

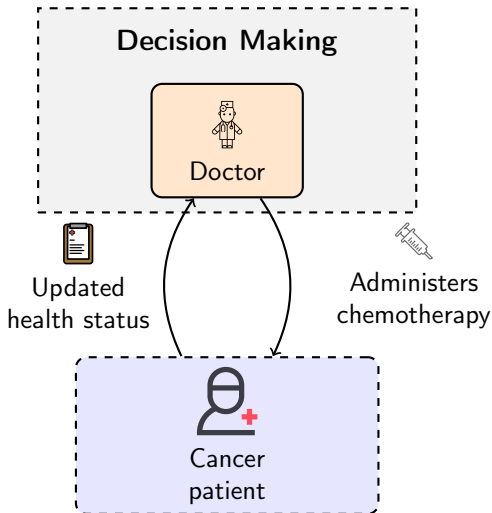
# Interpretability, Decision Trees, and Sequential Decision Making

Hector Kohler

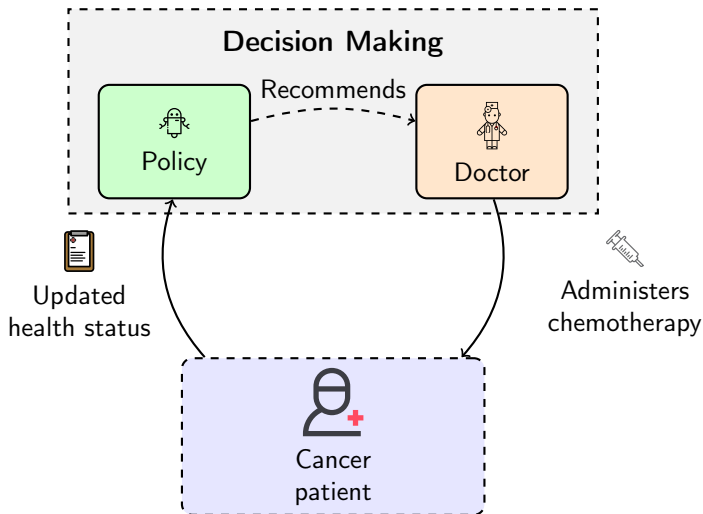
Supervised by Dr. Riad Akrou (HdR) and Prof. Philippe Preux (HdR)  
Université de Lille, CNRS, Inria, UMR CRISAL 9189, France

December 4, 2025

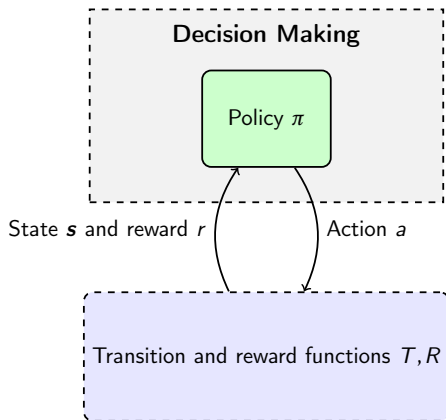
# Sequential decision making (SDM)



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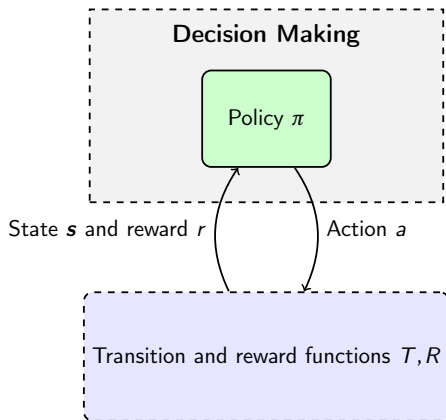


# Markov decision processes (MDPs) and reinforcement learning (RL)



Markov decision processes (Puterman 1994).

# Markov decision processes (MDPs) and reinforcement learning (RL)

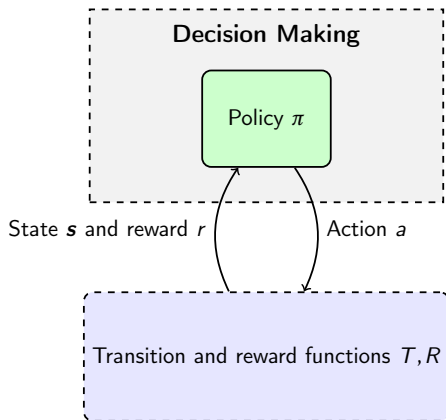


- RL (Sutton and Barto 1998) aims to find a policy,  $\pi : S \rightarrow A$  that maximizes:

$$J(\pi) = \mathbb{E}_{s_t \sim T} \left[ \sum_{t=0}^{\infty} \gamma^t R(s_t, \pi(s_t)) \right]$$

Markov decision processes (Puterman 1994).

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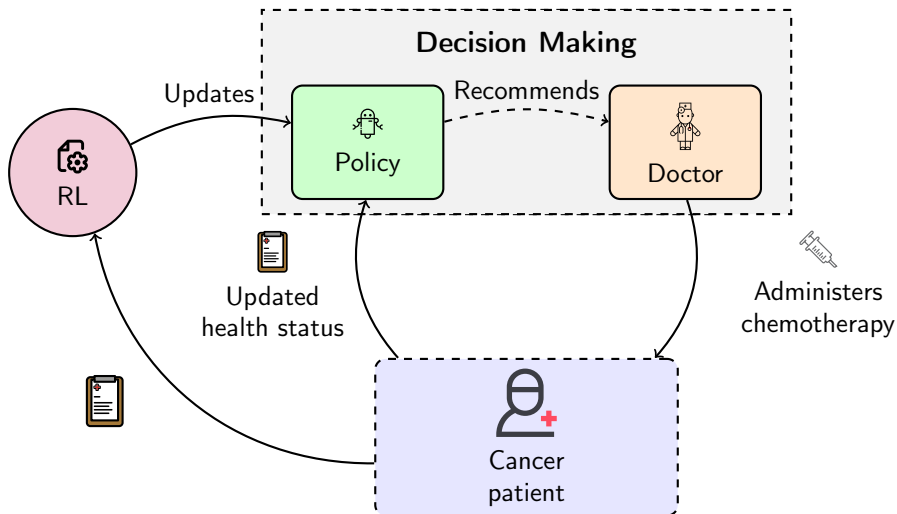
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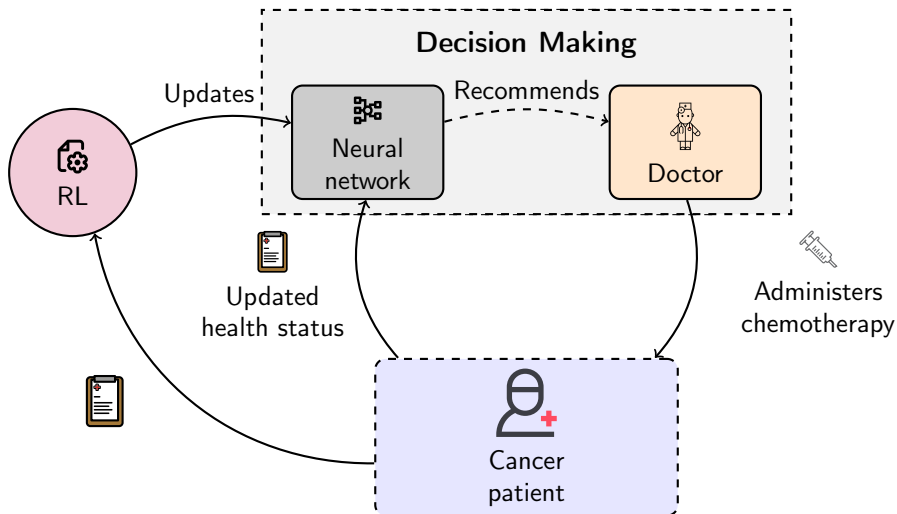
$$J(\pi) = \mathbb{E}_{s_t \sim T} \left[ \sum_{t=0}^{\infty} \gamma^t R(s_t, \pi(s_t)) \right]$$

- Lots of succesful (deep) RL algorithms (Schulman et al. 2017).

