# Interpretability, Decision Trees, and Sequential Decision Making

Hector Kohler

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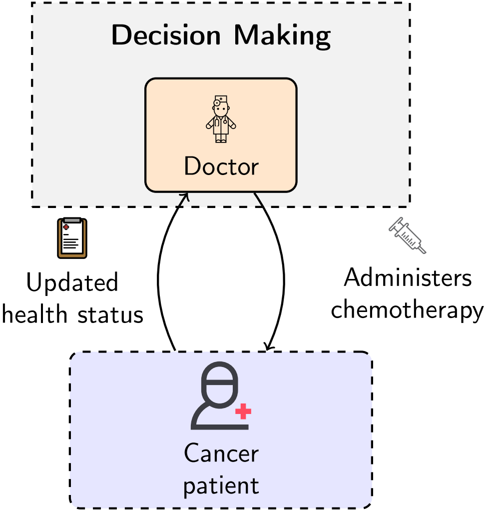
Université de Lille, CNRS, Inria, UMR CRIStAL 9189, France

December 2, 2025

# Sequential decision making (SDM)



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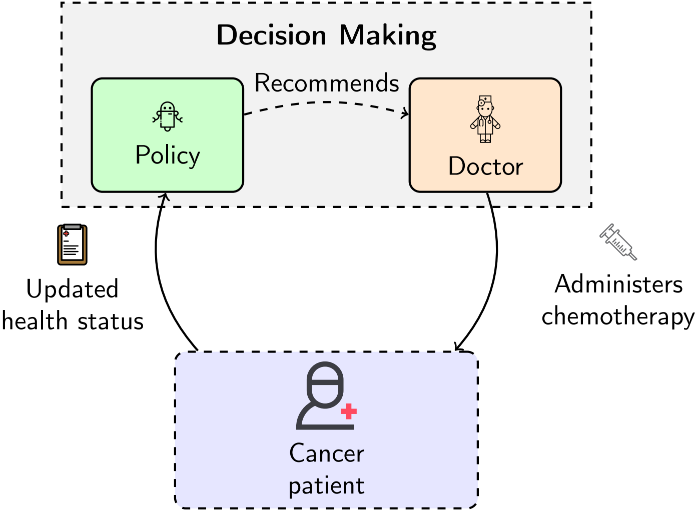
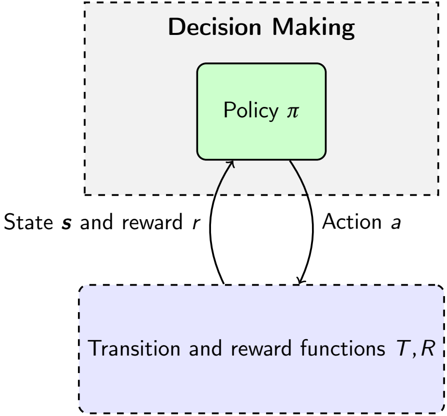


# Markov decision processes (MDPs) and reinforcement learning (RL)




Markov decision processes (Puterman 1994).

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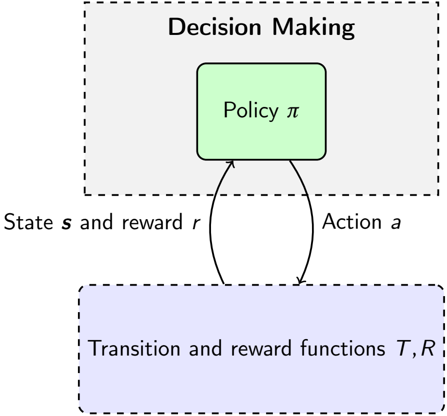


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- RL (Sutton and Barto 1998) aims to find a policy, $\pi : S \to A$ that maximizes:

$$J(\pi) = \mathop{\mathbb{E}}_{\boldsymbol{s}_t \sim T}\left[\sum_{t=0}^{\infty} \gamma^t R(s_t, \pi(s_t))\right]$$

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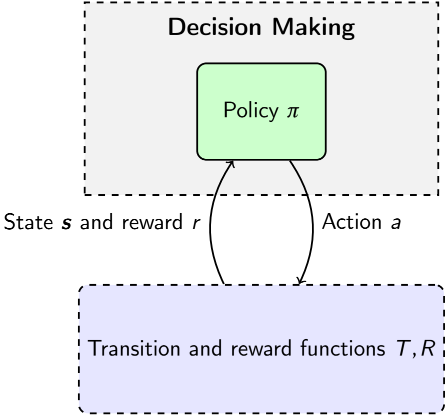


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- Lots of succesful RL algorithms (Schulman et al. 2017).

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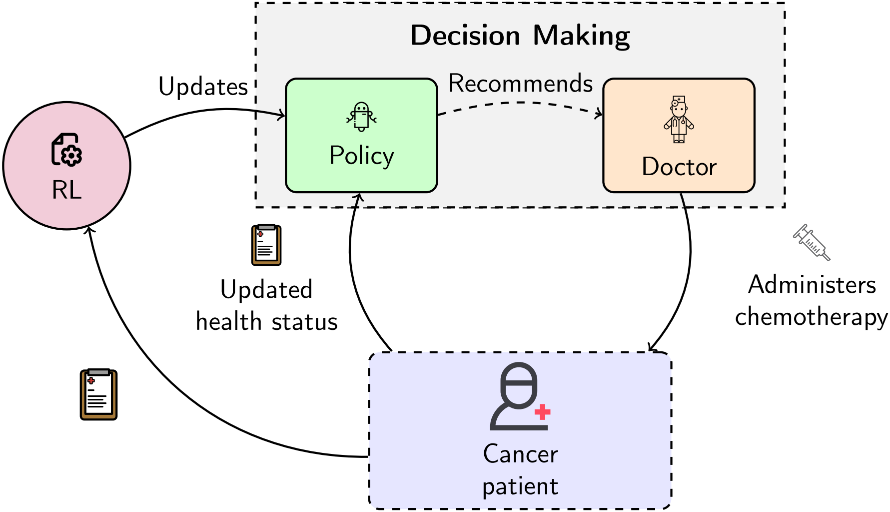

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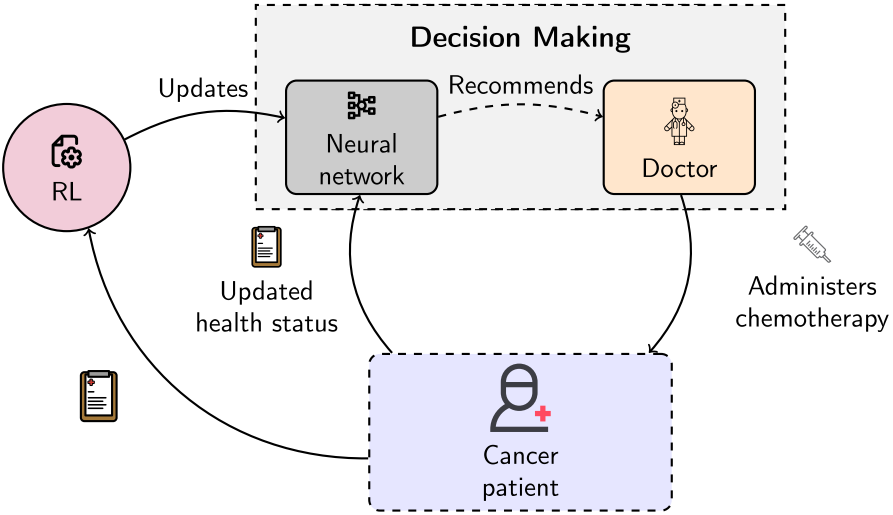

