Université de Lille
Inria

École doctorale **École Graduée MADIS-631**

Unité de recherche **Centre de Recherche en Informatique, Signal et Automatique de Lille**

Thèse présentée par **Hector Kohler**

Soutenue le **1er décembre 2025**

En vue de l'obtention du grade de docteur de l'Université de Lille et de l'Inria

Discipline **Informatique**

Spécialité **Informatique et Applications**

Arbres de Décision pour la Prise de Décision Séquentielle

Thèse dirigée par Philippe Preux   directeur
Riad Akrour   co-directeur

Composition du jury

Rapporteurs	René Descartes	professeur à l'IHP	
	Denis Diderot	directeur de recherche au CNRS	
Examinateurs	Victor Hugo	professeur à l'ENS Lyon	président du jury
	Sophie Germain	mcf à l'Université de Paris 13	
	Joseph Fourier	chargé de recherche à l'INRIA	
	Paul Verlaine	chargé de recherche hdr au CNRS	
Invité	George Sand		
Directeurs de thèse	Philippe Preux	professeur à l'Université de Lille	
	Riad Akrour	Inria	

Colophon

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University Department **Centre de Recherche en Informatique, Signal et Automatique de Lille**

Thesis defended by **Hector Kohler**

Defended on **December 1, 2025**

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Academic Field **Computer Science**

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Decision Trees for Sequential Decision Making

Thesis supervised by   Philippe Preux   Supervisor
Riad Akrour   Co-Supervisor

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Guest	George Sand		
Supervisors	Philippe Preux	Professor at Université de Lille	
	Riad Akrour	Inria	

ARBRES DE DÉCISION POUR LA PRISE DE DÉCISION SÉQUENTIELLE

Résumé

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Mots clés : apprentissage par renforcement, arbres de décision, interprétabilité, méthodologie

Decision Trees for Sequential Decision Making

Abstract

In this Ph.D. thesis, we study algorithms to learn decision trees for classification and sequential decision making. Decision trees are interpretable because humans can read through the decision tree computations from the root to the leaves. This makes decision trees the go-to model when human verification is required like in medicine applications. However, decision trees are non-differentiable making them hard to optimize unlike neural networks that can be trained efficiently with gradient descent. Existing interpretable reinforcement learning approaches usually learn soft trees (non-interpretable as is) or are ad-hoc (train a neural network then fit a tree to it) potentially missing better solutions.

In the first part of this manuscript, we aim to directly learn decision trees for a Markov decision process with reinforcement learning. In practice we show that this amounts to solving a partially observable Markov decision process. Most existing RL algorithms are not suited for POMDPs. This parallel between decision tree learning with RL and POMDPs solving help us understand why in practice it is often easier to obtain a non-interpretable expert policy–a neural network–and then distillate it into a tree rather than learning the decision tree from scratch.

The second contribution from this work arose from the observation that looking for a deicison tree classifier (or regressor) can be seen as sequentially adding nodes to a tree to maximize the accuracy of predictions. We thus formulate decision tree induction as sloving a Markov decision problem and propose a new state-of-the-art algorithm that can be trained with supervised example data and generalizes well to unseen data.

Work from the previous parts rely on the hypothesis that decision trees are indeed an interpretable model that humans can use in sensitive applications. But is it really the case? In the last part of this thesis, we attempt to answer some more general questions about interpretability: can we measure interpretability without humans? And are decision trees really more interpretable than neural networks?

Keywords: reinforcement learning, deicision trees, interpretability, methodology

Sommaire

Preliminary Concepts

What is Sequential Decision Making?

In this manuscript we are interested in sequential decision making. Humans engage in sequential decision making in all aspects of life. In medicine, doctors have to decide when to use chemotherapy next based on the patient's current health in order to heal (cite). In agriculture, agronomists have to decide when to fertilize next based on the current soil and weather conditions in order to maximize plant growth (cite). In automotive, the auto-pilot system has to decide how to steer the wheel next based on lidar sensors in order to maintain a safe trajectory (cite). In video games, a bot decides what attack to throw next based on the player's and its own state in order to provide the best entertainment (cite). Those sequential decision making processes exhibits key similarities : an agent takes actions based on some current information to achieve some goal. As computer scientists, we ought to design algorithms that can help humans during those sequential decision making processes. For example, a doctor could benefit from a computer program (cite) that would recommend the "best" treatment given the patient's state. Algorithms presented in this thesis use the formalism of Markov decision processes that we present next to model sequential decision making.

