

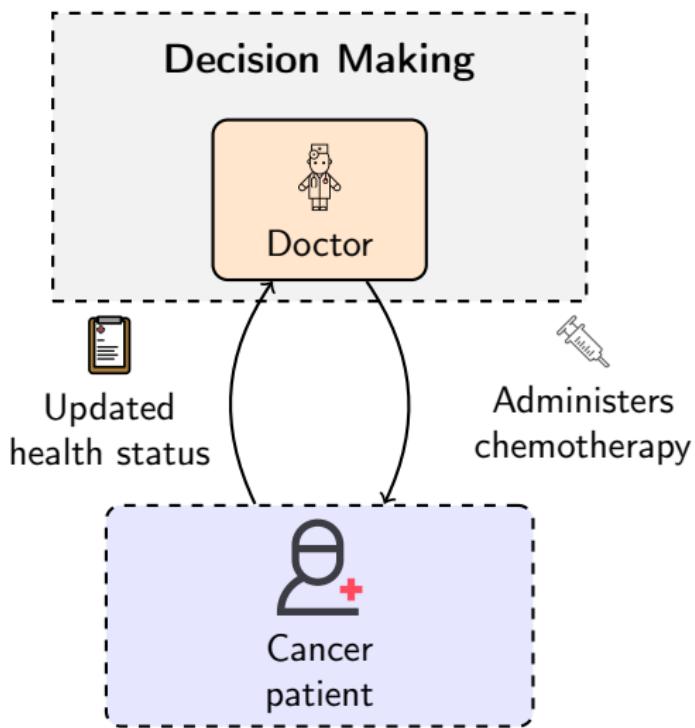
# Interpretability, Decision Trees, and Sequential Decision Making

Hector Kohler

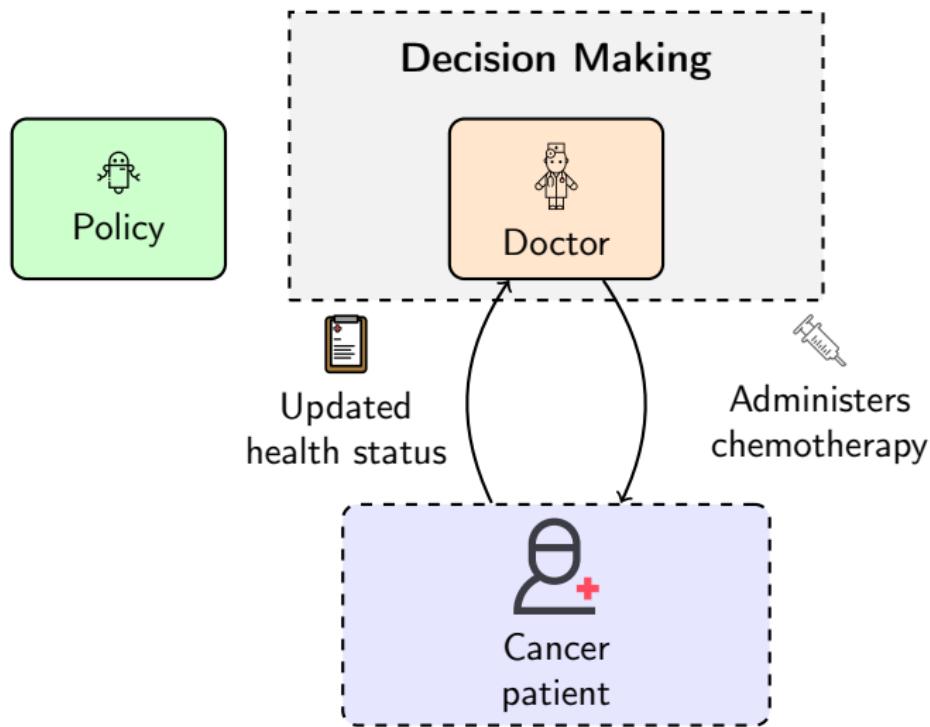
Supervised by Dr. Riad Akrour (HdR) and Prof. Philippe Preux (HdR)  
Université de Lille, CNRS, Inria, UMR CRIStAL 9189, France

November 27, 2025

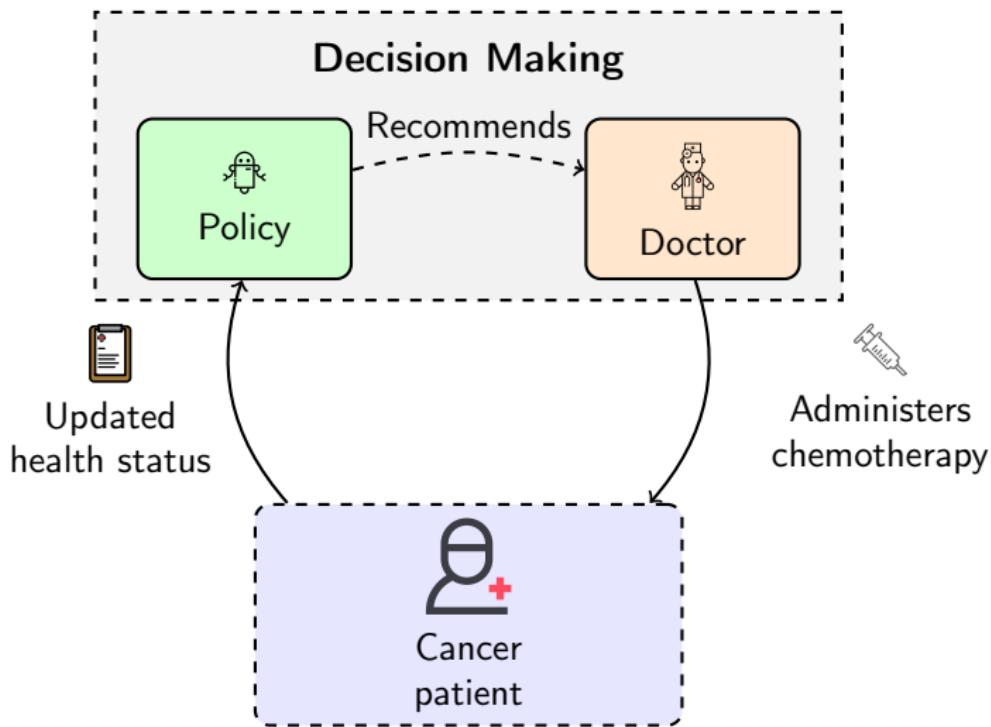
# Sequential decision making (SDM) and machine learning (ML)



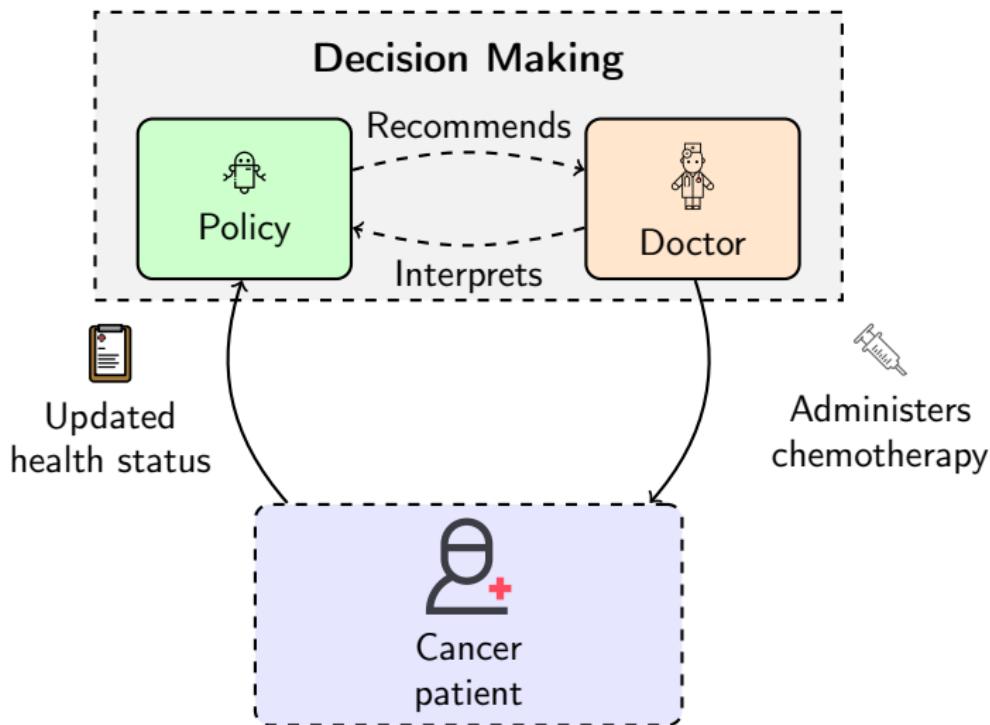
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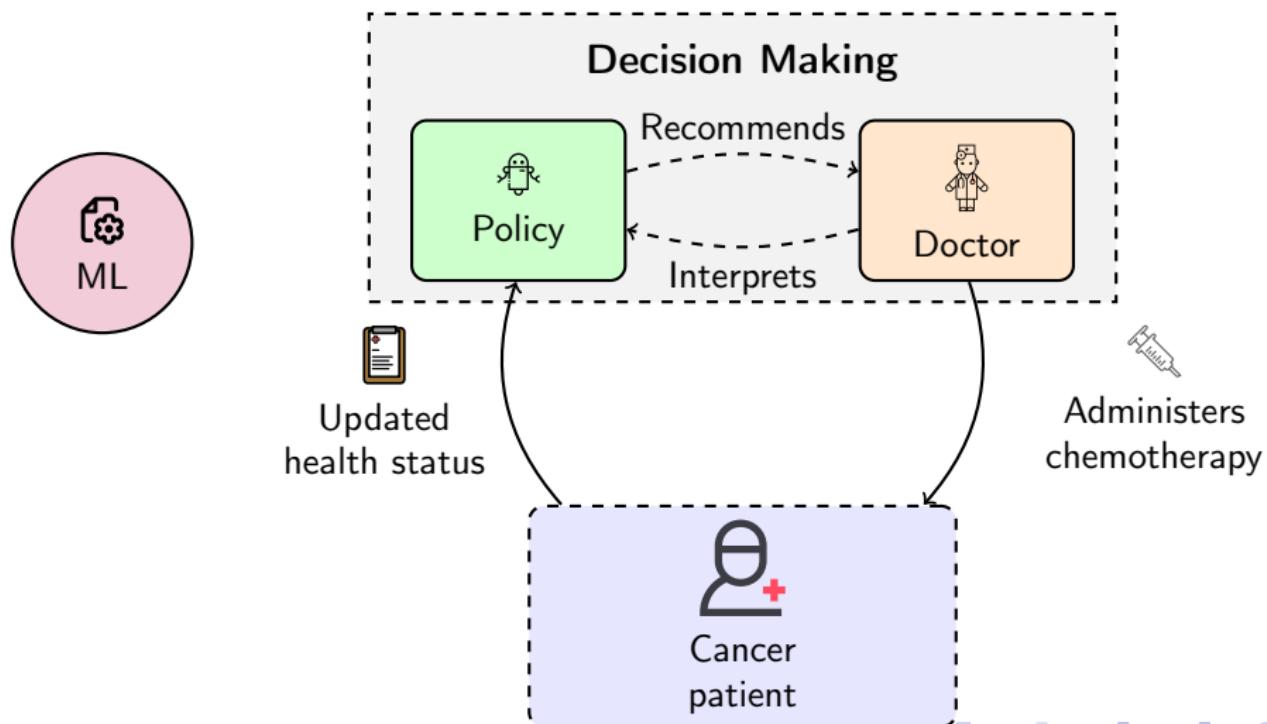
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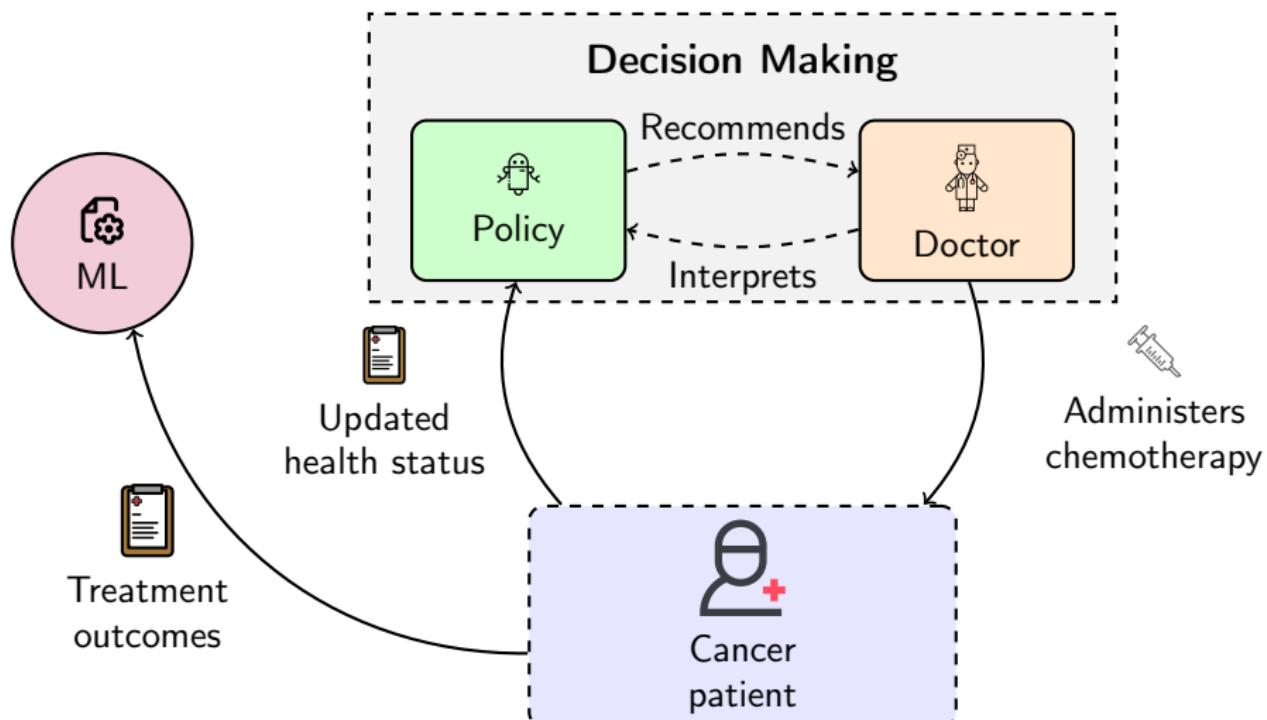
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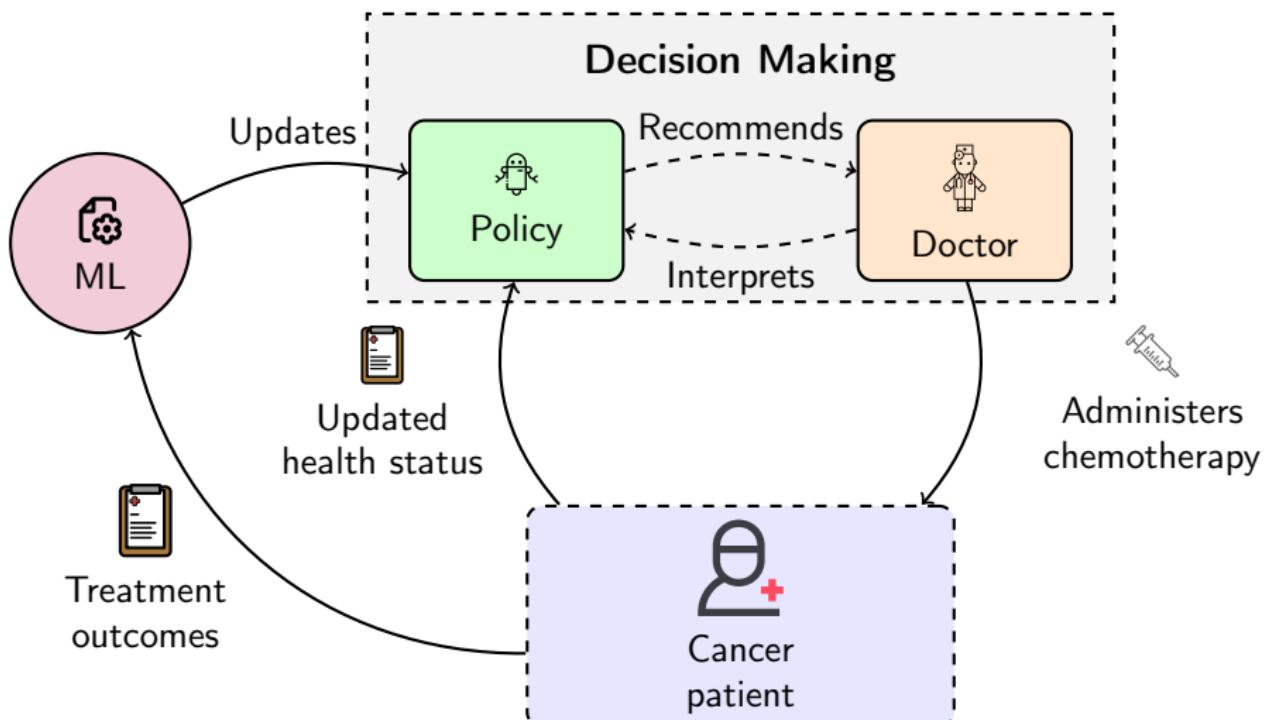
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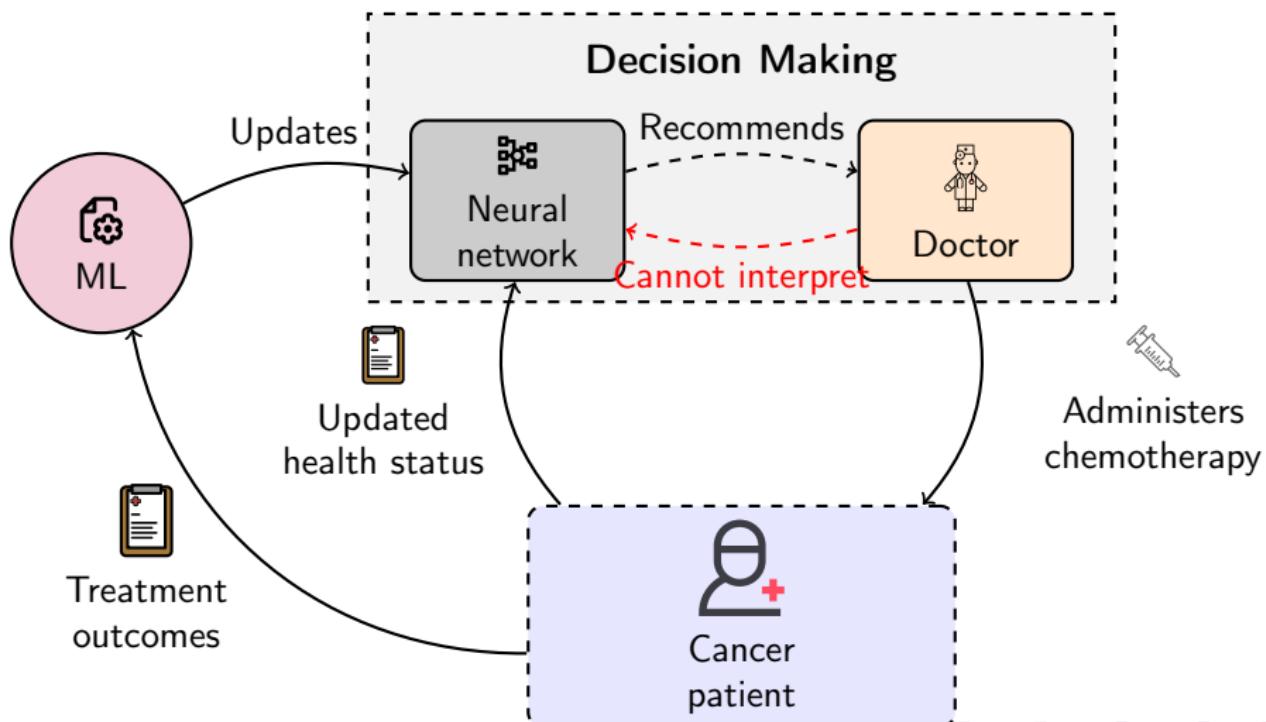
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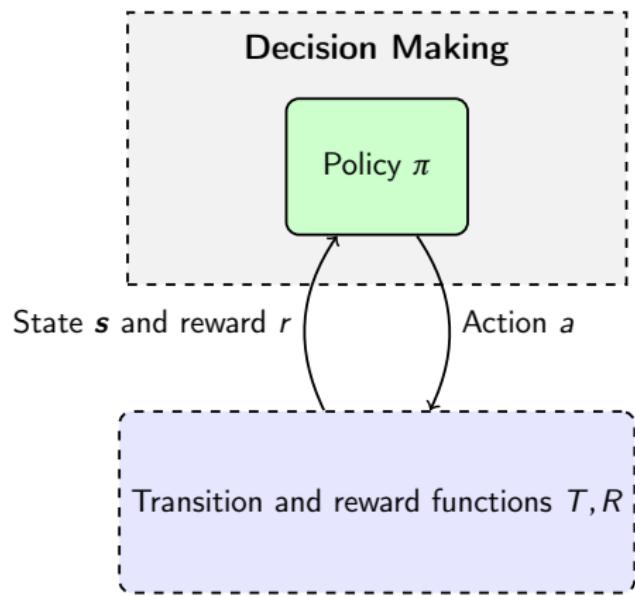
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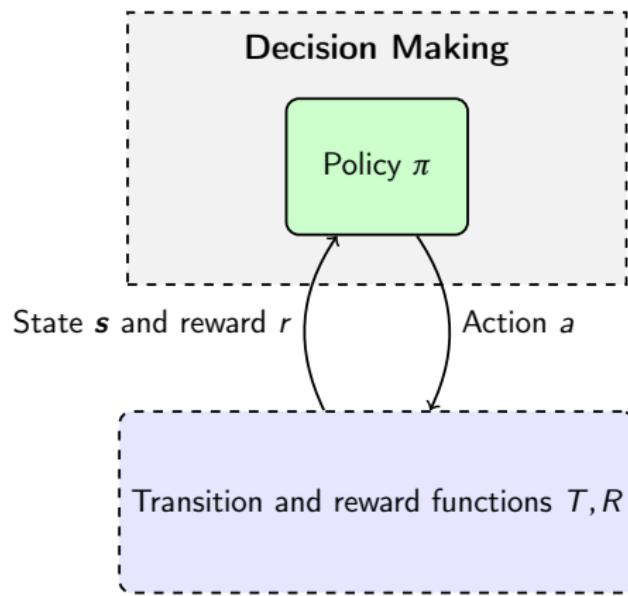