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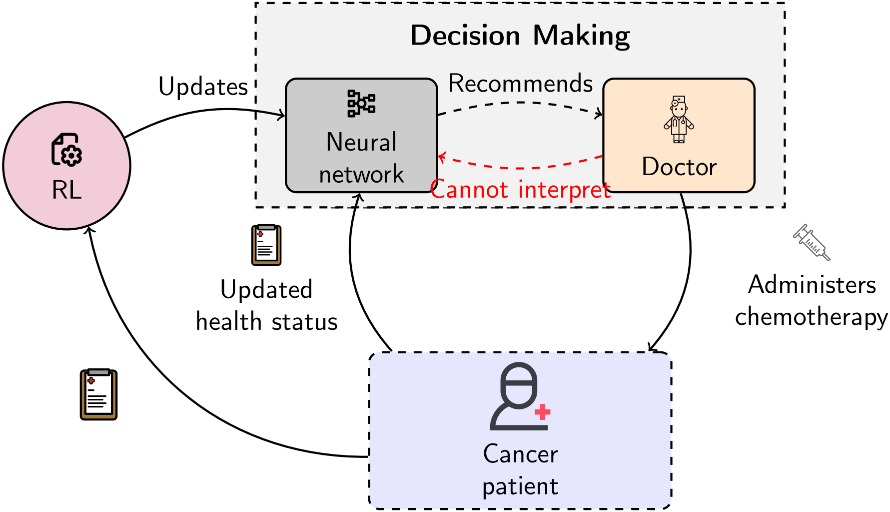

# Local vs. global interpretability (Glanois et al. 2024).

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Saliency maps of different MDP states (Greydanus et al. 2018).

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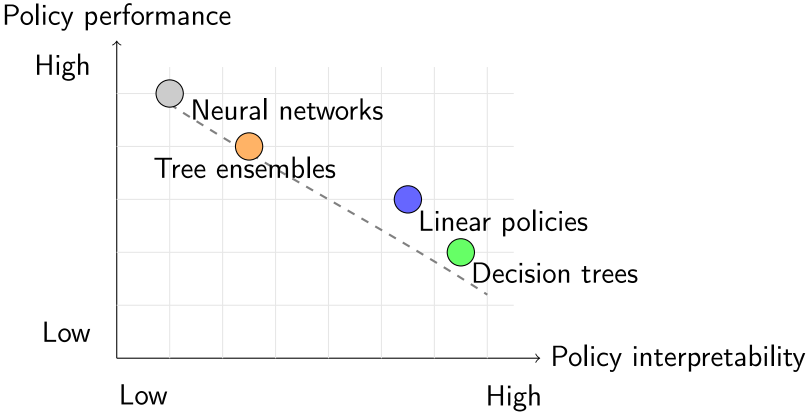


Global interpretation.

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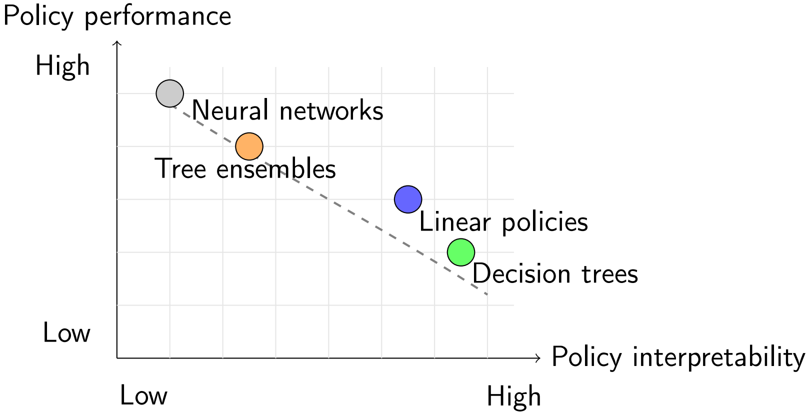
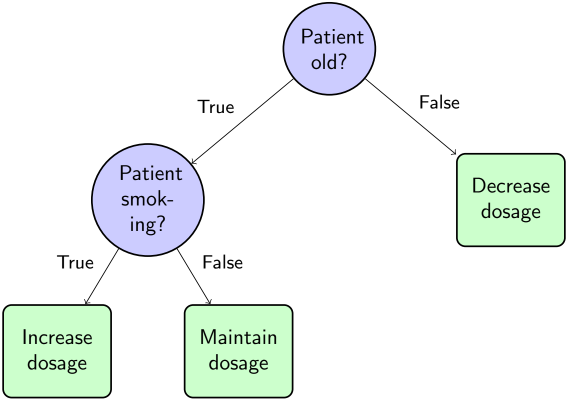



⚠**Multiple definitions (Lipton 2018).**

# Decision trees




A generic decision tree of depth $D = 2$.

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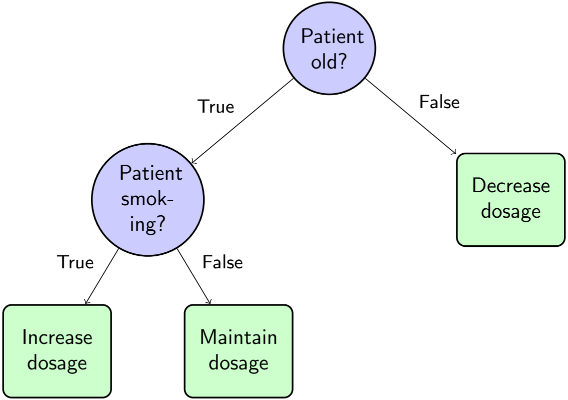


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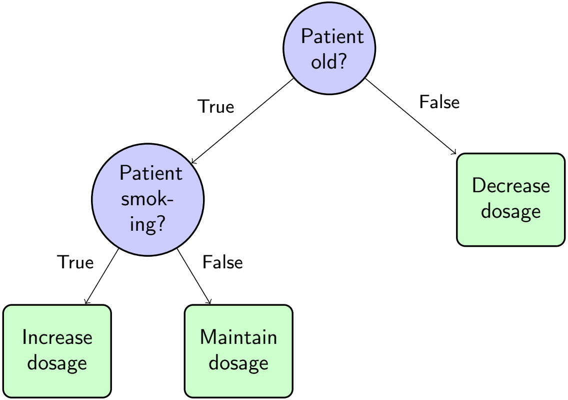


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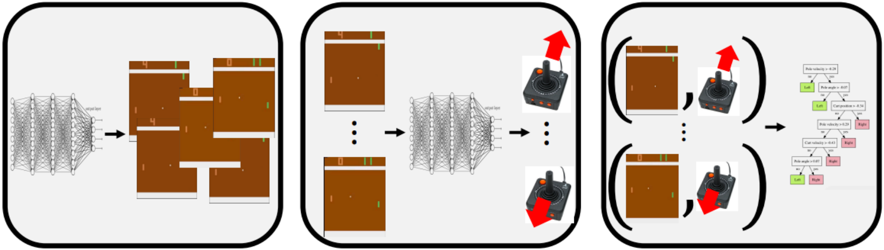
**What about SDM?**

# Imitation learning



**Step 1:** Use NN to generate states

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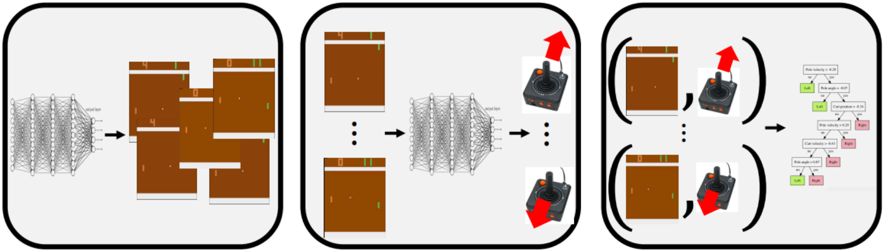

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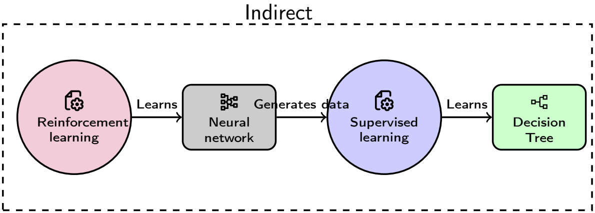
Most research focused on indirect learning of interpretable policies (Bastani, Pu, and Solar-Lezama 2018).