# Sequential decision making and reinforcement learning

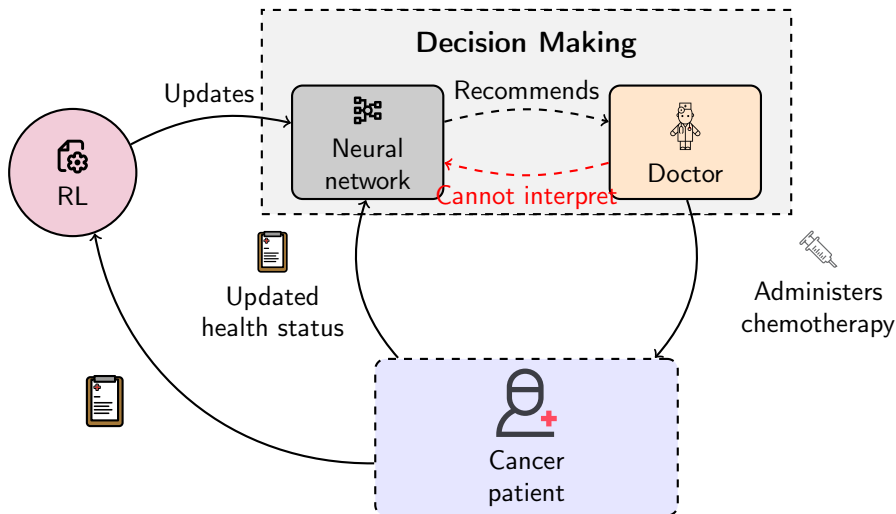


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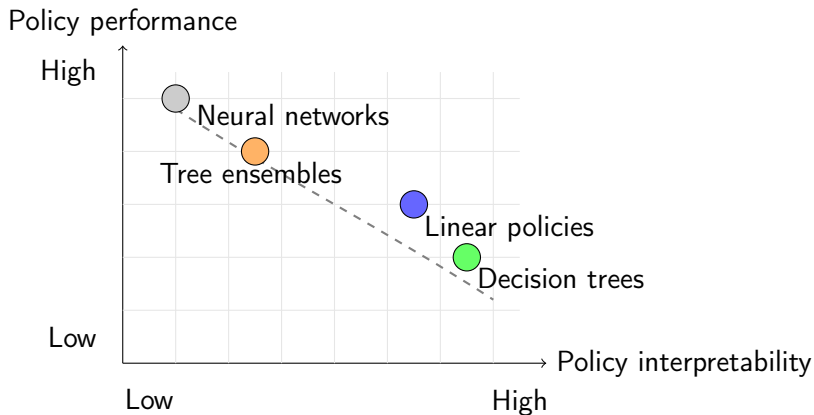
⚠ No consensus on definition (Lipton 2018).

# Interpretability in SDM (Glanois et al. 2024).



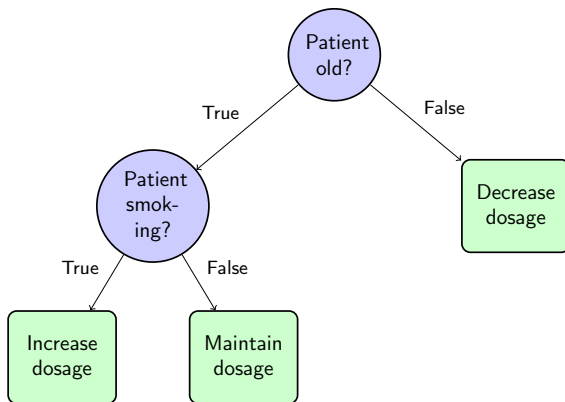
Local interpretability with saliency maps of different MDP states (Greydanus et al. 2018).

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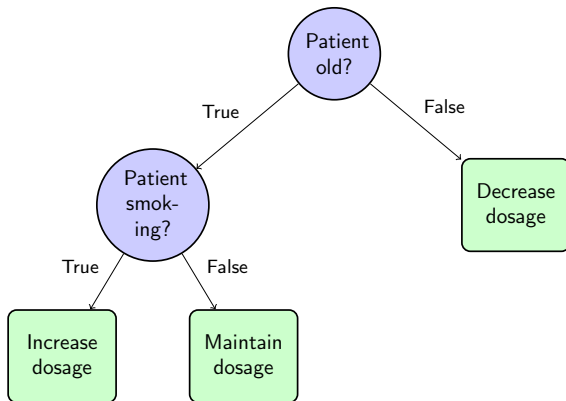
Global interpretability with intrinsically interpretable policies.

# Decision trees



A generic decision tree of depth  $D = 2$ .

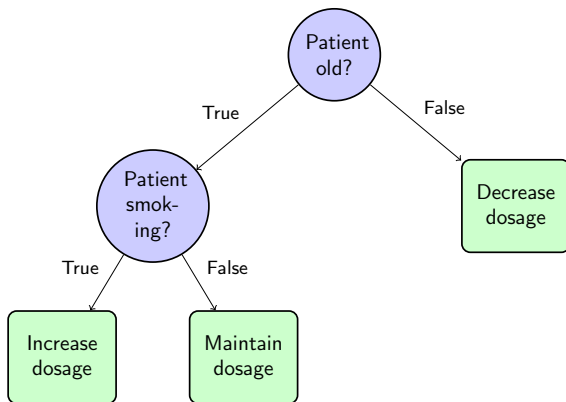
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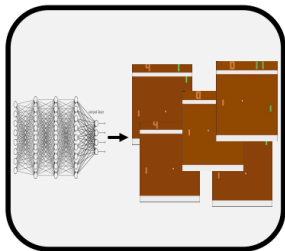


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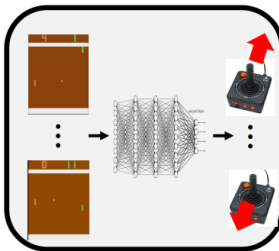
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**What about SDM?**

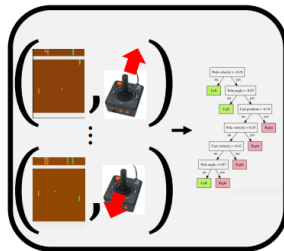
# Imitation learning



**Step 1:** Use NN to generate states



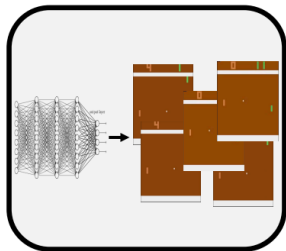
### Step 2: Use NN to obtain actions



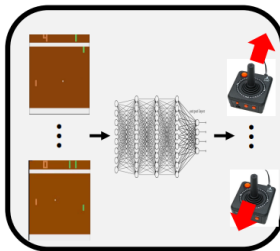
**Step 3:** Use supervised learning to train a decision tree



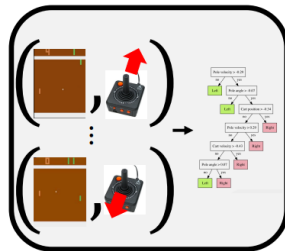
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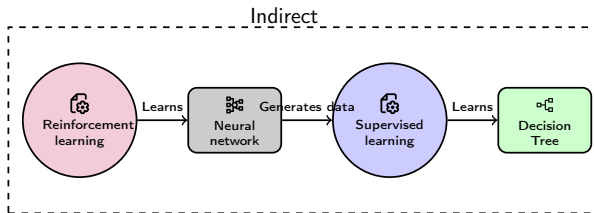
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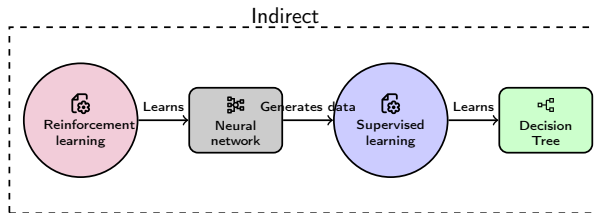
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Most research focused on indirect learning of interpretable policies (Bastani, Pu, and Solar-Lezama 2018).

# Two ways to get interpretable policies for SDM (Glanois et al. 2024)

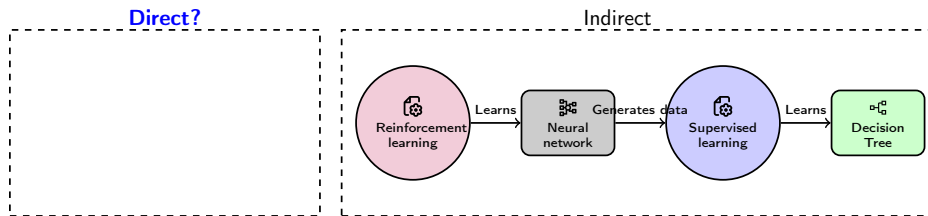


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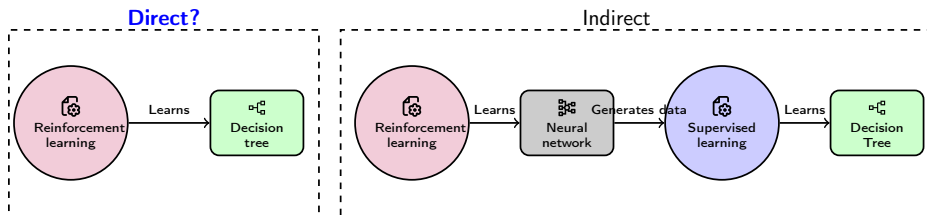
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- ② Can we leverage SDM to learn decision trees for classification/regression?
- ③ How to measure policy interpretability in SDM?

