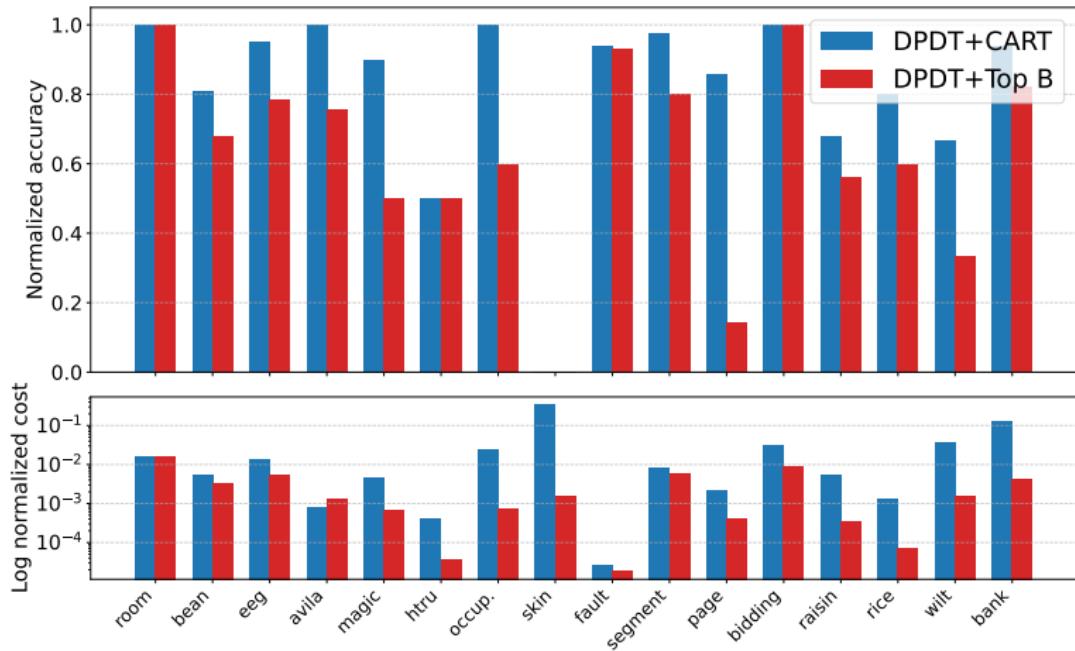
- DPDT trees can be as accurate as greedy trees.

Fast like greedy trees, accurate like optimal trees



- DPDT trees can be not worse than greedy trees.
- DPDT trees can be strictly better than greedy trees.

Fast like greedy trees, accurate like optimal trees



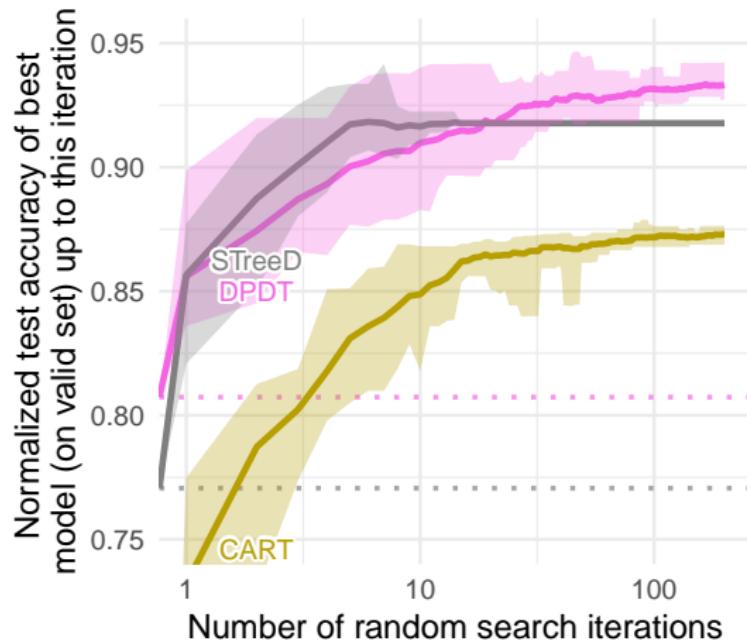
- DPDT trees can be not worse than greedy trees.
- DPDT trees can be strictly better than greedy trees.
- CART generates more diverse splits than Top B for DPDT.

Large scale evaluation of DPDT trees generalization

(Grinsztajn, Oyallon, and Varoquaux 2022)

Large scale evaluation of DPDT trees generalization

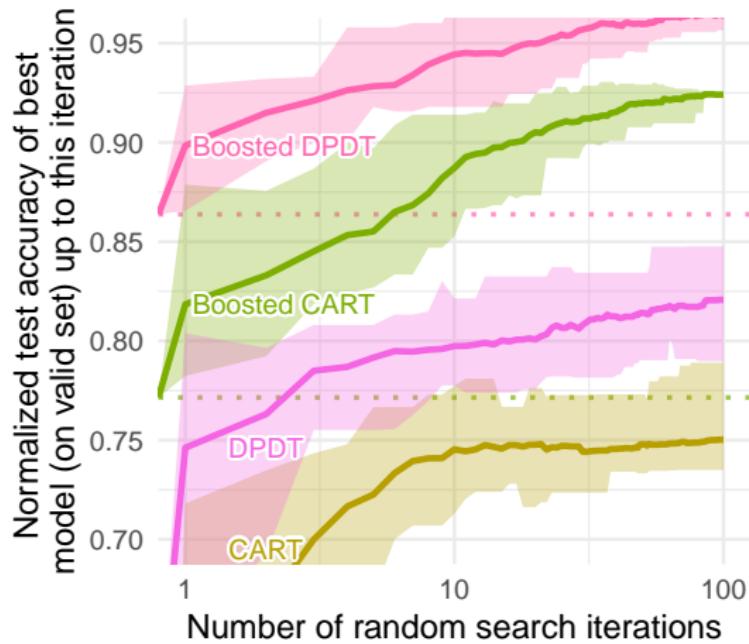
(Grinsztajn, Oyallon, and Varoquaux 2022)



DPDT depth-5 trees vs. other depth-5 trees

Large scale evaluation of DPDT trees generalization

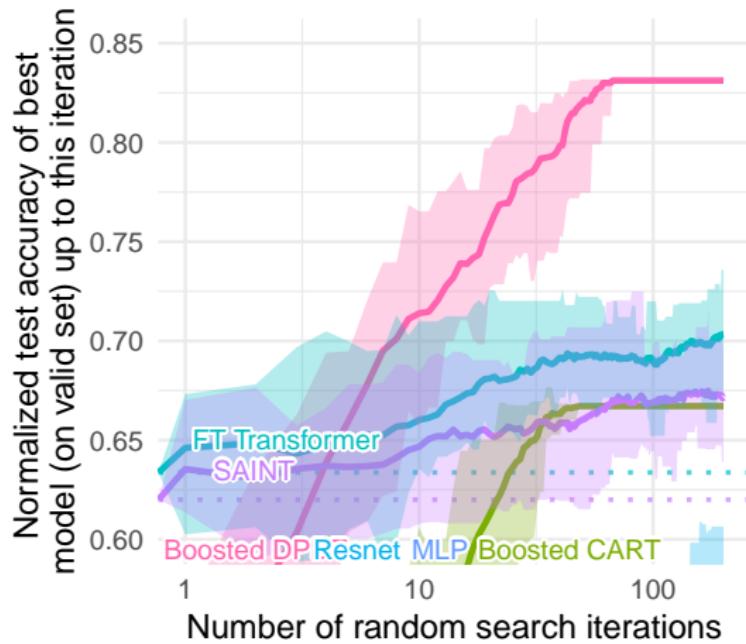
(Grinsztajn, Oyallon, and Varoquaux 2022)



Boosted DPDT vs. Boosted CART

Large scale evaluation of DPDT trees generalization

(Grinsztajn, Oyallon, and Varoquaux 2022)



Boosted DPDT vs. other classifiers

Perspectives

Perspectives

- New SOTA decision tree induction with dynamic programming in MDPs.

Perspectives

- New SOTA decision tree induction with dynamic programming in MDPs.
- What about using DPDT for indirect decision tree policy learning for SDM?

Perspectives

- New SOTA decision tree induction with dynamic programming in MDPs.
- What about using DPDT for indirect decision tree policy learning for SDM?
- What performances could we reach with an industry-grade implementation of XGboost+DPDT?

- New SOTA decision tree induction with dynamic programming in MDPs.
- What about using DPDT for indirect decision tree policy learning for SDM?
- What performances could we reach with an industry-grade implementation of XGboost+DPDT?

Let us take a step back

Q: Are decision trees really the most interpretable model?

How to measure policy interpretability?

The notion of *simulability* (Lipton 2018)

How to measure policy interpretability?

The notion of *simulability* (Lipton 2018)

- Interpretability \simeq time to reproduce the computations.

How to measure policy interpretability?

The notion of *simulability* (Lipton 2018)

- Interpretability \simeq time to reproduce the computations.
- Interpretability \simeq effort to read through the entire policy.

How to measure policy interpretability?

⚠ Without humans?

The notion of *simulability* (Lipton 2018)

- Interpretability \simeq time to reproduce the computations.
- Interpretability \simeq effort to read through the entire policy.

How to measure policy interpretability?

⚠ Without humans?