# Markov decision processes (MDPs) and reinforcement learning (RL)



Markov decision processes [Put94].

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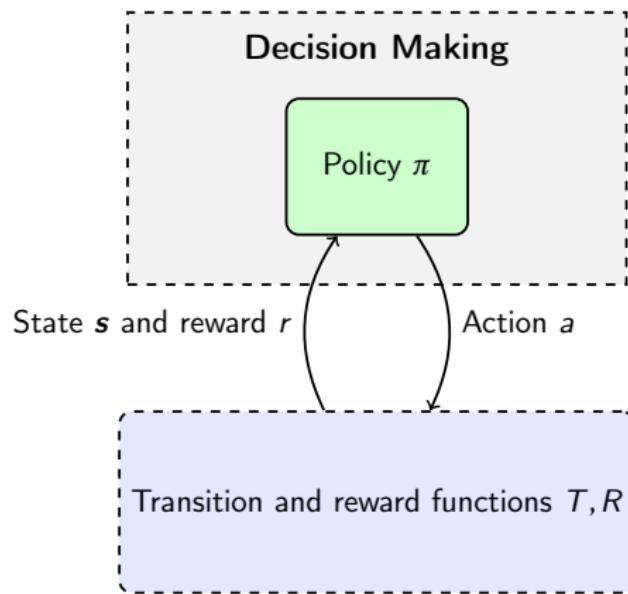


- RL [SB98] aims to find a policy,  $\pi : S \rightarrow A$  that maximizes:

$$\mathbb{E}_{s_t \sim T} \left[ \sum_{t=0}^{\infty} \gamma^t R(s_t, \pi(s_t)) \right]$$

Markov decision processes [Put94].

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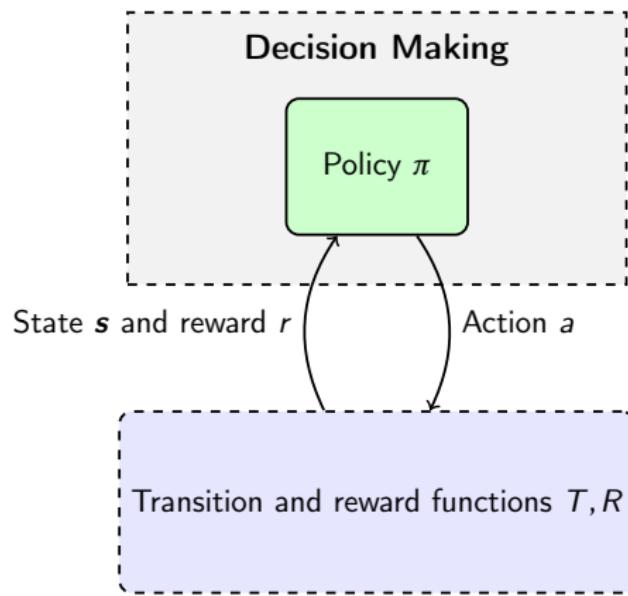
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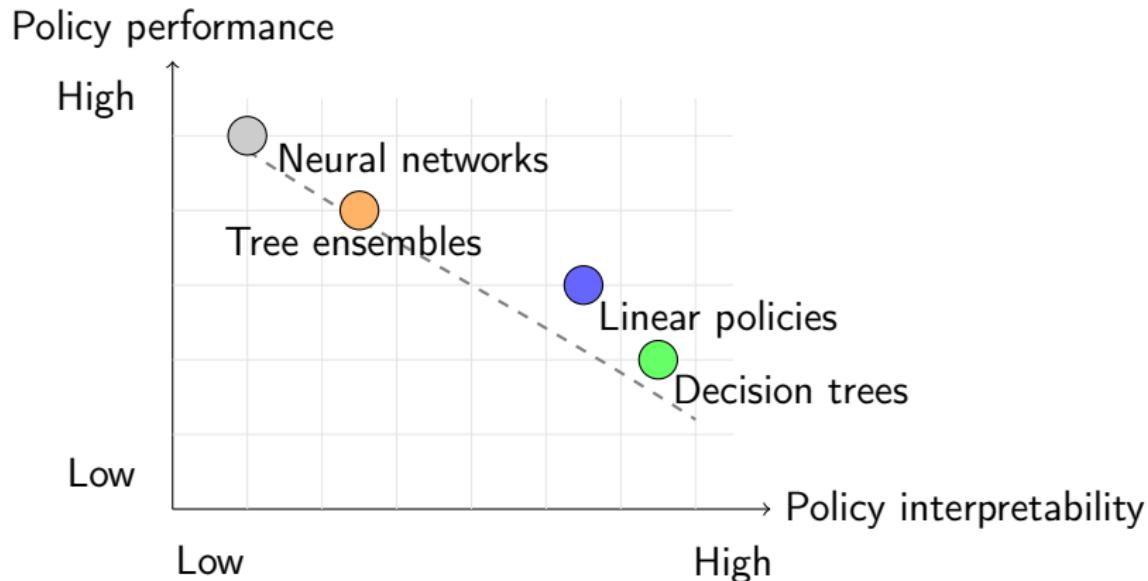
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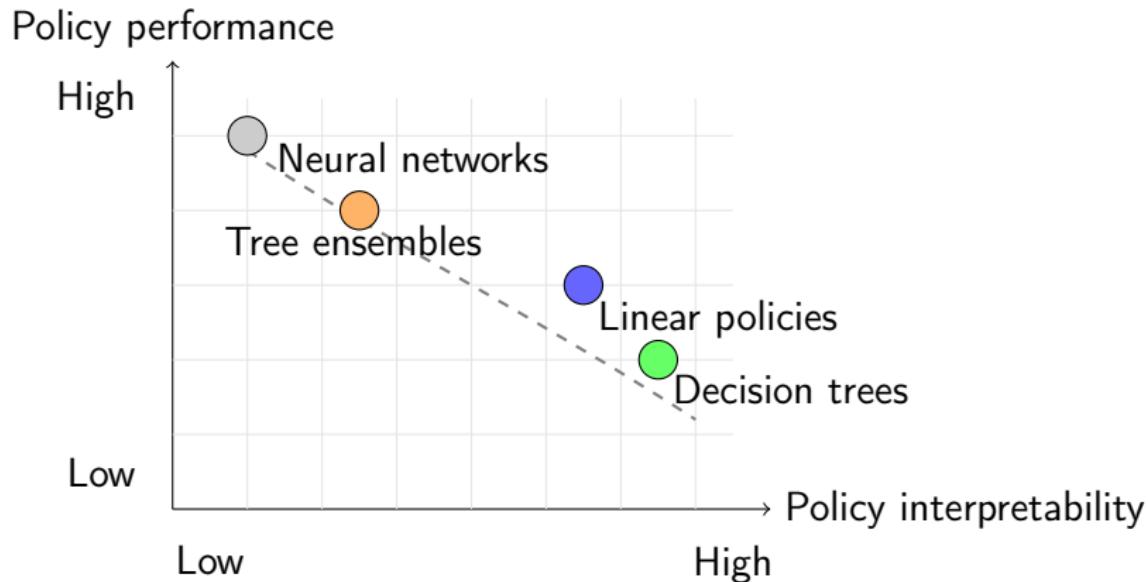
- Lots of successful RL algorithms [SB98; Mn+15; Sch+17].
- Few interpretability concerns.