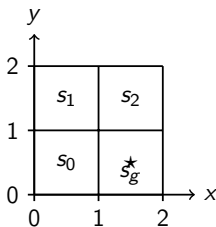
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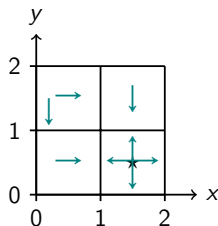
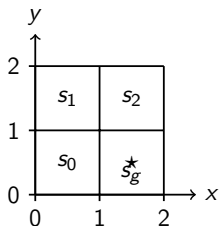


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# Grid world MDP and decision tree policies



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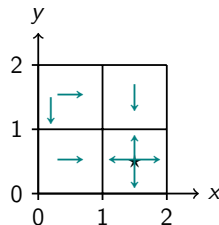
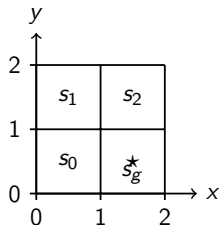


# Grid world MDP and decision tree policies

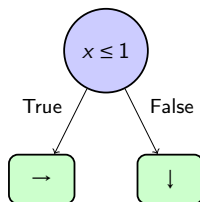


Grid world MDP and optimal actions.

# Grid world MDP and decision tree policies



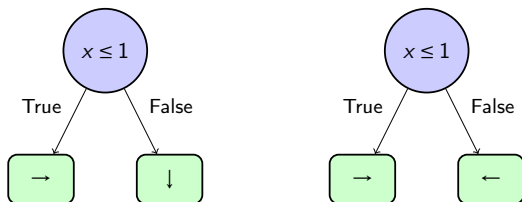
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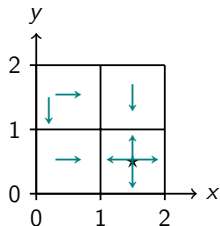
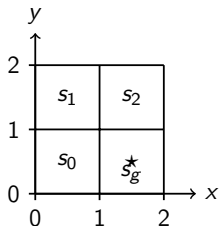
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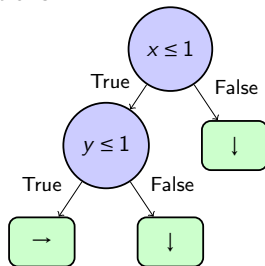
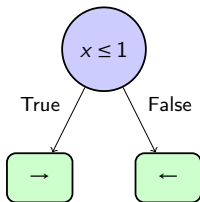
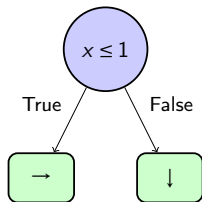
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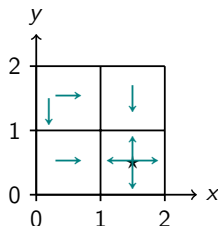
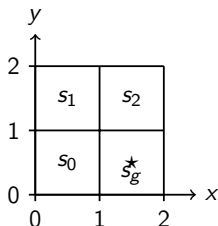
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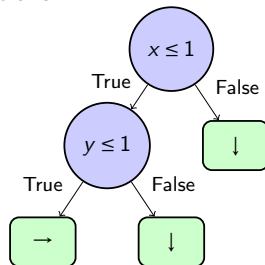
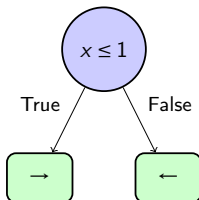
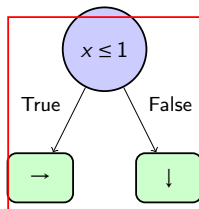
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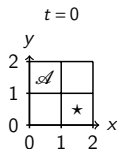
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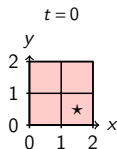
Decision tree policies with different interpretability-performance trade-offs.



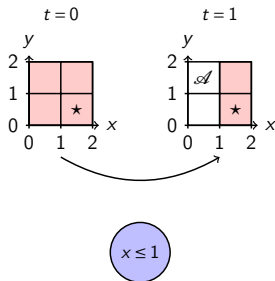
# Iterative bounding Markov decision processes (Topin et al. 2021)



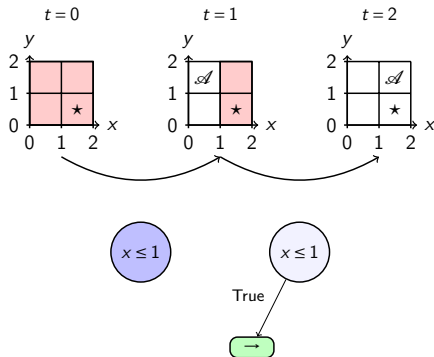
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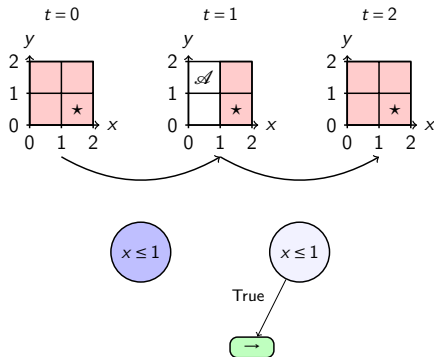
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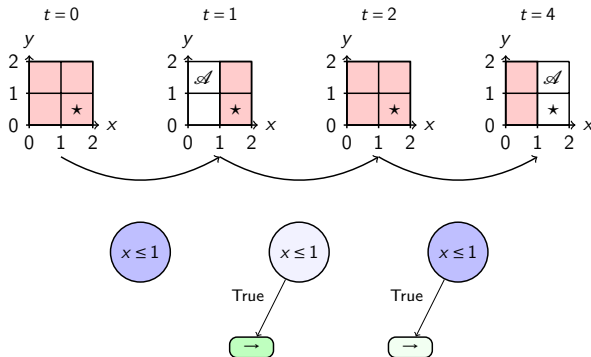
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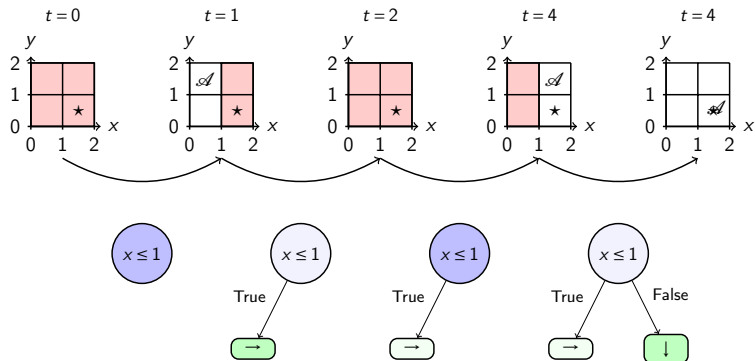
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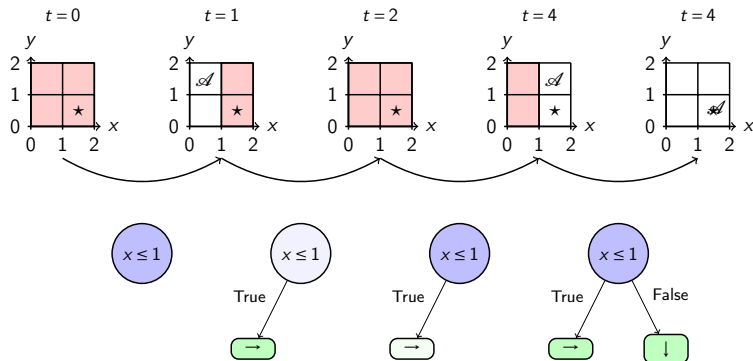
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- **⚠ IBMDP policies  $\pi_{po} : O \rightarrow A \cup A_{info}$  are decision tree policies for  $\mathcal{M}$ .**

# RL for memoryless policies in POMDPs

## RL for memoryless policies

- Finding the best deterministic and memoryless policy in a POMDP is NP-hard (Littman 1994)!
- The best memoryless policy can be stochastic (Singh, Jaakkola, and Jordan 1994).
- Value-based RL converges to sub-optimal solutions.