The notion of *simulability* (Lipton 2018)

- Interpretability \simeq **runtime in seconds?**
- Interpretability \simeq effort to read through the entire policy.

How to measure policy interpretability?

⚠ Without humans?

The notion of *simulability* (Lipton 2018)

- Interpretability \simeq **runtime in seconds?**
- Interpretability \simeq **size in bytes?**

How to measure policy interpretability?

⚠ Without humans?

The notion of *simulability* (Lipton 2018)

- Interpretability \simeq **runtime in seconds?**
- Interpretability \simeq **size in bytes?**

How to compare policy of different classes?

How to measure policy interpretability?

⚠ Without humans?

The notion of *simulability* (Lipton 2018)

- Interpretability \simeq **runtime in seconds?**
- Interpretability \simeq **size in bytes?**

How to compare policy of different classes?

- Different hardwares (CPUs vs GPUs).

How to measure policy interpretability?

⚠ Without humans?

The notion of *simulability* (Lipton 2018)

- Interpretability \simeq **runtime in seconds?**
- Interpretability \simeq **size in bytes?**

How to compare policy of different classes?

- Different hardwares (CPUs vs GPUs).
- Different implementations (matrix operations vs fully sequentially)
(Luo et al. 2024)

We propose policy unfolding

```
# Small ReLU MLP for Pendulum
def play(x):
    h_layer_0_0 = 1.68 * x[0] + -0.69 * x[1] + -0.74 * x[2] + -1.40
    h_layer_0_0 = max(0.0, h_layer_0_0)
    h_layer_0_1 = 0.20 * x[0] + 0.29 * x[1] + -0.021 * x[2] + 1.25
    h_layer_0_1 = max(0.0, h_layer_0_1)
    h_layer_0_2 = 0.33 * x[0] + -0.57 * x[1] + 0.47 * x[2] + 1.94
    h_layer_0_2 = max(0.0, h_layer_0_2)
    h_layer_0_3 = 1.39 * x[0] + 0.94 * x[1] + 0.50 * x[2] + -1.13
    h_layer_0_3 = max(0.0, h_layer_0_3)
    h_layer_1_0 = 1.16 * h_layer_0_0 + -1.59 * h_layer_0_1 + 0.95 * h_layer_0_2 +
        -1.22 * h_layer_0_3 + -0.54
    h_layer_1_0 = max(0.0, h_layer_1_0)
    h_layer_1_1 = -0.55 * h_layer_0_0 + 1.13 * h_layer_0_1 + -0.58 * h_layer_0_2 +
        + -0.72 * h_layer_0_3 + 1.56
    h_layer_1_1 = max(0.0, h_layer_1_1)
    h_layer_1_2 = 1.10 * h_layer_0_0 + -1.01 * h_layer_0_1 + 0.96 * h_layer_0_2 +
        -2.84 * h_layer_0_3 + -0.02
    h_layer_1_2 = max(0.0, h_layer_1_2)
    h_layer_1_3 = 0.27 * h_layer_0_0 + 0.44 * h_layer_0_1 + 0.39 * h_layer_0_2 +
        0.15 * h_layer_0_3 + -1.24
    h_layer_1_3 = max(0.0, h_layer_1_3)
    h_layer_2_0 = -2.80 * h_layer_1_0 + -0.60 * h_layer_1_1 + 3.07 * h_layer_1_2 +
        + -1.63 * h_layer_1_3 + -0.36
    y_0 = h_layer_2_0

    return [y_0]
```

Is time/size of unfolded policies a good proxy?

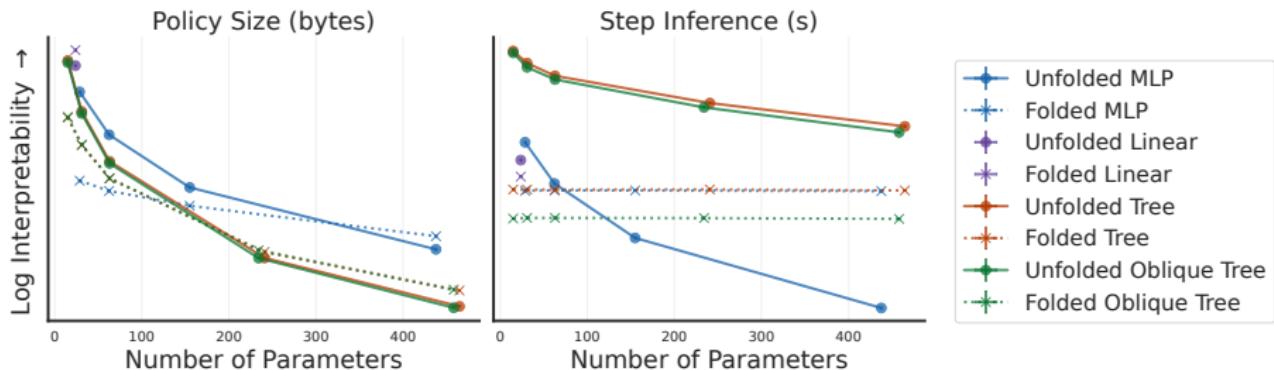
Setup

We imitate ~40000 expert policies from stable-baselines3 using various policy classes/nb parameters on various environments.

Is time/size of unfolded policies a good proxy?

Setup

We imitate ~40000 expert policies from stable-baselines3 using various policy classes/nb parameters on various environments.

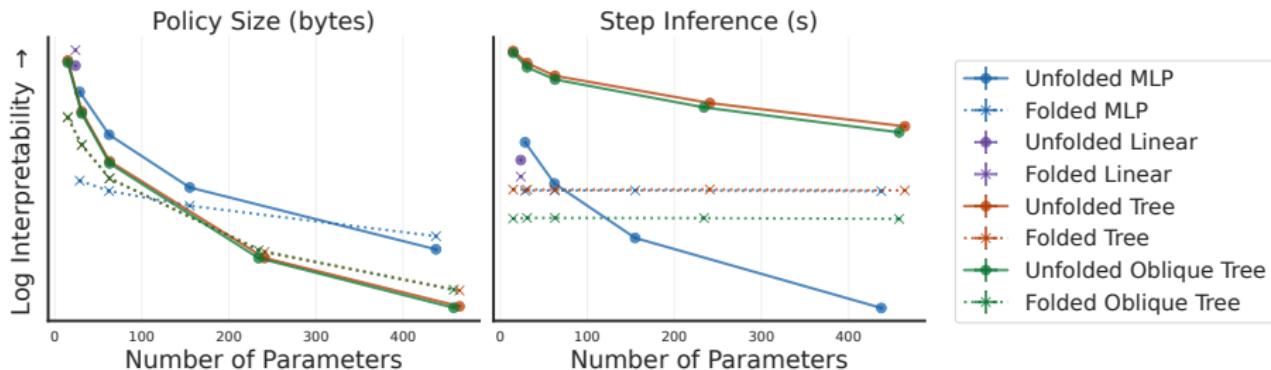


Aggregated policies interpretability on classic control environments

Is time/size of unfolded policies a good proxy?

Setup

We imitate ~40000 expert policies from stable-baselines3 using various policy classes/nb parameters on various environments.



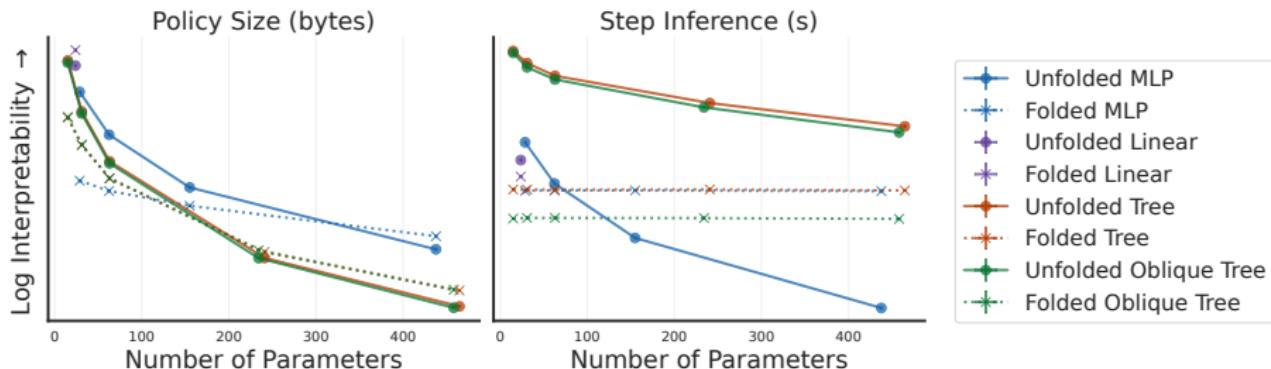
Aggregated policies interpretability on classic control environments

- Less parameters means more interpretability (Freitas 2014).

Is time/size of unfolded policies a good proxy?

Setup

We imitate ~40000 expert policies from stable-baselines3 using various policy classes/nb parameters on various environments.



Aggregated policies interpretability on classic control environments

- Less parameters means more interpretability ([Freitas 2014](#)).
- Not always true that "trees are more interpretable than neural networks".

