Université de Lille

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Numéro d'ordre : 42

Université de Lille
Inria

École doctorale **École Graduée MADIS-631**

Unité de recherche **Centre de Recherche en Informatique, Signal et Automatique de Lille**

Thèse présentée par **Hector Kohler**

Soutenue le **1er décembre 2025**

En vue de l'obtention du grade de docteur de l'Université de Lille et de l'Inria

Discipline **Informatique**

Spécialité **Informatique et Applications**

Arbres de Décision pour la Prise de Décision Séquentielle

Thèse dirigée par	Philippe Preux	directeur
	Riad Akrour	co-directeur

Composition du jury

Rapporteurs	René Descartes	professeur à l'IHP	
	Denis Diderot	directeur de recherche au CNRS	
Examinateurs	Victor Hugo	professeur à l'ENS Lyon	président du jury
	Sophie Germain	mcf à l'Université de Paris 13	
	Joseph Fourier	chargé de recherche à l'INRIA	
	Paul Verlaine	chargé de recherche hdr au CNRS	
Invité	George Sand		
Directeurs de thèse	Philippe Preux	professeur à l'Université de Lille	
	Riad Akrour	Inria	

Colophon

Mémoire de thèse intitulé « Arbres de Décision pour la Prise de Décision Séquentielle », écrit par Hector Kohler, achevé le 27 mai 2025, composé au moyen du système de préparation de document LaTeX et de la classe yathesis dédiée aux thèses préparées en France.

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Doctoral School **École Graduée MADIS-631**

University Department **Centre de Recherche en Informatique, Signal et Automatique de Lille**

Thesis defended by **Hector Kohler**

Defended on **December 1, 2025**

In order to become Doctor from Université de Lille and from Inria

Academic Field **Computer Science**

Speciality **Computer Science and Applications**

Decision Trees for Sequential Decision Making

Thesis supervised by

Philippe Preux	Supervisor
Riad Akrour	Co-Supervisor

Committee members

Referees	René Descartes	Professor at IHP	
	Denis Diderot	Senior Researcher at CNRS	
Examiners	Victor Hugo	Professor at ENS Lyon	Committee President
	Sophie Germain	Associate Professor at Université de Paris 13	
	Joseph Fourier	Junior Researcher at INRIA	
	Paul Verlaine	HDR Junior Researcher at CNRS	
Guest	George Sand		
Supervisors	Philippe Preux	Professor at Université de Lille	
	Riad Akrour	Inria	

Arbres de Décision pour la Prise de Décision Séquentielle

Résumé

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Mots clés : apprentissage par renforcement, arbres de décision, interprétabilité, méthodologie

Centre de Recherche en Informatique, Signal et Automatique de Lille
Université de Lille-Campus scientifique – Bâtiment ESPRIT – Avenue Henri Poincaré – 59655 Villeneuve d'Ascq

Decision Trees for Sequential Decision Making

Abstract

In this Ph.D. thesis, we study algorithms to learn decision trees for classification and sequential decision making. Decision trees are interpretable because humans can read through the decision tree computations from the root to the leaves. This makes decision trees the go-to model when human verification is required like in medicine applications. However, decision trees are non-differentiable making them hard to optimize unlike neural networks that can be trained efficiently with gradient descent. Existing interpretable reinforcement learning approaches usually learn soft trees (non-interpretable as is) or are ad-hoc (train a neural network then fit a tree to it) potentially missing better solutions.

In the first part of this manuscript, we aim to directly learn decision trees for a Markov decision process with reinforcement learning. In practice we show that this amounts to solving a partially observable Markov decision process. Most existing RL algorithms are not suited for POMDPs. This parallel between decision tree learning with RL and POMDPs solving help us understand why in practice it is often easier to obtain a non-interpretable expert policy–a neural network–and then distillate it into a tree rather than learning the decision tree from scratch.

The second contribution from this work arose from the observation that looking for a deicison tree classifier (or regressor) can be seen as sequentially adding nodes to a tree to maximize the accuracy of predictions. We thus formulate decision tree induction as sloving a Markov decision problem and propose a new state-of-the-art algorithm that can be trained with supervised example data and generalizes well to unseen data.

Work from the previous parts rely on the hypothesis that decision trees are indeed an interpretable model that humans can use in sensitive applications. But is it really the case? In the last part of this thesis, we attempt to answer some more general questions about interpretability: can we measure interpretability without humans? And are decision trees really more interpretable than neural networks?

Keywords: reinforcement learning, deicision trees, interpretability, methodology

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Preliminary Concepts

What is Sequential Decision Making?

