Interpretability, Decision Trees, and Sequential Decision Making

Hector Kohler

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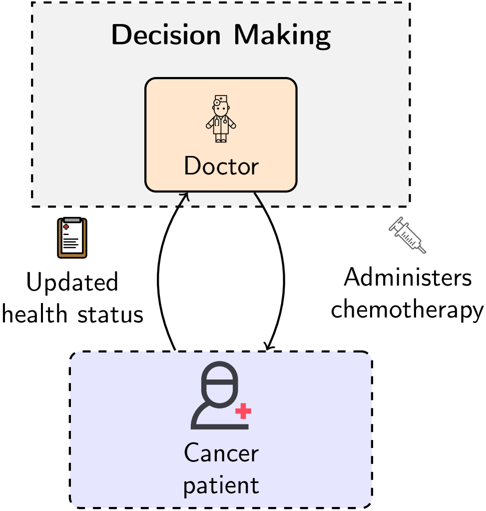
November 25, 2025

Sequential decision making (SDM) and machine learning (ML)



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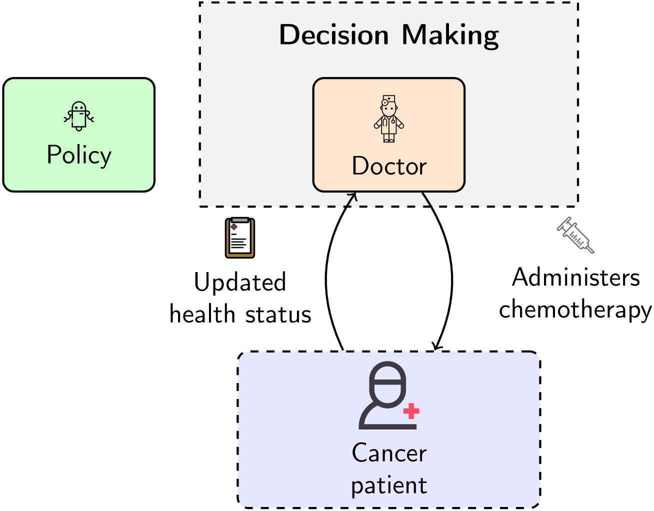

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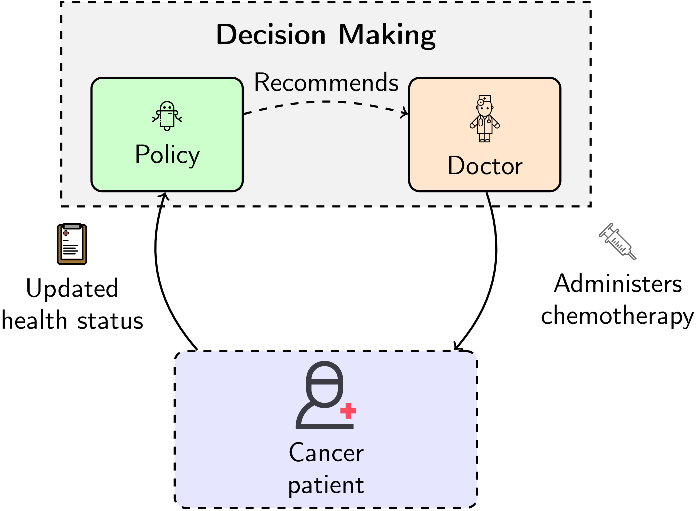

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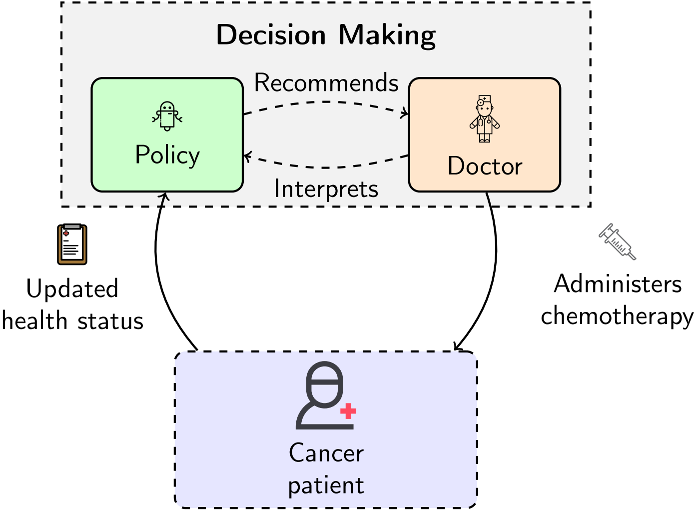

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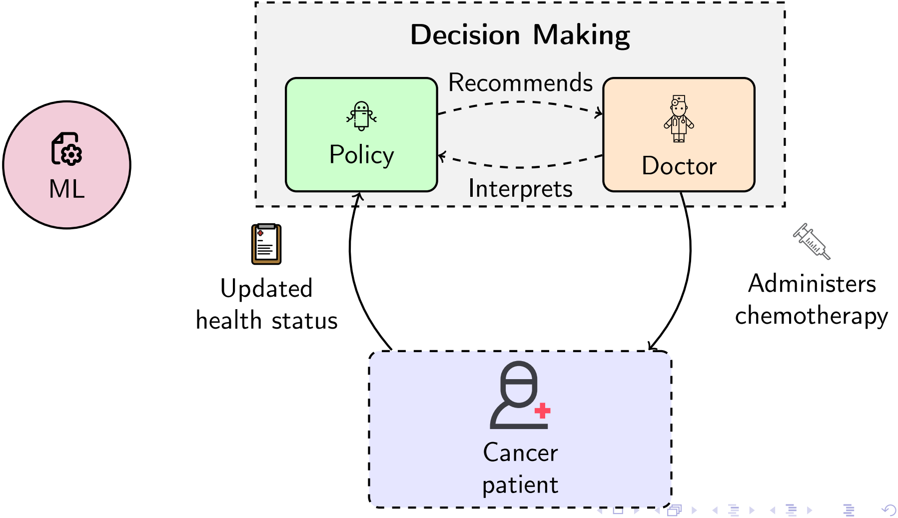

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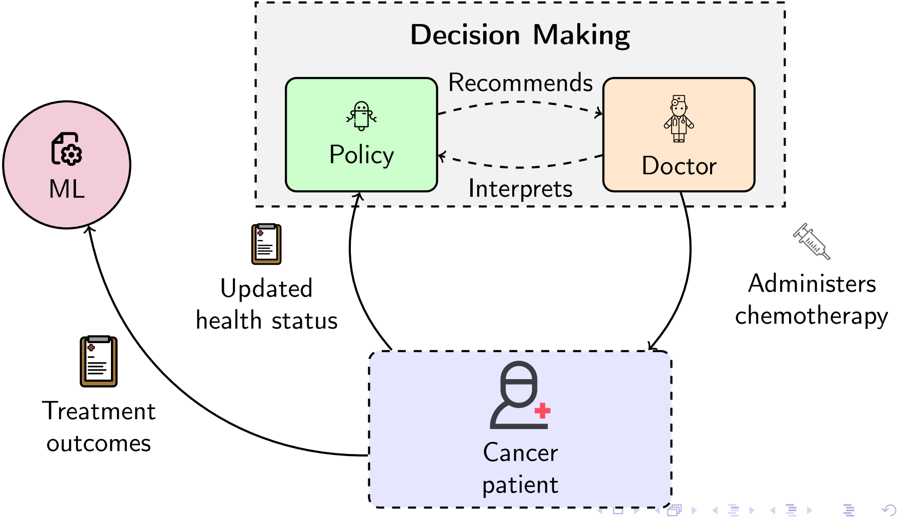

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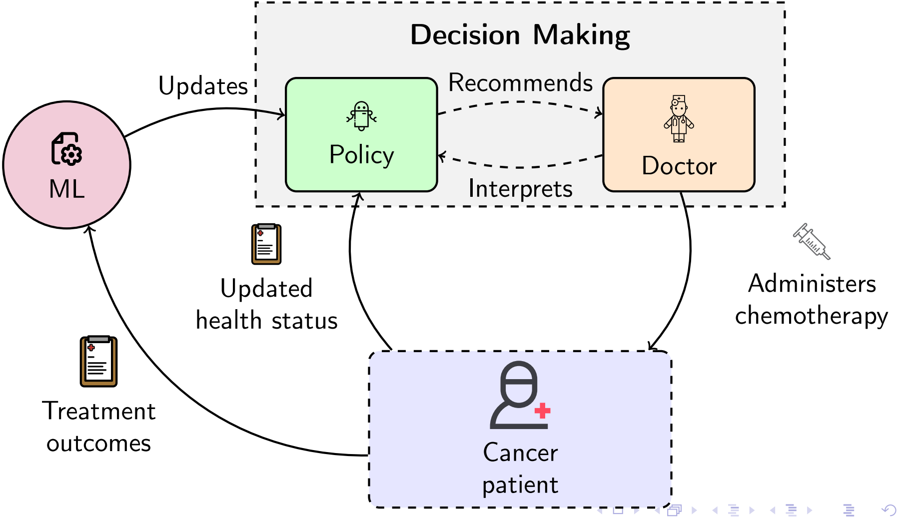