# Policy interpretability



**Heuristic** interpretability-performance trade-offs of different policy classes.

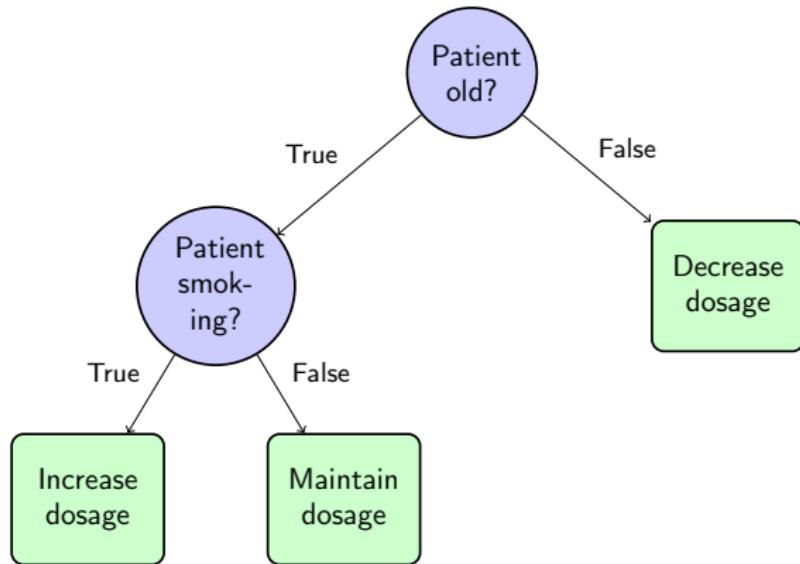
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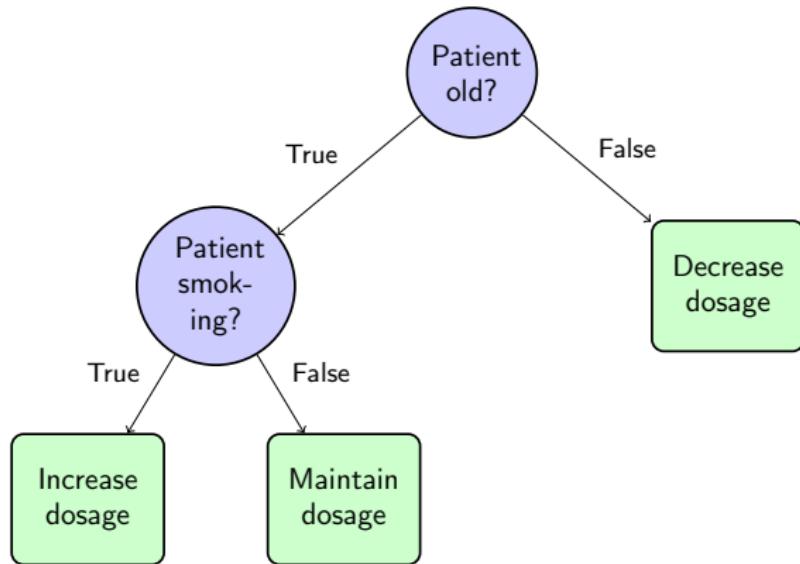
⚠ No definition of interpretability for machine learning models!

# Decision trees



A generic decision tree of depth  $D = 2$ .

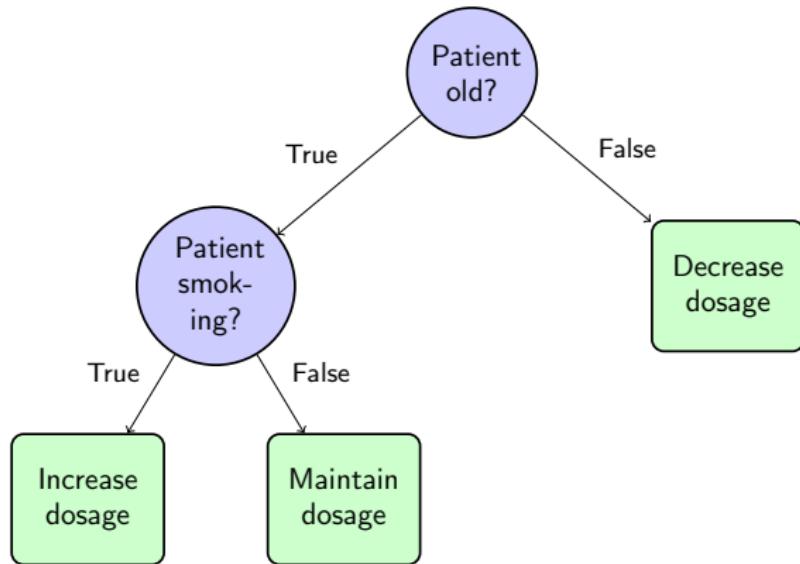
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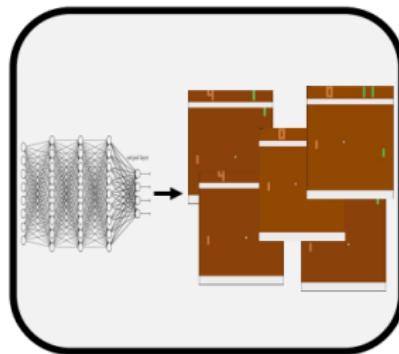


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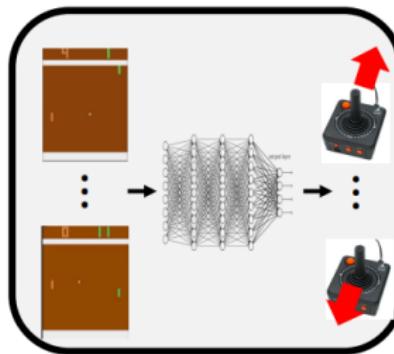
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What about SDM?

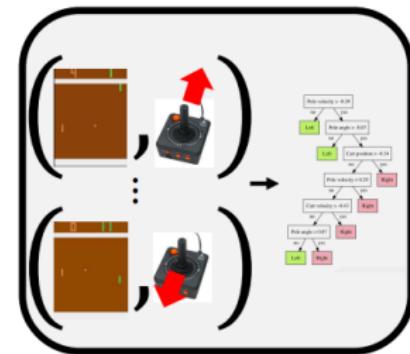
# Indirect approach: imitation learning



**Step 1:** Use NN to generate states

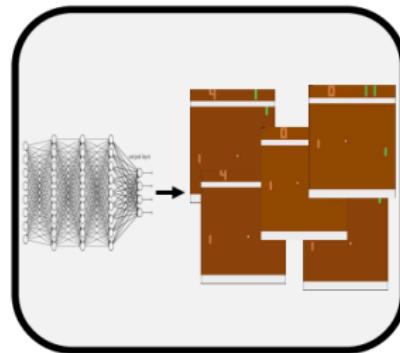


**Step 2:** Use NN to obtain actions

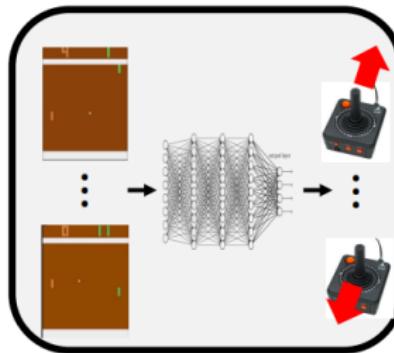


**Step 3:** Use supervised learning  
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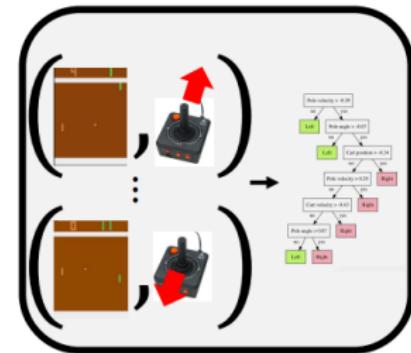
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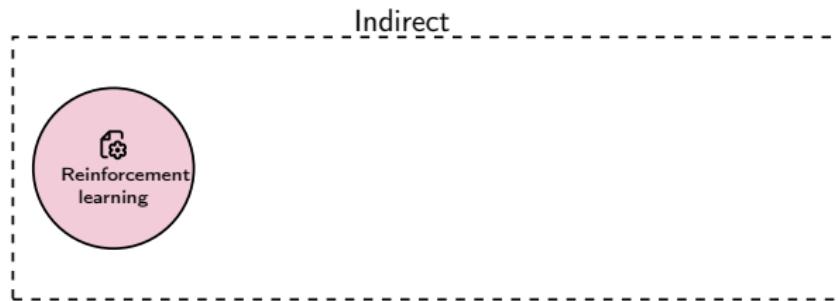
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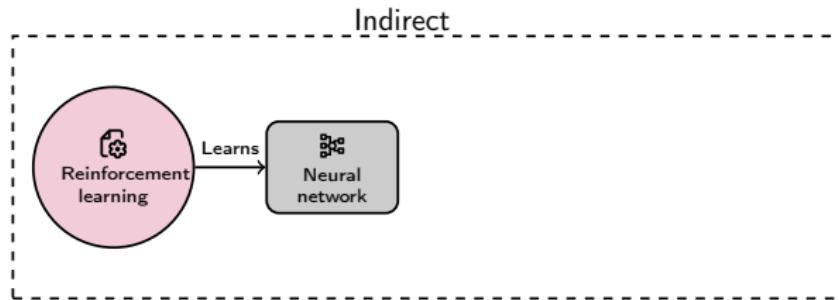
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Most research focused on indirect learning of interpretable policies [RGB10;  
BPS18; Ver+18; Mil+24].

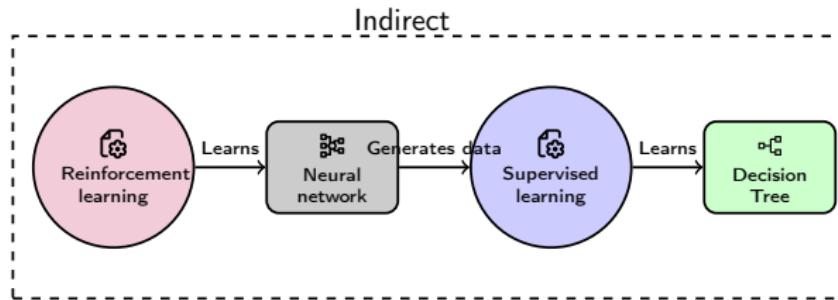
# Two ways to get interpretable policies for SDM



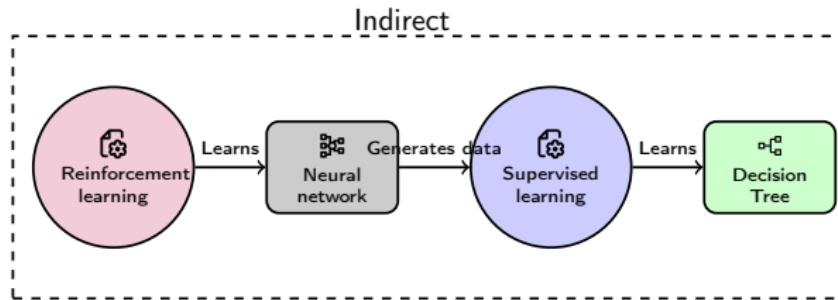
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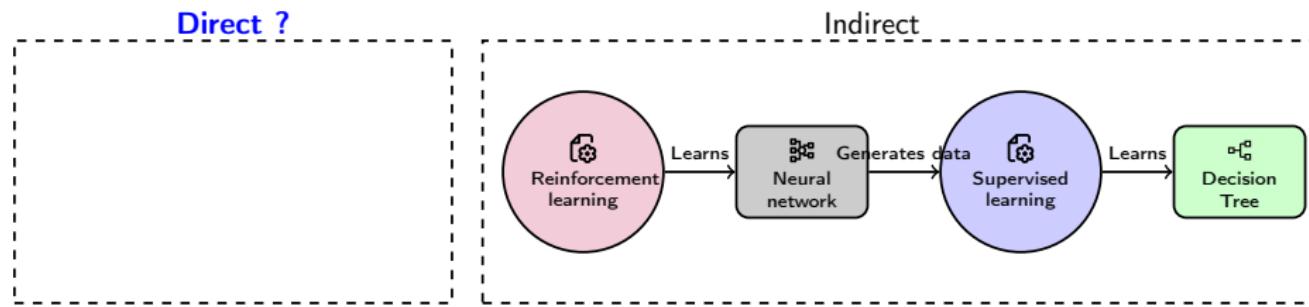


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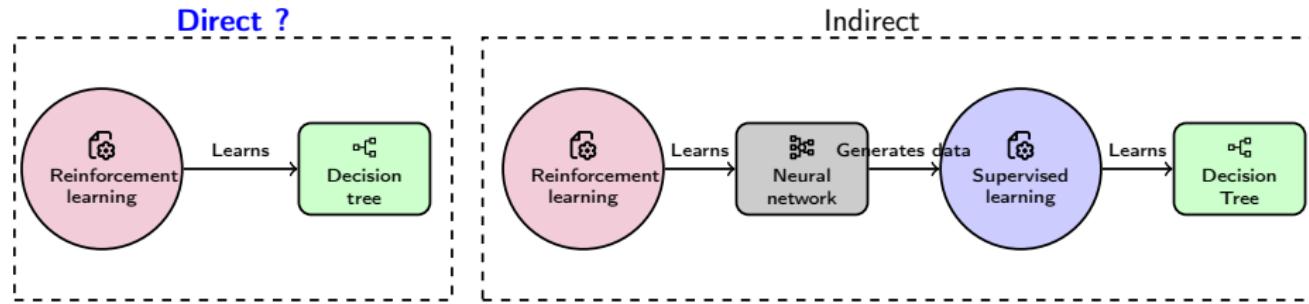
⚠ Policies obtained indirectly optimize a surrogate objective rather than a trade-off between interpretability and cumulative rewards.

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# Contributions

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- ② How to leverage DM to learn interpretable classifiers for supervised learning?
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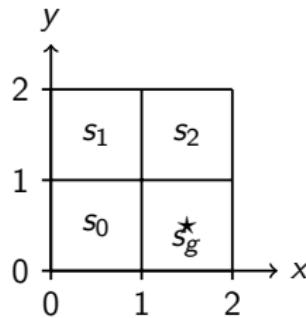
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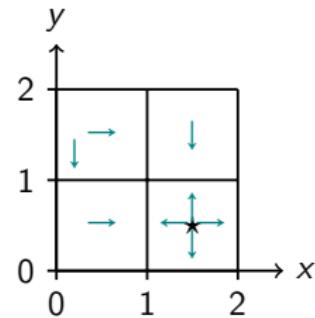
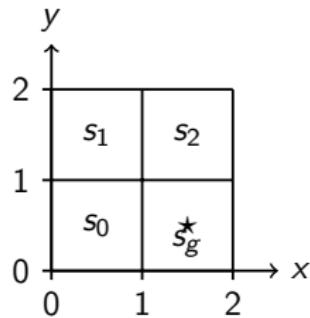
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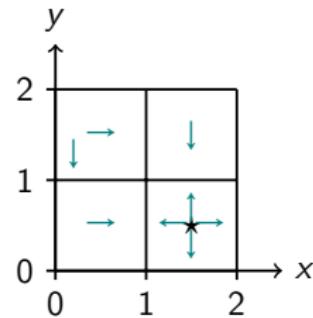
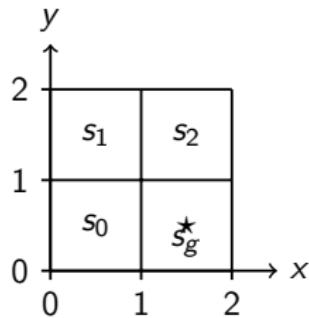
# Grid world MDP and decision tree policies



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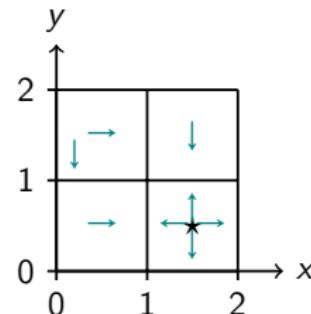
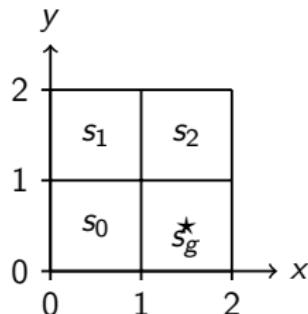


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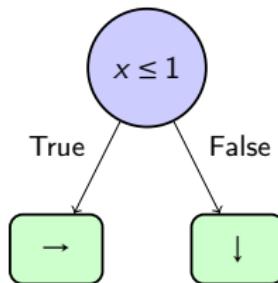


Grid world MDP and optimal actions.