Markov decision processes/problems

Exact solutions

Reinforcement learning

What is Interpretable Sequential Decision Making?

Why do we care about interpretability?

What are existing approaches for interpretable sequential decision making?

What are decision tree policies?

Première partie

A difficult problem : Learning Decision Trees for MDP

I have not failed. I've just found 10.000 ways that won't work.

Thomas A. Edison

Chapitre 1

A Decision Tree Policy for an MDP is a Policy for some Partially Observable MDP

1.1 How to Learn a Decision Tree Policy for an MDP ?

1.1.1 Imitation

1.1.2 Soft Trees

1.1.3 Iterative Bounding MDPs

1.2 How to solve Iterative Bounding MDPs ?

1.2.1 Asymmetric Reinforcement Learning

1.2.2 Learning a decision tree policy is solving a POMDP

1.3 Is it hard to properly learn a Decision Tree Policy for an MDP ?

1.3.1 POMDPs are way harder to solve than MDPs

1.3.2 Memoryless approaches to solve POMDPs seem uneffective

Chapitre 2

An attempt at Learning Decision Tree Policies with Reinforcement Learning

2.1 Grid Worlds

$$V(g) = \zeta \sum_{i=0}^{\infty} \gamma^{2i} + \sum_{i=0}^{\infty} \gamma^{2i+1}$$
$$V(0) = \zeta + \gamma^2 V(g)$$
$$V(1) = \zeta + \gamma^2 V(0)$$
$$\frac{1}{4}\gamma\frac{1}{1-\gamma} + \frac{1}{4}\frac{1}{1-\gamma} \leq \frac{1}{4}V(g) + \frac{2}{4}V(0) + \frac{1}{4}V(1)$$
$$\zeta \cdot \sum_{i=0}^{\infty} \gamma^i \leq \frac{1}{4}V(g) + \frac{2}{4}V(0) + \frac{1}{4}V(1)$$
$$\frac{1}{4}V(g) + \frac{1}{4}(\zeta + \gamma V(0)) + \frac{1}{4}(\zeta + \gamma V(1)) + \frac{1}{4}V(0) \leq \frac{1}{4}V(g) + \frac{2}{4}V(0) + \frac{1}{4}V(1)$$

2.1.1 Step-by-step derivation of the lower bound on ζ

Step 1 : Simplify the left side of the inequality

$$\frac{1}{4}\gamma\frac{1}{1-\gamma} + \frac{1}{4}\frac{1}{1-\gamma} = \frac{1}{4}\frac{1}{1-\gamma}(\gamma+1) \tag{2.1}$$
$$= \frac{\gamma+1}{4(1-\gamma)} \tag{2.2}$$

Step 2 : Express $V(g)$, $V(0)$, and $V(1)$ in simplified forms

$$V(g) = \zeta \sum_{i=0}^{\infty} \gamma^{2i} + \sum_{i=0}^{\infty} \gamma^{2i+1} \tag{2.3}$$

$$= \zeta \frac{1}{1-\gamma^2} + \gamma \frac{1}{1-\gamma^2} \tag{2.4}$$

$$= \frac{\zeta + \gamma}{1-\gamma^2} \tag{2.5}$$

$$V(0) = \zeta + \gamma^2 V(g) \tag{2.6}$$

$$= \zeta + \gamma^2 \frac{\zeta + \gamma}{1-\gamma^2} \tag{2.7}$$

$$= \frac{\zeta(1-\gamma^2) + \gamma^2(\zeta + \gamma)}{1-\gamma^2} \tag{2.8}$$

$$= \frac{\zeta + \gamma^3}{1-\gamma^2} \tag{2.9}$$

$$V(1) = \zeta + \gamma^2 V(0) \tag{2.10}$$

$$= \zeta + \gamma^2 \frac{\zeta + \gamma^3}{1-\gamma^2} \tag{2.11}$$

$$= \frac{\zeta(1-\gamma^2) + \gamma^2(\zeta + \gamma^3)}{1-\gamma^2} \tag{2.12}$$

$$= \frac{\zeta + \gamma^5}{1-\gamma^2} \tag{2.13}$$

Step 3 : Substitute into the right side of the inequality

$$\frac{1}{4}V(g)+\frac{2}{4}V(0)+\frac{1}{4}V(1)=\frac{1}{4}\frac{\zeta+\gamma}{1-\gamma^2}+\frac{1}{2}\frac{\zeta+\gamma^3}{1-\gamma^2}+\frac{1}{4}\frac{\zeta+\gamma^5}{1-\gamma^2} \tag{2.14}$$

$$=\frac{1}{4(1-\gamma^2)}[(\zeta+\gamma)+2(\zeta+\gamma^3)+(\zeta+\gamma^5)] \tag{2.15}$$

$$=\frac{4\zeta+\gamma+2\gamma^3+\gamma^5}{4(1-\gamma^2)} \tag{2.16}$$

Step 4 : Set up the inequality

$$\frac{\gamma+1}{4(1-\gamma)}\leq\frac{4\zeta+\gamma+2\gamma^3+\gamma^5}{4(1-\gamma^2)} \tag{2.17}$$