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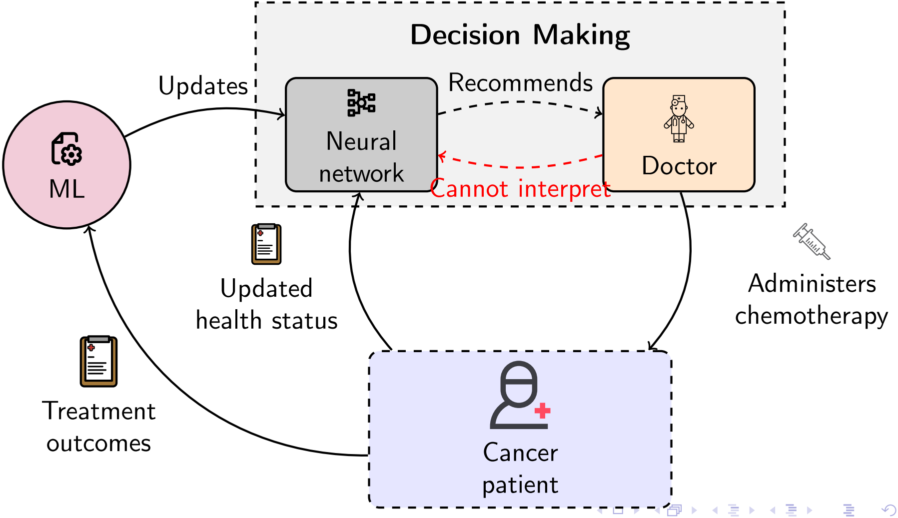
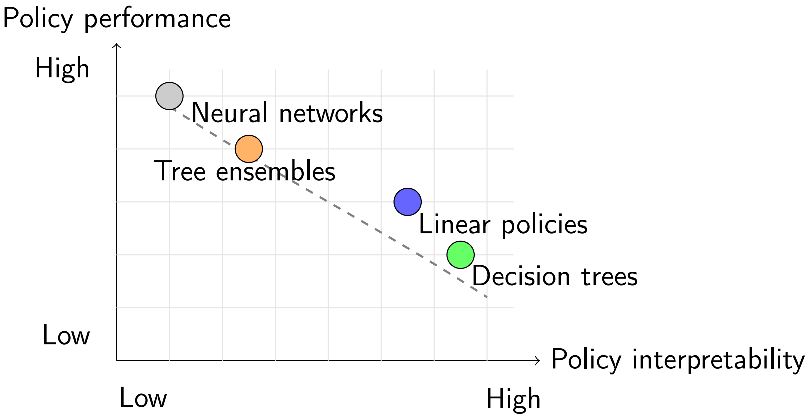
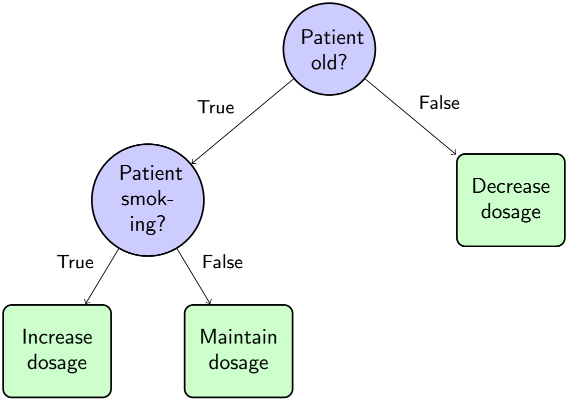


Policy interpretability



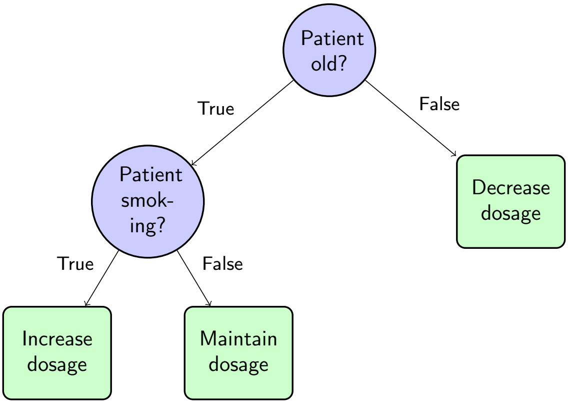
Heuristic interpretability-performance trade-offs of different policy classes. Interpretability is often presented in opposition to performances.

Decision trees



A generic decision tree of depth $D = 2$.

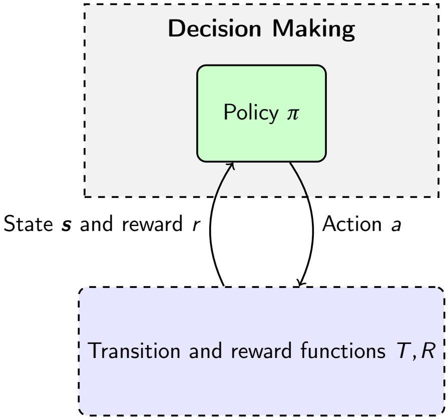
Decision trees



A generic decision tree of depth $D = 2$.

Successful algorithms for non-sequential learning: [Bre+84; BD17; Dem+22; VZ19; MMW22] ...What about SDM?

Markov decision processes (MDPs) and reinforcement learning (RL)



Markov decision processes [Put94].

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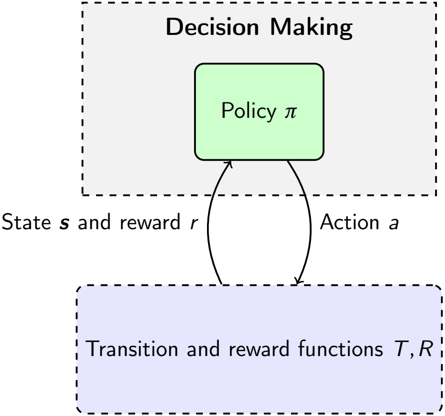

Markov decision processes [Put94].

- RL [SB98] aims to find a policy, $\pi : S \to A$ that maximizes:

$$\mathop{\mathbb{E}}_{\boldsymbol{s}_t \sim T}\left[\sum_{t=0}^{\infty} \gamma^t R(s_t, \pi(s_t))\right]$$

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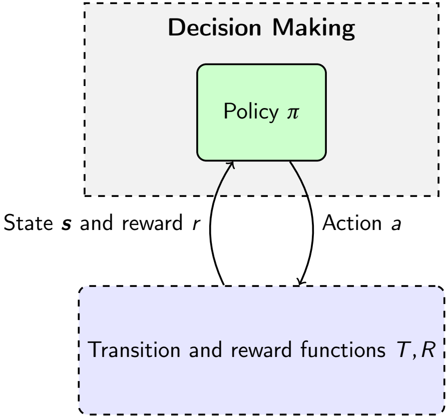

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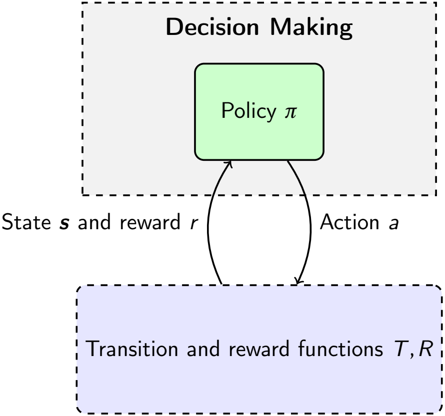

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- Lots of succesful RL algorithms [SB98; Mni+15; Sch+17].
- Few interpretability concerns.