Markov decision processes/problems

(figure) Markov decision processes (MDPs) were first introduced in the 1950s by Richard Bellman (cite). Informally, an MDP models how an agent acts over time to achieve its goal. At every timestep, the agent observes its current state,

e.g. a patient weight and tumor size, and takes an action, e.g. injects a certain amount of chemotherapy. When doing a certain action in a certain state, the agent gets a reward that helps it evaluate the quality of its action with respect to its goal, e.g., the tumor size decrease when the agent has to cure cancer. Finally, the agent is provided with a new state, e.g. the updated patient state, and repeats this process over time. Following Martin L. Puterman's book on MDPs (cite), we formally define as follows.

Definition (Markov decision process). *An MDP is a tuple $\mathcal{M} = \langle S, A, R, T, T_0 \rangle$ where :*

— *S is a finite set of states $s \in \mathbb{R}^n$ representing all possible configurations of the environment.*
— *A is a finite set of actions $a \in \mathbb{Z}^m$ available to the agent.*
— *$R : S \times A \to \mathbb{R}$ is the reward function that assigns a real-valued reward to each state-action pair.*
— *$T : S \times A \to \Delta(S)$ is the transition function that maps state-action pairs to probability distributions over next states, where $\Delta(S)$ denotes the probability simplex over S.*
— *$T_0 \in \Delta(S)$ is the initial distribution over states.*

Now we can also model the "goal" of the agent. Informally, the goal of an agent is to behave such that it gets as much reward as it can over time. For example, in the cancer treatment case, the best reward the agent can get is to completely get rid of the patient's tumor after some time. Furthermore, we want our agent to prefer behaviour that gets rid of the patient's tumor as fast as possible. We can formally model the agent's goal as an optimization problem as follows.

Definition (Markov decision problem). *Given an MDP $\mathcal{M} = \langle S, A, R, T, T_0 \rangle$, the goal of an agent following policy $\pi : S \to A$ is to maximize the expected discounted sum of rewards :*

$$J(\pi) = \mathbb{E}\left[\sum_{t=0}^{\infty} \gamma^t R(s_t, a_t) \mid s_0 \sim T_0, a_t = \pi(s_t), s_{t+1} \sim T(s_t, a_t)\right]$$

where $\gamma \in (0, 1)$ is the discount factor that controls the trade-off between immediate and future rewards.

Hence, algorithms presented in this manuscript aim to find solutions to Markov decision problems, i.e. the optimal policy : $\pi^\star = \underset{\pi}{\mathrm{argmax}} J(\pi)$ For the rest of this text, we will use an abuse of notation and denote both a Markov decision process and the associated Markov decision problem by MDP.

Exact solutions for Markov decision problems

It is possible to compute the exact optimal policy $\pi^\star$ using dynamic programming (cite). Indeed, one can leverage the Markov property to find for all states the best action to take based on the reward of upcoming states.

Definition (Value of a state). *The value of a state $s \in S$ under policy π is the expected discounted sum of rewards starting from state s and following policy π :*

$$V^\pi(s) = \mathbb{E}\left[\sum_{t=0}^{\infty} \gamma^t R(s_t, a_t) \mid s_0 = s, a_t = \pi(s_t), s_{t+1} \sim T(s_t, a_t)\right]$$

Applying the Markov property gives a recursive definition of the value of s under policy π :

$$V^\pi(s) = R(s, \pi(s)) + \gamma \sum_{s' \in S} T^{s'}(s, \pi(s)) V^\pi(s')$$

where $T^{s'}(s, \pi(s))$ is the probability of transitioning to state s' when taking action $\pi(s)$ in state s.