## Asymmetric RL (Pinto et al. 2017)

- Value-based  $\rightarrow$  learns  $Q(o, a)$  with TD targets  $U(s, a)$  (Baisero, Daley, and Amato 2022).
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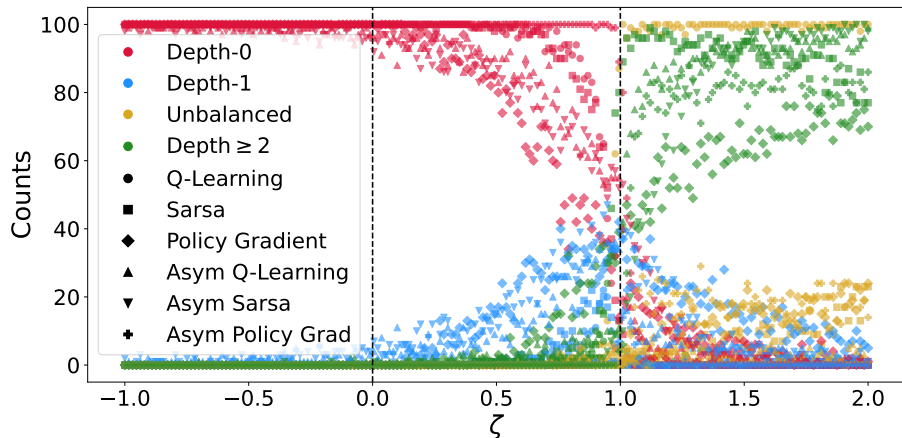
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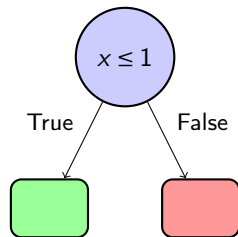
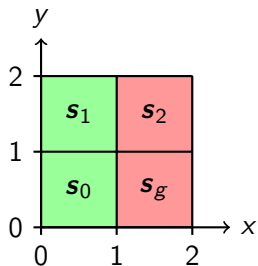
# Result: RL cannot retrieve optimal depth-1 trees for the grid world MDP



Distributions of tree policies learned with (asymmetric) RL algorithms as a function of the interpretability reward  $\zeta$ .

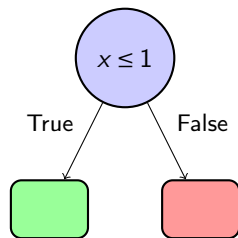
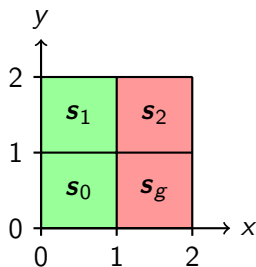


Direct RL of decision trees for classification tasks does not involve partial observability



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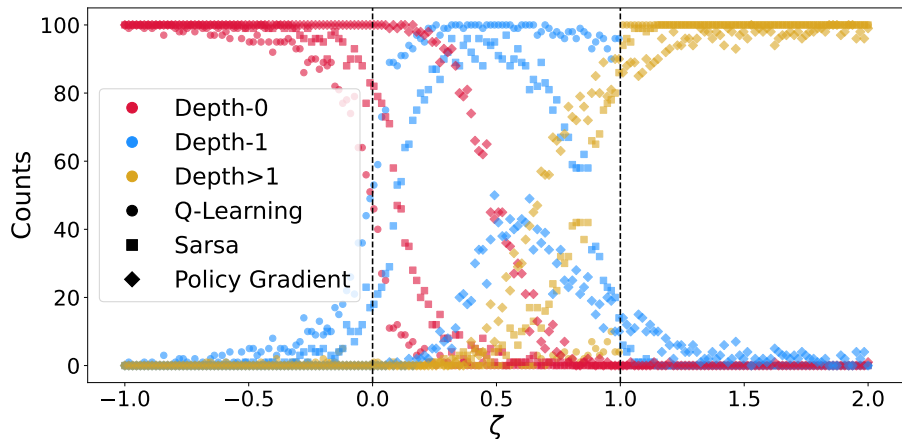
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Partial observations are sufficient statistics of the full states in IBMDPs for classification tasks.

# Result: RL can retrieve optimal depth-1 trees for the toy classification MDP



Distributions of tree policies learned with various RL algorithms.

# Perspectives for direct RL of decision tree policies.

- It seems that directly learning decision tree trading off interpretability and performances in MDPs can be difficult to achieve because of **partial observability**.
- Should we focus on indirect approaches? Hybrid approaches (Wu et al. 2020)?
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# Decision trees in supervised learning

- $N$  data points  $\{\mathbf{x}_i, y_i\}$ . Each  $\mathbf{x}_i$  is described by  $p$  features and has a label  $y_i \in \mathcal{Y}$ . We want to find a tree of depth at most  $D$ ,  $T \in \mathcal{T}_D$  that minimizes:

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- Trees **interpretable** and **competitive with neural nets** (Grinsztajn, Oyallon, and Varoquaux 2022).
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# Decision tree induction as solving MDPs

## Intuition

The induction of a decision tree is made of a sequence of decisions: at each node, we must decide whether it is better to split (a subset of) the training data, or to create a leaf node.

- S: data subsets.
- A: test or leaf nodes that can be added to the tree.
- R: interpretability term  $-\alpha$  and misclassifications.
- T: node traversals.

## Proposition (Objective Equivalence)

*Let  $\pi$  be a deterministic policy of the MDP. Then  $J_\alpha(\pi) = -\mathcal{L}_\alpha(E(\pi, s_0))$  where  $E$  is an algorithm that extracts a decision tree from  $\pi$  (Topin et al. 2021).*



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# Controlling the time complexity of decision tree induction

- Greedy algorithms consider only one candidate action in each state which is the test that minimizes some impurity criterion  
→ MDP state space size is  $O(2^D)$ .
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Top- $B$  greedy splits (Blanc et al. 2023), quantiles, random...

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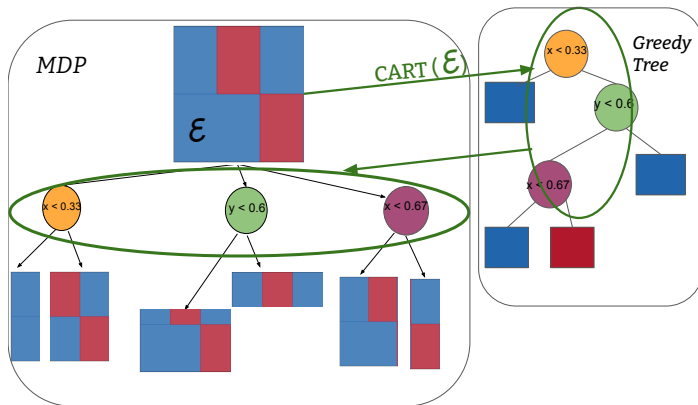
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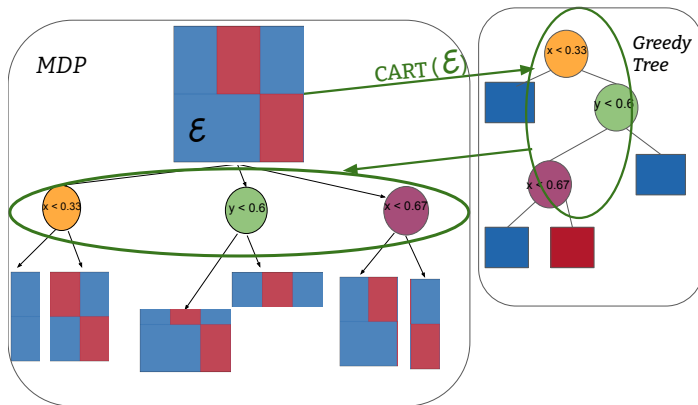


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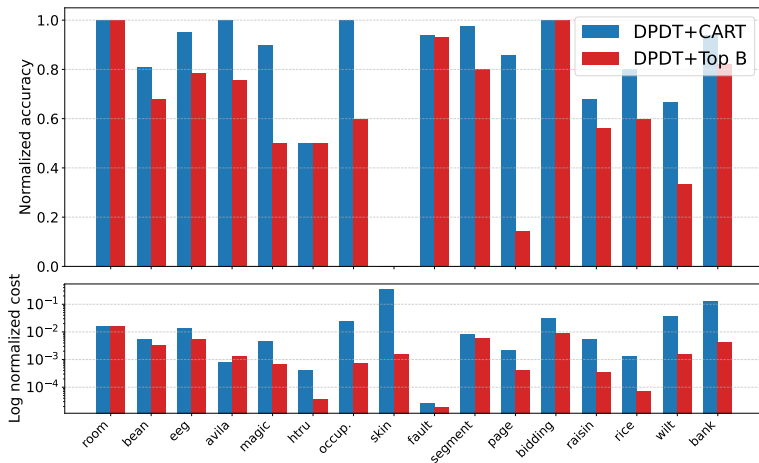
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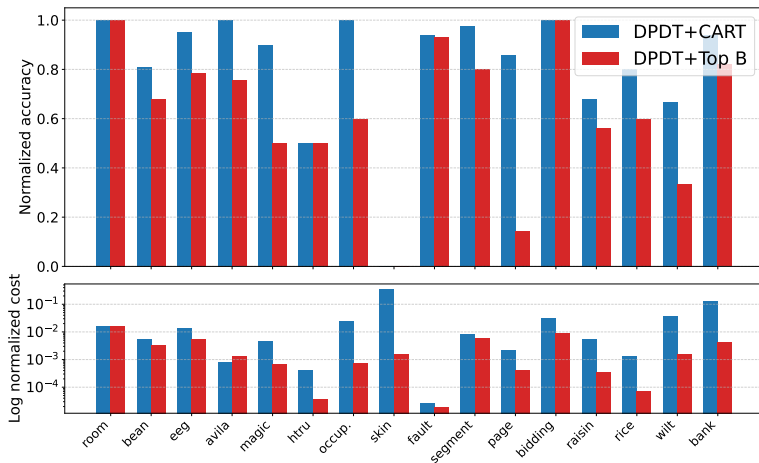
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# Fast like greedy trees, accurate like optimal trees