Two ways to get interpretable policies for SDM

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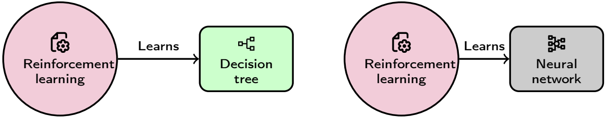

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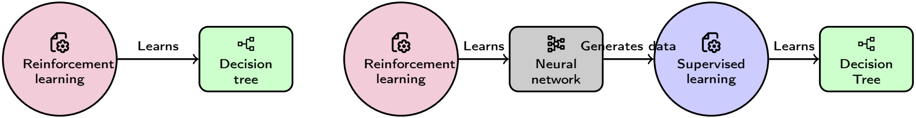

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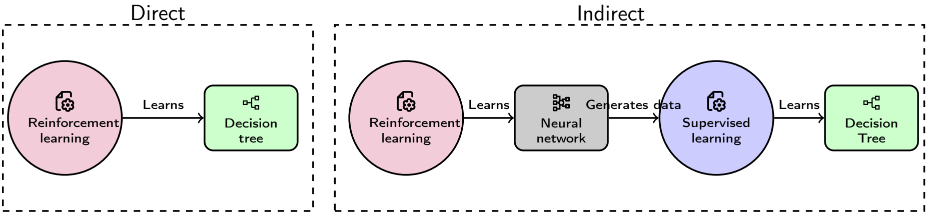

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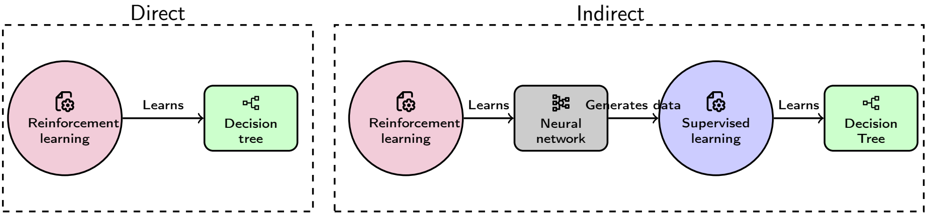
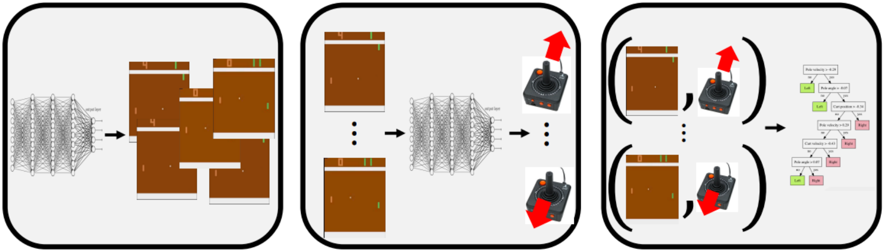


⚠Policies obtained indirectly optimize a surrogate objective rather than a trade-off between interpretability and cumulative rewards.

Indirect approach: imitation learning



Step 1: Use NN to generate states

Step 2: Use NN to obtain actions

Step 3: Use supervised learning to train a decision tree

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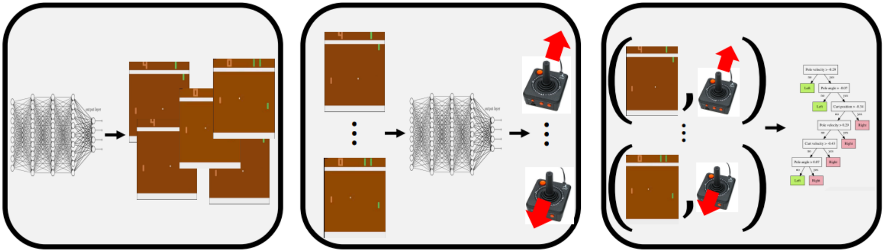

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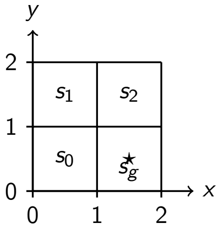
Most research focused on indirect learning of interpretable policies [RGB10; BPS18; Ver+18; Mil+24].

Grid world MDP and decision tree policies



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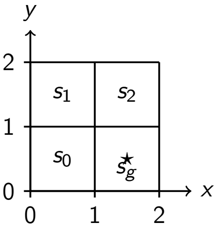

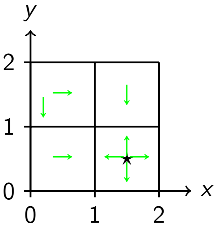

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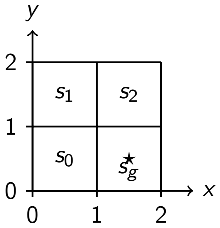

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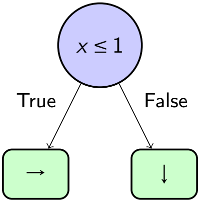

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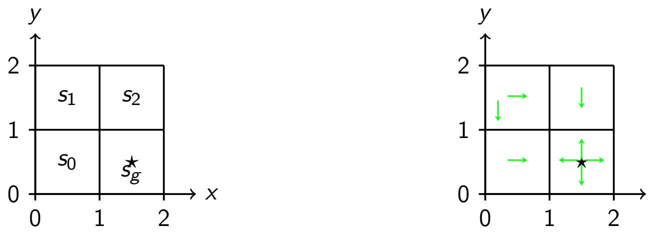

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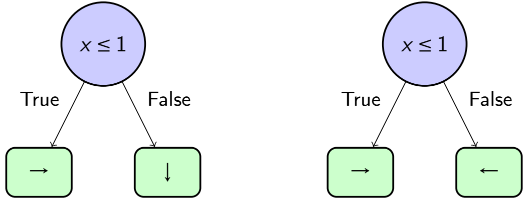

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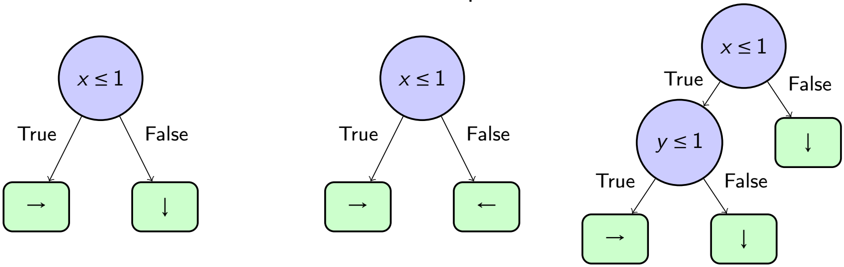

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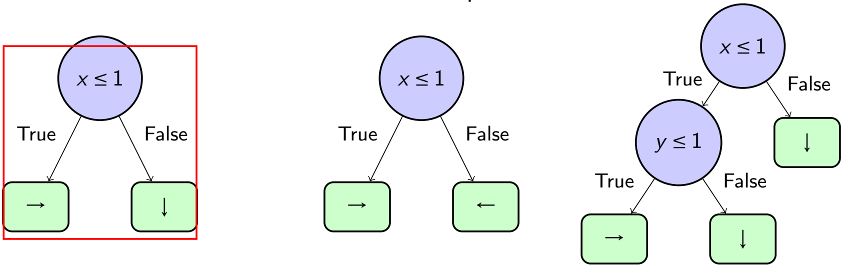

Decision tree policies with different interpretability-performance trade-offs.

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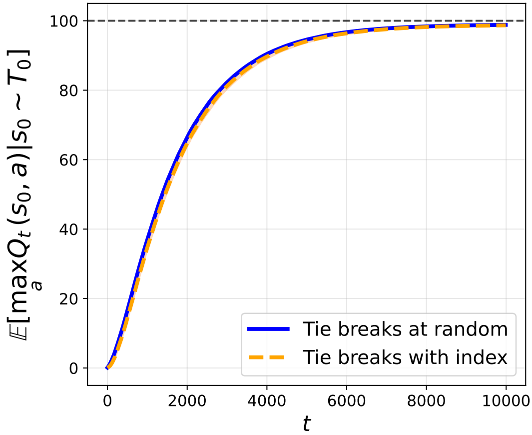

Sample complexity curve of Q-learning over 100 random seeds.

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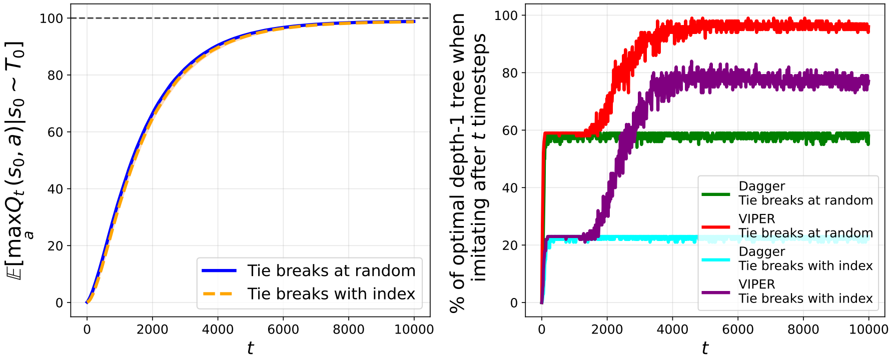

Sample complexity curve of Q-learning over 100 random seeds and performance of indirect interpretable methods when imitating the greedy policy with a tree at different Q-learning stages.