Definition (Optimal value of a state). *The optimal value of a state $s \in S$, $V^\star(s)$, is the value of state s when following the optimal policy : $V^{\pi^\star}(s)$.*

$$V^\star(s) = V^{\pi^\star}(s) = \max_\pi [J(\pi)]$$

Hence, the algorithms we study in the thesis can also be seen as solving the problem : $\pi^\star = \underset{\pi}{\mathrm{argmax}} \mathbb{E}[V^\pi(s_0)|s_0 \sim T_0]$. The well-known Value Iteration algorithm (algorithm) solves this problem exactly (cite).

More realistically, neither the transition kernel T nor the reward function R of the MDP are known, e.g., the doctor can't **know** how the tumor and the patient health will change after a dose of chemotherapy, it can only **observe** the change. This distinction between the information available to the agent is paralleled with the distinction between dynamic programming and reinforcement learning (RL) that we describe next.

Reinforcement learning of approximate solutions to MDPs

(figrure) Reinforcement learning algorithms popularized by Richard Sutton (figure) (cite) don't **compute** an optimal policy but rather **learn** an approximate one based on sequences of observations $(s_t, a_t, r_t, s_{t+1})_t$. RL algorithms usually fall into two categories : value-based (cite) and policy gradient (cite). The first group of RL algorithms computes an appromiation of $V^\star$ using temporal different learning (algorithms) (cite). The other class of RL algorithms leverage the policy gradient theorem (algorithm) (cite) to approximate $\pi^\star$. Both class of algorithms are known to converge to the optimal value or policy under some conditions (cite) and have known great successes in real-world appilcations (cite).

There are many other ways to learn policies such as simple random search (cite) or model-based reinforcement learning. However, not many algorithms consider the learning of policies that can be easily understood by humans which we discuss next and that is the core of this manuscript.

What is Interpretable Sequential Decision Making?

(figure)

Why do we care about interpretability?

(figure)

What are existing approaches for interpretable sequential decision making?

(figure)

What are decision tree policies?

(figure)

Why are decision tree policies harder to learn than decision tree classifiers?

Outline of the Thesis

(figure)

Première partie

A difficult problem : Learning Decision Trees for MDP

Chapitre 1

A Decision Tree Policy for an MDP is a Policy for some Partially Observable MDP

1.1 How to Learn a Decision Tree Policy for an MDP ?

1.1.1 Imitation

1.1.2 Soft Trees

1.1.3 Iterative Bounding MDPs

1.2 How to solve Iterative Bounding MDPs ?

1.2.1 Asymmetric Reinforcement Learning

1.2.2 Learning a decision tree policy is solving a POMDP

1.3 Is it hard to properly learn a Decision Tree Policy for an MDP ?

1.3.1 POMDPs are way harder to solve than MDPs

1.3.2 Memoryless approaches to solve POMDPs seem uneffective

An attempt at Learning Decision Tree Policies with Reinforcement Learning

2.1 Grid Worlds

$V(g) = \zeta \sum_{i=0}^{\infty} \gamma^{2i} + \sum_{i=0}^{\infty} \gamma^{2i+1}$

$V(0) = \zeta + \gamma^2 V(g)$

$V(1) = \zeta + \gamma^2 V(0)$

$\frac{1}{4}\gamma\frac{1}{1-\gamma} + \frac{1}{4}\frac{1}{1-\gamma} \leq \frac{1}{4}V(g) + \frac{2}{4}V(0) + \frac{1}{4}V(1)$

$\zeta \cdot \sum_{i=0}^{\infty} \gamma^i \leq \frac{1}{4}V(g) + \frac{2}{4}V(0) + \frac{1}{4}V(1)$

$\frac{1}{4}V(g) + \frac{1}{4}(\zeta + \gamma V(0)) + \frac{1}{4}(\zeta + \gamma V(1)) + \frac{1}{4}V(0) \leq \frac{1}{4}V(g) + \frac{2}{4}V(0) + \frac{1}{4}V(1)$

$\frac{1}{4}V(g) + \frac{1}{4}V(0) + \frac{1}{4}(\zeta + \gamma^2\zeta\sum_{i=0}^{\infty}\gamma^{2i}) + \frac{1}{4}(\zeta\sum_{i=0}^{\infty}\gamma^{2i}) \leq \frac{1}{4}V(g) + \frac{2}{4}V(0) + \frac{1}{4}V(1)$