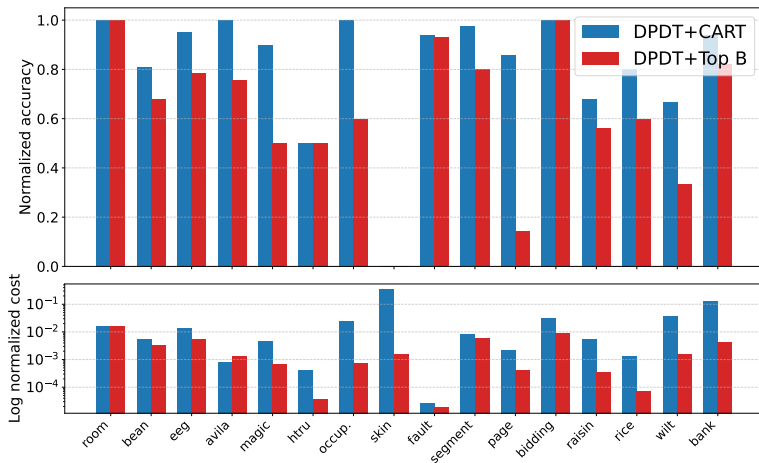
Train accuracies against cost for detph-3 trees.

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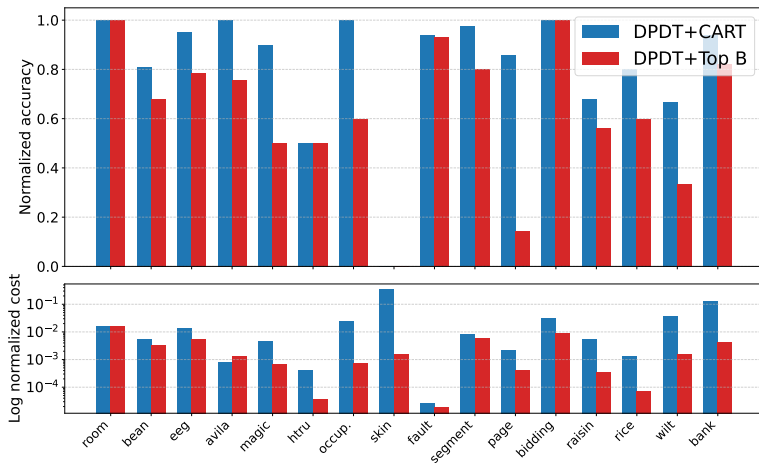
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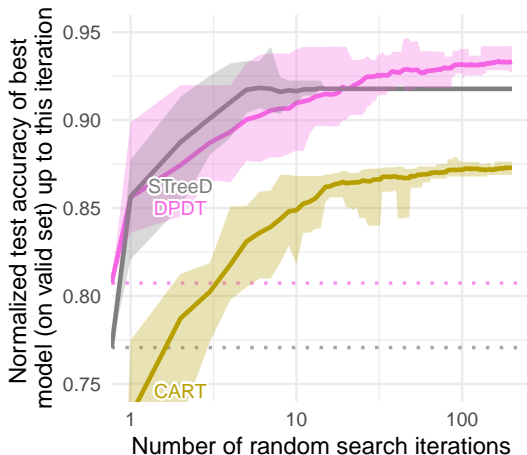
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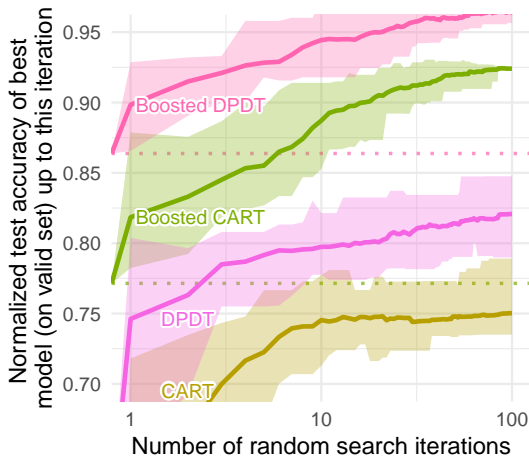


DPDT depth-5 trees vs. other depth-5 trees



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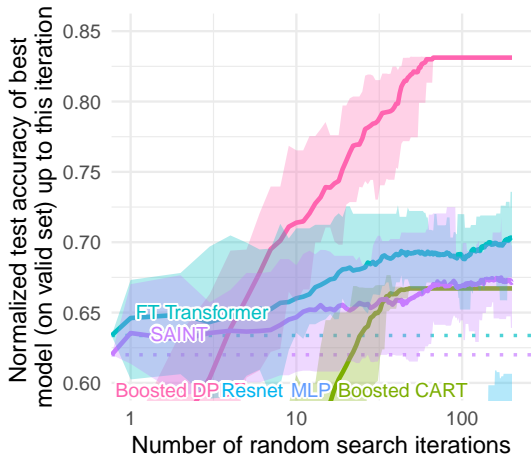
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Boosted DPDT vs. Boosted CART

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Boosted DPDT vs. other classifiers



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Let us take a step back

*Q: Are decision trees really the most interpretable model?*

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# How to measure policy interpretability?

## Challenges (Doshi-Velez and Kim 2017)

- No unified formal definition of policy interpretability.
- Measuring requires humans.

## The notion of *simulatability* (Lipton 2018)

- Interpretability  $\simeq$  **runtime in seconds?**
- Interpretability  $\simeq$  how much effort for a human to read through the entire policy.
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- No unified formal definition of policy interpretability.
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- Different hardwares (CPUs vs GPUs).
- Different implementations (matrix operations vs fully sequentially)  
(Luo et al. 2024)

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# We propose policy unfolding

```
# Small ReLU MLP for Pendulum
def play(x):
    h_layer_0_0 = 1.68 * x[0] + -0.69 * x[1] + -0.74 * x[2] + -1.40
    h_layer_0_0 = max(0.0, h_layer_0_0)
    h_layer_0_1 = 0.20 * x[0] + 0.29 * x[1] + -0.021 * x[2] + 1.25
    h_layer_0_1 = max(0.0, h_layer_0_1)
    h_layer_0_2 = 0.33 * x[0] + -0.57 * x[1] + 0.47 * x[2] + 1.94
    h_layer_0_2 = max(0.0, h_layer_0_2)
    h_layer_0_3 = 1.39 * x[0] + 0.94 * x[1] + 0.50 * x[2] + -1.13
    h_layer_0_3 = max(0.0, h_layer_0_3)
    h_layer_1_0 = 1.16 * h_layer_0_0 + -1.59 * h_layer_0_1 + 0.95 * h_layer_0_2 +
        -1.22 * h_layer_0_3 + -0.54
    h_layer_1_0 = max(0.0, h_layer_1_0)
    h_layer_1_1 = -0.55 * h_layer_0_0 + 1.13 * h_layer_0_1 + -0.58 * h_layer_0_2
        + -0.72 * h_layer_0_3 + 1.56
    h_layer_1_1 = max(0.0, h_layer_1_1)
    h_layer_1_2 = 1.10 * h_layer_0_0 + -1.01 * h_layer_0_1 + 0.96 * h_layer_0_2 +
        -2.84 * h_layer_0_3 + -0.02
    h_layer_1_2 = max(0.0, h_layer_1_2)
    h_layer_1_3 = 0.27 * h_layer_0_0 + 0.44 * h_layer_0_1 + 0.39 * h_layer_0_2 +
        0.15 * h_layer_0_3 + -1.24
    h_layer_1_3 = max(0.0, h_layer_1_3)
    h_layer_2_0 = -2.80 * h_layer_1_0 + -0.60 * h_layer_1_1 + 3.07 * h_layer_1_2
        + -1.63 * h_layer_1_3 + -0.36
    y_0 = h_layer_2_0

    return [y_0]
```

# Is time/size of unfolded policies a good proxy?

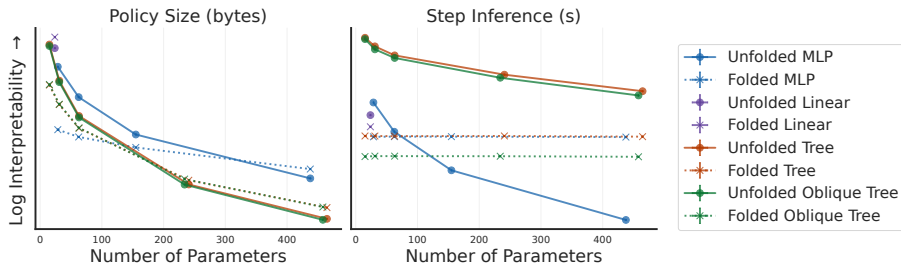
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Aggregated policies interpretability on classic control environments

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- What about floating points precision?
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# Conclusion: interpretable SDM is a difficult research topic

- Technical challenges: **Learning interpretable policies for SDM involves partial observability.**
  - Focus on indirect approaches and/or on POMDP research first?
  - **Created opportunities for new decision tree algos for classif/regression.**
- Fundamental challenges: **No consensus on interpretability definition.**
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Motivate interpretability by finding a real-world problem where interpretability is *really* necessary (Nagendran et al. 2024).