General perspectives

- Technical challenges: learning interpretable policies for SDM involves partial observability.
 - Focus on indirect approaches and/or on POMDP research first?
 - Created opportunities for new decision tree algos for classif/regression (DPDT).
- Fundamental challenges: no consensus on interpretability definition.
 - Keep exploring benchmarks for policy interpretability.
 - Discuss with the community (InterpPol workshop).
- Deep learning: Can we design deep learning layers that take datasets and output candidate splits?
- Combinatorial optimization: Can we formulate other combinatorial/NP-hard problems as MDPs and design other DPDT-like algorithms?
- Human-computer interaction: Can we do large scale human study of the ~40K programs interpretability?

General perspectives

- **Technical challenges:** learning interpretable policies for SDM involves partial observability.
 - Focus on indirect approaches and/or on POMDP research first?
 - Created opportunities for new decision tree algos for classif/regression (DPDT).
- Fundamental challenges: no concensus on interpretability definition.
 - Keep exploring benchmarks for policy interpretability.
 - Discuss with the community (InterpPol workshop).
- Deep learning: Can we design deep learning layers that take datasets and output candidate splits?
- Combinatorial optimization: Can we formulate other combinatorial/NP-hard problems as MDPs and design other DPDT-like algorithms?
- Human-computer interaction: Can we do large scale human study of the ~40K programs interpretability?

General perspectives

- Technical challenges: learning interpretable policies for SDM involves partial observability.
 - Focus on indirect approaches and/or on POMDP research first?
 - Created opportunities for new decision tree algos for classif/regression (DPDT).
- Fundamental challenges: no consensus on interpretability definition.
 - Keep exploring benchmarks for policy interpretability.
 - Discuss with the community (InterpPol workshop).
- Deep learning: Can we design deep learning layers that take datasets and output candidate splits?
- Combinatorial optimization: Can we formulate other combinatorial/NP-hard problems as MDPs and design other DPDT-like algorithms?
- Human-computer interaction: Can we do large scale human study of the ~40K programs interpretability?

General perspectives

- Technical challenges: learning interpretable policies for SDM involves partial observability.
 - Focus on indirect approaches and/or on POMDP research first?
 - Created opportunities for new decision tree algos for classif/regression (DPDT).
- Fundamental challenges: no consensus on interpretability definition.
 - Keep exploring benchmarks for policy interpretability.
 - Discuss with the community (InterpPol workshop).
- Deep learning: Can we design deep learning layers that take datasets and output candidate splits?
- Combinatorial optimization: Can we formulate other combinatorial/NP-hard problems as MDPs and design other DPDT-like algorithms?
- Human-computer interaction: Can we do large scale human study of the ~40K programs interpretability?

General perspectives

- **Technical challenges:** learning interpretable policies for SDM involves partial observability.
 - Focus on indirect approaches and/or on POMDP research first?
 - Created opportunities for new decision tree algos for classif/regression (DPDT).
- **Fundamental challenges:** no concensus on interpretability definition.
 - Keep exploring benchmarks for policy interpretability.
 - Discuss with the community (InterpPol workshop).
- **Deep learning:** Can we design deep learning layers that take datasets and output candidate splits?
- **Combinatorial optimization:** Can we formulate other combinatorial/NP-hard problems as MDPs and design other DPDT-like algorithms?
- **Human-computer interaction:** Can we do large scale human study of the ~40K programs interpretability?

General perspectives

- Technical challenges: learning interpretable policies for SDM involves partial observability.
 - Focus on indirect approaches and/or on POMDP research first?
 - Created opportunities for new decision tree algos for classif/regression (DPDT).
- Fundamental challenges: no consensus on interpretability definition.
 - Keep exploring benchmarks for policy interpretability.
 - Discuss with the community (InterpPol workshop).
- Deep learning: Can we design deep learning layers that take datasets and output candidate splits?
- Combinatorial optimization: Can we formulate other combinatorial/NP-hard problems as MDPs and design other DPDT-like algorithms?
- Human-computer interaction: Can we do large scale human study of the ~40K programs interpretability?

General perspectives

- Technical challenges: learning interpretable policies for SDM involves partial observability.
 - Focus on indirect approaches and/or on POMDP research first?
 - Created opportunities for new decision tree algos for classif/regression (DPDT).
- Fundamental challenges: no consensus on interpretability definition.
 - Keep exploring benchmarks for policy interpretability.
 - Discuss with the community (InterpPol workshop).
- Deep learning: Can we design deep learning layers that take datasets and output candidate splits?
- Combinatorial optimization: Can we formulate other combinatorial/NP-hard problems as MDPs and design other DPDT-like algorithms?
- Human-computer interaction: Can we do large scale human study of the ~40K programs interpretability?

General perspectives

- Technical challenges: learning interpretable policies for SDM involves partial observability.
 - Focus on indirect approaches and/or on POMDP research first?
 - Created opportunities for new decision tree algos for classif/regression (DPDT).
- Fundamental challenges: no consensus on interpretability definition.
 - Keep exploring benchmarks for policy interpretability.
 - Discuss with the community (InterpPol workshop).
- Deep learning: Can we design deep learning layers that take datasets and output candidate splits?
- Combinatorial optimization: Can we formulate other combinatorial/NP-hard problems as MDPs and design other DPDT-like algorithms?
- Human-computer interaction: Can we do large scale human study of the ~40K programs interpretability?

General perspectives

- Technical challenges: learning interpretable policies for SDM involves partial observability.
 - Focus on indirect approaches and/or on POMDP research first?
 - Created opportunities for new decision tree algos for classif/regression (DPDT).
- Fundamental challenges: no consensus on interpretability definition.
 - Keep exploring benchmarks for policy interpretability.
 - Discuss with the community (InterpPol workshop).
- Deep learning: Can we design deep learning layers that take datasets and output candidate splits?
- Combinatorial optimization: Can we formulate other combinatorial/NP-hard problems as MDPs and design other DPDT-like algorithms?
- Human-computer interaction: Can we do large scale human study of the ~40K programs interpretability?

General perspectives

- Technical challenges: learning interpretable policies for SDM involves partial observability.
 - Focus on indirect approaches and/or on POMDP research first?
 - Created opportunities for new decision tree algos for classif/regression (DPDT).
- Fundamental challenges: no consensus on interpretability definition.
 - Keep exploring benchmarks for policy interpretability.
 - Discuss with the community (InterpPol workshop).
- Deep learning: Can we design deep learning layers that take datasets and output candidate splits?
- Combinatorial optimization: Can we formulate other combinatorial/NP-hard problems as MDPs and design other DPDT-like algorithms?
- Human-computer interaction: Can we do large scale human study of the ~40K programs interpretability?

-  Baisero, Andrea and Christopher Amato (2022). "Unbiased Asymmetric Reinforcement Learning under Partial Observability". In: *Proceedings of the 21st International Conference on Autonomous Agents and Multiagent Systems*. AAMAS '22. Virtual Event, New Zealand: International Foundation for Autonomous Agents and Multiagent Systems, pp. 44–52. ISBN: 9781450392136.
-  Baisero, Andrea, Brett Daley, and Christopher Amato (Jan. 2022). "Asymmetric DQN for partially observable reinforcement learning". In: *Proceedings of the Thirty-Eighth Conference on Uncertainty in Artificial Intelligence*. Ed. by James Cussens and Kun Zhang. Vol. 180. Proceedings of Machine Learning Research. PMLR, pp. 107–117. URL: <https://proceedings.mlr.press/v180/baisero22a.html>.
-  Bastani, Osbert, Yewen Pu, and Armando Solar-Lezama (2018). "Verifiable Reinforcement Learning via Policy Extraction". In.
-  Bertsimas, Dimitris and Jack Dunn (2017). "Optimal classification trees". In: *Machine Learning* 106, pp. 1039–1082.
-  Blanc, Guy et al. (2023). "Harnessing the power of choices in decision tree learning". In: *Advances in Neural Information Processing Systems* 36, pp. 80220–80232.

-  Breiman, L et al. (1984). *Classification and Regression Trees*. Wadsworth.
-  Doshi-Velez, Finale and Been Kim (2017). "Towards A Rigorous Science of Interpretable Machine Learning". In: arXiv: 1702.08608 [stat.ML]. URL: <https://arxiv.org/abs/1702.08608>.
-  Freitas, Alex A. (Mar. 2014). "Comprehensible classification models: a position paper". In: *SIGKDD Explor. Newslett.* 15.1, pp. 1–10. ISSN: 1931-0145. DOI: 10.1145/2594473.2594475. URL: <https://doi.org/10.1145/2594473.2594475>.
-  Glanois, Claire et al. (2024). "A survey on interpretable reinforcement learning". In: *Machine Learning*, pp. 1–44.
-  Greydanus, Sam et al. (2018). *Visualizing and Understanding Atari Agents*.
-  Grinsztajn, Léo, Edouard Oyallon, and Gaël Varoquaux (2022). "Why do tree-based models still outperform deep learning on typical tabular data?" In: *Advances in neural information processing systems* 35, pp. 507–520.

 Lipton, Zachary C. (2018). "The Mythos of Model Interpretability: In machine learning, the concept of interpretability is both important and slippery.". In: *Queue* 16.3, pp. 31–57.

 Littman, Michael L. (1994). "Memoryless policies: theoretical limitations and practical results". In: *Proceedings of the Third International Conference on Simulation of Adaptive Behavior: From Animals to Animats 3: From Animals to Animats 3*. SAB94. Brighton, United Kingdom: MIT Press, pp. 238–245. ISBN: 0262531224.

 Luo, Lirui et al. (2024). "End-to-End Neuro-Symbolic Reinforcement Learning with Textual Explanations". In: *International Conference on Machine Learning (ICML)*.