# Two ways to get interpretable policies for SDM (Glanois et al. 2024)



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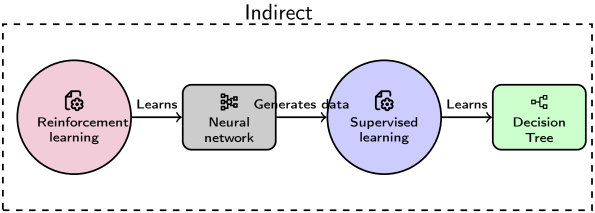


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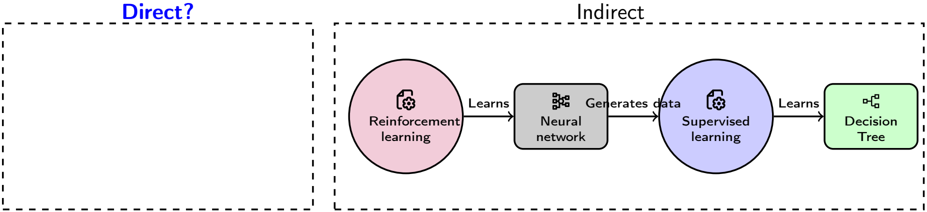


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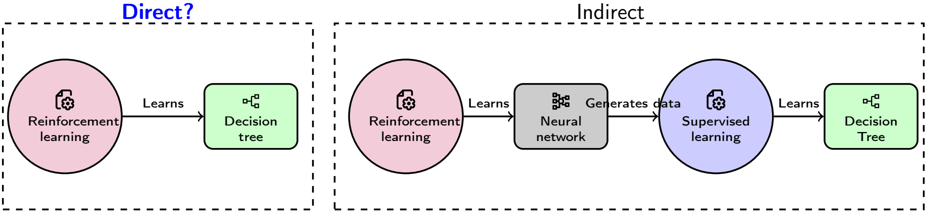


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2. Can we leverage SDM to learn decision trees for classification/regression?
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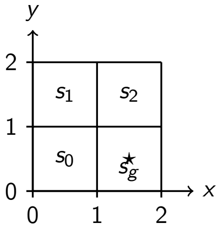

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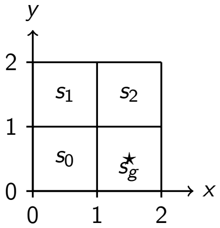


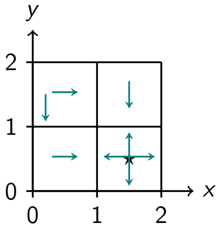

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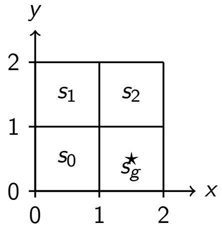


Grid world MDP and optimal actions.

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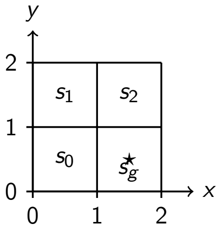


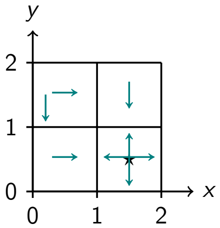


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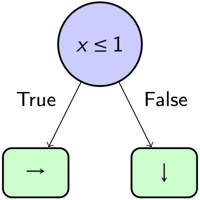

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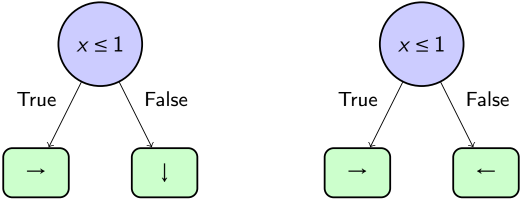

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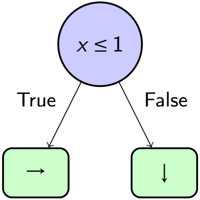


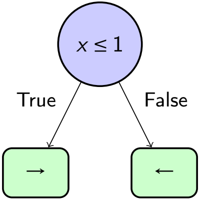


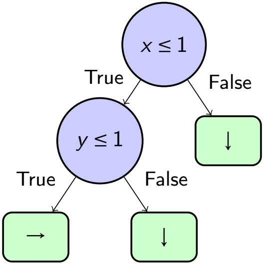

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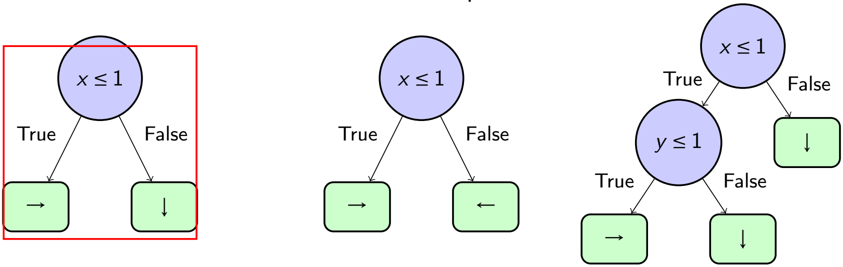
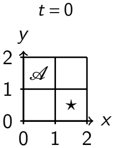



Decision tree policies with different interpretability-performance trade-offs.

# Iterative bounding Markov decision processes (Topin et al. 2021)



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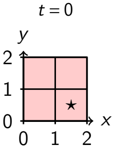

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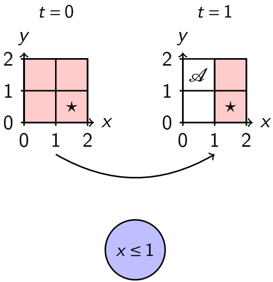

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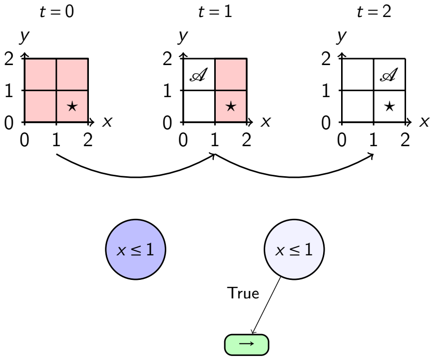

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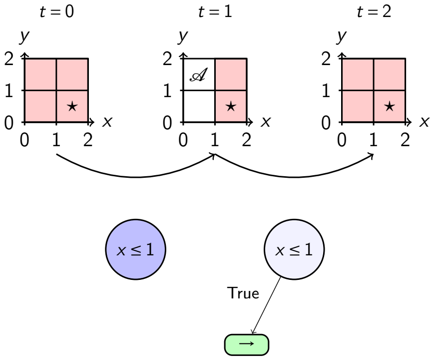