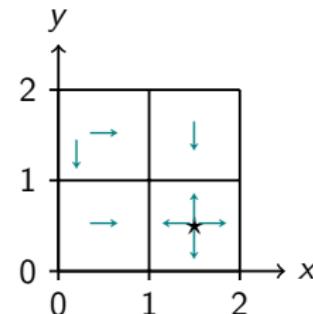
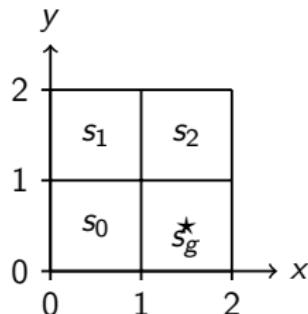
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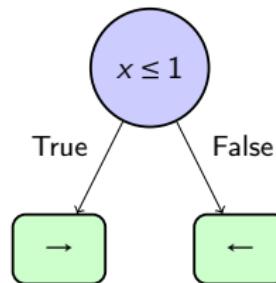
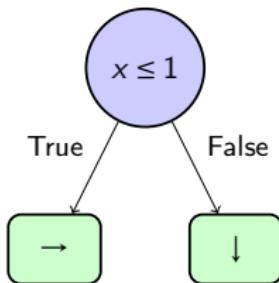
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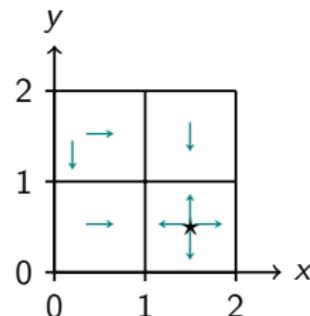
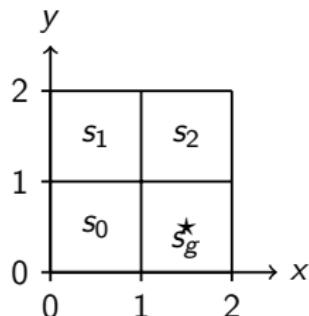
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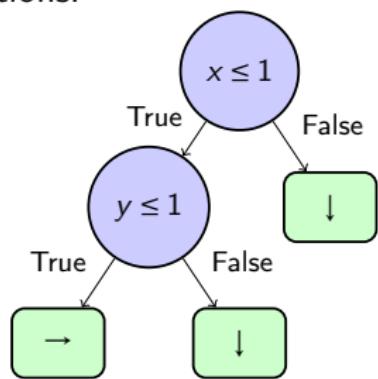
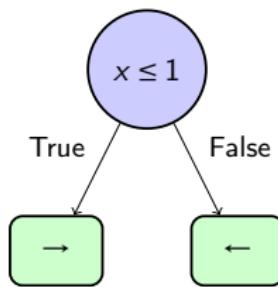
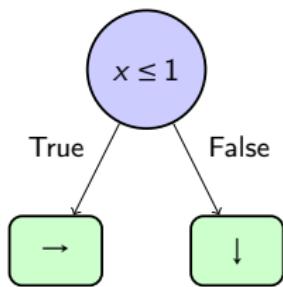
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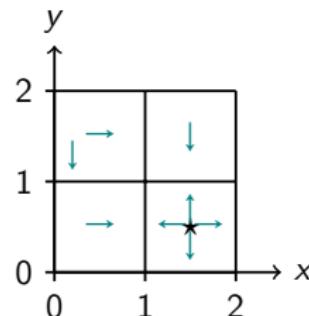
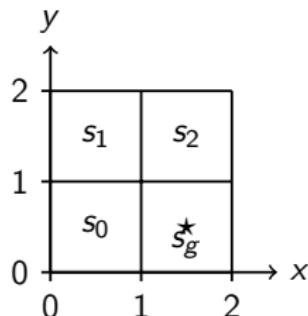
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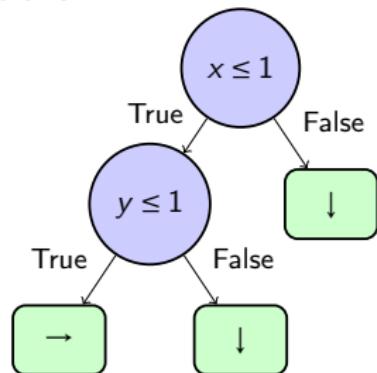
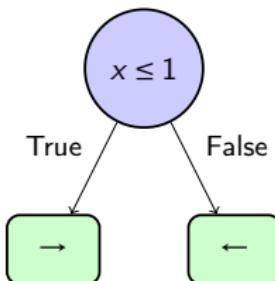
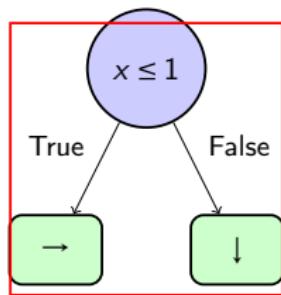
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Decision tree policies with different interpretability-performance trade-offs.

# Direct RL of decision tree policies with iterative bounding Markov decision processes

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Given an MDP  $\mathcal{M} \langle S, A, R, T \rangle$ , an associated iterative bounding Markov decision process (IBMDP, [Top+21])  $\mathcal{M}_{IB}$  is an MDP:

$$\langle \underbrace{S \times O}_{\text{Augmented state space}}, \underbrace{A \cup A_{info}}_{\text{Augmented action space}}, \underbrace{(R, \zeta)}_{\text{Augmented rewards}}, \underbrace{(T_{info}, T)}_{\text{Augmented transitions}} \rangle$$

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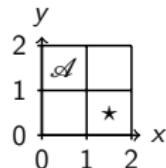
## IBMDPs promises

- No need to design new algorithm: we can use RL.
- IBMDP rewards trade-off naturally interpretability and performances.

# Grid world IBMDP example

$t = 0$

$$s_t = (0.5, 1.5)$$

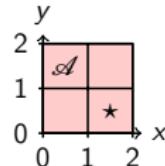


# Grid world IBMDP example

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$$o_t = (0, 2, 0, 2)$$

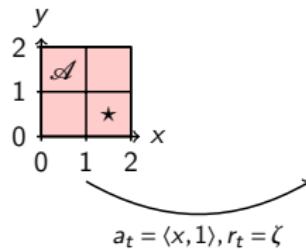


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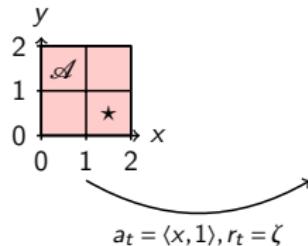
$$a_t = \langle x, 1 \rangle, r_t = \zeta$$

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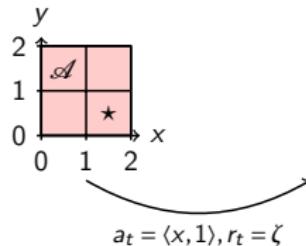
$$x \leq 1$$

# Grid world IBMDP example

$$t = 0$$

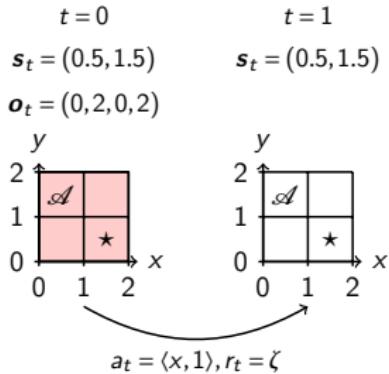
$$s_t = (0.5, 1.5)$$

$$o_t = (0, 2, 0, 2)$$



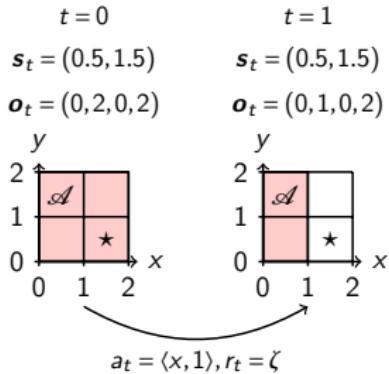
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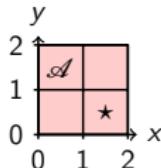
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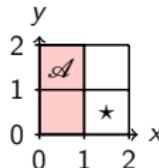
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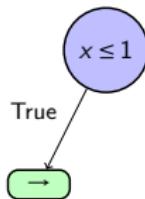
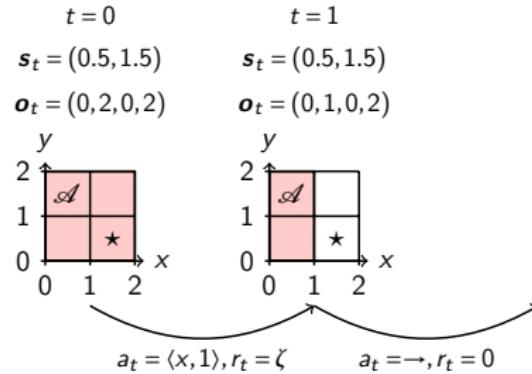


$$a_t = \langle x, 1 \rangle, r_t = \zeta$$

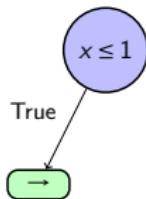
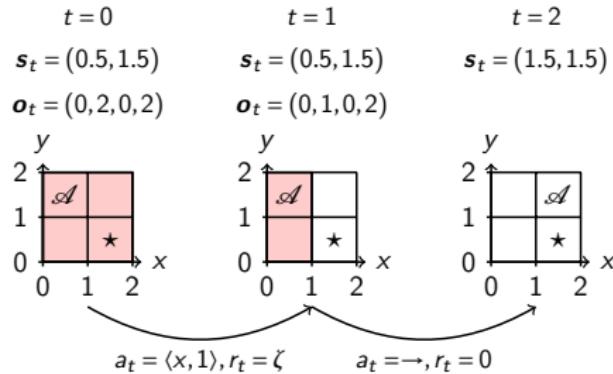
$$a_t = \rightarrow, r_t = 0$$

A light blue circle containing the inequality  $x \leq 1$ .