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# Broader perspectives

- **Deep learning:** Can we design deep learning layers that take datasets and output candidate splits?
- **Combinatorial optimization:** Can we formulate other combinatorial/NP-hard problems as MDPs and design other DPDT-like algorithms?
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Result: for similar problems, RL struggles more when there is partial observability

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	POIBMDP
	POIBMDP-asym
	IBMDP
	MDP

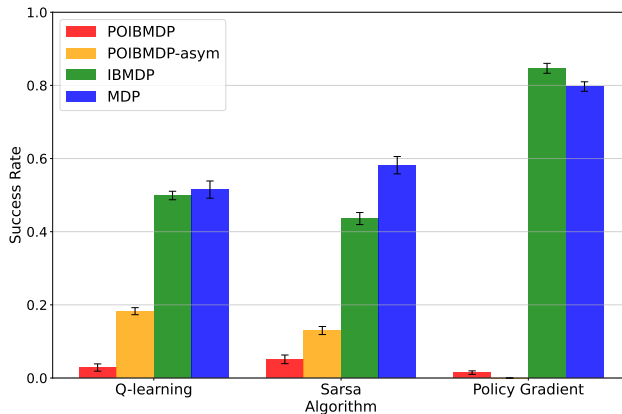
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Success rates over thousands of RL runs with varying hyperparameters when learning different policies in the same IBMDP<sup>1</sup>.

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<sup>1</sup>We also observed similar results on classic controls and variants of the grid world MDP.

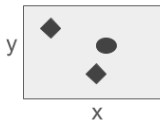
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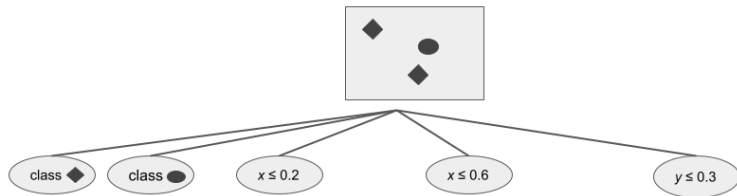
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# Decision tree induction as solving MDPs



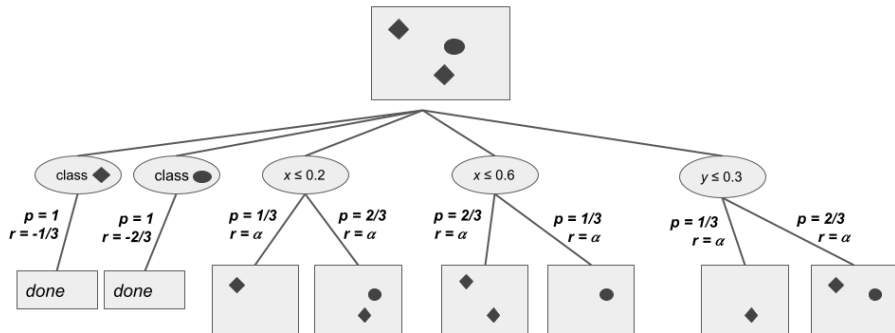
Example of decision tree induction as an MDP.

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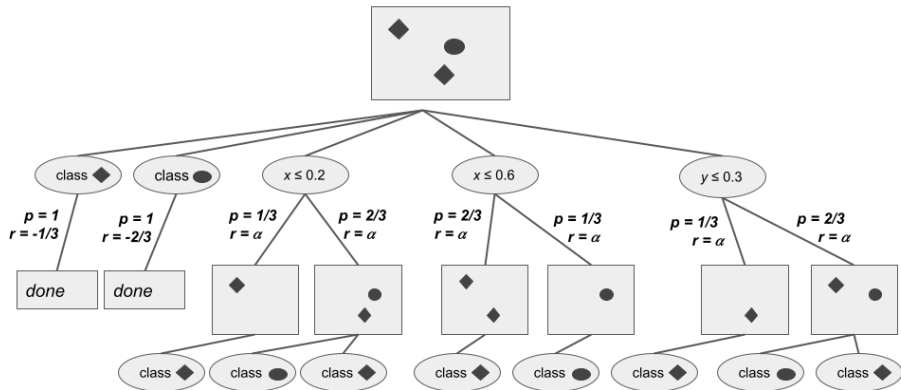
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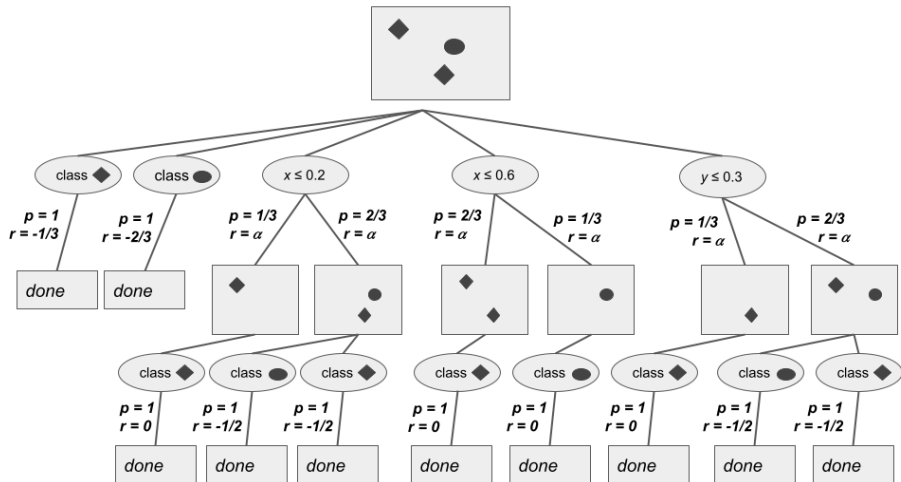
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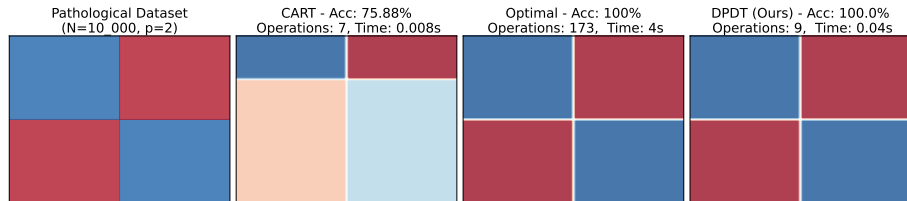
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# Fast like greedy trees, accurate like optimal trees



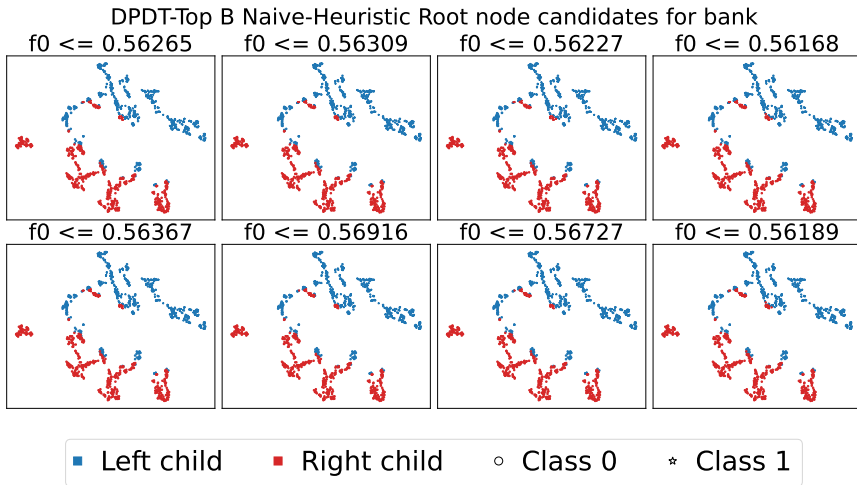
Comparison of greedy, optimal, and DPDT depth-2 trees on the checkersboard dataset.