Contributions

1. How difficult is it to directly optimize a trade-off of interpretability and performance in SDM?
2. How to leverage sequential decision making to learn interpretable classifiers for supervised learning?
3. How to measure policy interpretability in sequential decision making?

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Direct RL of decision tree policies with iterative boundoing Markov decision processes

Given an MDP $\mathcal{M}$ $\langle S, A, R, T\rangle$, an associated iterative bounding Markov decision process (IBMDP, [Top+21]) $\mathcal{M}_{IB}$ is a tuple:

$$\langle \overbrace{S \times O}^{\text{Augmented state space}}, \underbrace{A \cup A_{info}}_{\text{Augmented action space}}, \overbrace{(R, \zeta)}^{\text{Augmented rewards}}, \underbrace{(T_{info}, T, T_0)}_{\text{Augmented transitions}} \rangle$$

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- No need to design new algorithm: we can use deep RL.

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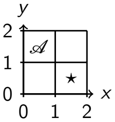
IBMDPs promises

- No need to design new algorithm: we can use deep RL.
- IBMDP rewards trade-off naturally interpretability and performances.

Grid world IBMDP example

$$t = 0$$

$$\boldsymbol{s}_t = (0.5, 1.5)$$

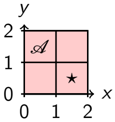


Grid world IBMDP example

$t = 0$

$\boldsymbol{s}_t = (0.5, 1.5)$

$\boldsymbol{o}_t = (0, 2, 0, 2)$



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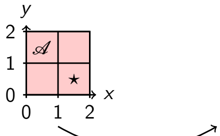

$a_t = \langle x, 1 \rangle, r_t = \zeta$

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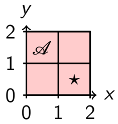

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$x \leq 1$

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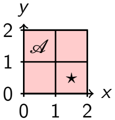

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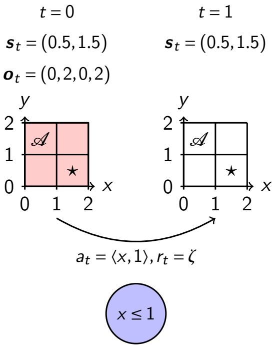

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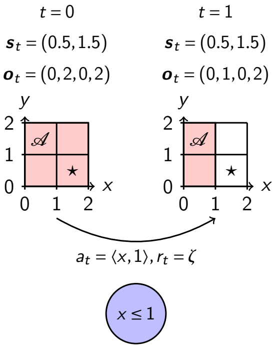

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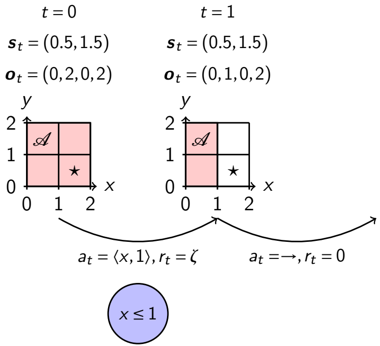

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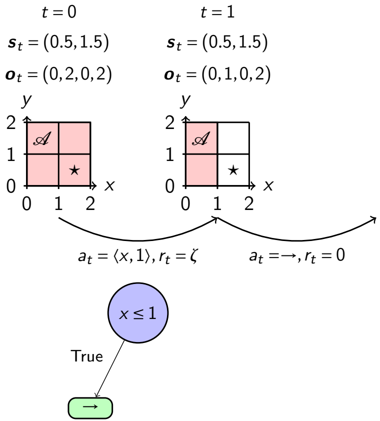
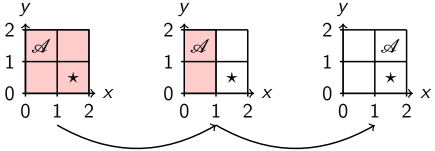
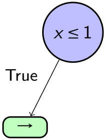


Grid world IBMDP example

$t=0$	$t=1$	$t=2$
$\boldsymbol{s}_t = (0.5, 1.5)$	$\boldsymbol{s}_t = (0.5, 1.5)$	$\boldsymbol{s}_t = (1.5, 1.5)$
$\boldsymbol{o}_t = (0, 2, 0, 2)$	$\boldsymbol{o}_t = (0, 1, 0, 2)$	



$a_t = \langle x, 1 \rangle, r_t = \zeta$ $\qquad$ $a_t = \rightarrow, r_t = 0$



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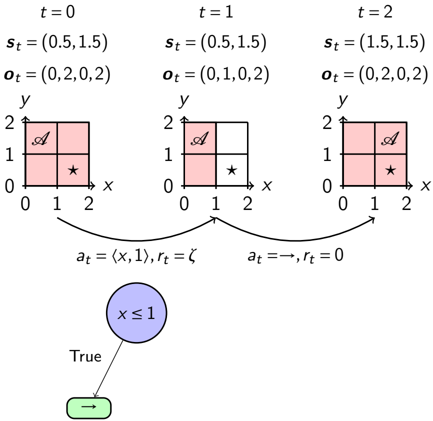

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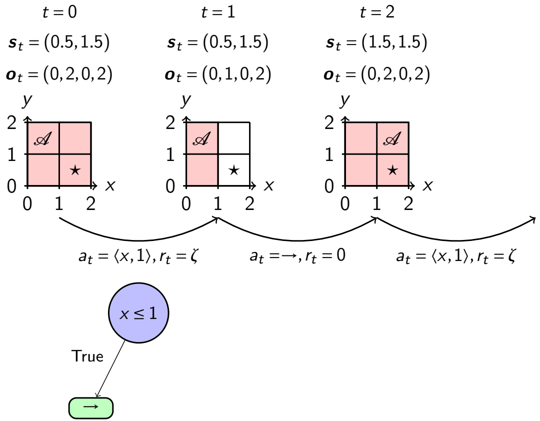

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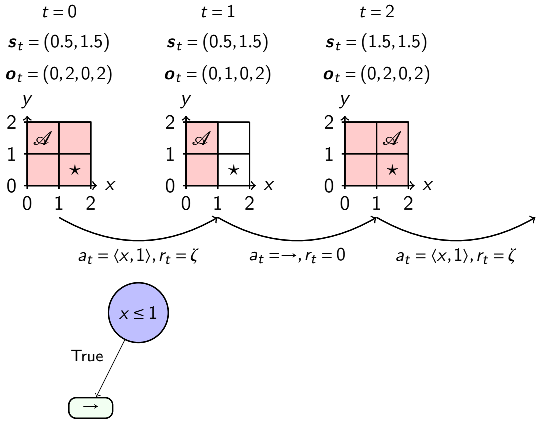
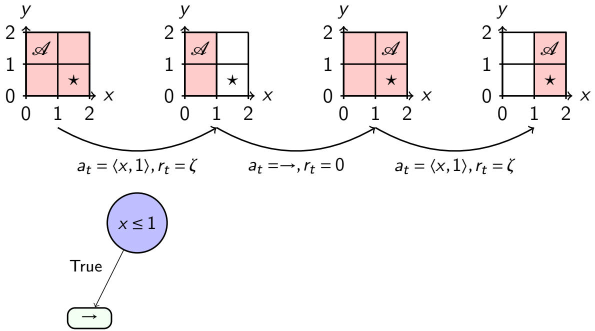


Grid world IBMDP example

$t=0$	$t=1$	$t=2$	$t=4$
$\boldsymbol{s}_t=(0.5,1.5)$	$\boldsymbol{s}_t=(0.5,1.5)$	$\boldsymbol{s}_t=(1.5,1.5)$	$\boldsymbol{s}_t=(1.5,1.5)$
$\boldsymbol{o}_t=(0,2,0,2)$	$\boldsymbol{o}_t=(0,1,0,2)$	$\boldsymbol{o}_t=(0,2,0,2)$	$\boldsymbol{o}_t=(1,2,0,2)$