2.1.1 Step-by-step derivation of the lower bound on ζ

Step 1 : Simplify the left side of the inequality

$$\frac{1}{4}\gamma\frac{1}{1-\gamma} + \frac{1}{4}\frac{1}{1-\gamma} = \frac{1}{4}\frac{1}{1-\gamma}(\gamma+1) \tag{2.1}$$

$$= \frac{\gamma+1}{4(1-\gamma)} \tag{2.2}$$

Step 2 : Express $V(g)$**,** $V(0)$**, and** $V(1)$ **in simplified forms**

$$V(g) = \zeta \sum_{i=0}^{\infty} \gamma^{2i} + \sum_{i=0}^{\infty} \gamma^{2i+1} \tag{2.3}$$

$$= \zeta \frac{1}{1-\gamma^2} + \gamma \frac{1}{1-\gamma^2} \tag{2.4}$$

$$= \frac{\zeta + \gamma}{1-\gamma^2} \tag{2.5}$$

$$V(0) = \zeta + \gamma^2 V(g) \tag{2.6}$$

$$= \zeta + \gamma^2 \frac{\zeta + \gamma}{1-\gamma^2} \tag{2.7}$$

$$= \frac{\zeta(1-\gamma^2) + \gamma^2(\zeta+\gamma)}{1-\gamma^2} \tag{2.8}$$

$$= \frac{\zeta + \gamma^3}{1-\gamma^2} \tag{2.9}$$

$$V(1) = \zeta + \gamma^2 V(0) \tag{2.10}$$

$$= \zeta + \gamma^2 \frac{\zeta + \gamma^3}{1-\gamma^2} \tag{2.11}$$

$$= \frac{\zeta(1-\gamma^2) + \gamma^2(\zeta+\gamma^3)}{1-\gamma^2} \tag{2.12}$$

$$= \frac{\zeta + \gamma^5}{1-\gamma^2} \tag{2.13}$$

Step 3 : Substitute into the right side of the inequality

$$\frac{1}{4}V(g) + \frac{2}{4}V(0) + \frac{1}{4}V(1) = \frac{1}{4}\frac{\zeta+\gamma}{1-\gamma^2} + \frac{1}{2}\frac{\zeta+\gamma^3}{1-\gamma^2} + \frac{1}{4}\frac{\zeta+\gamma^5}{1-\gamma^2} \tag{2.14}$$

$$= \frac{1}{4(1-\gamma^2)}[(\zeta+\gamma) + 2(\zeta+\gamma^3) + (\zeta+\gamma^5)] \tag{2.15}$$

$$= \frac{4\zeta + \gamma + 2\gamma^3 + \gamma^5}{4(1-\gamma^2)} \tag{2.16}$$

Step 4 : Set up the inequality

$$\frac{\gamma+1}{4(1-\gamma)} \le \frac{4\zeta + \gamma + 2\gamma^3 + \gamma^5}{4(1-\gamma^2)} \tag{2.17}$$

Step 5 : Use the identity $1 - \gamma^2 = (1-\gamma)(1+\gamma)$

$$\frac{\gamma+1}{4(1-\gamma)} \le \frac{4\zeta + \gamma + 2\gamma^3 + \gamma^5}{4(1-\gamma)(1+\gamma)} \tag{2.18}$$

Step 6 : Multiply both sides by $4(1-\gamma)$

$$\gamma + 1 \le \frac{4\zeta + \gamma + 2\gamma^3 + \gamma^5}{1+\gamma} \tag{2.19}$$

Step 7 : Multiply both sides by $(1+\gamma)$

$$(\gamma+1)(1+\gamma) \le 4\zeta + \gamma + 2\gamma^3 + \gamma^5 \tag{2.20}$$

$$(\gamma+1)^2 \le 4\zeta + \gamma + 2\gamma^3 + \gamma^5 \tag{2.21}$$

Step 8 : Expand and rearrange

$$\gamma^2 + 2\gamma + 1 \le 4\zeta + \gamma + 2\gamma^3 + \gamma^5 \tag{2.22}$$

$$4\zeta \ge \gamma^2 + 2\gamma + 1 - \gamma - 2\gamma^3 - \gamma^5 \tag{2.23}$$

$$4\zeta \ge \gamma^2 + \gamma + 1 - 2\gamma^3 - \gamma^5 \tag{2.24}$$

$$\zeta \ge \frac{\gamma^2 + \gamma + 1 - 2\gamma^3 - \gamma^5}{4} \tag{2.25}$$

Therefore, we obtain a **lower bound** on ζ :

$$\zeta \geq \frac{\gamma^2 + \gamma + 1 - 2\gamma^3 - \gamma^5}{4} \tag{2.26}$$

where $0 < \gamma < 1$.