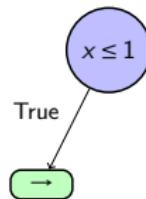
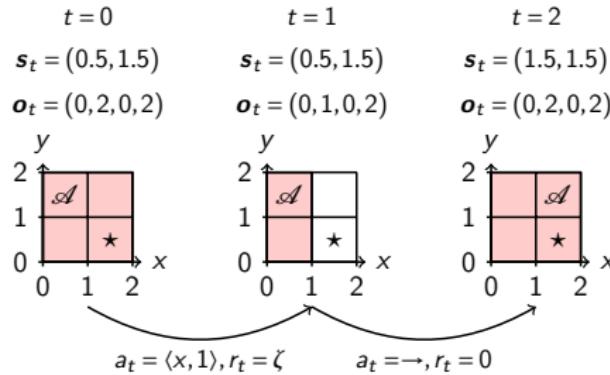
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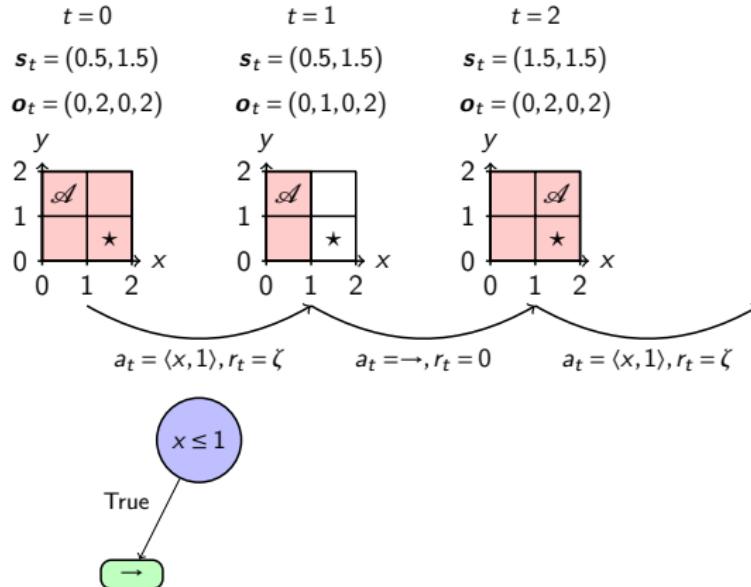
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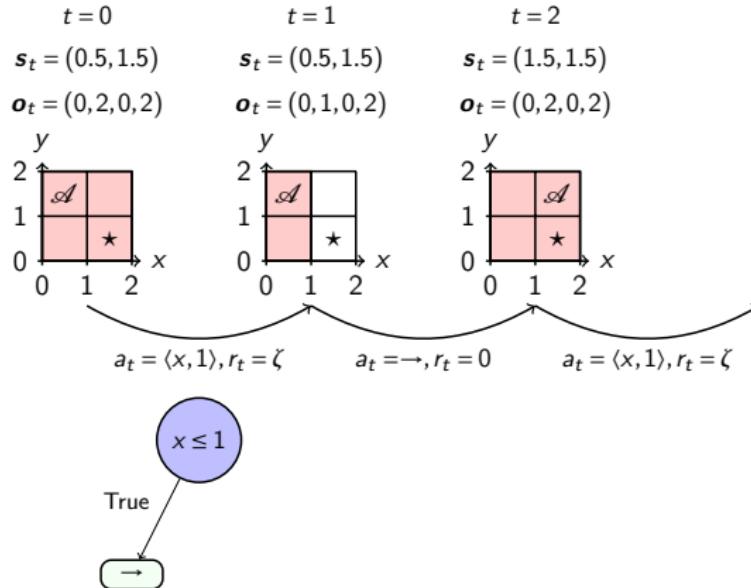
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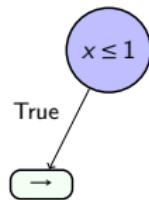
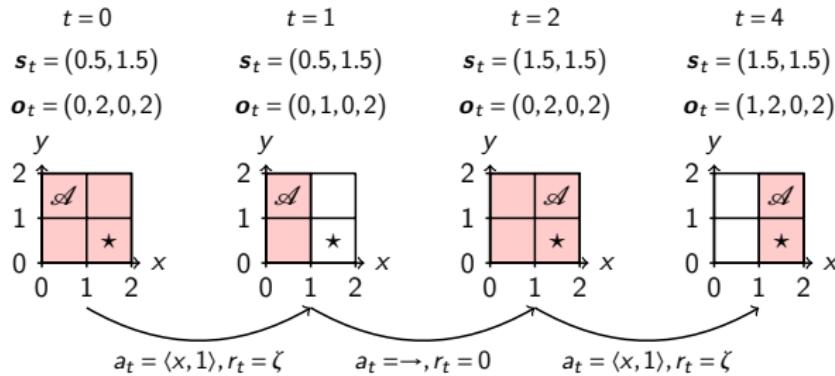
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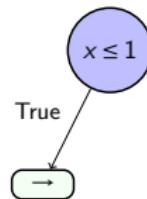
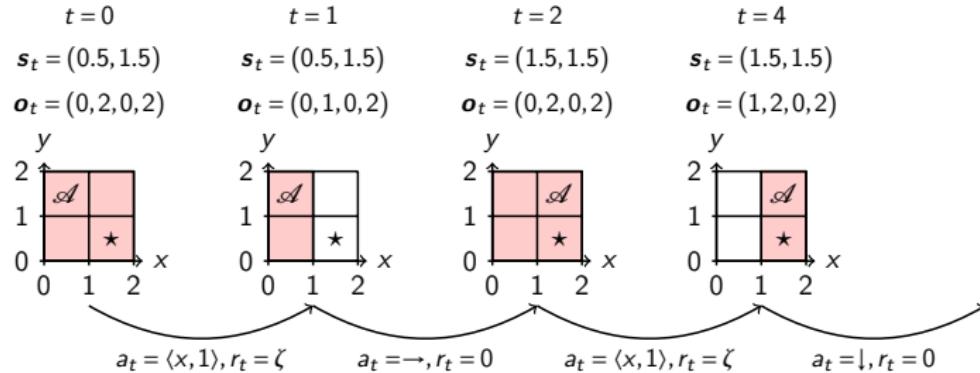
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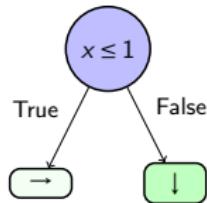
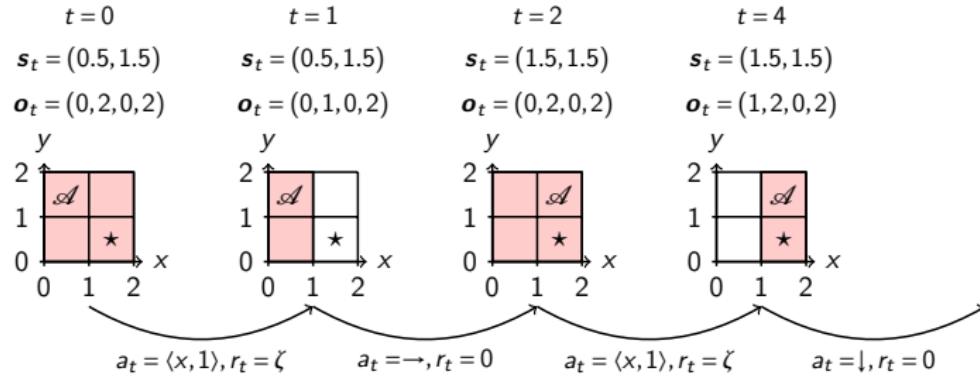
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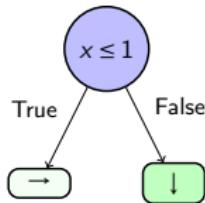
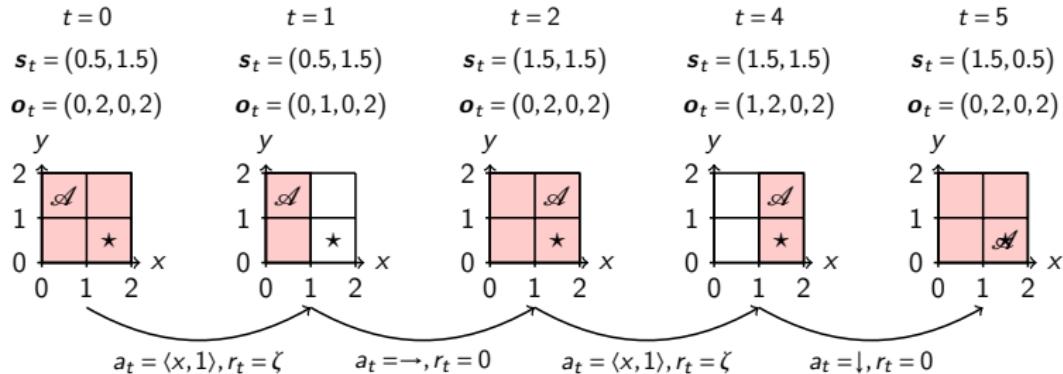
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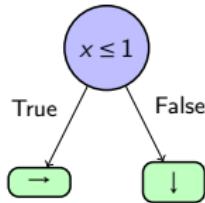
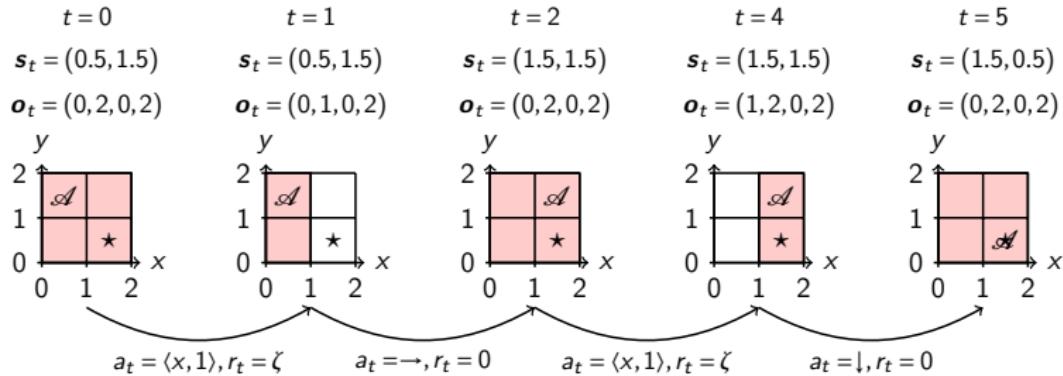
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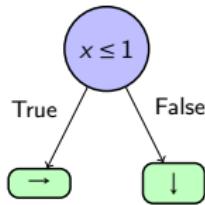
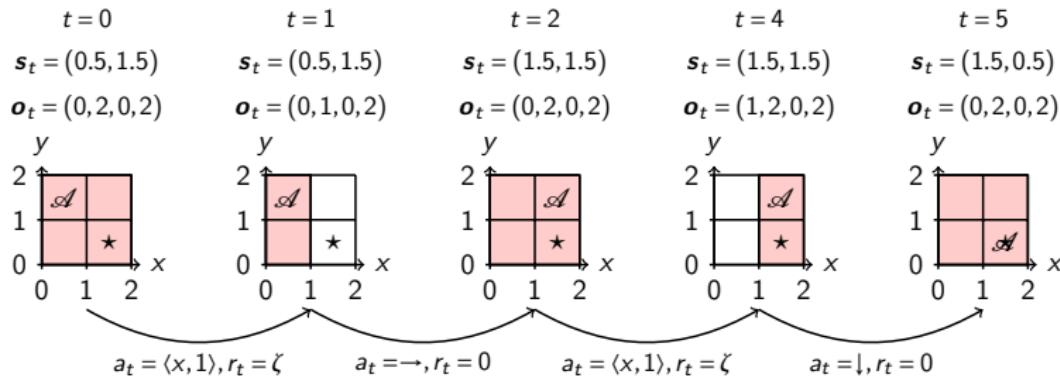
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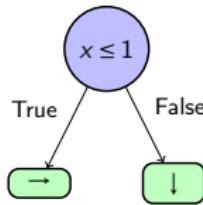
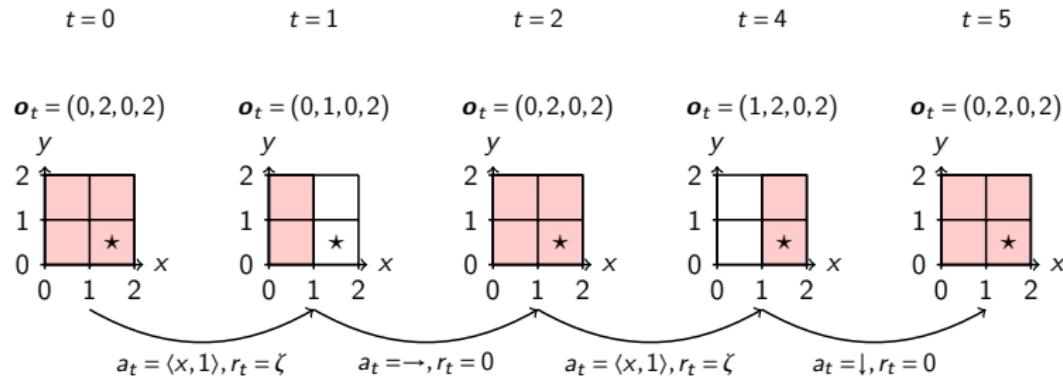


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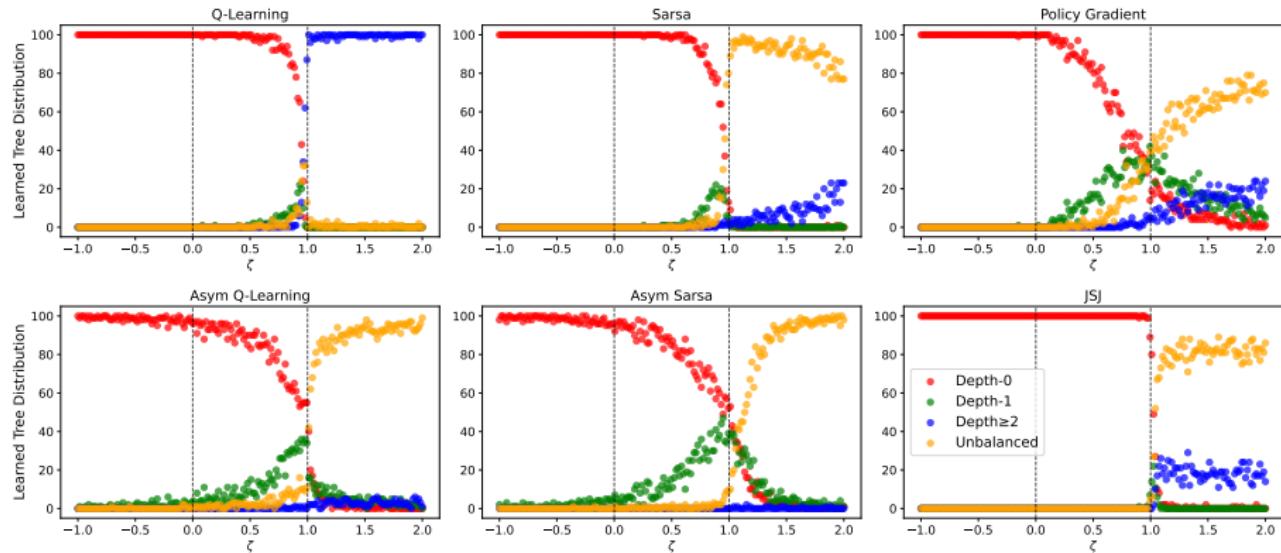
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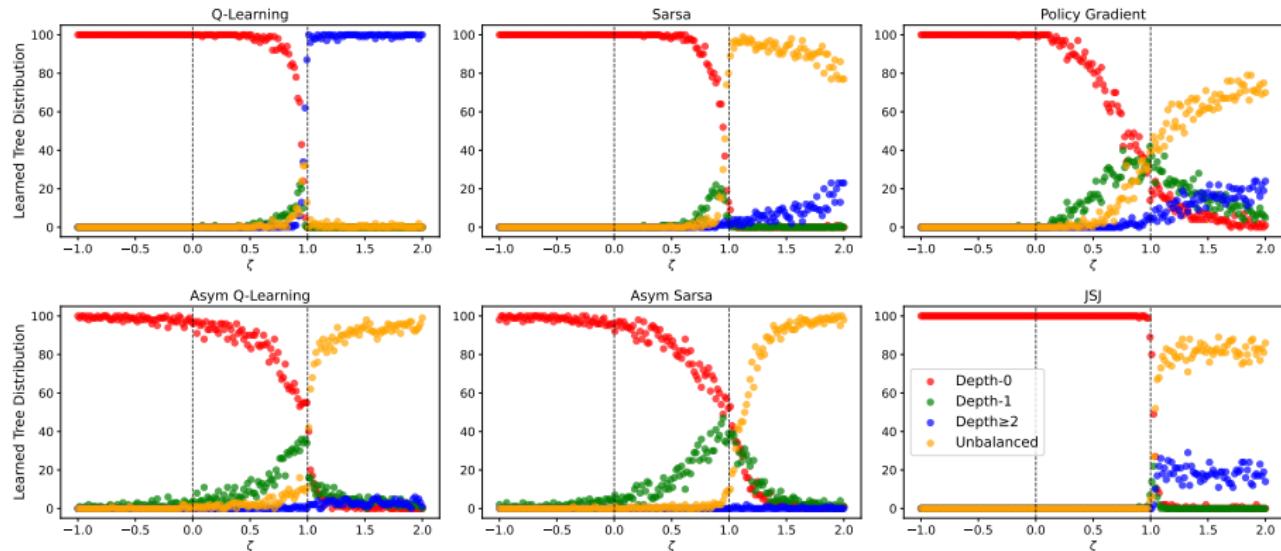
Hard problem

Finding the best **deterministic** and **partially observable** policy is NP-hard  
[Lit94]!