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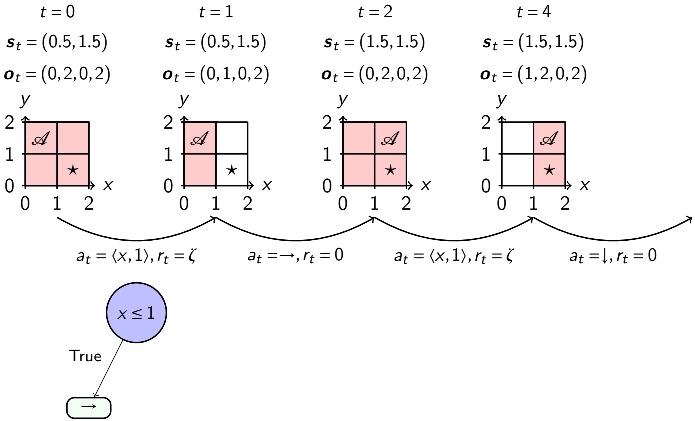

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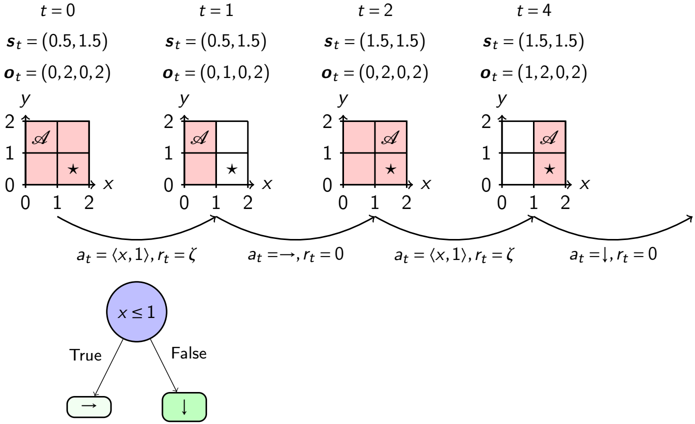

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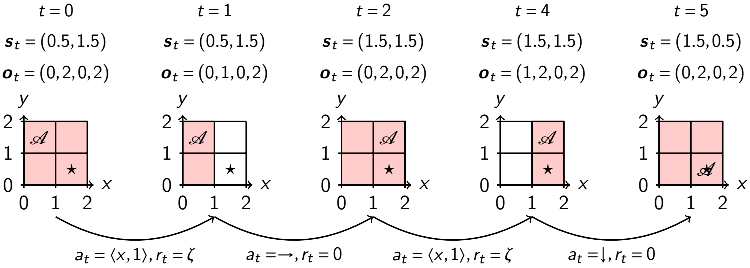

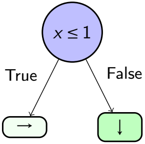

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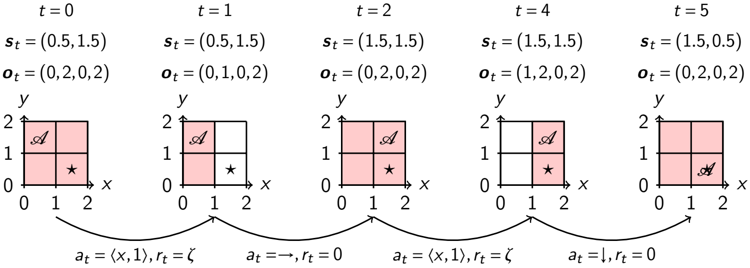

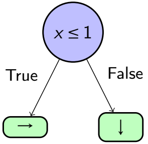

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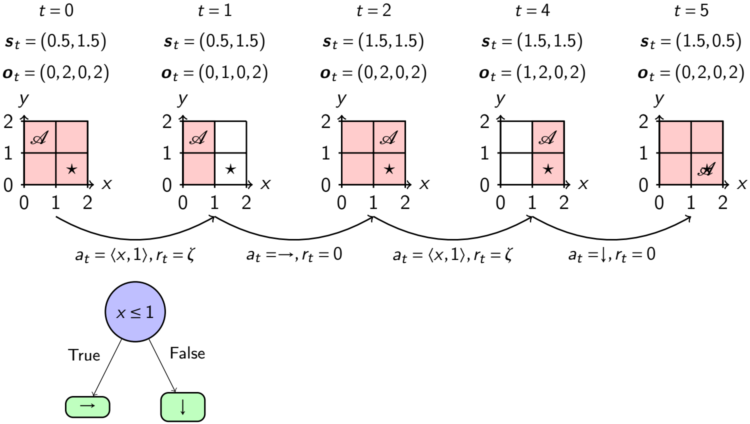

- **Determinstic** and **partially observable** policies $\implies$ decision trees.

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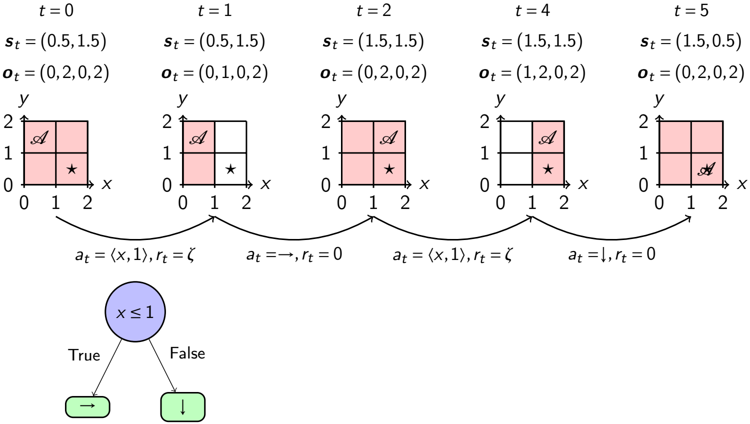

- **Determinstic** and **partially observable** policies $\Longrightarrow$ decision trees.
- ⚠Finding the best **deterministic** and **partially observable** policy is NP-hard [Lit94]!

Re-formulation

Q: Can we use reinforcement learning to directly optimize trade-offs of performance and interpretability in SDM?

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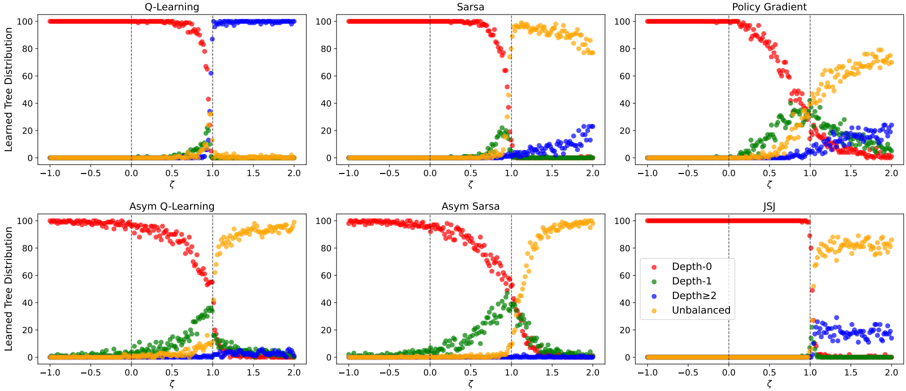
$$\approx$$

*Q: How does RL perform for optimizing **deterministic** and **partially observable** policies in IBMDPs?*

Result: RL cannot retrieve optimal depth-1 trees for the grid world MDP



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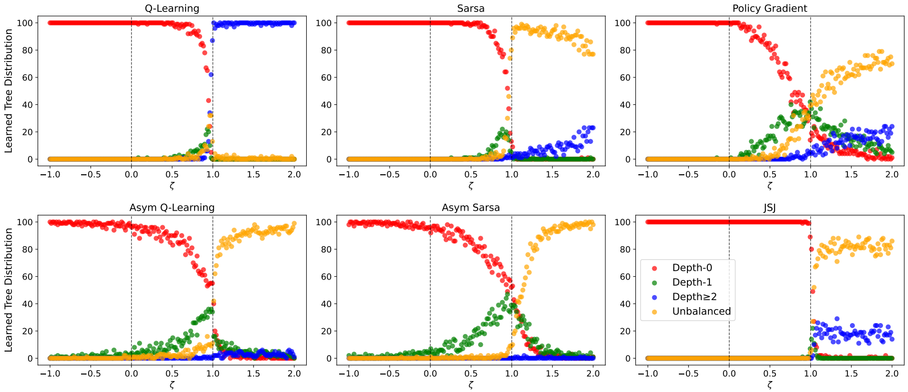
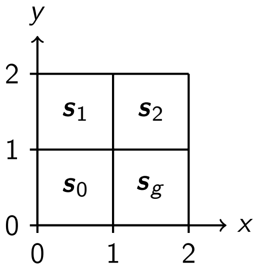


Distributions of tree policies learned with (asymmetric) RL algorithms [SB98; SJJ94; LS98; BA22; BDA22] across 100 seeds as a function of the interpretability reward ζ.