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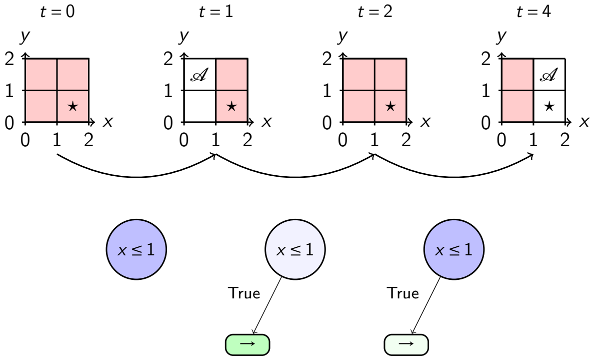

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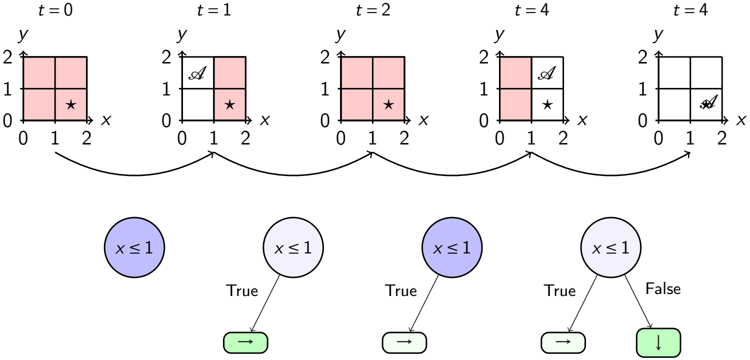

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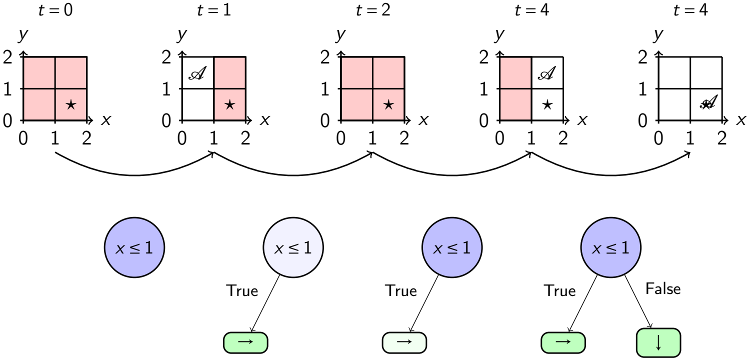

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- ⚠**IBMDP policies** $\pi_{po} : O \to A \cup A_{info}$ **are decision tree policies for** $\mathscr{M}$**.**

# RL for memoryless policies in POMDPs

## RL for memoryless policies

- Finding the best **deterministic** and memoryless policy in a POMDP is NP-hard (Littman 1994)!
- The best memoryless policy can be stochastic (S. P. Singh, Jaakkola, and Jordan 1994).
- Value-based RL converges to sub-optimal solutions (Loch and S. P. Singh 1998).

## Asymmetric RL

- Access to hidden states during training but not at execution (Pinto et al. 2017).
- Value-based → learns $Q(o, a)$ with TD targets $U(s, a)$ (Baisero, Daley, and Amato 2022).
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# RL for memoryless policies in POMDPs

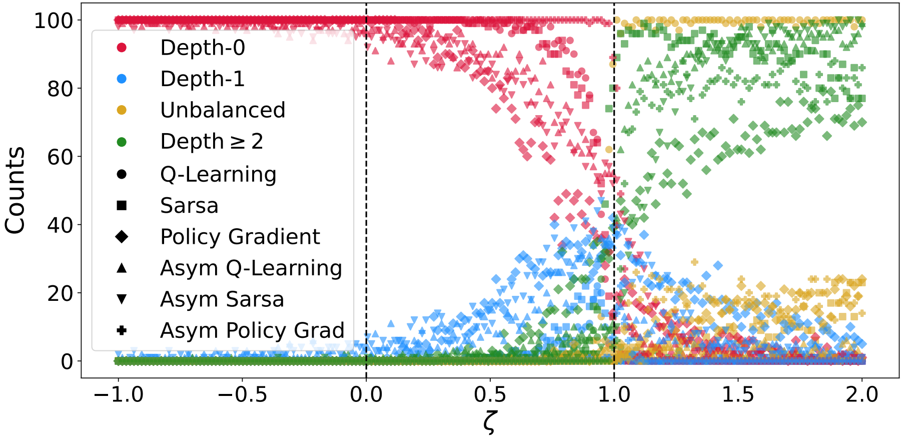
## RL for memoryless policies

- Finding the best **deterministic** and memoryless policy in a POMDP is NP-hard (Littman 1994)!
- The best memoryless policy can be stochastic (S. P. Singh, Jaakkola, and Jordan 1994).
- Value-based RL converges to sub-optimal solutions (Loch and S. P. Singh 1998).

## Asymmetric RL

- Access to hidden states during training but not at execution (Pinto et al. 2017).
- Value-based → learns $Q(o, a)$ with TD targets $U(s, a)$ (Baisero, Daley, and Amato 2022).
- Actor-critic[a] → policy gradient on $\pi(o, a)$ using a critic $V(s)$ (Baisero and Amato 2022).

# Result: RL cannot retrieve optimal depth-1 trees for the grid world MDP




Distributions of tree policies learned with (asymmetric) RL algorithms as a function of the interpretability reward $\zeta$.

# Result: for similar problems, RL struggles more when there is partial observability


Success rates over thousands of RL runs with varying hyperparameters when learning different policies in the same IBMDP[1].

[1] We also observed similar results on classic controls and variants of the grid world MDP.

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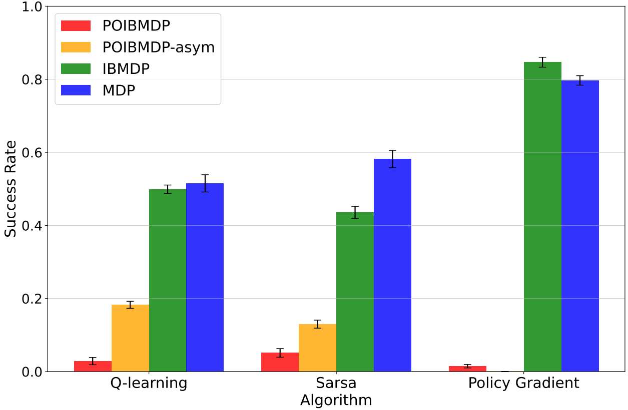


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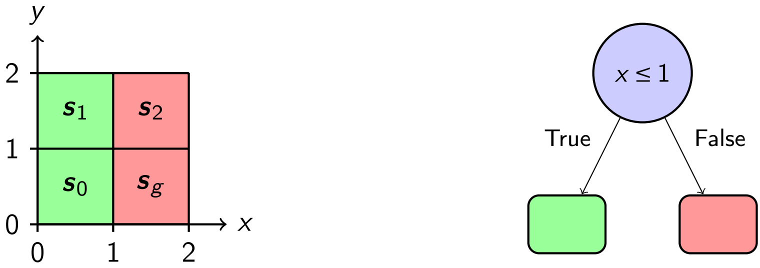


Classification MDP and the unique optimal depth-1 tree.