 Marton, Sascha et al. (2025). "Mitigating Information Loss in Tree-Based Reinforcement Learning via Direct Optimization". In: URL: <https://openreview.net/forum?id=qpXctF2aLZ>.

 Milani, Stephanie et al. (Apr. 2024). "Explainable Reinforcement Learning: A Survey and Comparative Review". In: *ACM Comput. Surv.* 56.7. ISSN: 0360-0300. DOI: 10.1145/3616864. URL: <https://doi.org/10.1145/3616864>.

-  Pinto, Lerrel et al. (2017). *Asymmetric Actor Critic for Image-Based Robot Learning*. arXiv: 1710.06542 [cs.R0]. URL: <https://arxiv.org/abs/1710.06542>.
-  Puterman, Martin L. (1994). *Markov Decision Processes: Discrete Stochastic Dynamic Programming*. John Wiley & Sons.
-  Schulman, John et al. (2017). "Proximal policy optimization algorithms". In: *arXiv preprint arXiv:1707.06347*.
-  Sigaud, Olivier and Olivier Buffet (2013). "Partially Observable Markov Decision Processes". In: *Markov Decision Processes in Artificial Intelligence*. John Wiley Sons, Ltd. Chap. 7, pp. 185–228. ISBN: 9781118557426. DOI:
<https://doi.org/10.1002/9781118557426.ch7>. eprint: <https://onlinelibrary.wiley.com/doi/pdf/10.1002/9781118557426.ch7>. URL: <https://onlinelibrary.wiley.com/doi/abs/10.1002/9781118557426.ch7>.
-  Singh, Satinder P., Tommi S. Jaakkola, and Michael I. Jordan (1994). "Learning without state-estimation in partially observable Markovian decision processes". In: *Proceedings of the Eleventh International Conference on International Conference on Machine Learning*. ICML'94.

New Brunswick, NJ, USA: Morgan Kaufmann Publishers Inc., pp. 284–292. ISBN: 1558603352.

-  Sutton, Richard S. and Andrew G. Barto (1998). *Reinforcement Learning: An Introduction*. Cambridge, MA: The MIT Press.
-  Topin, Nicholay et al. (2021). “Iterative bounding mdps: Learning interpretable policies via non-interpretable methods”. In: *Proceedings of the AAAI Conference on Artificial Intelligence* 35, pp. 9923–9931.
-  Wu, Mike et al. (Apr. 2020). “Regional Tree Regularization for Interpretability in Deep Neural Networks”. In: 34, pp. 6413–6421. DOI: 10.1609/aaai.v34i04.6112. URL: <https://ojs.aaai.org/index.php/AAAI/article/view/6112>.

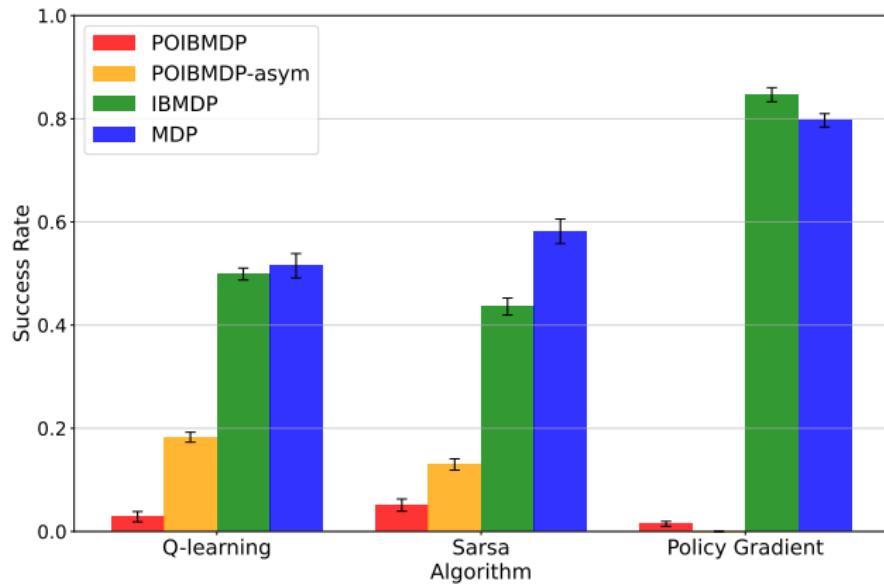
Result: for similar problems, RL struggles more when there is partial observability



Success rates over thousands of RL runs with varying hyperparameters when learning different policies in the same IBMDP¹.

¹We also observed similar results on classic controls and variants of the grid world MDP.

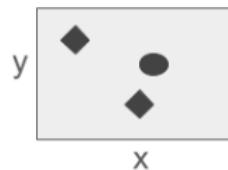
Result: for similar problems, RL struggles more when there is partial observability



Success rates over thousands of RL runs with varying hyperparameters when learning different policies in the same IBMMDP¹.

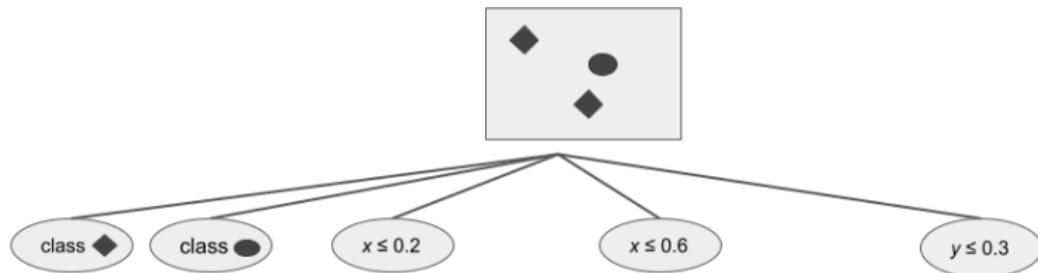
¹We also observed similar results on classic controls and variants of the grid world MDP.

Decision tree induction as solving MDPs



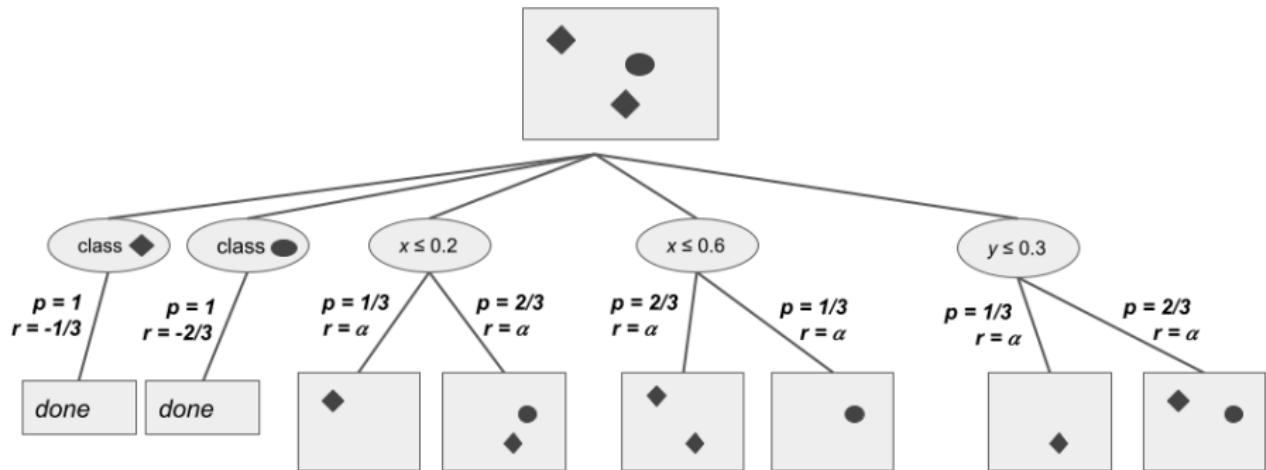
Example of decision tree induction as an MDP.

Decision tree induction as solving MDPs



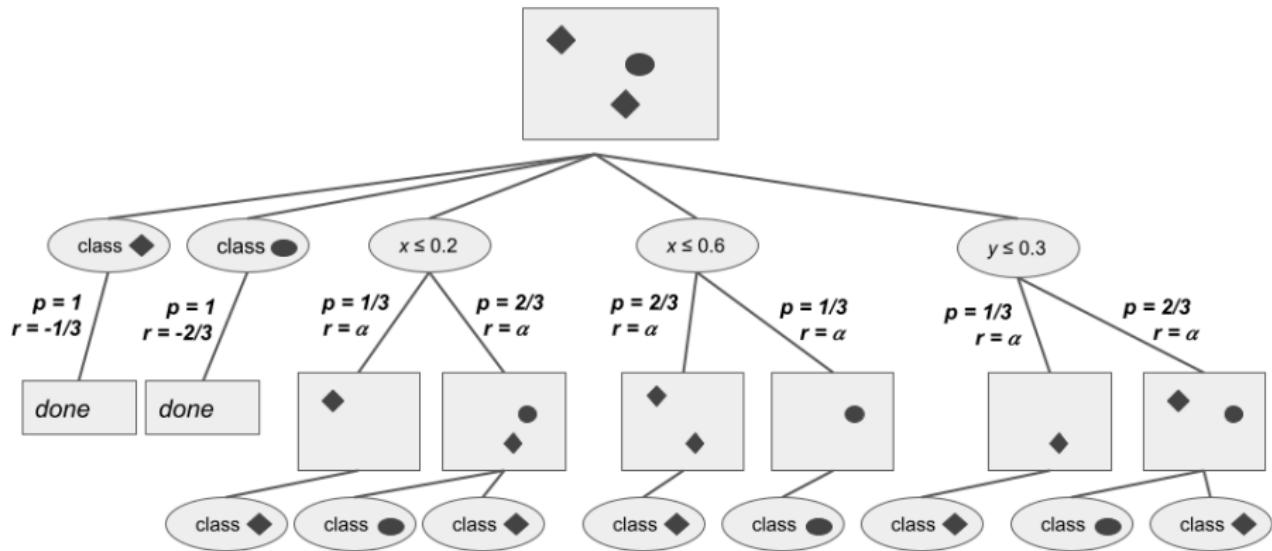
Example of decision tree induction as an MDP.