Interesting sub-class of MDPs: classification MDPs



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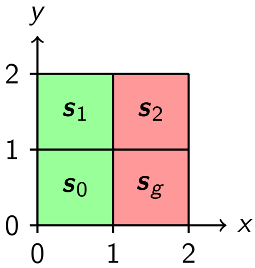

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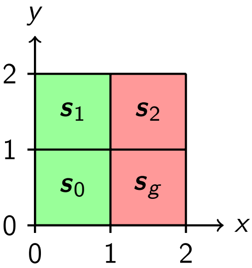

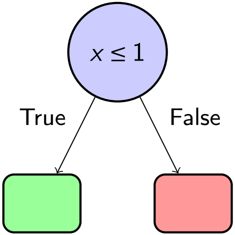

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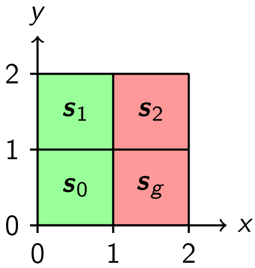
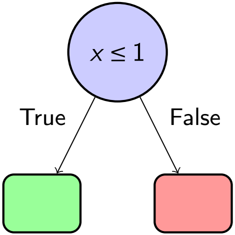
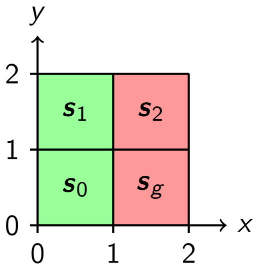
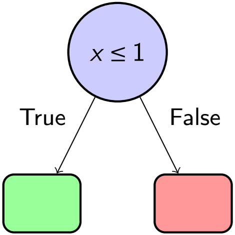




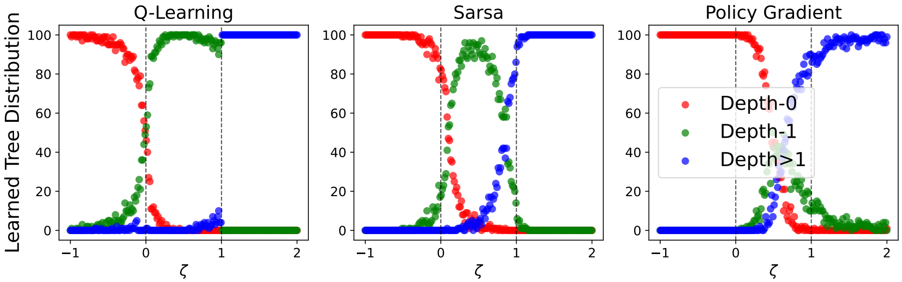
Classification MDP and the unique optimal depth-1 tree.





Classification MDP and the unique optimal depth-1 tree.
We show that deterministic partially observable policies for classification IBMDPs (⇔ decision tree policies) are in fact Markovian.

Result: RL can retrieve optimal depth-1 trees for the toy classification MDPs



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Decision trees in supervised learning

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- N data points. Each x_i is described by p features and has a label $y_i \in \mathcal{Y}$.

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- In between optimal and greedy?

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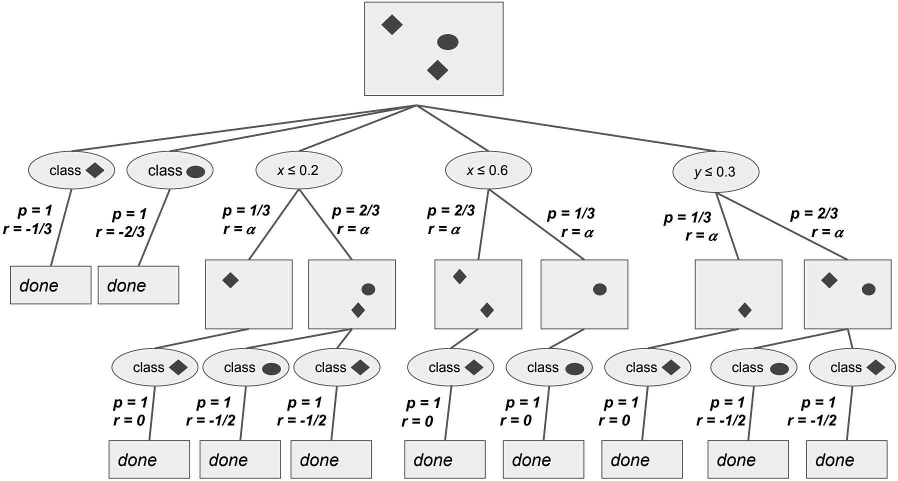

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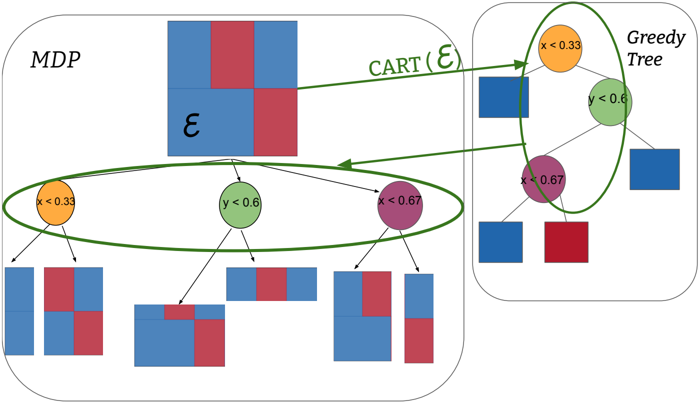
How to choose the B candidate actions/splits?

Dynamic Programming Decision Trees (DPDT)[1]



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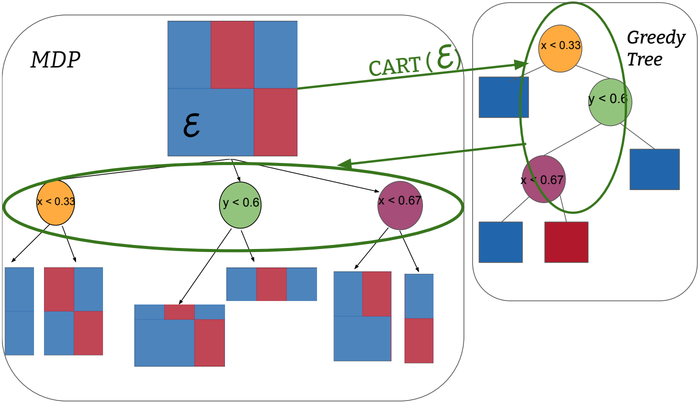

DPDT was presented at the 31st ACM SIGKDD conference.

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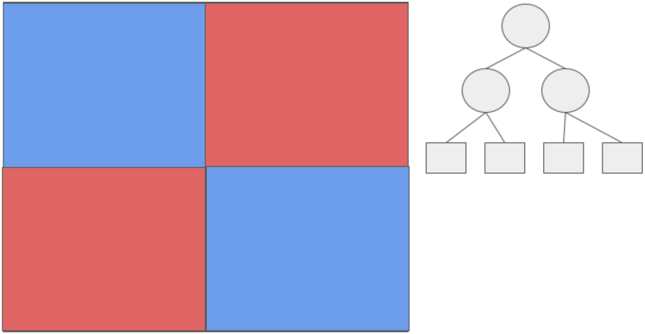
Theorem (DPDT trees can be strictly better than greedy trees)

There exist a depth budget D and a dataset for which DPDT trees are strictly better than greedy trees.