# Result: RL cannot retrieve optimal depth-1 trees for the grid world MDP

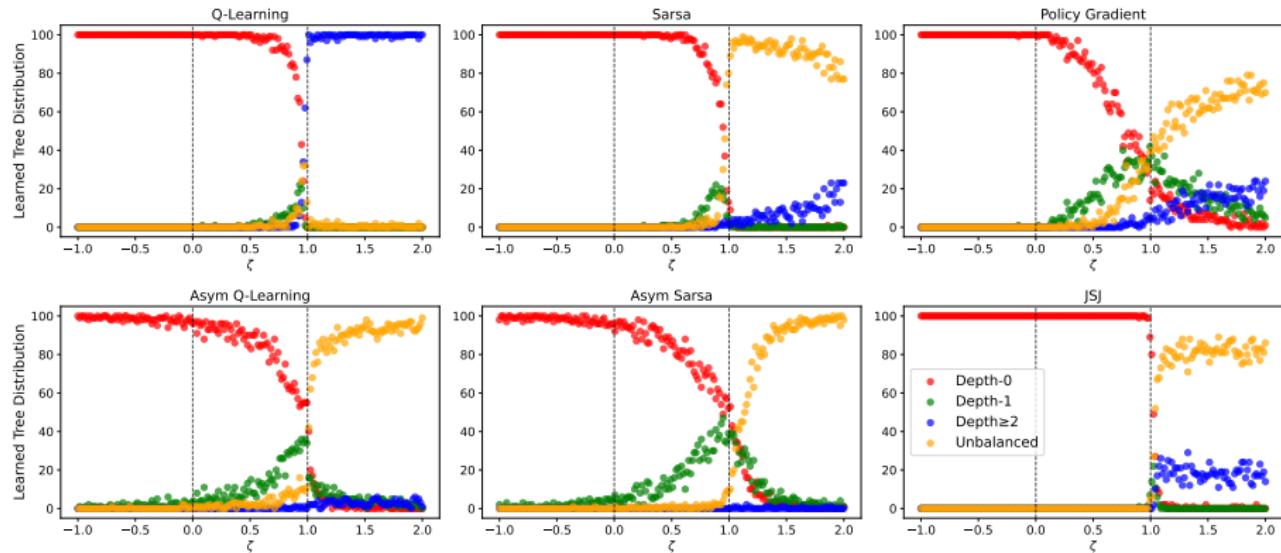


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Distributions of tree policies learned with (asymmetric) RL algorithms [SB98; SJ94; LS98; BA22; BDA22] as a function of the interpretability reward  $\zeta$ .

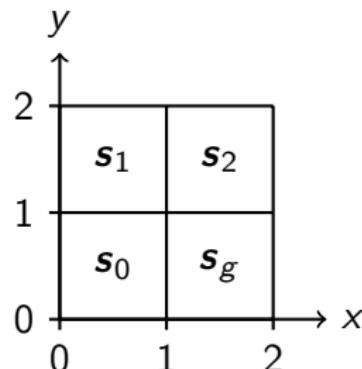
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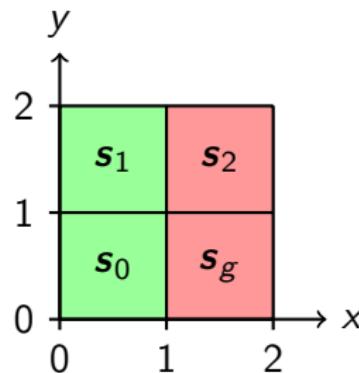
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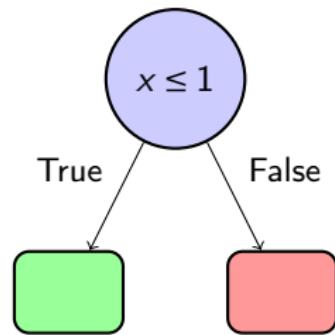
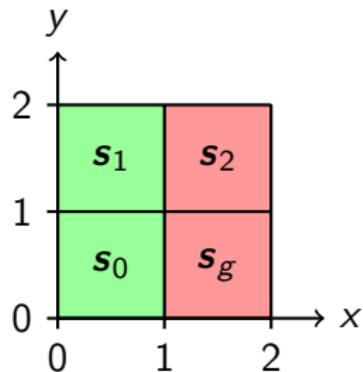


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Classification MDP and the unique optimal depth-1 tree.

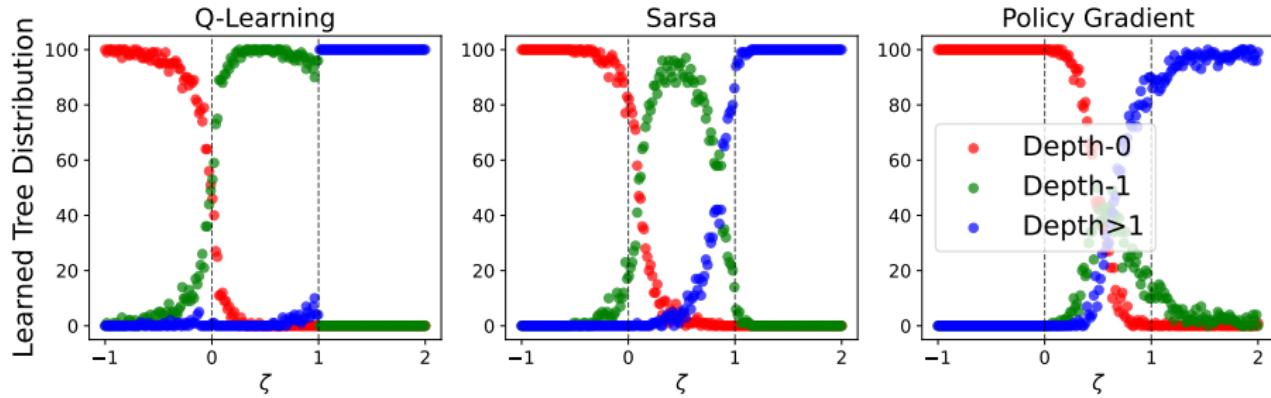
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Classification MDP and the unique optimal depth-1 tree.

Deterministic partially observable policies for classification IBMDPs  
( $\Leftrightarrow$  decision tree policies) are in fact Markovian.

Result: RL can retrieve optimal depth-1 trees for the toy classification MDP



Distributions of tree policies learned with various RL algorithms.

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*Q: Can we leverage SDM to design new decision tree induction algorithms for the supervised learning setting? A: Yes!*

# Decision trees in supervised learning

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- $N$  data points  $\{\mathbf{x}_i, y_i\}$ . Each  $\mathbf{x}_i$  is described by  $p$  features and has a label  $y_i \in \mathcal{Y}$ . We want to find a tree of depth at most  $D$   $T \in \mathcal{T}_D$  that minimizes:

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- In between optimal and greedy?

# Decision tree induction as solving MDPs

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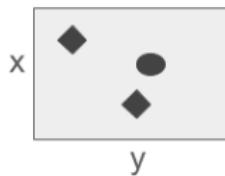
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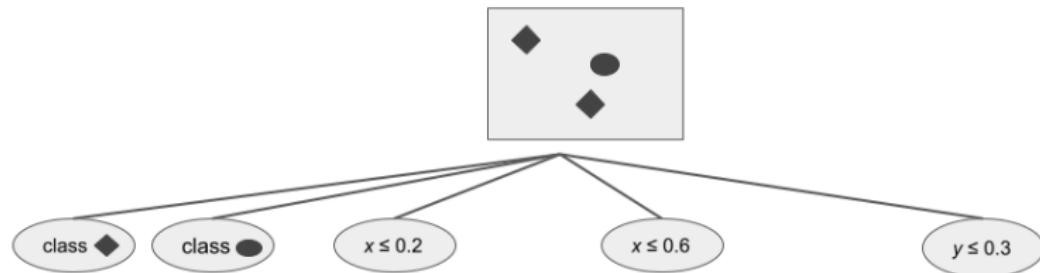
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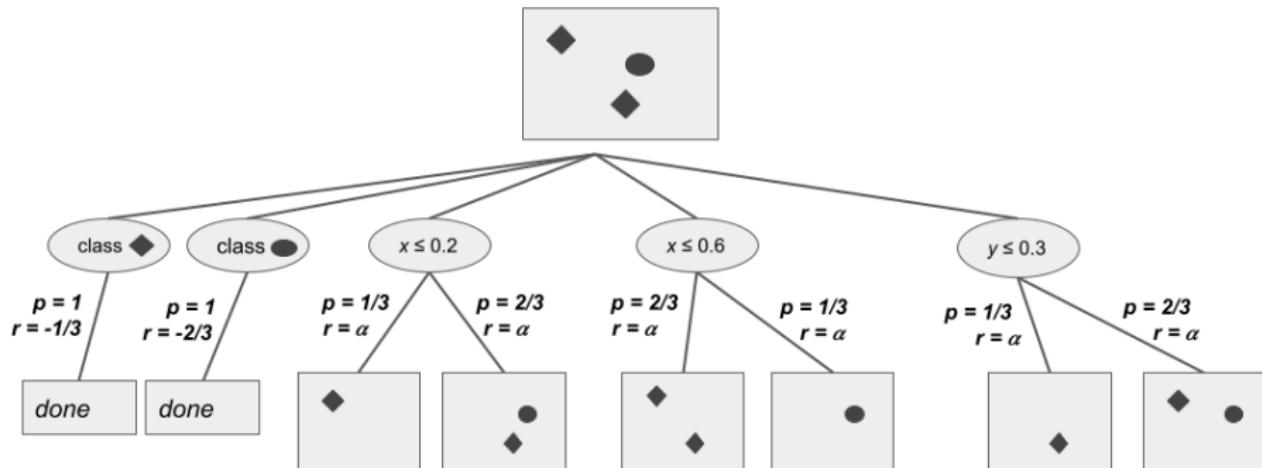
Example of decision tree induction as an MDP.

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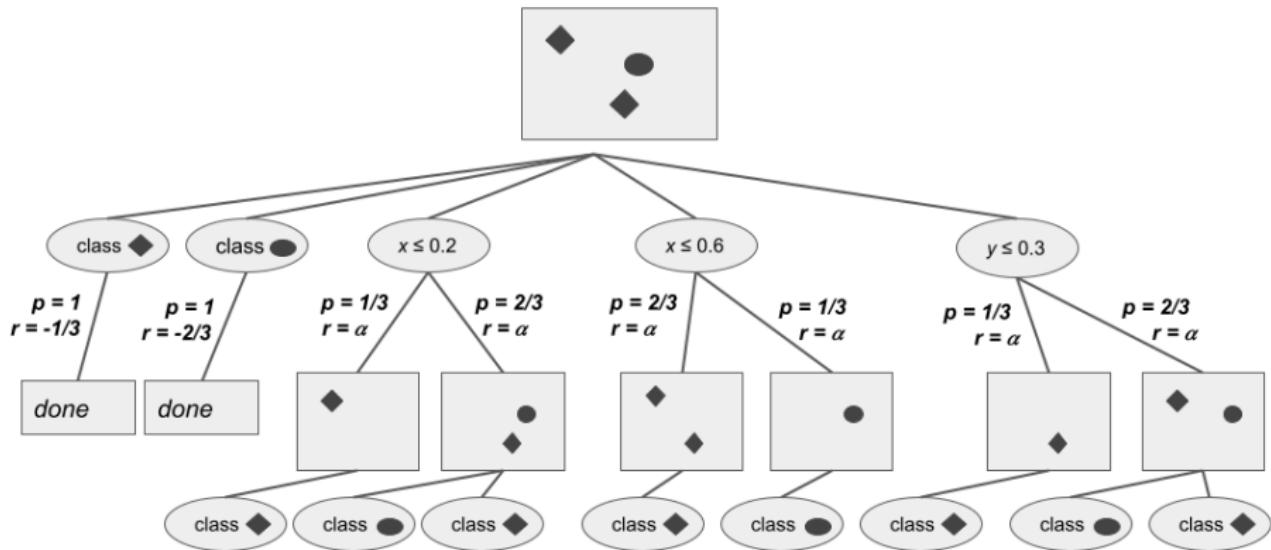
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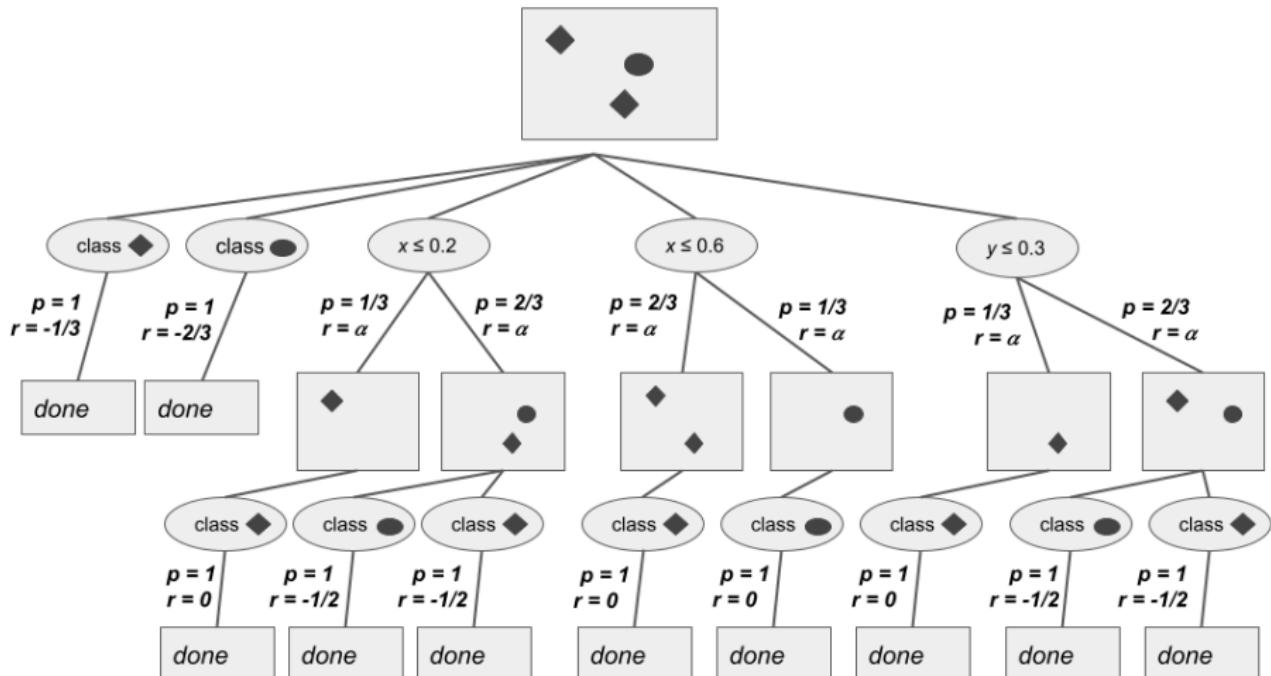
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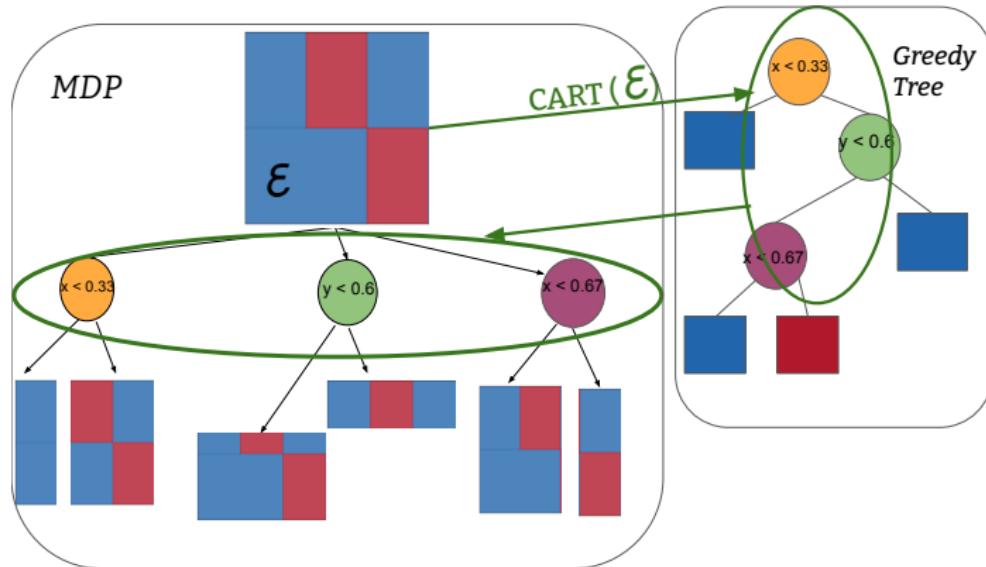
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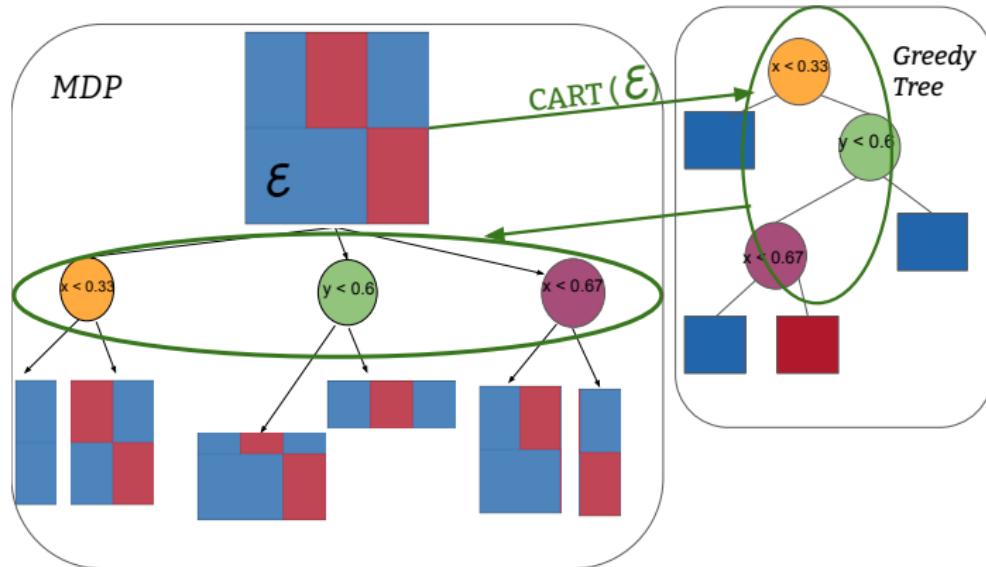
How to choose the  $B$  candidate actions/splits?