Decision tree induction as solving MDPs



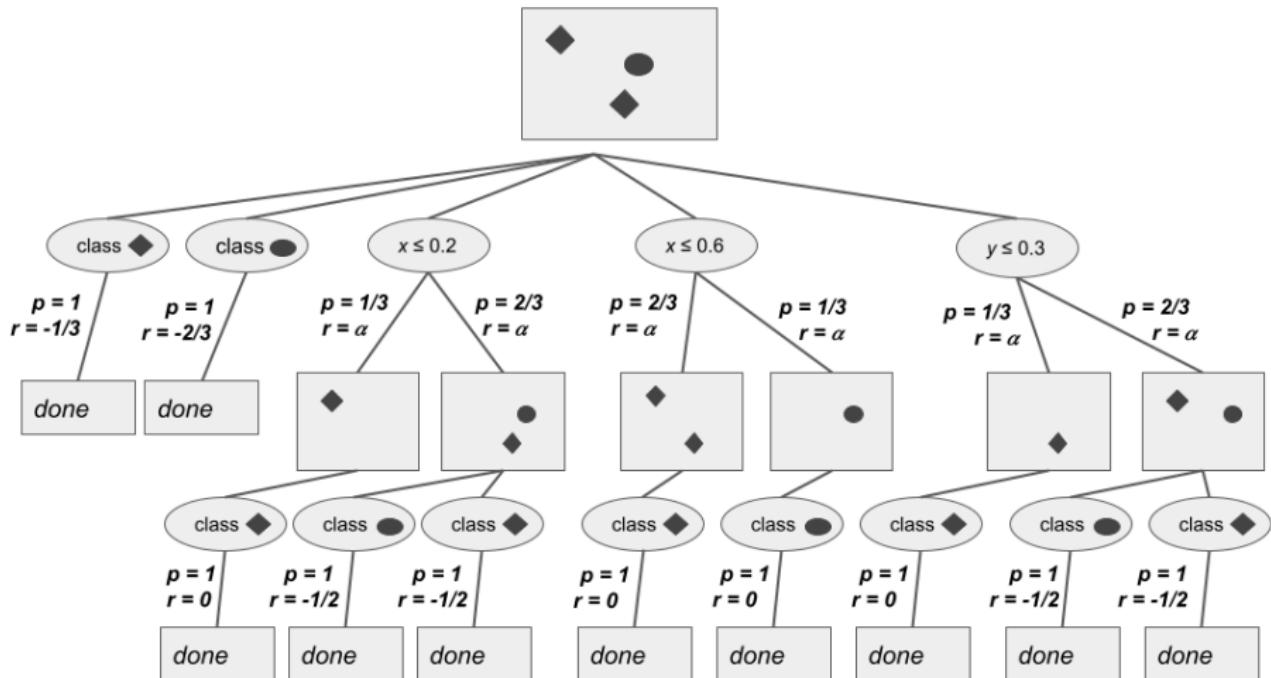
Example of decision tree induction as an MDP.

Decision tree induction as solving MDPs



Example of decision tree induction as an MDP.

Decision tree induction as solving MDPs



Example of decision tree induction as an MDP.

Fast like greedy trees, accurate like optimal trees



Comparison of greedy, optimal, and DPDT depth-2 trees on the checkersboard dataset.

Fast like greedy trees, accurate like optimal trees

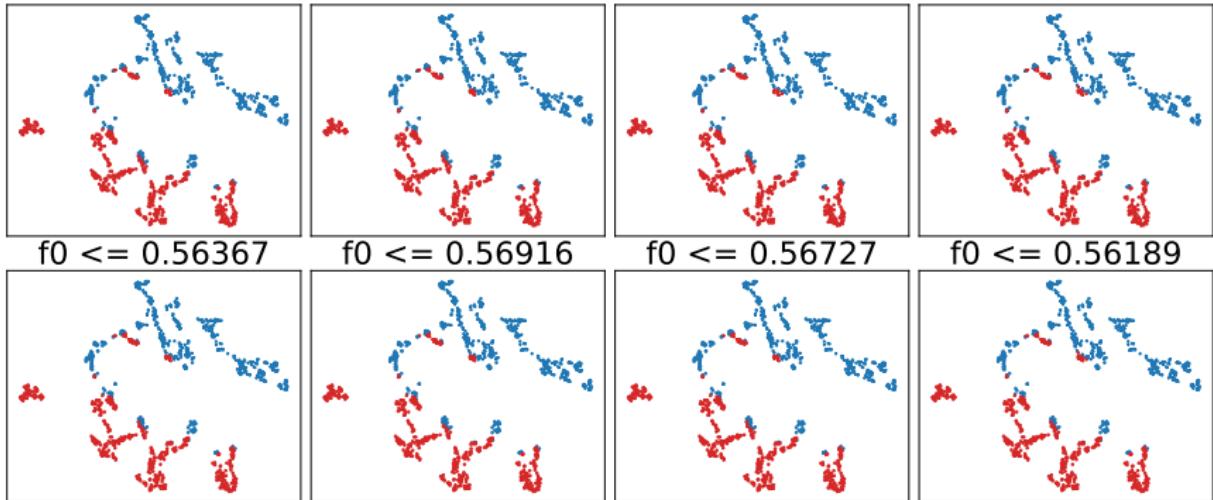
Comparison of accuracies and operations for depth-3 trees.

Dataset	Accuracy						Operations					
	Opt	Greedy	DPDT				Opt	Greedy	DPDT			
			CART ⁻	CART ⁺	TopB ⁻	TopB ⁺			CART ⁻	CART ⁺	TopB ⁻	TopB ⁺
room	0.992	0.968	0.991	0.992	0.990	0.992	10^6	15	286	16100	111	16100
bean	0.871	0.777	0.812	0.853	0.804	0.841	$5 \cdot 10^6$	15	295	25900	112	16800
eeg	0.708	0.666	0.689	0.706	0.684	0.699	$2 \cdot 10^6$	13	289	26000	95	11000
avila	0.585	0.532	0.574	0.585	0.563	0.572	$3 \cdot 10^7$	9	268	24700	60	38900
magic	0.831	0.801	0.822	0.828	0.807	0.816	$6 \cdot 10^6$	15	298	28000	70	4190
htru	0.981	0.979	0.979	0.980	0.979	0.980	$6 \cdot 10^7$	15	295	25300	55	2180
occup.	0.994	0.989	0.991	0.994	0.990	0.992	$7 \cdot 10^5$	13	280	16300	33	510
skin	0.969	0.966	0.966	0.966	0.966	0.966	$7 \cdot 10^4$	15	301	23300	20	126
fault	0.682	0.553	0.672	0.674	0.672	0.673	$9 \cdot 10^8$	13	295	24200	111	16800
segment	0.887	0.574	0.812	0.879	0.786	0.825	$2 \cdot 10^6$	7	220	16300	68	11400
page	0.971	0.964	0.970	0.970	0.964	0.965	10^7	15	298	22400	701	4050
bidding	0.993	0.981	0.985	0.993	0.985	0.993	$3 \cdot 10^5$	13	256	9360	58	2700
raisin	0.894	0.869	0.879	0.886	0.875	0.883	$4 \cdot 10^6$	15	295	20900	48	1440
rice	0.938	0.933	0.934	0.937	0.933	0.936	$2 \cdot 10^7$	15	298	25500	49	1470
wilt	0.996	0.993	0.994	0.995	0.994	0.994	$3 \cdot 10^5$	13	274	11300	33	465
bank	0.983	0.933	0.971	0.980	0.951	0.974	$6 \cdot 10^4$	13	271	7990	26	256