2.1.2   Step-by-step derivation of the upper bound on ζ

Starting from the inequality :

$$\frac{1}{4}V(g) + \frac{1}{4}(\zeta + \gamma V(0)) + \frac{1}{4}(\zeta + \gamma V(1)) + \frac{1}{4}V(0) \leq \frac{1}{4}V(g) + \frac{2}{4}V(0) + \frac{1}{4}V(1)$$

Step 1 : Cancel the $\frac{1}{4}V(g)$ terms from both sides

$$\frac{1}{4}(\zeta + \gamma V(0)) + \frac{1}{4}(\zeta + \gamma V(1)) + \frac{1}{4}V(0) \leq \frac{2}{4}V(0) + \frac{1}{4}V(1) \tag{2.27}$$

Step 2 : Expand the left side

$$\frac{1}{4}\zeta + \frac{1}{4}\gamma V(0) + \frac{1}{4}\zeta + \frac{1}{4}\gamma V(1) + \frac{1}{4}V(0) \leq \frac{2}{4}V(0) + \frac{1}{4}V(1) \tag{2.28}$$

Step 3 : Combine like terms

$$\frac{1}{2}\zeta + \frac{1}{4}\gamma V(0) + \frac{1}{4}\gamma V(1) \leq \frac{2}{4}V(0) - \frac{1}{4}V(0) + \frac{1}{4}V(1) \tag{2.29}$$

$$\frac{1}{2}\zeta + \frac{1}{4}\gamma V(0) + \frac{1}{4}\gamma V(1) \leq \frac{1}{4}V(0) + \frac{1}{4}V(1) \tag{2.30}$$

Step 4 : Factor out common terms

$$\frac{1}{2}\zeta \leq \frac{1}{4}V(0) + \frac{1}{4}V(1) - \frac{1}{4}\gamma V(0) - \frac{1}{4}\gamma V(1) \tag{2.31}$$

$$\frac{1}{2}\zeta \leq \frac{1}{4}V(0)(1 - \gamma) + \frac{1}{4}V(1)(1 - \gamma) \tag{2.32}$$

$$\frac{1}{2}\zeta \leq \frac{1 - \gamma}{4}(V(0) + V(1)) \tag{2.33}$$

$$\zeta \leq \frac{1 - \gamma}{2}(V(0) + V(1)) \tag{2.34}$$

Step 5 : Substitute the expressions for $V(0)$ and $V(1)$

$$V(0) + V(1) = \frac{\zeta + \gamma^3}{1 - \gamma^2} + \frac{\zeta + \gamma^5}{1 - \gamma^2} \tag{2.35}$$

$$= \frac{2\zeta + \gamma^3 + \gamma^5}{1 - \gamma^2} \tag{2.36}$$

Step 6 : Substitute back into the inequality

$$\zeta \leq \frac{1 - \gamma}{2} \cdot \frac{2\zeta + \gamma^3 + \gamma^5}{1 - \gamma^2} \tag{2.37}$$

$$= \frac{(1 - \gamma)(2\zeta + \gamma^3 + \gamma^5)}{2(1 - \gamma^2)} \tag{2.38}$$