Step 5 : Use the identity $1-\gamma^2=(1-\gamma)(1+\gamma)$

$$\frac{\gamma+1}{4(1-\gamma)}\leq\frac{4\zeta+\gamma+2\gamma^3+\gamma^5}{4(1-\gamma)(1+\gamma)} \tag{2.18}$$

Step 6 : Multiply both sides by $4(1-\gamma)$

$$\gamma+1\leq\frac{4\zeta+\gamma+2\gamma^3+\gamma^5}{1+\gamma} \tag{2.19}$$

Step 7 : Multiply both sides by $(1+\gamma)$

$$(\gamma+1)(1+\gamma)\leq 4\zeta+\gamma+2\gamma^3+\gamma^5 \tag{2.20}$$

$$(\gamma+1)^2\leq 4\zeta+\gamma+2\gamma^3+\gamma^5 \tag{2.21}$$

Step 8 : Expand and rearrange

$$\gamma^2+2\gamma+1\leq 4\zeta+\gamma+2\gamma^3+\gamma^5 \tag{2.22}$$

$$4\zeta\geq\gamma^2+2\gamma+1-\gamma-2\gamma^3-\gamma^5 \tag{2.23}$$

$$4\zeta\geq\gamma^2+\gamma+1-2\gamma^3-\gamma^5 \tag{2.24}$$

$$\zeta\geq\frac{\gamma^2+\gamma+1-2\gamma^3-\gamma^5}{4} \tag{2.25}$$

Therefore, we obtain a **lower bound** on ζ :

$$\zeta \geq \frac{\gamma^2 + \gamma + 1 - 2\gamma^3 - \gamma^5}{4} \tag{2.26}$$

where $0 < \gamma < 1$.

2.1.2 Step-by-step derivation of the upper bound on ζ

Starting from the inequality :

$$\frac{1}{4}V(g) + \frac{1}{4}(\zeta + \gamma V(0)) + \frac{1}{4}(\zeta + \gamma V(1)) + \frac{1}{4}V(0) \leq \frac{1}{4}V(g) + \frac{2}{4}V(0) + \frac{1}{4}V(1)$$

Step 1 : Cancel the $\frac{1}{4}V(g)$ terms from both sides

$$\frac{1}{4}(\zeta + \gamma V(0)) + \frac{1}{4}(\zeta + \gamma V(1)) + \frac{1}{4}V(0) \leq \frac{2}{4}V(0) + \frac{1}{4}V(1) \tag{2.27}$$

Step 2 : Expand the left side

$$\frac{1}{4}\zeta + \frac{1}{4}\gamma V(0) + \frac{1}{4}\zeta + \frac{1}{4}\gamma V(1) + \frac{1}{4}V(0) \leq \frac{2}{4}V(0) + \frac{1}{4}V(1) \tag{2.28}$$

Step 3 : Combine like terms

$$\frac{1}{2}\zeta + \frac{1}{4}\gamma V(0) + \frac{1}{4}\gamma V(1) \leq \frac{2}{4}V(0) - \frac{1}{4}V(0) + \frac{1}{4}V(1) \tag{2.29}$$

$$\frac{1}{2}\zeta + \frac{1}{4}\gamma V(0) + \frac{1}{4}\gamma V(1) \leq \frac{1}{4}V(0) + \frac{1}{4}V(1) \tag{2.30}$$