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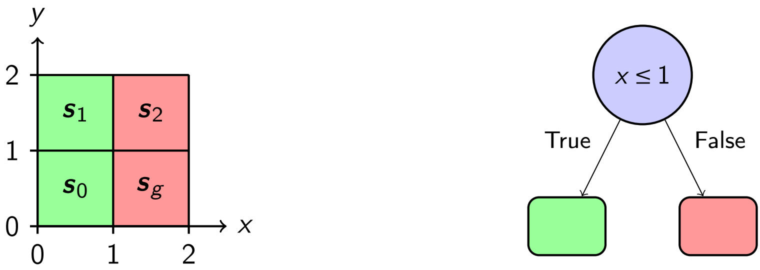
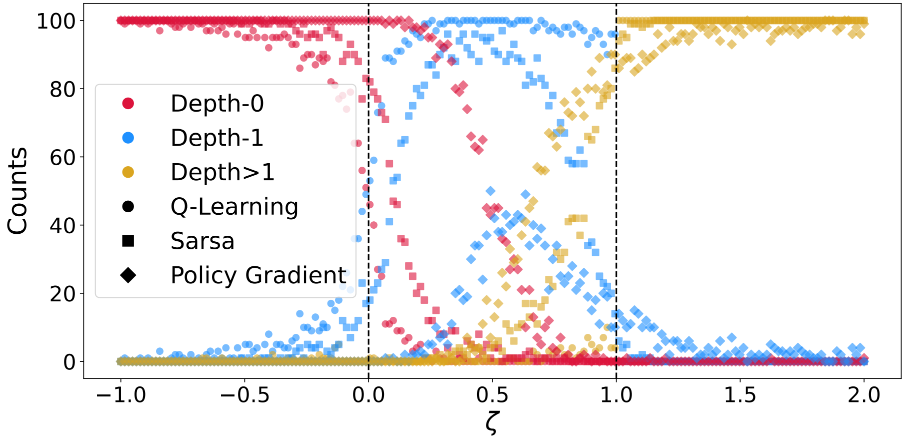



Classification MDP and the unique optimal depth-1 tree.

**Partial observations are sufficient statistics off the full states in classification IBMDPs.**

# Result: RL can retrieve optimal depth-1 trees for the toy classification MDP




Distributions of tree policies learned with various RL algorithms.

# Perspectives for direct RL of decision tree policies.

- It seems that directly learning decision tree trading off interpretability and performances in MDPs can be difficult to achieve because of **partial observability**.
- Should we focus on indirect approaches? Hybrid approaches (Wu et al. 2020)?
- Fixing the policy tree structure a priori (parametric trees, (Marton et al. 2025))?

For classification MDPs, decision tree policies are fully Markovian in associated IBMDPs

*Q: Can we leverage SDM to design new decision tree induction algorithms for classification/regression?*

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# Decision trees in supervised learning

- $N$ data points $\{\boldsymbol{x}_i, y_i\}$. Each $\boldsymbol{x}_i$ is described by $p$ features and has a label $y_i \in \mathscr{Y}$. We want to find a tree of depth at most $D$, $T \in \mathscr{T}_D$ that minimizes:

$$\mathscr{L}_\alpha(T) = \frac{1}{N}\sum_{i=1}^{N} \ell(y_i, T(x_i)) + \alpha C(T)$$

- Trees **interpretable** and **competitive with neural nets** (Grinsztajn, Oyallon, and Varoquaux 2022).
- Greedy algorithms **sub-optimal accuracy**, but $\boldsymbol{O}(2^{\boldsymbol{D}})$ operations (Breiman et al. 1984) .
- Optimal algorithms, **optimal accuracy**, but $\boldsymbol{O}((2\boldsymbol{Np})^{\boldsymbol{D}})$ operations (NP-hard) (Bertsimas and Dunn 2017).
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# Decision tree induction as solving MDPs

## Intuition

The induction of a decision tree is made of a sequence of decisions: at each node, we must decide whether it is better to split (a subset of) the training data, or to create a leaf node.

- S: data subsets.
- A: test or leaf nodes that can be added to the tree.
- R: interpretability term $-\alpha$ and accuracies.
- T: node traversals.

## Proposition (Objective Equivalence)

*Let $\pi$ be a deterministic policy of the MDP. Then $J_\alpha(\pi) = -\mathcal{L}_\alpha(E(\pi, s_0))$ where $E$ is an algorithm that extracts a decision tree from $\pi$ (Topin et al. 2021).*

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- Greedy algorithms consider only one candidate action in each state which is the test that minimizes some impurity criterion
  → **MDP state space size is $O(2^D)$**.
- Optimal algorithms consider all possible actions in each state
  → **MDP state space size is $O((2Np)^D)$**.
- **Dynamic Programming Decision Trees (DPDT)**: Let's choose candidate actions adaptively
  → for each MDP state consider $B$ actions: **state space size is $O((2B)^D)$**.

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Top-B greedy splits (Blanc et al. 2023), quantiles, random. . .

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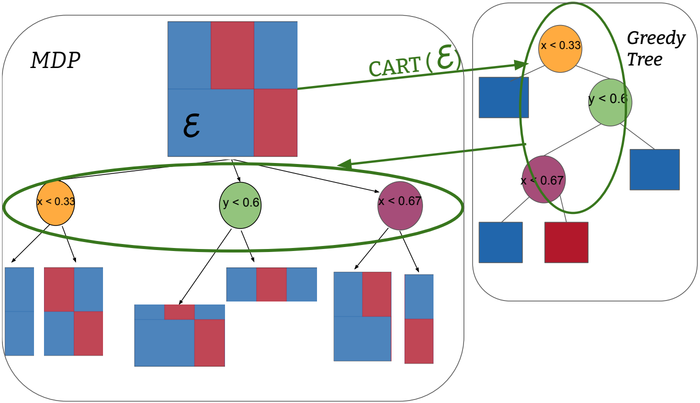


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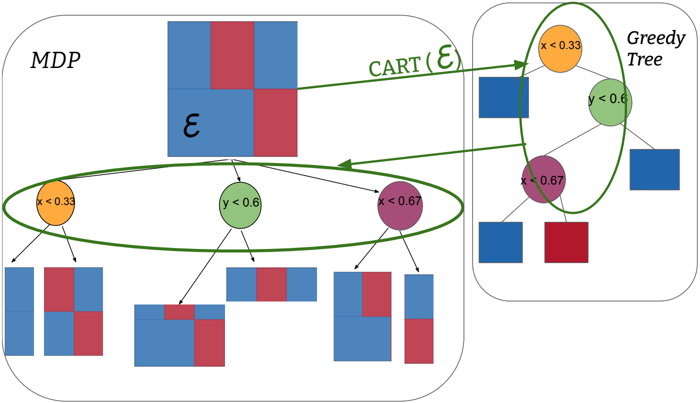


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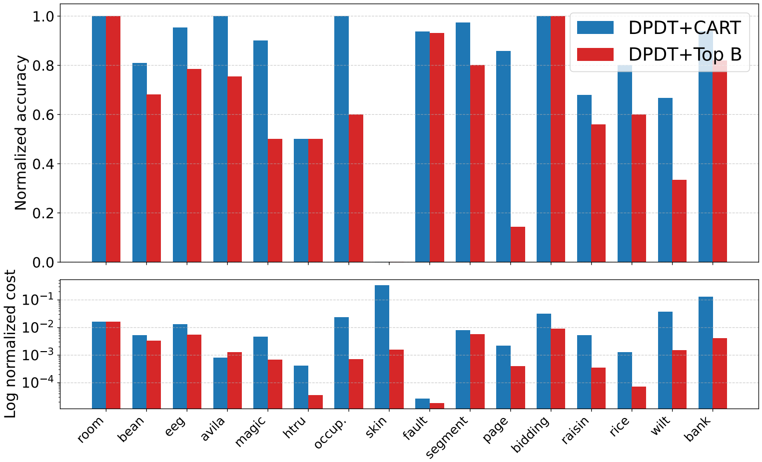
# Fast like greedy trees, accurate like optimal trees




Train accuracies against cost for detph-3 trees.