# Fast like greedy trees, accurate like optimal trees

Comparison of accuracies and operations for depth-3 trees.

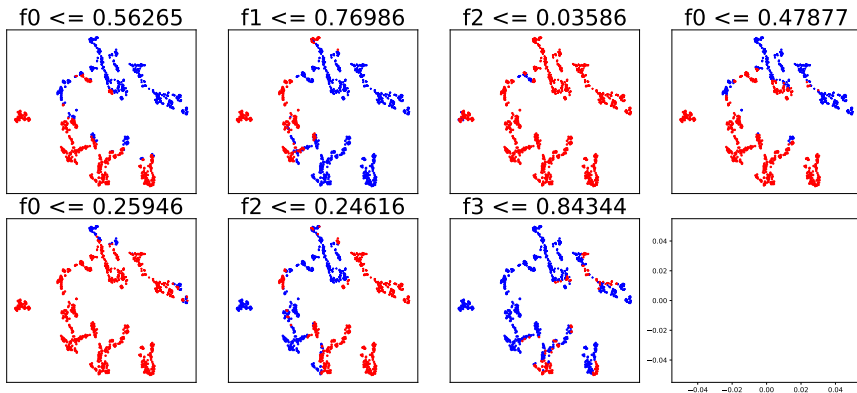
Dataset	Accuracy						Operations					
	Opt	Greedy	CART <sup>-</sup>	CART <sup>+</sup>	TopB <sup>-</sup>	TopB <sup>+</sup>	Opt	Greedy	CART <sup>-</sup>	CART <sup>+</sup>	TopB <sup>-</sup>	TopB <sup>+</sup>
room	0.992	0.968	0.991	0.992	0.990	0.992	$10^6$	15	286	16100	111	16100
bean	0.871	0.777	0.812	0.853	0.804	0.841	$5 \cdot 10^6$	15	295	25900	112	16800
eeg	0.708	0.666	0.689	0.706	0.684	0.699	$2 \cdot 10^6$	13	289	26000	95	11000
avila	0.585	0.532	0.574	0.585	0.563	0.572	$3 \cdot 10^7$	9	268	24700	60	38900
magic	0.831	0.801	0.822	0.828	0.807	0.816	$6 \cdot 10^6$	15	298	28000	70	4190
htru	0.981	0.979	0.979	0.980	0.979	0.980	$6 \cdot 10^7$	15	295	25300	55	2180
occup.	0.994	0.989	0.991	0.994	0.990	0.992	$7 \cdot 10^5$	13	280	16300	33	510
skin	0.969	0.966	0.966	0.966	0.966	0.966	$7 \cdot 10^4$	15	301	23300	20	126
fault	0.682	0.553	0.672	0.674	0.672	0.673	$9 \cdot 10^8$	13	295	24200	111	16800
segment	0.887	0.574	0.812	0.879	0.786	0.825	$2 \cdot 10^6$	7	220	16300	68	11400
page	0.971	0.964	0.970	0.970	0.964	0.965	$10^7$	15	298	22400	701	4050
bidding	0.993	0.981	0.985	0.993	0.985	0.993	$3 \cdot 10^5$	13	256	9360	58	2700
raisin	0.894	0.869	0.879	0.886	0.875	0.883	$4 \cdot 10^6$	15	295	20900	48	1440
rice	0.938	0.933	0.934	0.937	0.933	0.936	$2 \cdot 10^7$	15	298	25500	49	1470
wilt	0.996	0.993	0.994	0.995	0.994	0.994	$3 \cdot 10^5$	13	274	11300	33	465
bank	0.983	0.933	0.971	0.980	0.951	0.974	$6 \cdot 10^4$	13	271	7990	26	256

# CART generates more diverse splits than Top B



# CART generates more diverse splits than Top B

DPDT-CART-Heuristic Root node candidates for bank



■ Left child   ■ Right child   ○ Class 0   ☆ Class 1

# Why generating candidate splits with CART?

## Theorem (DPDT trees are not worse than greedy trees)

*The greedy tree is always a solution of the MDPs we solve. Because we solve the MDPs exactly with DP, if the greedy tree is the best solution, DPDT will find it.*

## Theorem (DPDT trees can be strictly better than greedy trees)

*There exist a depth budget  $D$  and a dataset for which DPDT trees are strictly better than greedy trees.<sup>a</sup>*

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<sup>a</sup>cf. checkersboard dataset.

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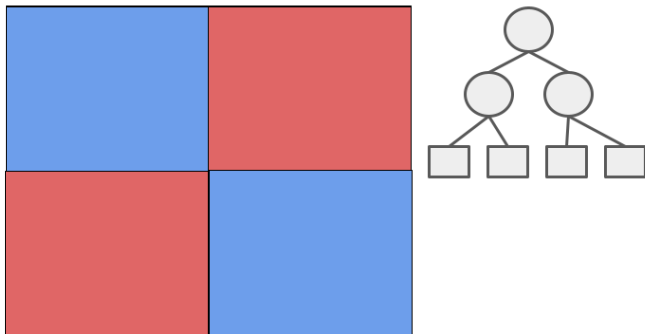
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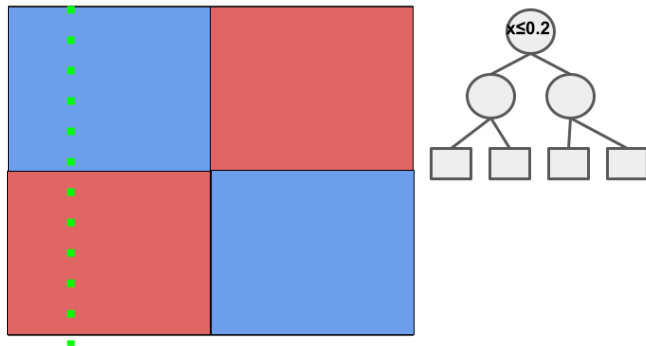
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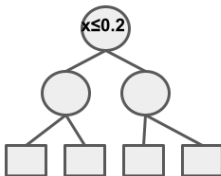
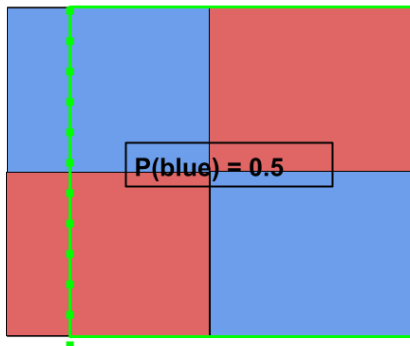
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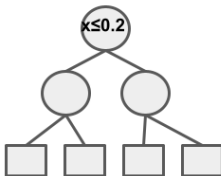
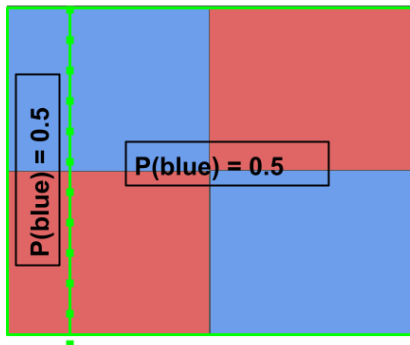
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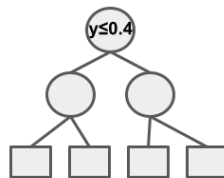
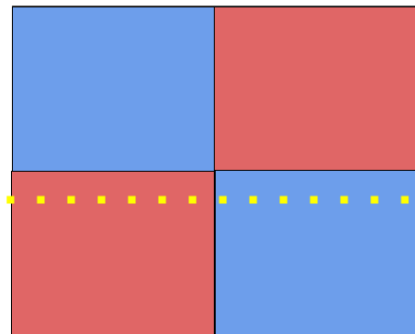
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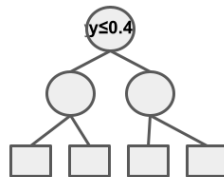
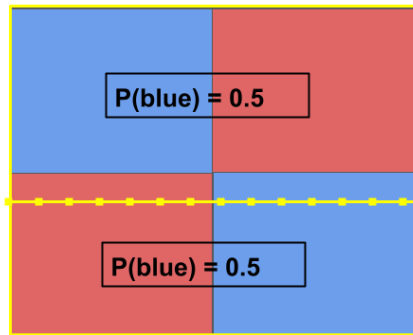
# DPDT trees can be strictly better than greedy trees



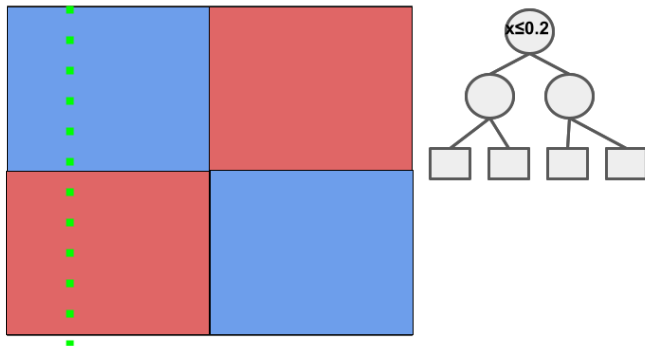
# DPDT trees can be strictly better than greedy trees