DPDT trees can be strictly better than greedy trees



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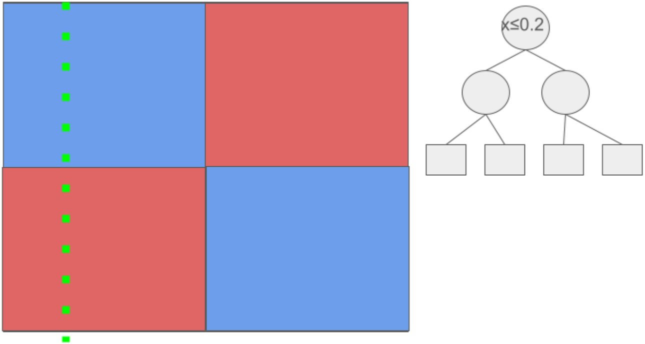

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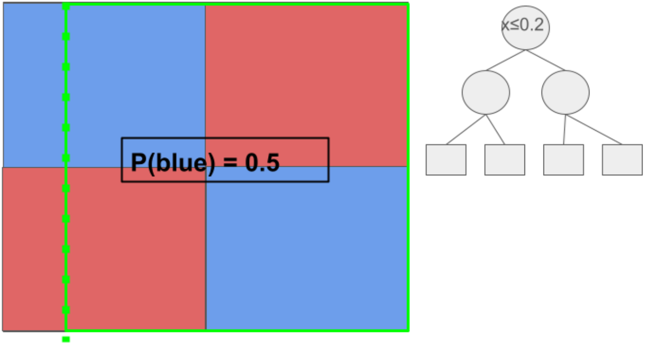

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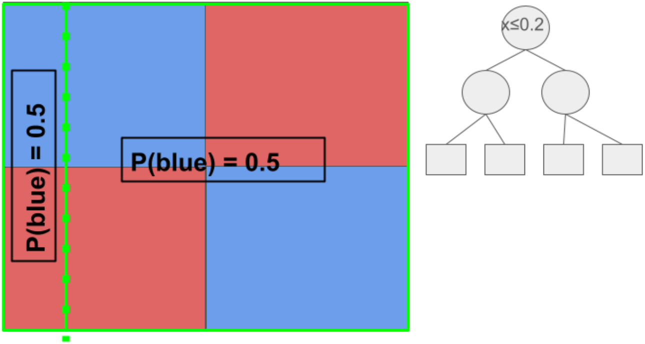

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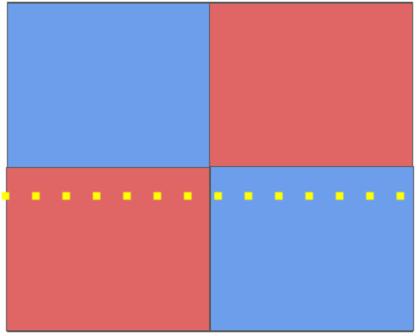

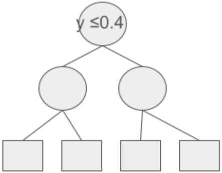

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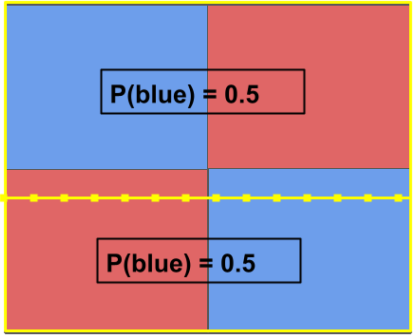

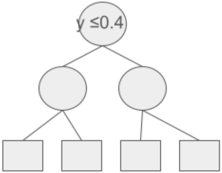

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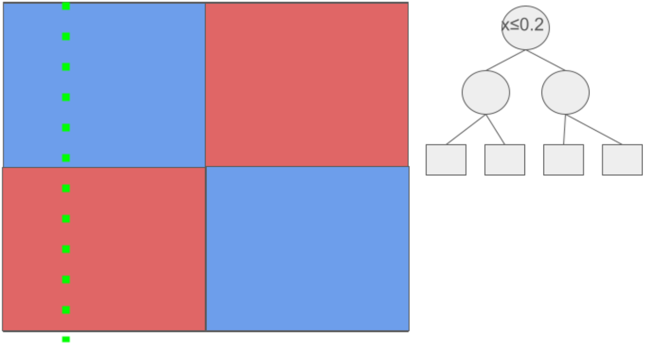

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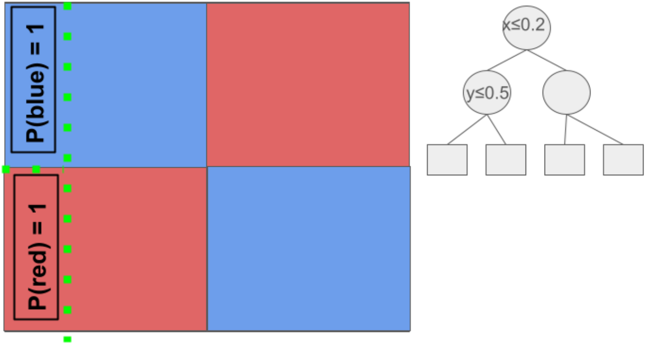

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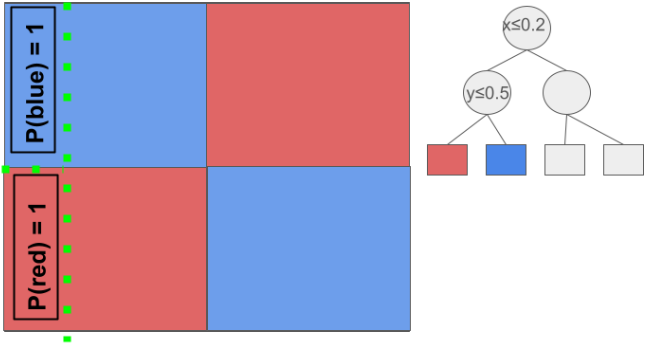

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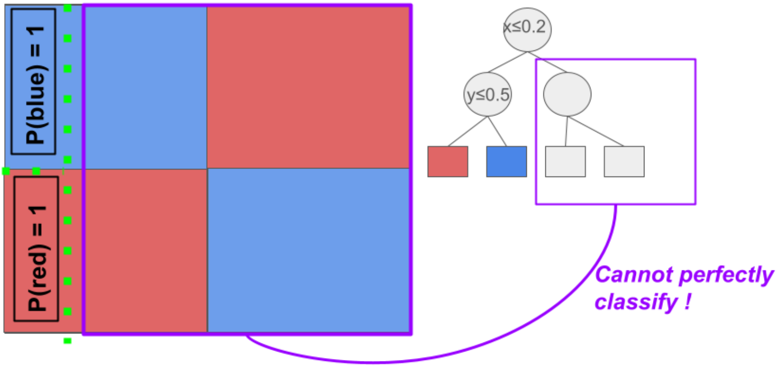

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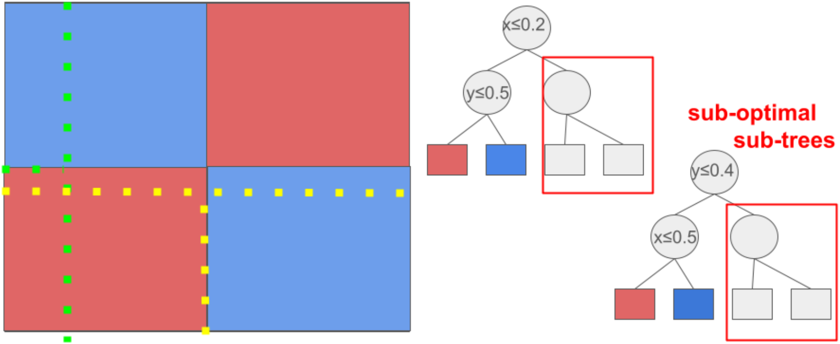

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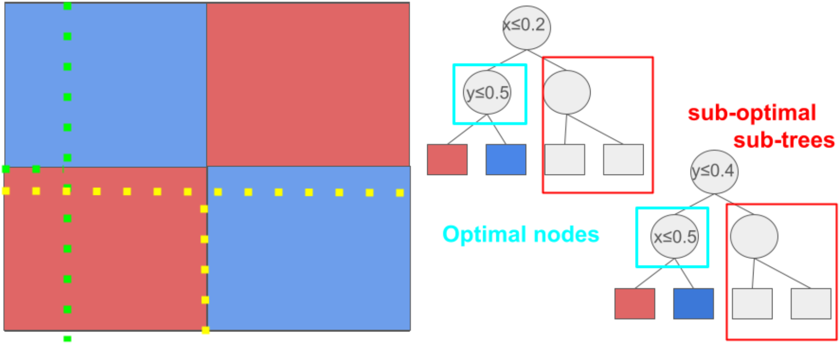
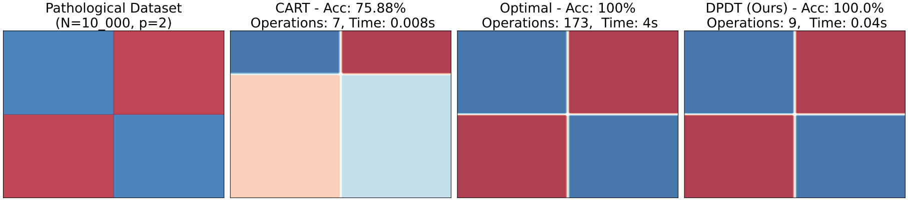


Fast like greedy trees, accurate like optimal trees

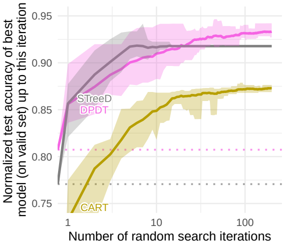


Comparing tree accuracy to complexity

Train accuracy and operation count when learning depth-3 decision trees.