CART generates more diverse splits than Top B

DPDT-Top B Naive-Heuristic Root node candidates for bank

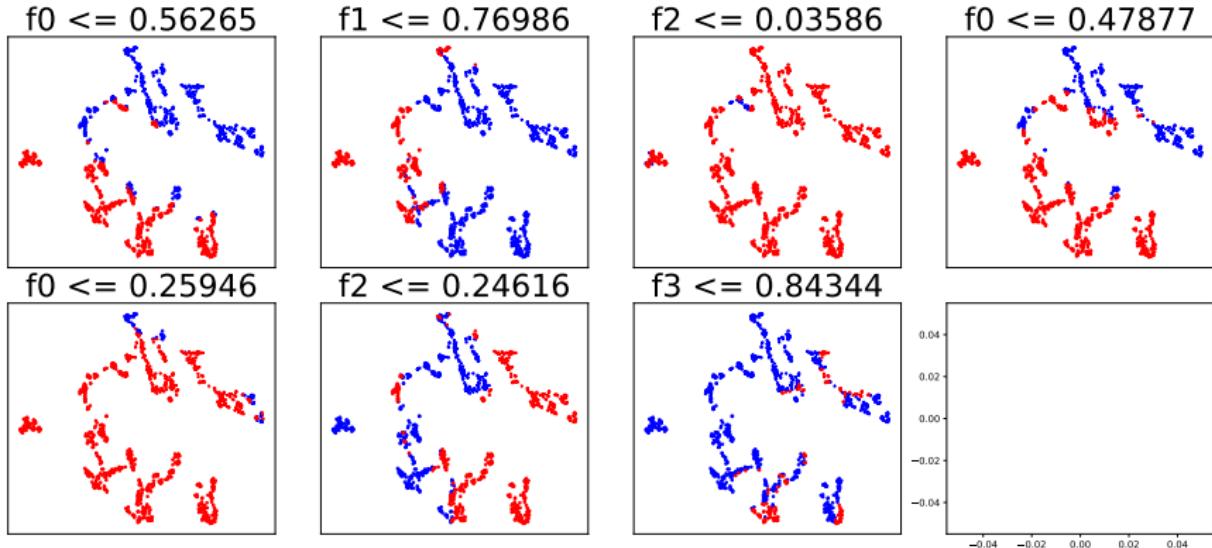
$f_0 \leq 0.56265$ $f_0 \leq 0.56309$ $f_0 \leq 0.56227$ $f_0 \leq 0.56168$



- Left child
- Right child
- Class 0
- * Class 1

CART generates more diverse splits than Top B

DPDT-CART-Heuristic Root node candidates for bank



- Left child
- Right child
- Class 0
- * Class 1

Why generating candidate splits with CART?

Theorem (DPDT trees are not worse than greedy trees)

The greedy tree is always a solution of the MDPs we solve. Because we solve the MDPs exactly with DP, if the greedy tree is the best solution, DPDT will find it.

Theorem (DPDT trees can be strictly better than greedy trees)

There exist a depth budget D and a dataset for which DPDT trees are strictly better than greedy trees.^a

^acf. checkersboard dataset.

Why generating candidate splits with CART?

Theorem (DPDT trees are not worse than greedy trees)

The greedy tree is always a solution of the MDPs we solve. Because we solve the MDPs exactly with DP, if the greedy tree is the best solution, DPDT will find it.

Theorem (DPDT trees can be strictly better than greedy trees)

There exist a depth budget D and a dataset for which DPDT trees are strictly better than greedy trees.^a

^acf. checkersboard dataset.

Why generating candidate splits with CART?

Theorem (DPDT trees are not worse than greedy trees)

The greedy tree is always a solution of the MDPs we solve. Because we solve the MDPs exactly with DP, if the greedy tree is the best solution, DPDT will find it.

Theorem (DPDT trees can be strictly better than greedy trees)

There exist a depth budget D and a dataset for which DPDT trees are strictly better than greedy trees.^a

^acf. checkersboard dataset.

Why generating candidate splits with CART?

Theorem (DPDT trees are not worse than greedy trees)

The greedy tree is always a solution of the MDPs we solve. Because we solve the MDPs exactly with DP, if the greedy tree is the best solution, DPDT will find it.

Theorem (DPDT trees can be strictly better than greedy trees)

There exist a depth budget D and a dataset for which DPDT trees are strictly better than greedy trees.^a

^acf. checkersboard dataset.

Why generating candidate splits with CART?

Theorem (DPDT trees are not worse than greedy trees)

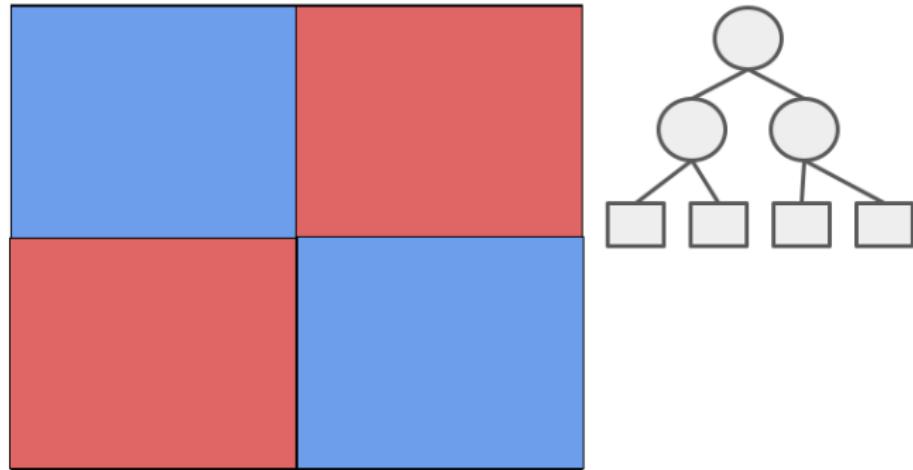
The greedy tree is always a solution of the MDPs we solve. Because we solve the MDPs exactly with DP, if the greedy tree is the best solution, DPDT will find it.

Theorem (DPDT trees can be strictly better than greedy trees)

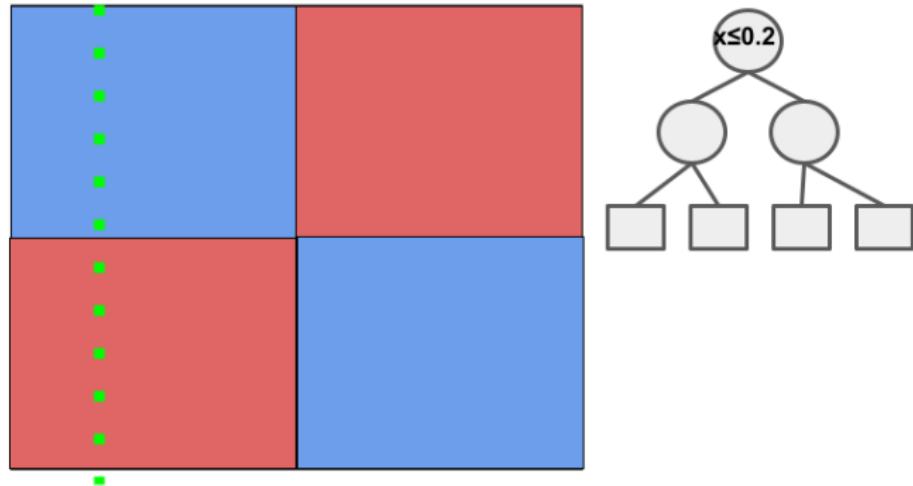
There exist a depth budget D and a dataset for which DPDT trees are strictly better than greedy trees.^a

^acf. checkersboard dataset.

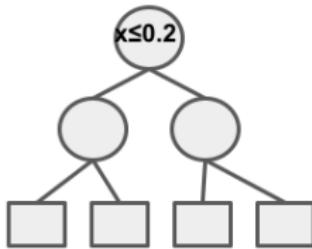
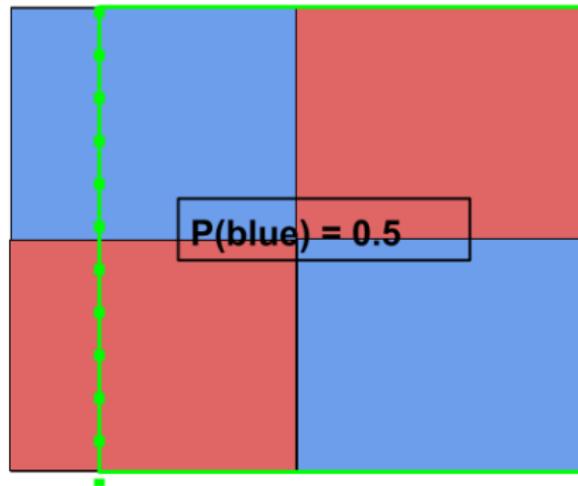
DPDT trees can be strictly better than greedy trees



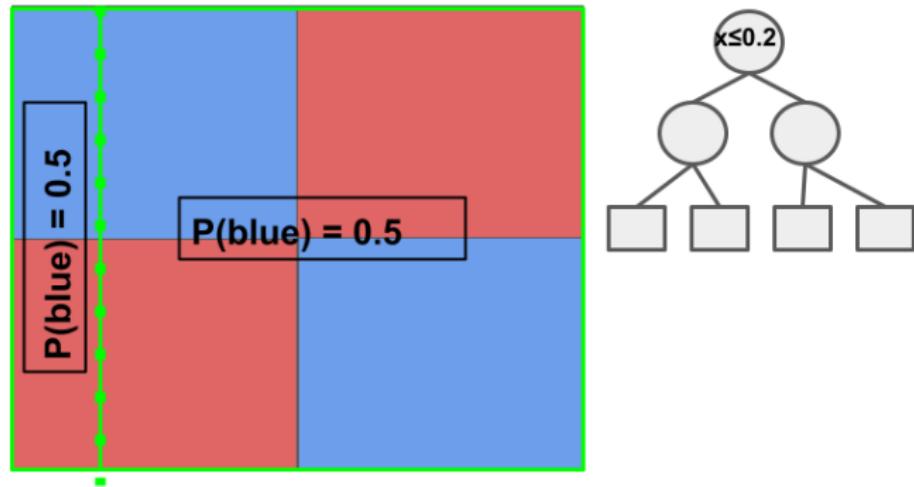
DPDT trees can be strictly better than greedy trees