# DPDT trees can be strictly better than greedy trees

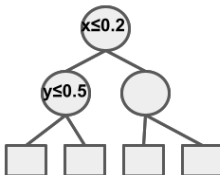
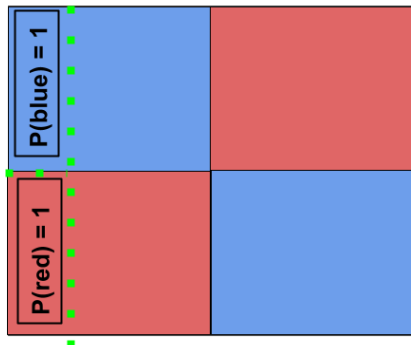


# DPDT trees can be strictly better than greedy trees

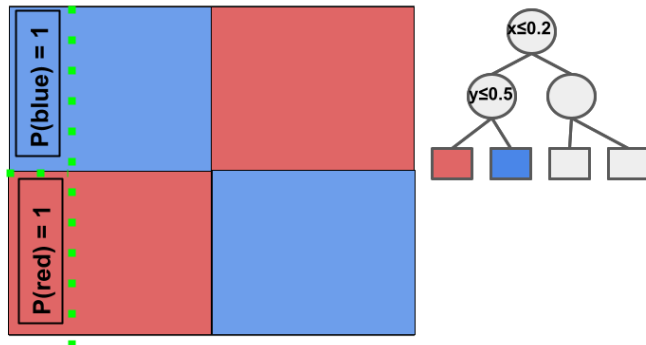




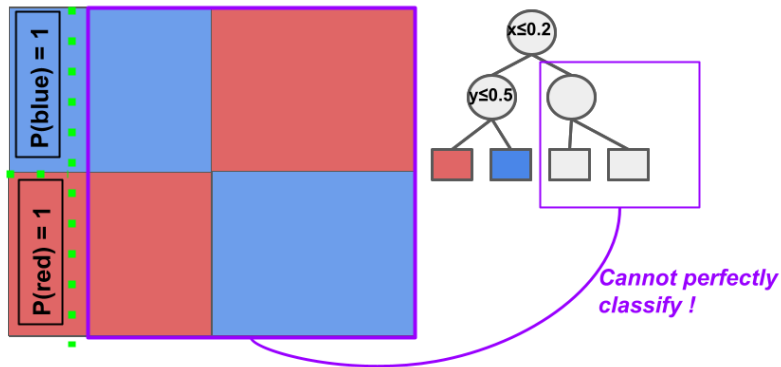
# DPDT trees can be strictly better than greedy trees



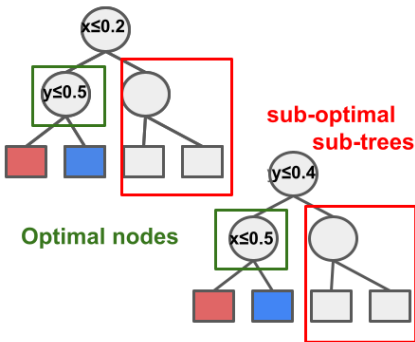
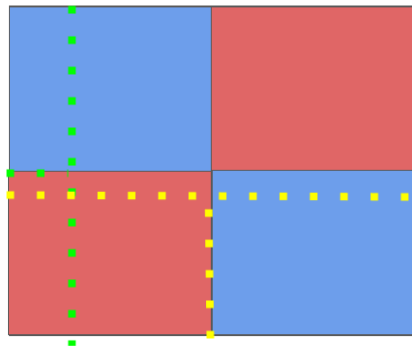
# DPDT trees can be strictly better than greedy trees



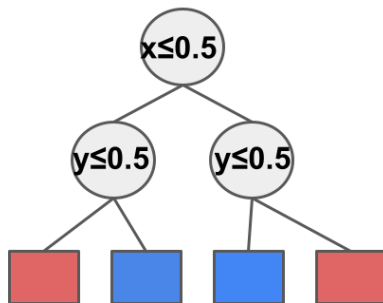
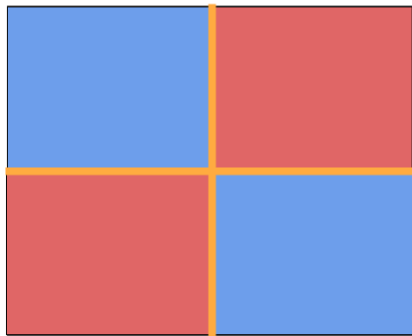
# DPDT trees can be strictly better than greedy trees



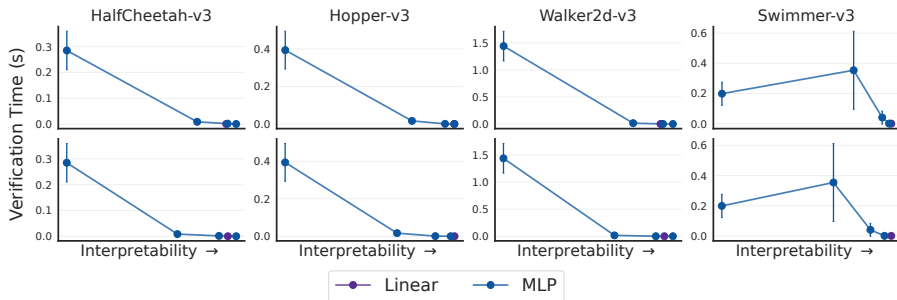
# DPDT trees can be strictly better than greedy trees



# DPDT trees can be strictly better than greedy trees

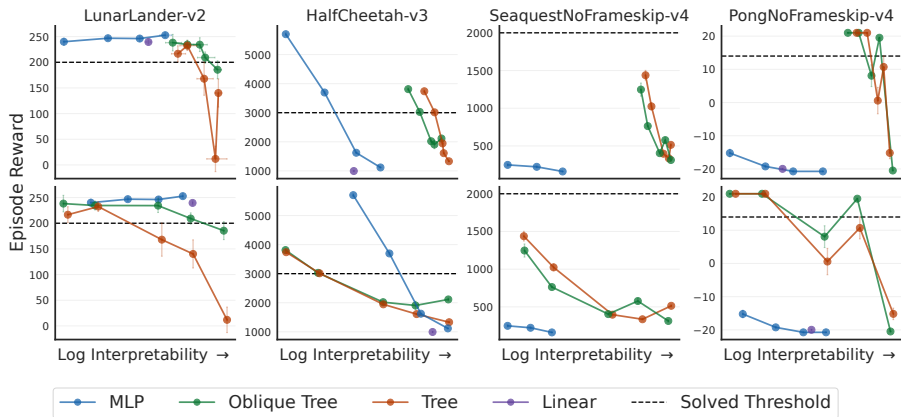


# Result: verification time does scale with step inference time



Verification time as a function of policy interpretability. Top row, interpretability is measured with step inference times. Bottom row, the interpretability is measured with policy size.

# Result: there is no dominating policy class for all environments



Interpretability-Performance trade-offs. Top row, interpretability is measured with step inference times. Bottom row, the interpretability is measured with policy size.

# We propose policy unfolding

```
# Decision tree for Mountain Car
def play(x):
    if x[1] <= -0.2597:
        if x[1] <= -0.6378:
            return 0
        else:
            if x[0] <= -1.0021:
                return 2
            else:
                return 0
    else:
        if x[1] <= -0.0508:
            if x[0] <= 0.2979:
                if x[0] <= 0.0453:
                    return 2
                else:
                    if x[1] <=
-0.2156:
                        return 0
                    else:
                        return 2
            else:
                return 0
        else:
            return 2
```

```
# Small ReLU MLP for Pendulum
def play(x):
    h_layer_0_0 = 1.238*x[0]+0.971*x
    [1]
    +0.430*x[2]+0.933
    h_layer_0_0 = max(0, h_layer_0_0
    )
    h_layer_0_1 = -1.221*x[0]+1.001
    *x[1]-0.423*x[2]
    +0.475
    h_layer_0_1 = max(0, h_layer_0_1
    )
    h_layer_1_0 = -0.109*h_layer_0_0
    -0.377*h_layer_0_1
    +1.694
    h_layer_1_0 = max(0, h_layer_1_0
    )
    h_layer_1_1 = -3.024*h_layer_0_0
    -1.421*h_layer_0_1
    +1.530
    h_layer_1_1 = max(0, h_layer_1_1
    )
    h_layer_2_0 = -1.790*h_layer_1_0
    +2.840*h_layer_1_1
    +0.658
    y_0 = h_layer_2_0
    return [y_0]
```