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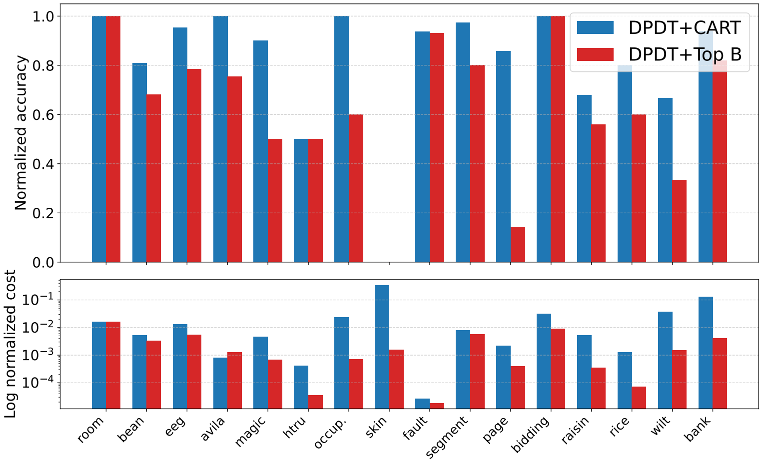


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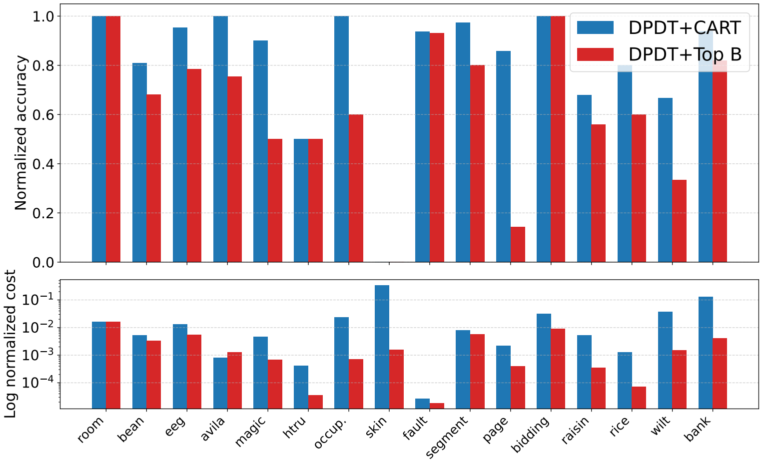


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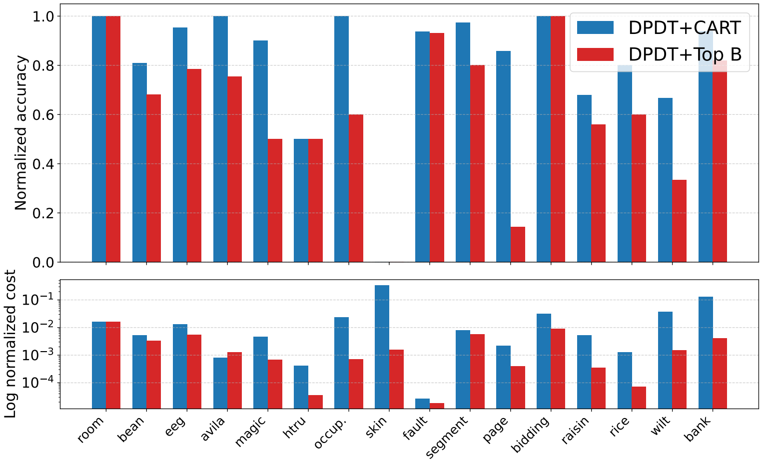


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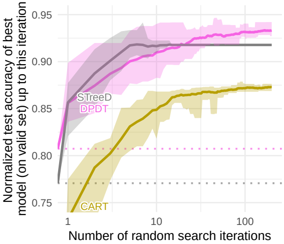
- CART generates more diverse splits than Top B for DPDT.

# Large scale evaluation of DPDT trees generalization

(Grinsztajn, Oyallon, and Varoquaux 2022)




DPDT depth-5 trees vs.
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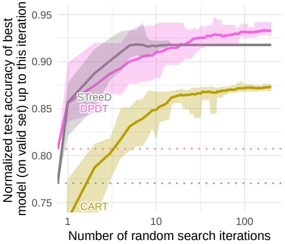


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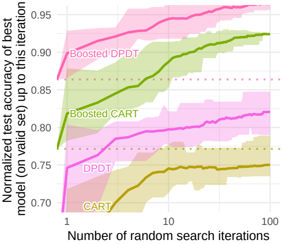


Boosted DPDT vs. Boosted CART

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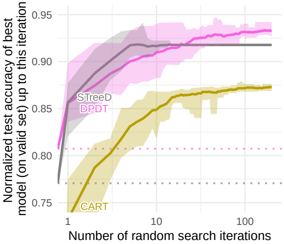


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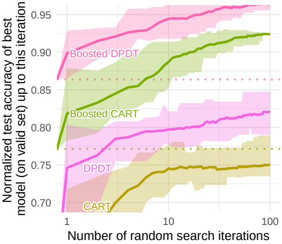
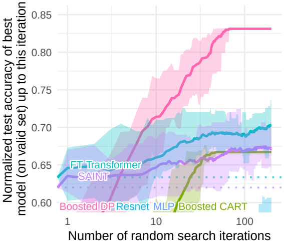



Boosted DPDT vs. Boosted CART




Boosted DPDT vs. other classifiers

# Perspectives

- New SOTA decision tree induction with dynamic programming in MDPs.
- What about using DPDT for indirect decision tree policy learning for SDM?
- What performances could we reach with an industry-grade implementation of XGboost+DPDT?

**Let us take a step back**

*Q: Are decision trees really the most interpretable model?*

**A: It depends.**

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# We propose policy unfolding

```
# Small ReLU MLP for Pendulum
def play(x):
    h_layer_0_0 = 1.68 * x[0] + -0.69 * x[1] + -0.74 * x[2] + -1.40
    h_layer_0_0 = max(0.0, h_layer_0_0)
    h_layer_0_1 = 0.20 * x[0] + 0.29 * x[1] + -0.021 * x[2] + 1.25
    h_layer_0_1 = max(0.0, h_layer_0_1)
    h_layer_0_2 = 0.33 * x[0] + -0.57 * x[1] + 0.47 * x[2] + 1.94
    h_layer_0_2 = max(0.0, h_layer_0_2)
    h_layer_0_3 = 1.39 * x[0] + 0.94 * x[1] + 0.50 * x[2] + -1.13
    h_layer_0_3 = max(0.0, h_layer_0_3)
    h_layer_1_0 = 1.16 * h_layer_0_0 + -1.59 * h_layer_0_1 + 0.95 * h_layer_0_2 +
       -1.22 * h_layer_0_3 + -0.54
    h_layer_1_0 = max(0.0, h_layer_1_0)
    h_layer_1_1 = -0.55 * h_layer_0_0 + 1.13 * h_layer_0_1 + -0.58 * h_layer_0_2
     + -0.72 * h_layer_0_3 + 1.56
    h_layer_1_1 = max(0.0, h_layer_1_1)
    h_layer_1_2 = 1.10 * h_layer_0_0 + -1.01 * h_layer_0_1 + 0.96 * h_layer_0_2 +
       -2.84 * h_layer_0_3 + -0.02
    h_layer_1_2 = max(0.0, h_layer_1_2)
    h_layer_1_3 = 0.27 * h_layer_0_0 + 0.44 * h_layer_0_1 + 0.39 * h_layer_0_2 +
     0.15 * h_layer_0_3 + -1.24
    h_layer_1_3 = max(0.0, h_layer_1_3)
    h_layer_2_0 = -2.80 * h_layer_1_0 + -0.60 * h_layer_1_1 + 3.07 * h_layer_1_2
     + -1.63 * h_layer_1_3 + -0.36
    y_0 = h_layer_2_0

    return [y_0]
```

# Is policy unfolding really necessary?

## Setup

We imitate $\sim 40000$ expert policies from `stable-baselines3` using various policy classes/nb parameters on various environments.