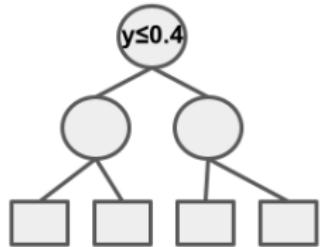
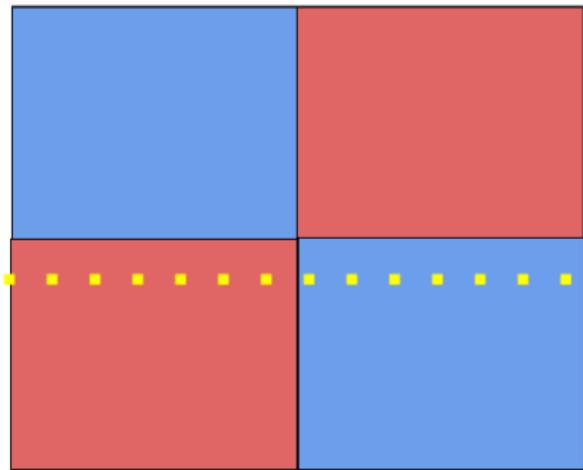
DPDT trees can be strictly better than greedy trees



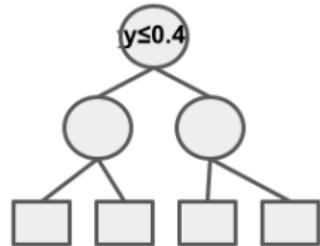
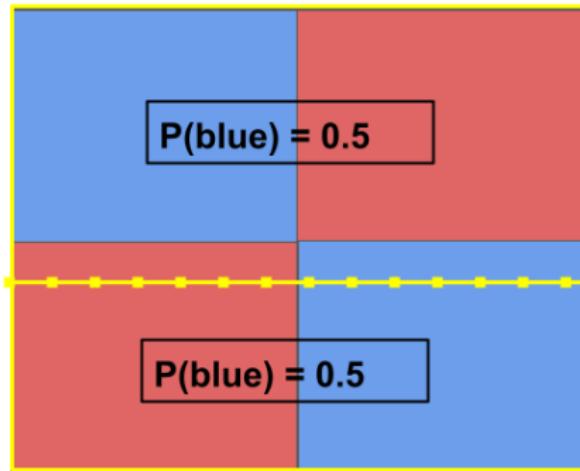
DPDT trees can be strictly better than greedy trees



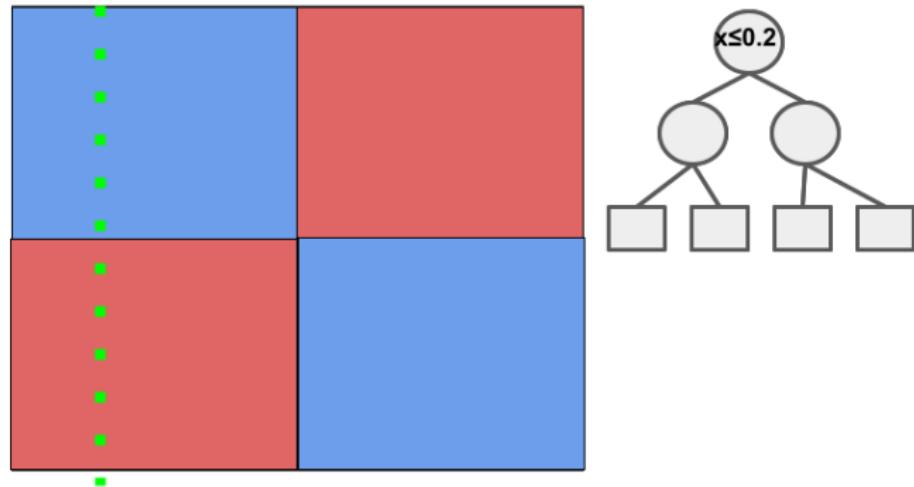
DPDT trees can be strictly better than greedy trees



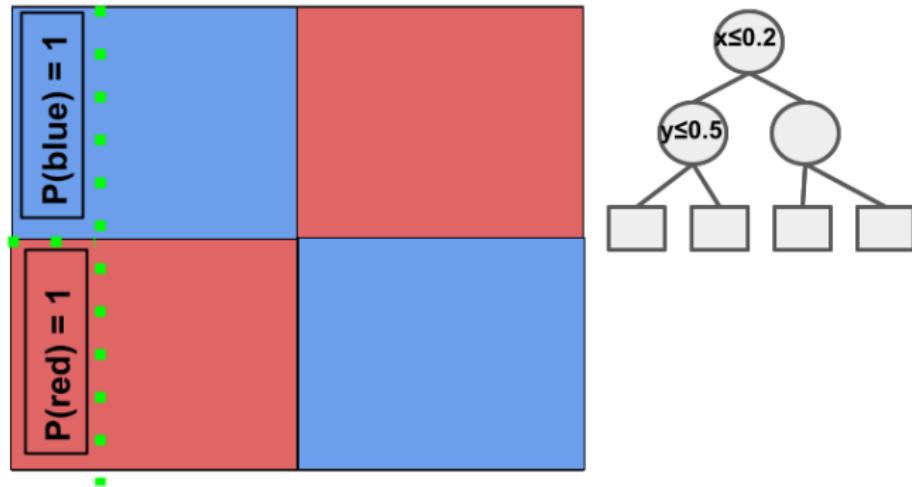
DPDT trees can be strictly better than greedy trees



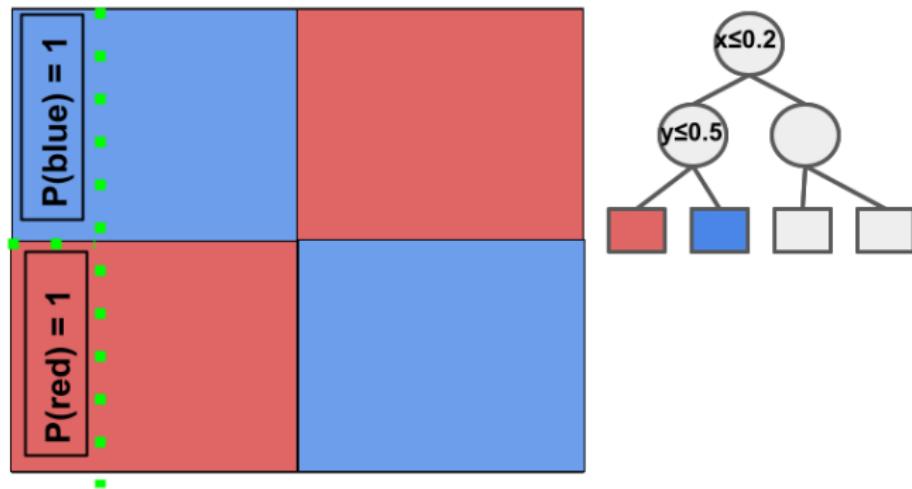
DPDT trees can be strictly better than greedy trees



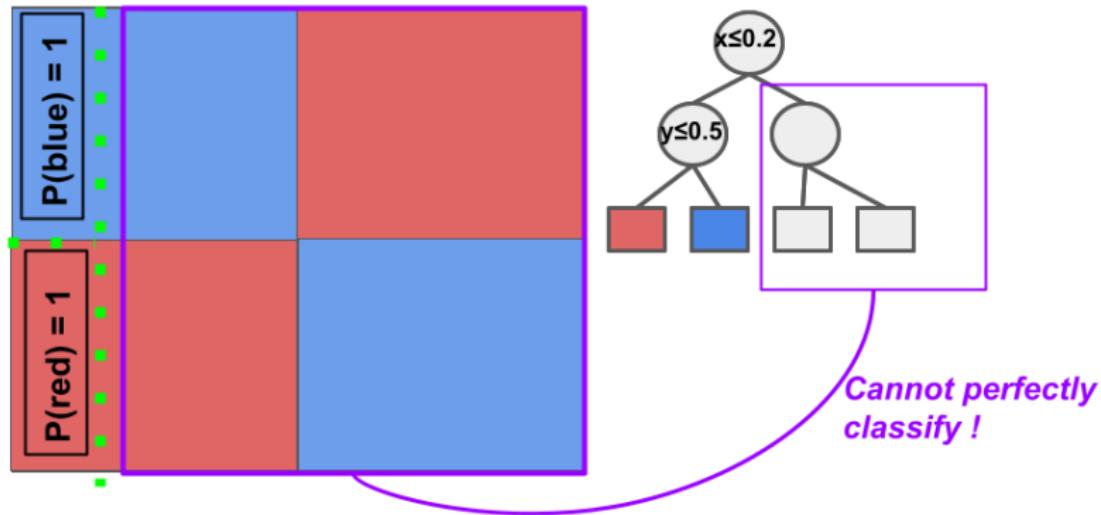
DPDT trees can be strictly better than greedy trees