Step 7 : Use the identity $1 - \gamma^2 = (1 - \gamma)(1 + \gamma)$

$$\zeta \leq \frac{(1 - \gamma)(2\zeta + \gamma^3 + \gamma^5)}{2(1 - \gamma)(1 + \gamma)} \tag{2.39}$$

$$= \frac{2\zeta + \gamma^3 + \gamma^5}{2(1 + \gamma)} \tag{2.40}$$

Step 8 : Multiply both sides by $2(1 + \gamma)$

$$2(1 + \gamma)\zeta \leq 2\zeta + \gamma^3 + \gamma^5 \tag{2.41}$$

$$2\zeta + 2\gamma\zeta \leq 2\zeta + \gamma^3 + \gamma^5 \tag{2.42}$$

$$2\gamma\zeta \leq \gamma^3 + \gamma^5 \tag{2.43}$$

$$\zeta \leq \frac{\gamma^3 + \gamma^5}{2\gamma} \tag{2.44}$$

$$\zeta \leq \frac{\gamma^2 + \gamma^4}{2} \tag{2.45}$$

Therefore, we obtain an **upper bound** on ζ :

$$\zeta \leq \frac{\gamma^2 + \gamma^4}{2} \tag{2.46}$$

Combined bounds :

$$\frac{\gamma^2 + \gamma + 1 - 2\gamma^3 - \gamma^5}{4} \leq \zeta \leq \frac{\gamma^2 + \gamma^4}{2} \tag{2.47}$$

where $0 < \gamma < 1$.

2.1.3 Step-by-step derivation for the third inequality

Starting from the inequality :

$$\zeta \cdot \sum_{i=0}^{\infty} \gamma^i \leq \frac{1}{4}V(g) + \frac{2}{4}V(0) + \frac{1}{4}V(1)$$

Step 1 : Simplify the left side using the geometric series

$$\zeta \cdot \sum_{i=0}^{\infty} \gamma^i = \zeta \cdot \frac{1}{1-\gamma} \tag{2.48}$$

$$= \frac{\zeta}{1-\gamma} \tag{2.49}$$

Step 2 : Use the previously derived expression for the right side From our earlier calculation :

$$\frac{1}{4}V(g) + \frac{2}{4}V(0) + \frac{1}{4}V(1) = \frac{4\zeta + \gamma + 2\gamma^3 + \gamma^5}{4(1-\gamma^2)} \tag{2.50}$$

Step 3 : Set up the inequality

$$\frac{\zeta}{1-\gamma} \leq \frac{4\zeta + \gamma + 2\gamma^3 + \gamma^5}{4(1-\gamma^2)} \tag{2.51}$$

Step 4 : Use the identity $1 - \gamma^2 = (1-\gamma)(1+\gamma)$

$$\frac{\zeta}{1-\gamma} \leq \frac{4\zeta + \gamma + 2\gamma^3 + \gamma^5}{4(1-\gamma)(1+\gamma)} \tag{2.52}$$

Step 5 : Multiply both sides by $(1 - \gamma)$

$$\zeta \leq \frac{4\zeta + \gamma + 2\gamma^3 + \gamma^5}{4(1 + \gamma)} \tag{2.53}$$

Step 6 : Multiply both sides by $4(1 + \gamma)$

$$4(1 + \gamma)\zeta \leq 4\zeta + \gamma + 2\gamma^3 + \gamma^5 \tag{2.54}$$

$$4\zeta + 4\gamma\zeta \leq 4\zeta + \gamma + 2\gamma^3 + \gamma^5 \tag{2.55}$$

Step 7 : Subtract 4ζ **from both sides**

$$4\gamma\zeta \leq \gamma + 2\gamma^3 + \gamma^5 \tag{2.56}$$

$$\zeta \leq \frac{\gamma + 2\gamma^3 + \gamma^5}{4\gamma} \tag{2.57}$$

$$\zeta \leq \frac{1 + 2\gamma^2 + \gamma^4}{4} \tag{2.58}$$

Therefore, we obtain another **upper bound** on ζ :

$$\zeta \leq \frac{1 + 2\gamma^2 + \gamma^4}{4} \tag{2.59}$$

Final combined bounds from all three inequalities :

$$\frac{\gamma^2 + \gamma + 1 - 2\gamma^3 - \gamma^5}{4} \leq \zeta \leq \min\left\{\frac{\gamma^2 + \gamma^4}{2}, \frac{1 + 2\gamma^2 + \gamma^4}{4}\right\} \tag{2.60}$$

where $0 < \gamma < 1$.