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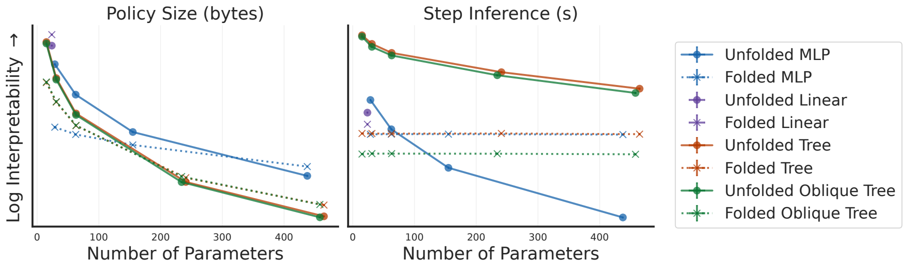


Aggregated policies interpretability on classic control environments

# Perspectives

- Beliefs such as "trees are more interpretable than neural networks" should be used with caution.
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- What about the cost of basic operations ($\times$ vs. $+$ vs. $\geq$)?

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- **Technical challenges:** **Learning interpretable policies for SDM involves partial observability.**
  → Focus on indirect approaches and/or on POMDP research first?
  → **Created opportunities for new decision tree algos for classif/regression.**
- **Fundamental challenges:** **No concensus on interpretability definition**.
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Motivate interpretability by finding a real-world problem where interpretability is *really* necessary (Nagendran et al. 2024).

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# Broader perspectives

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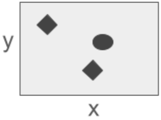
# Decision tree induction as solving MDPs




Example of decision tree induction as an MDP.

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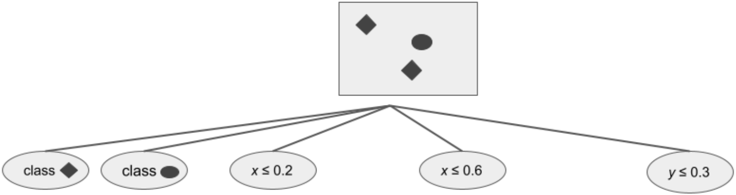


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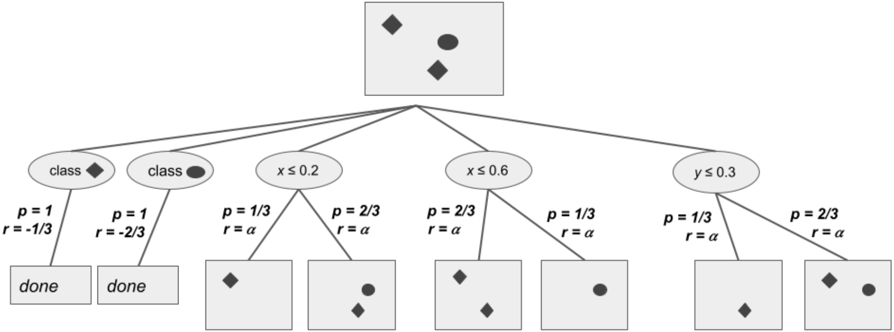


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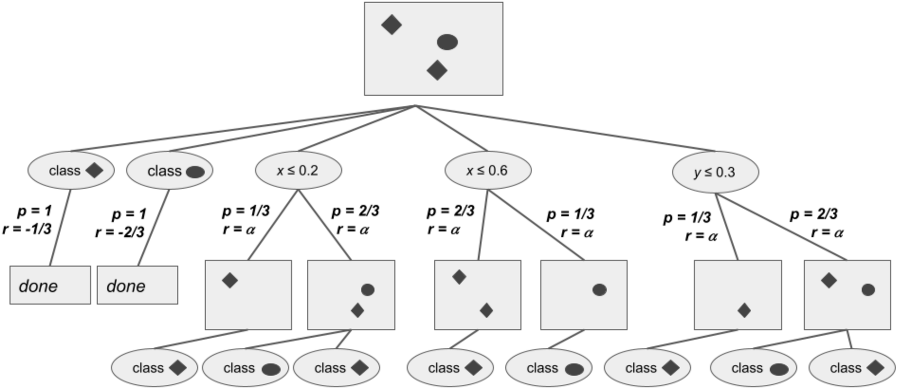


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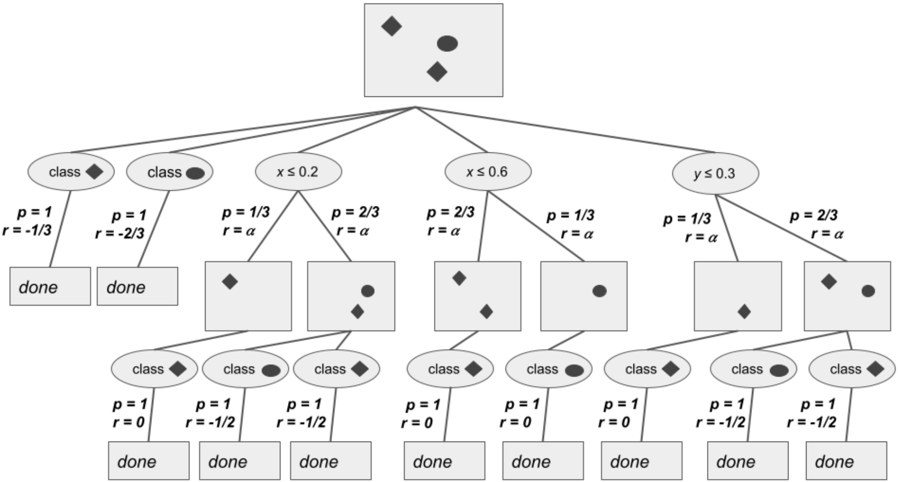


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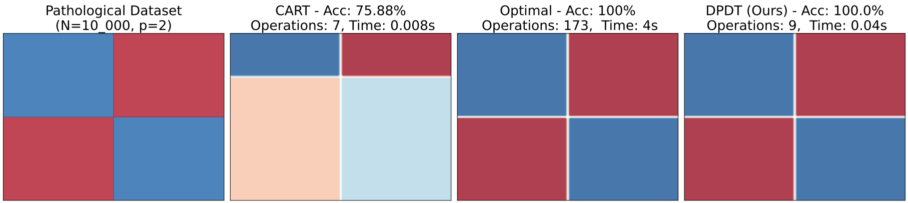
# Fast like greedy trees, accurate like optimal trees



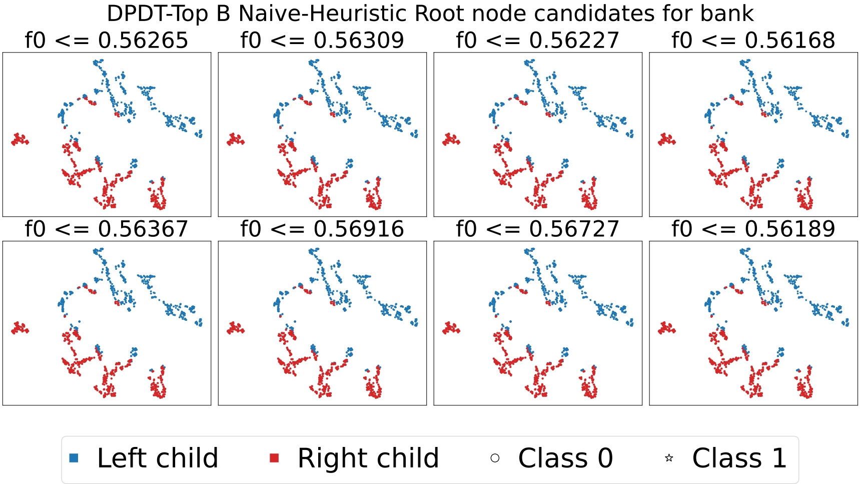

Comparison of greedy, optimal, and DPDT depth-2 trees on the checkersboard dataset.