Step 4 : Factor out common terms

$$\frac{1}{2}\zeta \leq \frac{1}{4}V(0) + \frac{1}{4}V(1) - \frac{1}{4}\gamma V(0) - \frac{1}{4}\gamma V(1) \tag{2.31}$$

$$\frac{1}{2}\zeta \leq \frac{1}{4}V(0)(1-\gamma) + \frac{1}{4}V(1)(1-\gamma) \tag{2.32}$$

$$\frac{1}{2}\zeta \leq \frac{1-\gamma}{4}(V(0) + V(1)) \tag{2.33}$$

$$\zeta \leq \frac{1-\gamma}{2}(V(0) + V(1)) \tag{2.34}$$

Step 5 : Substitute the expressions for $V(0)$ and $V(1)$

$$V(0) + V(1) = \frac{\zeta + \gamma^3}{1 - \gamma^2} + \frac{\zeta + \gamma^5}{1 - \gamma^2} \tag{2.35}$$

$$= \frac{2\zeta + \gamma^3 + \gamma^5}{1 - \gamma^2} \tag{2.36}$$

Step 6 : Substitute back into the inequality

$$\zeta \leq \frac{1 - \gamma}{2} \cdot \frac{2\zeta + \gamma^3 + \gamma^5}{1 - \gamma^2} \tag{2.37}$$

$$= \frac{(1 - \gamma)(2\zeta + \gamma^3 + \gamma^5)}{2(1 - \gamma^2)} \tag{2.38}$$

Step 7 : Use the identity $1 - \gamma^2 = (1 - \gamma)(1 + \gamma)$

$$\zeta \leq \frac{(1 - \gamma)(2\zeta + \gamma^3 + \gamma^5)}{2(1 - \gamma)(1 + \gamma)} \tag{2.39}$$

$$= \frac{2\zeta + \gamma^3 + \gamma^5}{2(1 + \gamma)} \tag{2.40}$$

Step 8 : Multiply both sides by $2(1 + \gamma)$

$$2(1 + \gamma)\zeta \leq 2\zeta + \gamma^3 + \gamma^5 \tag{2.41}$$

$$2\zeta + 2\gamma\zeta \leq 2\zeta + \gamma^3 + \gamma^5 \tag{2.42}$$

$$2\gamma\zeta \leq \gamma^3 + \gamma^5 \tag{2.43}$$

$$\zeta \leq \frac{\gamma^3 + \gamma^5}{2\gamma} \tag{2.44}$$

$$\zeta \leq \frac{\gamma^2 + \gamma^4}{2} \tag{2.45}$$

Therefore, we obtain an **upper bound** on ζ :

$$\zeta \leq \frac{\gamma^2 + \gamma^4}{2} \tag{2.46}$$

Combined bounds :

$$\frac{\gamma^2+\gamma+1-2\gamma^3-\gamma^5}{4} \leq \zeta \leq \frac{\gamma^2+\gamma^4}{2} \tag{2.47}$$

where $0 < \gamma < 1$.

2.1.3 Step-by-step derivation for the third inequality

Starting from the inequality :

$$\zeta \cdot \sum_{i=0}^{\infty} \gamma^i \leq \frac{1}{4}V(g) + \frac{2}{4}V(0) + \frac{1}{4}V(1)$$

Step 1 : Simplify the left side using the geometric series

$$\zeta \cdot \sum_{i=0}^{\infty} \gamma^i = \zeta \cdot \frac{1}{1-\gamma} \tag{2.48}$$

$$= \frac{\zeta}{1-\gamma} \tag{2.49}$$

Step 2 : Use the previously derived expression for the right side From our earlier calculation :

$$\frac{1}{4}V(g) + \frac{2}{4}V(0) + \frac{1}{4}V(1) = \frac{4\zeta+\gamma+2\gamma^3+\gamma^5}{4(1-\gamma^2)} \tag{2.50}$$

Step 3 : Set up the inequality

$$\frac{\zeta}{1-\gamma} \leq \frac{4\zeta+\gamma+2\gamma^3+\gamma^5}{4(1-\gamma^2)} \tag{2.51}$$

Step 4 : Use the identity $1-\gamma^2 = (1-\gamma)(1+\gamma)$

$$\frac{\zeta}{1-\gamma} \leq \frac{4\zeta+\gamma+2\gamma^3+\gamma^5}{4(1-\gamma)(1+\gamma)} \tag{2.52}$$

Step 5 : Multiply both sides by $(1-\gamma)$

$$\zeta \leq \frac{4\zeta + \gamma + 2\gamma^3 + \gamma^5}{4(1+\gamma)} \tag{2.53}$$