# Dynamic Programming Decision Trees (DPDT)<sup>1</sup>



<sup>1</sup>Because states are entire datasets, we implement DPDT with a depth-first search to limit the space complexity.

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DPDT was presented at the 31st ACM SIGKDD conference.

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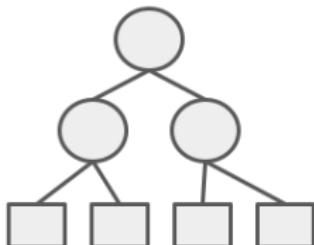
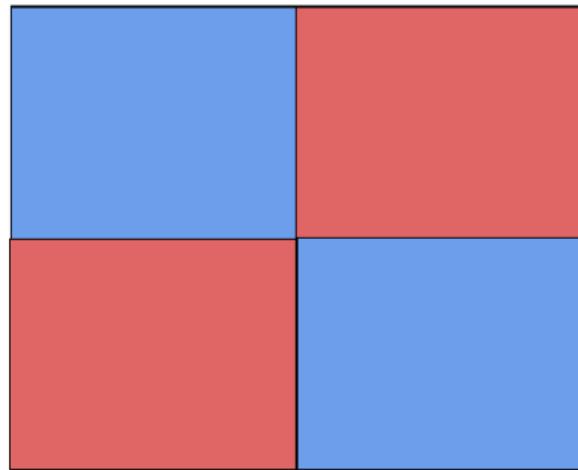
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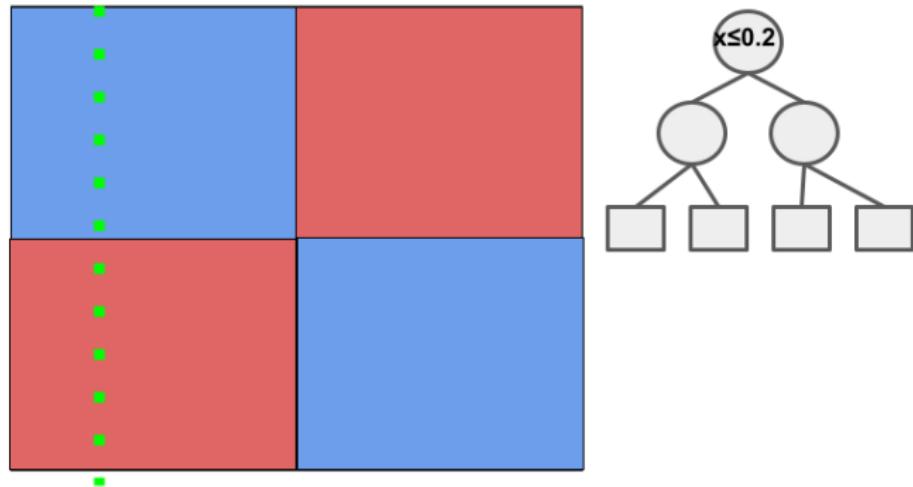
Theorem (DPDT trees can be strictly better than greedy trees)

*There exist a depth budget  $D$  and a dataset for which DPDT trees are strictly better than greedy trees.*

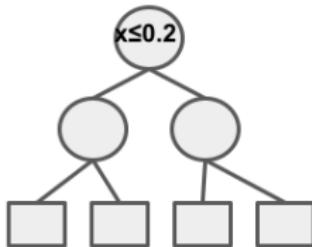
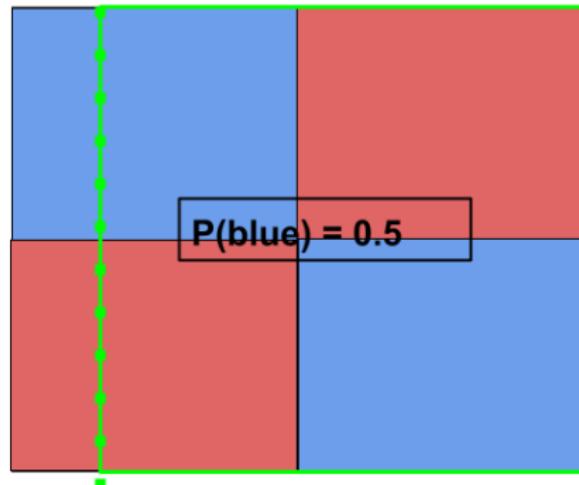
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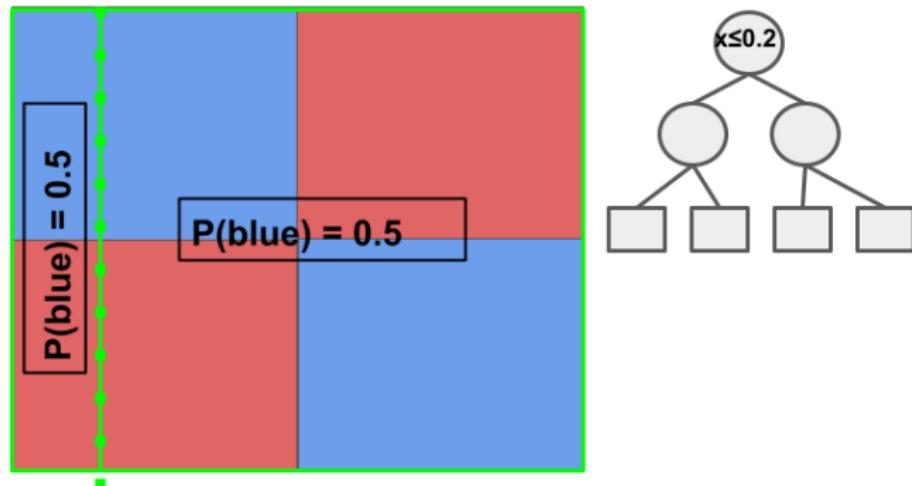
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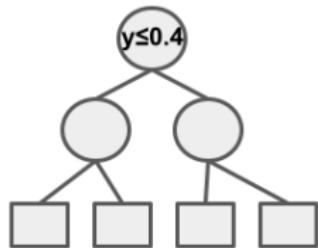
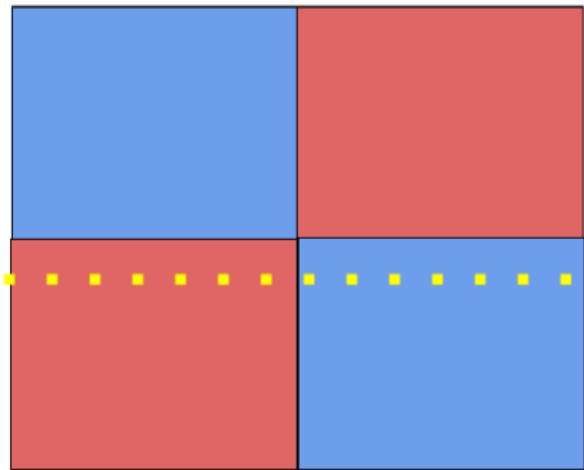
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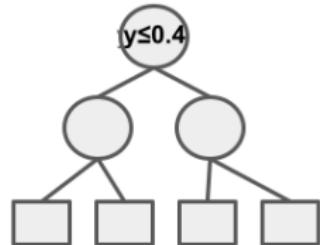
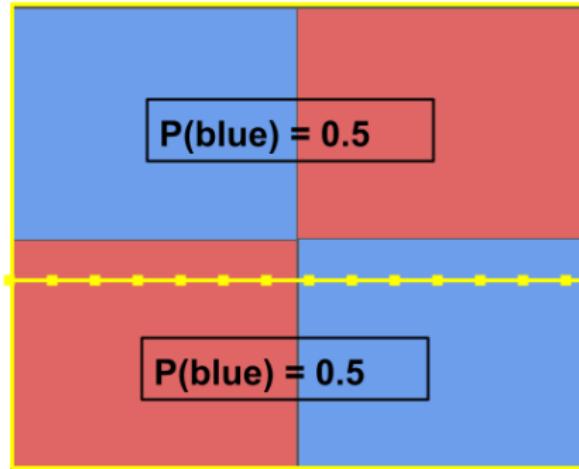
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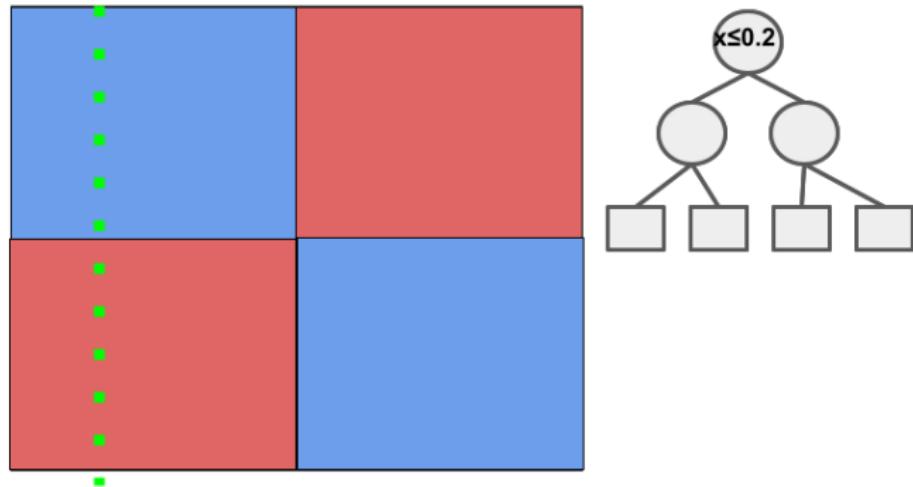
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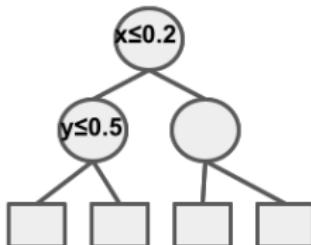
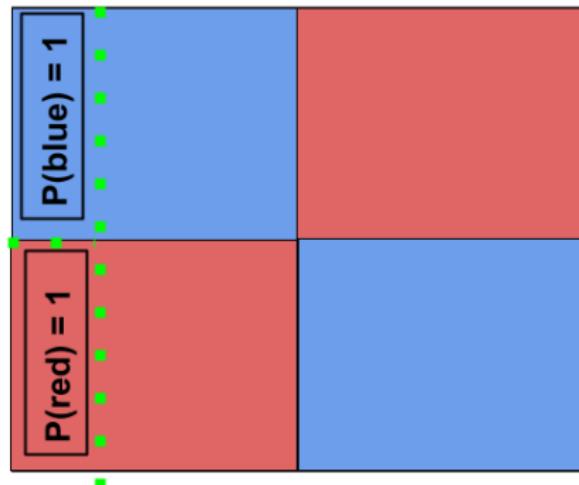
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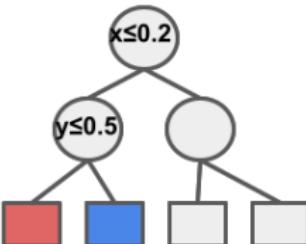
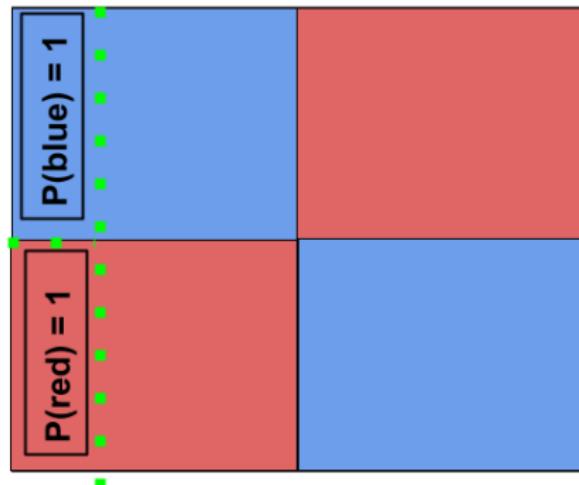
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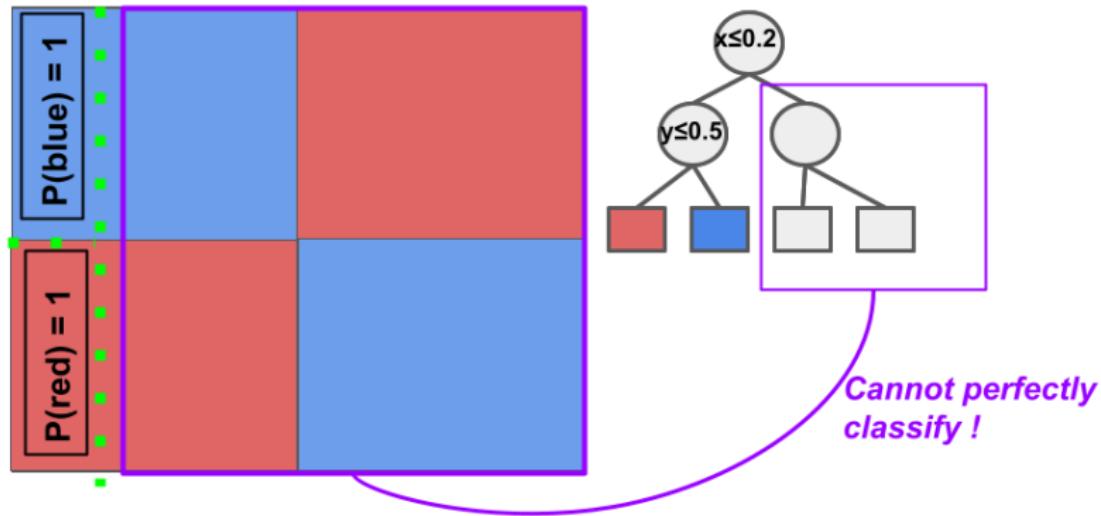
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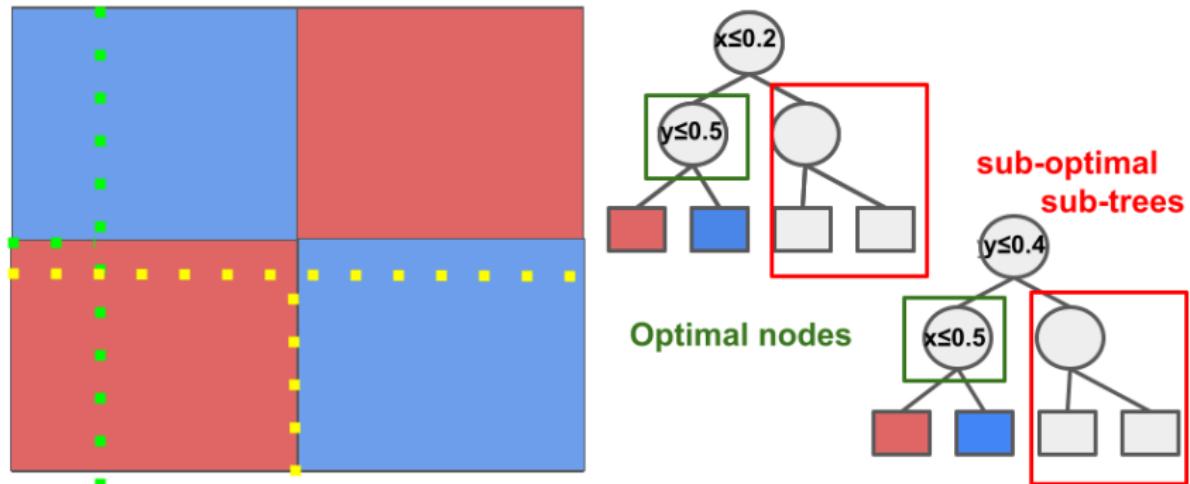
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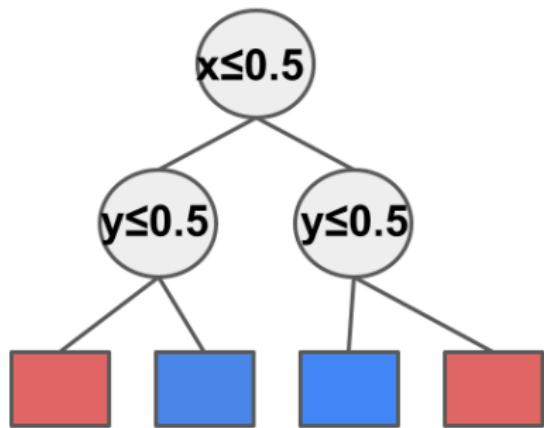
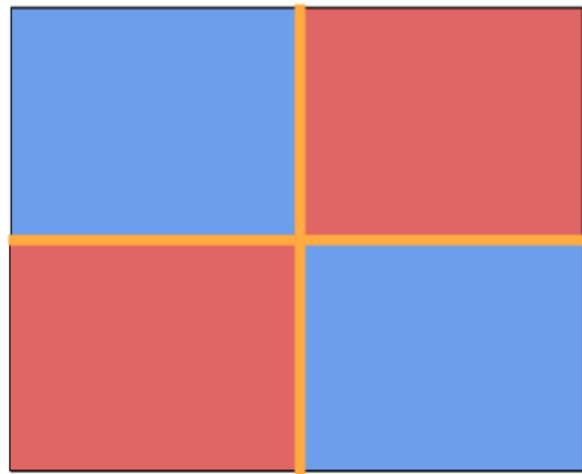
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# Fast like greedy trees, accurate like optimal trees