			Accuracy				Operations			
Dataset	N	p	Opt Quant-BnB	Greedy CART	DPDT light	DPDT full	Opt Quant-BnB	Greedy CART	DPDT light	DPDT full
room	8103	16	**0.992**	0.968	0.991	**0.992**	10^6	15	286	16100
bean	10888	16	**0.871**	0.777	0.812	0.853	$5{\cdot}10^6$	15	295	25900
eeg	11984	14	**0.708**	0.666	0.689	0.706	$2{\cdot}10^6$	13	289	26000
avila	10430	10	**0.585**	0.532	0.574	**0.585**	$3{\cdot}10^7$	9	268	24700
magic	15216	10	**0.831**	0.801	0.822	0.828	$6{\cdot}10^6$	15	298	28000
htru	14318	8	**0.981**	0.979	0.979	0.980	$6{\cdot}10^7$	15	295	25300
occup.	8143	5	**0.994**	0.989	0.991	**0.994**	$7{\cdot}10^5$	13	280	16300
skin	196045	3	**0.969**	0.966	0.966	0.966	$7{\cdot}10^4$	15	301	23300
fault	1552	27	**0.682**	0.553	0.672	0.674	$9{\cdot}10^8$	13	295	24200
segment	1848	18	**0.887**	0.574	0.812	0.879	$2{\cdot}10^6$	7	220	16300
page	4378	10	**0.971**	0.964	0.970	0.970	10^7	15	298	22400
bidding	5056	9	**0.993**	0.981	0.985	**0.993**	$3{\cdot}10^5$	13	256	9360
raisin	720	7	**0.894**	0.869	0.879	0.886	$4{\cdot}10^6$	15	295	20900
rice	3048	7	**0.938**	0.933	0.934	0.937	$2{\cdot}10^7$	15	298	25500
wilt	4339	5	**0.996**	0.993	0.994	0.995	$3{\cdot}10^5$	13	274	11300
bank	1097	4	**0.983**	0.933	0.971	0.980	$6{\cdot}10^4$	13	271	7990

DPDT trees generalization



DPDT depth-5 trees vs. other detph-5 trees

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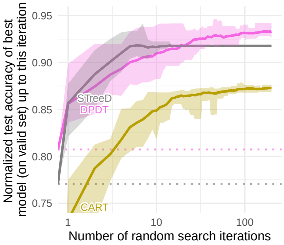

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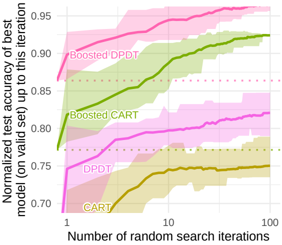
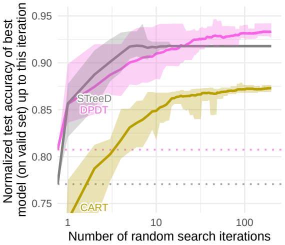
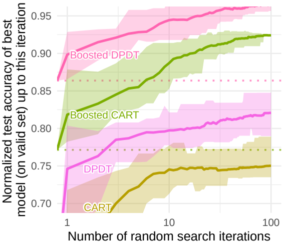
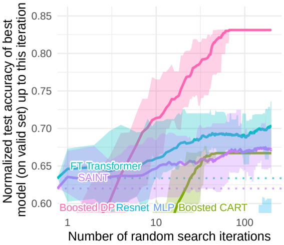


Boosted DPDT vs. Boosted CART







DPDT depth-5 trees vs. other detph-5 trees

Boosted DPDT vs. Boosted CART

Boosted DPDT vs. other classifiers

Perspectives

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Q: Are decision trees really the most interpretable model?

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Q: Are decision trees really the most interpretable model?

A: It depends.

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- Less parameters mean more interpretability [Fre14; Lav99].
- Time to formally verify a policy decreases with interpretability [Bar+20].

A methodology to measure policy interpretability without humans

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- Different hardwares (CPUs vs GPUs).
- Different implementations (matrix operations vs fully sequentially) ...

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```
# Decision tree for Mountain Car
def play(x):
    if x[1] <= -0.2597:
        if x[1] <= -0.6378:
            return 0
        else:
            if x[0] <= -1.0021:
                return 2
            else:
                return 0
    else:
        if x[1] <= -0.0508:
            if x[0] <= 0.2979:
                if x[0] <= 0.0453:
                    return 2
                else:
                    if x[1] <=
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```

```
# Small ReLU MLP for Pendulum
def play(x):
    h_layer_0_0 = 1.238*x[0]+0.971*x
     [1]
                  +0.430*x[2]+0.933
    h_layer_0_0 = max(0, h_layer_0_0
     )
    h_layer_0_1 = -1.221*x[0]+1.001
                  *x[1]-0.423*x[2]
                  +0.475
    h_layer_0_1 = max(0, h_layer_0_1
     )
    h_layer_1_0 = -0.109*h_layer_0_0
                  -0.377*h_layer_0_1
                  +1.694
    h_layer_1_0 = max(0, h_layer_1_0
     )
    h_layer_1_1 = -3.024*h_layer_0_0
                  -1.421*h_layer_0_1
                  +1.530
    h_layer_1_1 = max(0, h_layer_1_1
     )

    h_layer_2_0 = -1.790*h_layer_1_0
                  +2.840*h_layer_1_1
                  +0.658
    y_0 = h_layer_2_0
    return [y_0]
```

Empirical validation

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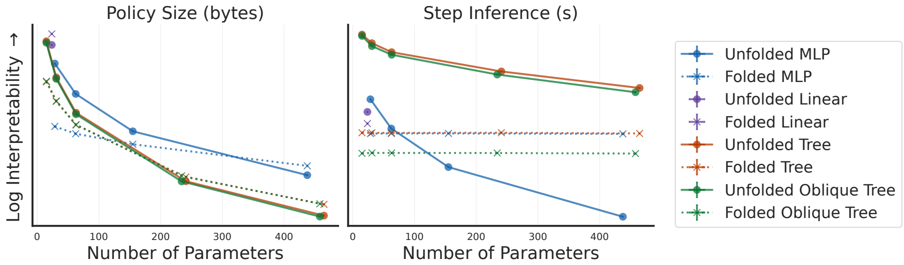
Setup

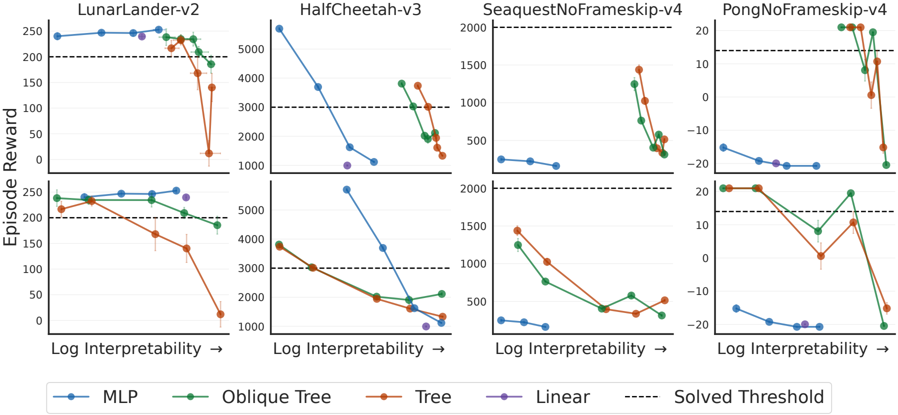
We imitate ∼ 40000 expert policies from `stable-baselines3` using various policy classes/nb parameters on various environments.

Result: unfolding policies is necessary to respect consensus



Aggregated policies interpretability on classic control environments

Result: there is no dominating policy class for all environments



Interpretability-Performance trade-offs. Top row, interpretability is measured with step inference times. Bottom row, the interpretability is measured with policy size.

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- Tree-like policy classes can have good inductive bias (e.g. Atari).
- Can a human study confirm our results?
- What about (very) big models?
- Can we use our policy programs as low level skills (hierarchical RL)?

Conclusion: interpretable machine learning is a difficult research topic

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My hope

Motivate interpretability by finding a real-world problem where interpretability is *really* necessary [Nag+24].

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