Step 6 : Multiply both sides by $4(1+\gamma)$

$$4(1+\gamma)\zeta \leq 4\zeta + \gamma + 2\gamma^3 + \gamma^5 \tag{2.54}$$

$$4\zeta + 4\gamma\zeta \leq 4\zeta + \gamma + 2\gamma^3 + \gamma^5 \tag{2.55}$$

Step 7 : Subtract 4ζ **from both sides**

$$4\gamma\zeta \leq \gamma + 2\gamma^3 + \gamma^5 \tag{2.56}$$

$$\zeta \leq \frac{\gamma + 2\gamma^3 + \gamma^5}{4\gamma} \tag{2.57}$$

$$\zeta \leq \frac{1 + 2\gamma^2 + \gamma^4}{4} \tag{2.58}$$

Therefore, we obtain another **upper bound** on ζ :

$$\zeta \leq \frac{1 + 2\gamma^2 + \gamma^4}{4} \tag{2.59}$$

Final combined bounds from all three inequalities :

$$\frac{\gamma^2 + \gamma + 1 - 2\gamma^3 - \gamma^5}{4} \leq \zeta \leq \min\left\{\frac{\gamma^2 + \gamma^4}{2}, \frac{1 + 2\gamma^2 + \gamma^4}{4}\right\} \tag{2.60}$$

where $0 < \gamma < 1$.

2.2 Q-Learning

2.3 Preferences over Decision Tree Policies

2.4 Results

Chapitre 3

Conclusion

3.1 What happens when the MDP's transitions are independent of the current state ?

Deuxième partie

An easier problem : Learning Decision Trees for MDPs that are Classification tasks

Chapitre 4

DPDT-intro

Chapitre 5

DPDT-paper

Chapitre 6

Conclusion

Troisième partie

Beyond Decision Trees : what can be done with other Interpretable Policies ?

Conclusion générale

Annexe A

Documents juridiques

Cette partie regroupe les documents juridiques officiels.

A.1 Licence sous laquelle est publié notre travail

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A.2 Transposition de la licence précédente en droit français

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Annexe B

Programmes informatiques

Les listings suivants sont au cœur de notre travail.

Listing B.1 – Il est l'heure

```
#include <stdio.h>
int heures, minutes, secondes;

/*******************************************************/
/*                                                     */
/*             print_heure                             */
/*                                                     */
/*   But:                                              */
/*       Imprime l'heure                               */
/*                                                     */
/*   Interface:                                        */
/*       Utilise les variables globales                */
/*           heures, minutes, secondes                 */
/*                                                     */
/*******************************************************/

void print_heure(void)
{
  printf("Il est %d heure",heures);
  if (heures > 1) printf("s");
  printf(" %d minute",minutes);
  if (minutes > 1) printf("s");
  printf(" %d seconde",secondes);
  if (secondes > 1) printf("s");
```

```
25 ␣␣printf("\n");
26 }
```

Listing B.2 – Factorielle

```
1 int factorielle(int n)
2 {
3   if (n > 2) return n * factorielle(n - 1);
4   return n;
5 }
```

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Arbres de Décision pour la Prise de Décision Séquentielle

Résumé

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Mots clés : apprentissage par renforcement, arbres de décision, interprétabilité, méthodologie

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Centre de Recherche en Informatique, Signal et Automatique de Lille
Université de Lille-Campus scientifique – Bâtiment ESPRIT – Avenue Henri Poincaré – 59655 Villeneuve d'Ascq

Decision Trees for Sequential Decision Making

Abstract

In this Ph.D. thesis, we study algorithms to learn decision trees for classification and sequential decision making. Decision trees are interpretable because humans can read through the decision tree computations from the root to the leaves. This makes decision trees the go-to model when human verification is required like in medicine applications. However, decision trees are non-differentiable making them hard to optimize unlike neural networks that can be trained efficiently with gradient descent. Existing interpretable reinforcement learning approaches usually learn soft trees (non-interpretable as is) or are ad-hoc (train a neural network then fit a tree to it) potentially missing better solutions.

In the first part of this manuscript, we aim to directly learn decision trees for a Markov decision process with reinforcement learning. In practice we show that this amounts to solving a partially observable Markov decision process. Most existing RL algorithms are not suited for POMDPs. This parallel between decision tree learning with RL and POMDPs solving help us understand why in practice it is often easier to obtain a non-interpretable expert policy–a neural network–and then distillate it into a tree rather than learning the decision tree from scratch.

The second contribution from this work arose from the observation that looking for a deicison tree classifier (or regressor) can be seen as sequentially adding nodes to a tree to maximize the accuracy of predictions. We thus formulate decision tree induction as sloving a Markov decision problem and propose a new state-of-the-art algorithm that can be trained with supervised example data and generalizes well to unseen data.

Work from the previous parts rely on the hypothesis that decision trees are indeed an interpretable model that humans can use in sensitive applications. But is it really the case? In the last part of this thesis, we attempt to answer some more general questions about interpretability: can we measure interpretability without humans? And are decision trees really more interpretable than neural networks?

Keywords: reinforcement learning, deicision trees, interpretability, methodology