2.1.4   Step-by-step derivation for the fourth inequality

Starting from the inequality :

$$\frac{1}{4}V(g) + \frac{1}{4}V(0) + \frac{1}{4}(\zeta + \gamma^2\zeta\sum_{i=0}^{\infty}\gamma^{2i}) + \frac{1}{4}(\zeta\sum_{i=0}^{\infty}\gamma^{2i}) \leq \frac{1}{4}V(g) + \frac{2}{4}V(0) + \frac{1}{4}V(1)$$

Step 1 : Cancel the $\frac{1}{4}V(g)$ terms from both sides

$$\frac{1}{4}V(0) + \frac{1}{4}(\zeta + \gamma^2\zeta\sum_{i=0}^{\infty}\gamma^{2i}) + \frac{1}{4}(\zeta\sum_{i=0}^{\infty}\gamma^{2i}) \leq \frac{2}{4}V(0) + \frac{1}{4}V(1) \qquad (2.61)$$

Step 2 : Simplify using the geometric series $\sum_{i=0}^{\infty}\gamma^{2i} = \frac{1}{1-\gamma^2}$

$$\frac{1}{4}V(0) + \frac{1}{4}\left(\zeta + \gamma^2\zeta\frac{1}{1-\gamma^2}\right) + \frac{1}{4}\left(\zeta\frac{1}{1-\gamma^2}\right) \leq \frac{2}{4}V(0) + \frac{1}{4}V(1) \qquad (2.62)$$

Step 3 : Factor out common terms

$$\frac{1}{4}V(0) + \frac{1}{4}\zeta\left(1 + \frac{\gamma^2}{1-\gamma^2}\right) + \frac{1}{4}\zeta\frac{1}{1-\gamma^2} \leq \frac{2}{4}V(0) + \frac{1}{4}V(1) \qquad (2.63)$$

Step 4 : Simplify the coefficient of ζ

$$1 + \frac{\gamma^2}{1-\gamma^2} + \frac{1}{1-\gamma^2} = \frac{1-\gamma^2+\gamma^2+1}{1-\gamma^2} = \frac{2}{1-\gamma^2} \qquad (2.64)$$

So the inequality becomes :

$$\frac{1}{4}V(0) + \frac{1}{4}\zeta\frac{2}{1-\gamma^2} \leq \frac{2}{4}V(0) + \frac{1}{4}V(1) \qquad (2.65)$$

$$\frac{1}{4}V(0) + \frac{\zeta}{2(1-\gamma^2)} \leq \frac{1}{2}V(0) + \frac{1}{4}V(1) \qquad (2.66)$$

Step 5 : Rearrange to isolate the ζ term

$$\frac{\zeta}{2(1-\gamma^2)} \leq \frac{1}{2}V(0) - \frac{1}{4}V(0) + \frac{1}{4}V(1) \qquad (2.67)$$

$$\frac{\zeta}{2(1-\gamma^2)} \leq \frac{1}{4}V(0) + \frac{1}{4}V(1) \qquad (2.68)$$

$$\zeta \leq \frac{(1-\gamma^2)}{2}(V(0)+V(1)) \qquad (2.69)$$

Step 6 : Substitute the expressions for $V(0)$ and $V(1)$

$$V(0) + V(1) = \frac{\zeta + \gamma^3}{1 - \gamma^2} + \frac{\zeta + \gamma^5}{1 - \gamma^2} \tag{2.70}$$

$$= \frac{2\zeta + \gamma^3 + \gamma^5}{1 - \gamma^2} \tag{2.71}$$

Step 7 : Substitute back into the inequality

$$\zeta \le \frac{(1 - \gamma^2)}{2} \cdot \frac{2\zeta + \gamma^3 + \gamma^5}{1 - \gamma^2} \tag{2.72}$$

$$= \frac{2\zeta + \gamma^3 + \gamma^5}{2} \tag{2.73}$$

Step 8 : Multiply both sides by 2

$$2\zeta \le 2\zeta + \gamma^3 + \gamma^5 \tag{2.74}$$

$$0 \le \gamma^3 + \gamma^5 \tag{2.75}$$

$$0 \le \gamma^3(1 + \gamma^2) \tag{2.76}$$

Since $0 < \gamma < 1$, we have $\gamma^3 > 0$ and $(1 + \gamma^2) > 0$, so this inequality is always satisfied. This means the fourth inequality does not provide an additional constraint on ζ.