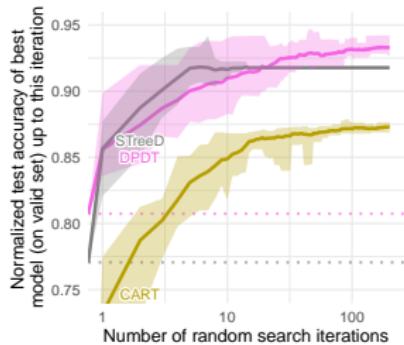
Comparison of greedy, optimal, and DPDT depth-2 trees on the checkersboard dataset.

# Comparing tree accuracy to complexity

Train accuracy and operation count when learning depth-3 decision trees.

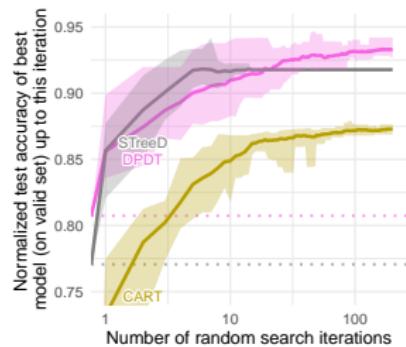
Dataset	N	p	Accuracy				Operations			
			Opt Quant-BnB	Greedy CART	DPDT light	DPDT full	Opt Quant-BnB	Greedy CART	DPDT light	DPDT full
room	8103	16	0.992	0.968	0.991	0.992	$10^6$	15	286	16100
bean	10888	16	0.871	0.777	0.812	0.853	$5 \cdot 10^6$	15	295	25900
eeg	11984	14	0.708	0.666	0.689	0.706	$2 \cdot 10^6$	13	289	26000
avila	10430	10	0.585	0.532	0.574	0.585	$3 \cdot 10^7$	9	268	24700
magic	15216	10	0.831	0.801	0.822	0.828	$6 \cdot 10^6$	15	298	28000
htru	14318	8	0.981	0.979	0.979	0.980	$6 \cdot 10^7$	15	295	25300
occup.	8143	5	0.994	0.989	0.991	0.994	$7 \cdot 10^5$	13	280	16300
skin	196045	3	0.969	0.966	0.966	0.966	$7 \cdot 10^4$	15	301	23300
fault	1552	27	0.682	0.553	0.672	0.674	$9 \cdot 10^8$	13	295	24200
segment	1848	18	0.887	0.574	0.812	0.879	$2 \cdot 10^6$	7	220	16300
page	4378	10	0.971	0.964	0.970	0.970	$10^7$	15	298	22400
bidding	5056	9	0.993	0.981	0.985	0.993	$3 \cdot 10^5$	13	256	9360
raisin	720	7	0.894	0.869	0.879	0.886	$4 \cdot 10^6$	15	295	20900
rice	3048	7	0.938	0.933	0.934	0.937	$2 \cdot 10^7$	15	298	25500
wilt	4339	5	0.996	0.993	0.994	0.995	$3 \cdot 10^5$	13	274	11300
bank	1097	4	0.983	0.933	0.971	0.980	$6 \cdot 10^4$	13	271	7990

# DPDT trees generalization

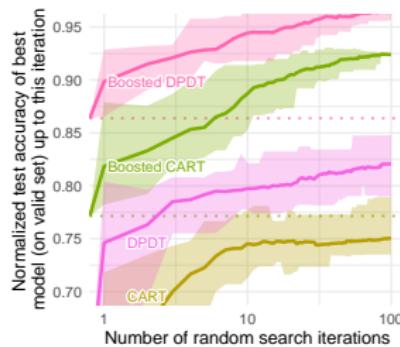


DPDT depth-5 trees vs.  
other depth-5 trees

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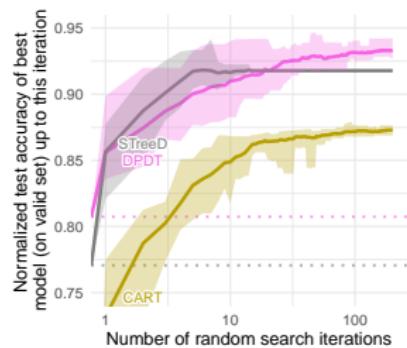


DPDT depth-5 trees vs.  
other depth-5 trees



Boosted DPDT vs. Boosted  
CART

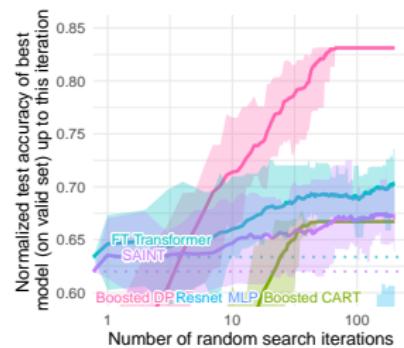
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Boosted DPDT vs. Boosted  
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Boosted DPDT vs. other  
classifiers

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**A: It depends.**

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```
# Decision tree for Mountain Car
def play(x):
    if x[1] <= -0.2597:
        if x[1] <= -0.6378:
            return 0
        else:
            if x[0] <= -1.0021:
                return 2
            else:
                return 0
    else:
        if x[1] <= -0.0508:
            if x[0] <= 0.2979:
                if x[0] <= 0.0453:
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```
# Small ReLU MLP for Pendulum
def play(x):
    h_layer_0_0 = 1.238*x[0]+0.971*x
    [1]
                           +0.430*x[2]+0.933
    h_layer_0_0 = max(0, h_layer_0_0
    )
    h_layer_0_1 = -1.221*x[0]+1.001
                           *x[1]-0.423*x[2]
                           +0.475
    h_layer_0_1 = max(0, h_layer_0_1
    )
    h_layer_1_0 = -0.109*h_layer_0_0
                           -0.377*h_layer_0_1
                           +1.694
    h_layer_1_0 = max(0, h_layer_1_0
    )
    h_layer_1_1 = -3.024*h_layer_0_0
                           -1.421*h_layer_0_1
                           +1.530
    h_layer_1_1 = max(0, h_layer_1_1
    )
    h_layer_2_0 = -1.790*h_layer_1_0
                           +2.840*h_layer_1_1
                           +0.658
    y_0 = h_layer_2_0
    return [y_0]
```

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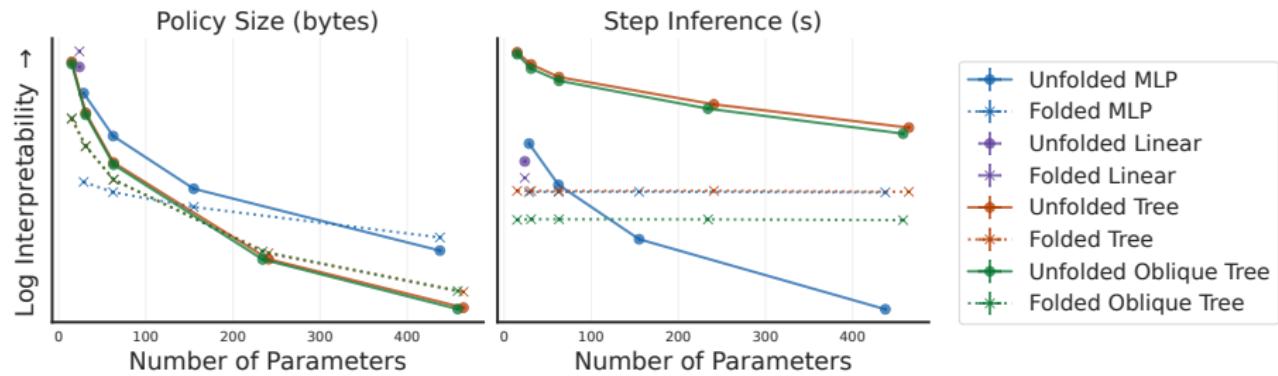
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## Setup

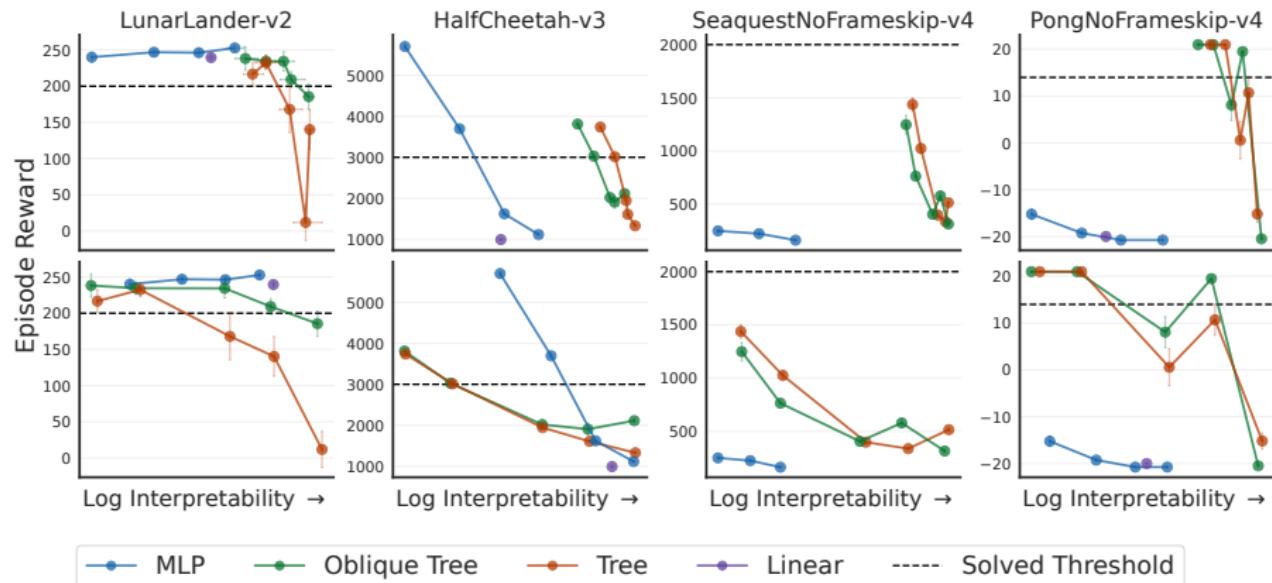
We imitate ~40000 expert policies from stable-baselines3 using various policy classes/nb parameters on various environments.

# Result: unfolding policies is necessary to respect consensus



Aggregated policies interpretability on classic control environments

Result: there is no dominating policy class for all environments



Interpretability-Performance trade-offs. Top row, interpretability is measured with step inference times. Bottom row, the interpretability is measured with policy size.

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- What about (very) big models?
- Can we use our policy programs as low level skills (hierarchical RL)?

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## My hope

Motivate interpretability by finding a real-world problem where interpretability is *really* necessary [Nag+24].

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- **Teaching:** Can we use unfolded policies (and interpretability) for teaching?

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