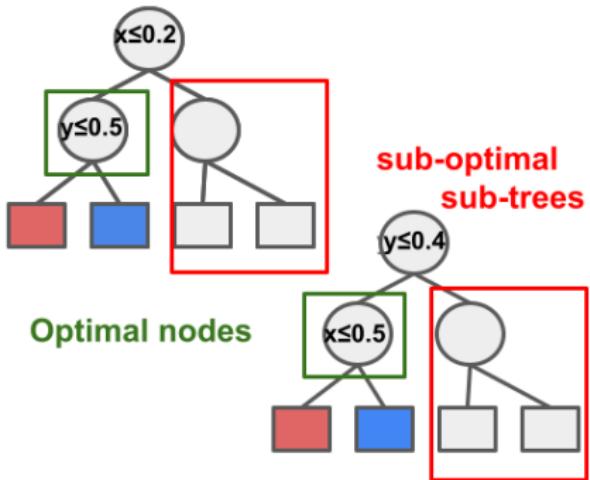
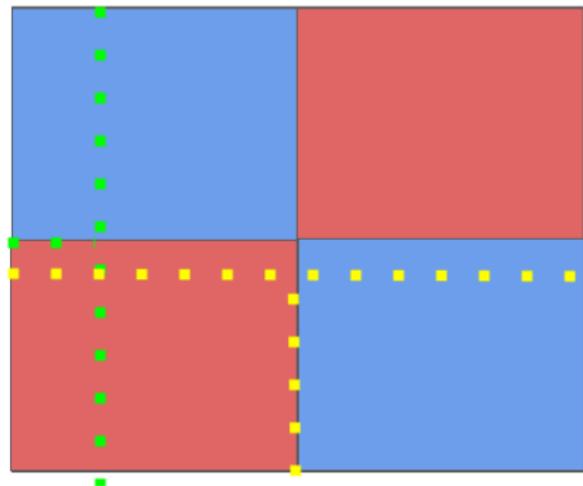
DPDT trees can be strictly better than greedy trees



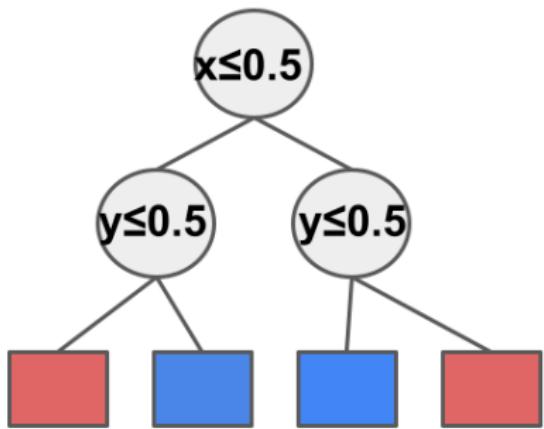
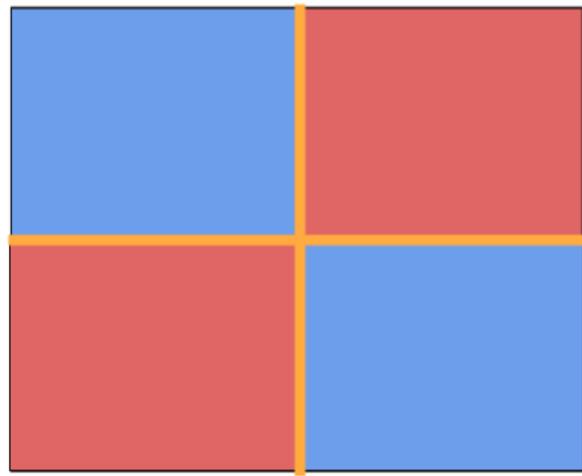
DPDT trees can be strictly better than greedy trees



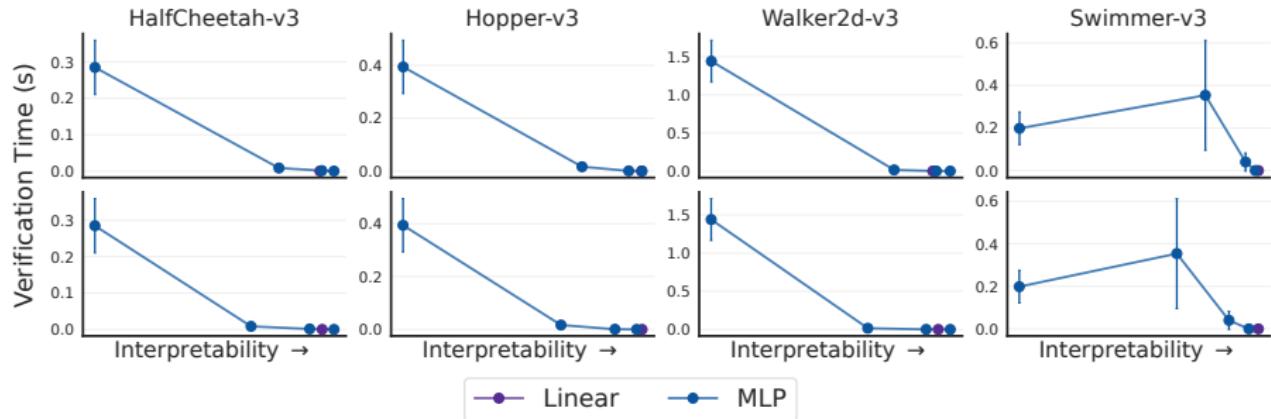
DPDT trees can be strictly better than greedy trees



DPDT trees can be strictly better than greedy trees

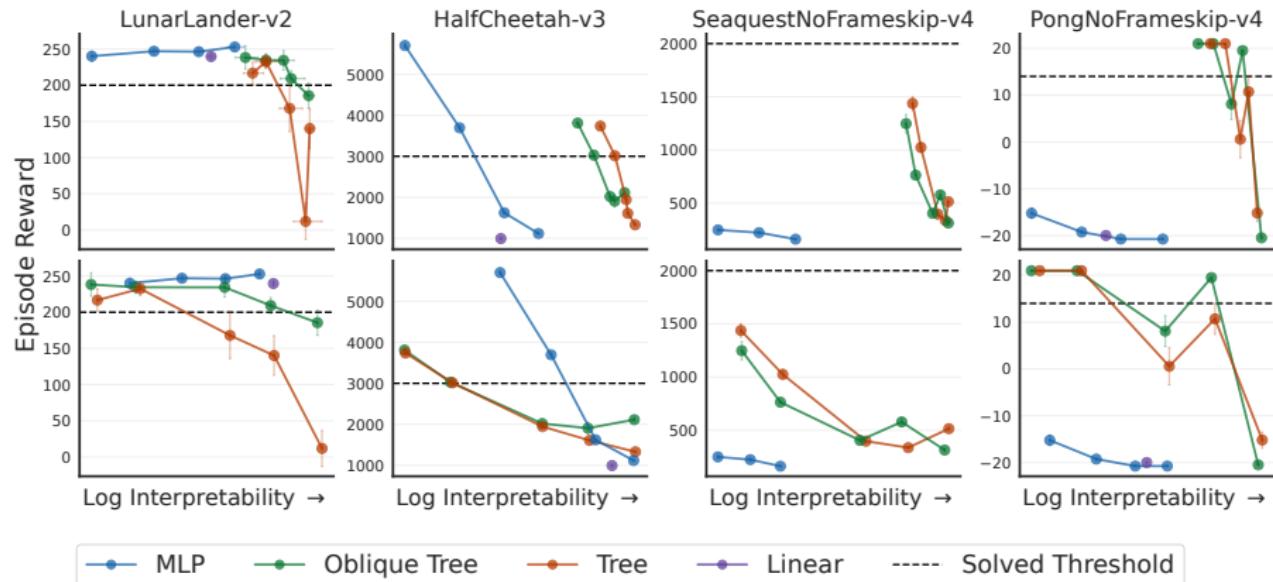


Result: verification time does scale with step inference time



Verification time as a function of policy interpretability. Top row, interpretability is measured with step inference times. Bottom row, the interpretability is measured with policy size.

Result: there is no dominating policy class for all environments



Interpretability-Performance trade-offs. Top row, interpretability is measured with step inference times. Bottom row, the interpretability is measured with policy size.

We propose policy unfolding

```
# Decision tree for Mountain Car
def play(x):
    if x[1] <= -0.2597:
        if x[1] <= -0.6378:
            return 0
        else:
            if x[0] <= -1.0021:
                return 2
            else:
                return 0
    else:
        if x[1] <= -0.0508:
            if x[0] <= 0.2979:
                if x[0] <= 0.0453:
                    return 2
                else:
                    if x[1] <=
-0.2156:
                        return 0
                    else:
                        return 2
            else:
                return 0
        else:
            return 2
```

```
# Small ReLU MLP for Pendulum
def play(x):
    h_layer_0_0 = 1.238*x[0]+0.971*x
    [1]
                           +0.430*x[2]+0.933
    h_layer_0_0 = max(0, h_layer_0_0
    )
    h_layer_0_1 = -1.221*x[0]+1.001
                           *x[1]-0.423*x[2]
                           +0.475
    h_layer_0_1 = max(0, h_layer_0_1
    )
    h_layer_1_0 = -0.109*h_layer_0_0
                           -0.377*h_layer_0_1
                           +1.694
    h_layer_1_0 = max(0, h_layer_1_0
    )
    h_layer_1_1 = -3.024*h_layer_0_0
                           -1.421*h_layer_0_1
                           +1.530
    h_layer_1_1 = max(0, h_layer_1_1
    )
    h_layer_2_0 = -1.790*h_layer_1_0
                           +2.840*h_layer_1_1
                           +0.658
    y_0 = h_layer_2_0
    return [y_0]
```