Updated final bounds from all four inequalities :

$$\frac{\gamma^2 + \gamma + 1 - 2\gamma^3 - \gamma^5}{4} \le \zeta \le \min\left\{\frac{\gamma^2 + \gamma^4}{2}, \frac{1 + 2\gamma^2 + \gamma^4}{4}\right\} \tag{2.77}$$

where $0 < \gamma < 1$. The fourth inequality is automatically satisfied and does not further constrain the bounds.

2.2 Q-Learning

2.3 Preferences over Decision Tree Policies

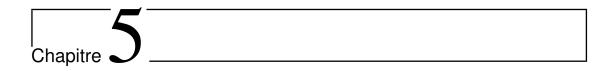## 2.4 Results

Conclusion

3.1 What happens when the MDP's transitions are independent of the current state?

Deuxième partie

An easier problem : Learning Decision Trees for MDPs that are Classification tasks

DPDT-intro

DPDT-paper

Chapitre 6

Conclusion

Troisième partie

Beyond Decision Trees : what can be done with other Interpretable Policies ?

Conclusion générale

Annexe A

Documents juridiques

Cette partie regroupe les documents juridiques officiels.

A.1 Licence sous laquelle est publié notre travail

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A.2 Transposition de la licence précédente en droit français

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B

Annexe

Programmes informatiques

Les listings suivants sont au cœur de notre travail.

Listing B.1 – Il est l'heure

```c
#include <stdio.h>
int heures, minutes, secondes;

/*****************************************************/
/*                                                 */
/*              print_heure                        */
/*                                                 */
/*    But:                                         */
/*        Imprime l'heure_____*/
/*_____*/
/*___Interface:_____*/
/*_____Utilise_les_variables_globales_____*/
/*_____heures,_minutes,_secondes_____*/
/*_____*/
/*****************************************************/

void print_heure(void)
{
  printf("Il est %d heure",heures);
  if (heures > 1) printf("s");
  printf(" %d minute",minutes);
  if (minutes > 1) printf("s");
  printf(" %d seconde",secondes);
  if (secondes > 1) printf("s");
```

```
25   ␣␣printf("\n");
26   }
```

Listing B.2 – Factorielle

```
1   int factorielle(int n)
2   {
3      if (n > 2) return n * factorielle(n - 1);
4      return n;
5   }
```

Table des matières

Résumé

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Mots clés : apprentissage par renforcement, arbres de décision, interprétabilité, méthodologie

Decision Trees for Sequential Decision Making

Abstract

In this Ph.D. thesis, we study algorithms to learn decision trees for classification and sequential decision making. Decision trees are interpretable because humans can read through the decision tree computations from the root to the leaves. This makes decision trees the go-to model when human verification is required like in medicine applications. However, decision trees are non-differentiable making them hard to optimize unlike neural networks that can be trained efficiently with gradient descent. Existing interpretable reinforcement learning approaches usually learn soft trees (non-interpretable as is) or are ad-hoc (train a neural network then fit a tree to it) potentially missing better solutions.

In the first part of this manuscript, we aim to directly learn decision trees for a Markov decision process with reinforcement learning. In practice we show that this amounts to solving a partially observable Markov decision process. Most existing RL algorithms are not suited for POMDPs. This parallel between decision tree learning with RL and POMDPs solving help us understand why in practice it is often easier to obtain a non-interpretable expert policy–a neural network–and then distillate it into a tree rather than learning the decision tree from scratch.

The second contribution from this work arose from the observation that looking for a deicison tree classifier (or regressor) can be seen as sequentially adding nodes to a tree to maximize the accuracy of predictions. We thus formulate decision tree induction as sloving a Markov decision problem and propose a new state-of-the-art algorithm that can be trained with supervised example data and generalizes well to unseen data.

Work from the previous parts rely on the hypothesis that decision trees are indeed an interpretable model that humans can use in sensitive applications. But is it really the case? In the last part of this thesis, we attempt to answer some more general questions about interpretability: can we measure interpretability without humans? And are decision trees really more interpretable than neural networks?

Keywords: reinforcement learning, deicision trees, interpretability, methodology