# Fast like greedy trees, accurate like optimal trees

Comparison of accuracies and operations for depth-3 trees.

| | Accuracy | | | | | | Operations | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Dataset** | Opt | Greedy | DPDT | | | | Opt | Greedy | DPDT | | | |
| | | | CART$^-$ | CART$^+$ | TopB$^-$ | TopB$^+$ | | | CART$^-$ | CART$^+$ | TopB$^-$ | TopB$^+$ |
| room | **0.992** | 0.968 | 0.991 | **0.992** | 0.990 | **0.992** | $10^6$ | 15 | 286 | 16100 | 111 | 16100 |
| bean | **0.871** | 0.777 | 0.812 | 0.853 | 0.804 | 0.841 | $5 \cdot 10^6$ | 15 | 295 | 25900 | 112 | 16800 |
| eeg | **0.708** | 0.666 | 0.689 | 0.706 | 0.684 | 0.699 | $2 \cdot 10^6$ | 13 | 289 | 26000 | 95 | 11000 |
| avila | **0.585** | 0.532 | 0.574 | **0.585** | 0.563 | 0.572 | $3 \cdot 10^7$ | 9 | 268 | 24700 | 60 | 38900 |
| magic | **0.831** | 0.801 | 0.822 | 0.828 | 0.807 | 0.816 | $6 \cdot 10^6$ | 15 | 298 | 28000 | 70 | 4190 |
| htru | **0.981** | 0.979 | 0.979 | 0.980 | 0.979 | 0.980 | $6 \cdot 10^7$ | 15 | 295 | 25300 | 55 | 2180 |
| occup. | **0.994** | 0.989 | 0.991 | **0.994** | 0.990 | 0.992 | $7 \cdot 10^5$ | 13 | 280 | 16300 | 33 | 510 |
| skin | **0.969** | 0.966 | 0.966 | 0.966 | 0.966 | 0.966 | $7 \cdot 10^4$ | 15 | 301 | 23300 | 20 | 126 |
| fault | **0.682** | 0.553 | 0.672 | 0.674 | 0.672 | 0.673 | $9 \cdot 10^8$ | 13 | 295 | 24200 | 111 | 16800 |
| segment | **0.887** | 0.574 | 0.812 | 0.879 | 0.786 | 0.825 | $2 \cdot 10^6$ | 7 | 220 | 16300 | 68 | 11400 |
| page | **0.971** | 0.964 | 0.970 | 0.970 | 0.964 | 0.965 | $10^7$ | 15 | 298 | 22400 | 701 | 4050 |
| bidding | **0.993** | 0.981 | 0.985 | **0.993** | 0.985 | **0.993** | $3 \cdot 10^5$ | 13 | 256 | 9360 | 58 | 2700 |
| raisin | **0.894** | 0.869 | 0.879 | 0.886 | 0.875 | 0.883 | $4 \cdot 10^6$ | 15 | 295 | 20900 | 48 | 1440 |
| rice | **0.938** | 0.933 | 0.934 | 0.937 | 0.933 | 0.936 | $2 \cdot 10^7$ | 15 | 298 | 25500 | 49 | 1470 |
| wilt | **0.996** | 0.993 | 0.994 | 0.995 | 0.994 | 0.994 | $3 \cdot 10^5$ | 13 | 274 | 11300 | 33 | 465 |
| bank | **0.983** | 0.933 | 0.971 | 0.980 | 0.951 | 0.974 | $6 \cdot 10^4$ | 13 | 271 | 7990 | 26 | 256 |

# CART generates more diverse splits than Top B



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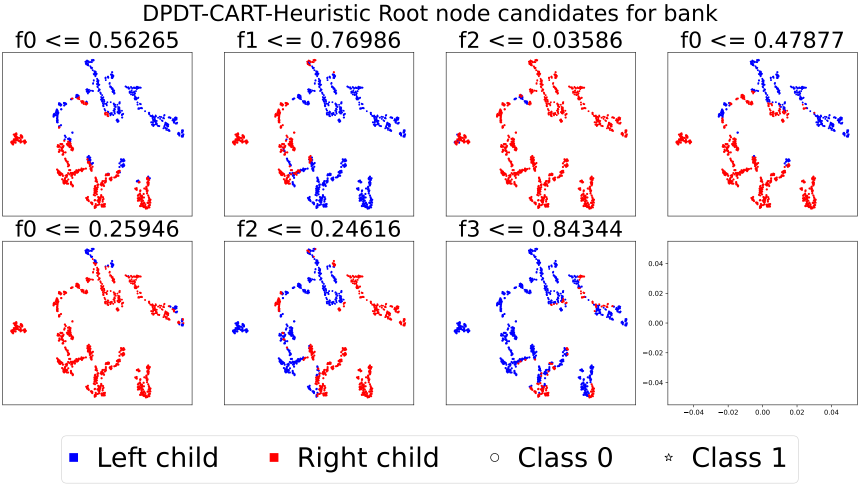

# Why generating candidate splits with CART?

**Theorem (DPDT trees are not worse than greedy trees)**

*The greedy tree is always a solution of the MDPs we solve. Because we solve the MDPs exactly with DP, if the greedy tree is the best solution, DPDT will find it.*

**Theorem (DPDT trees can be strictly better than greedy trees)**

*There exist a depth budget $D$ and a dataset for which DPDT trees are strictly better than greedy trees.*[a]

[a] cf. checkersboard dataset.

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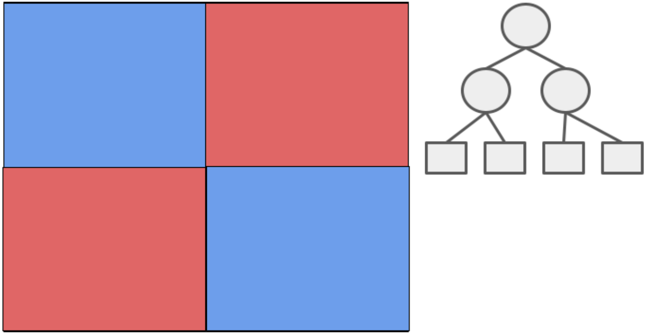

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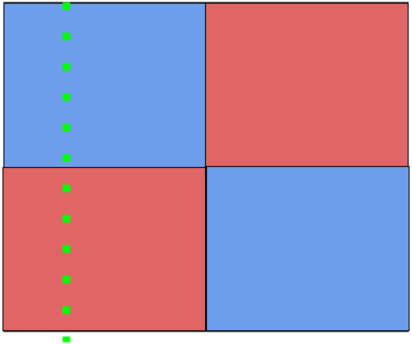

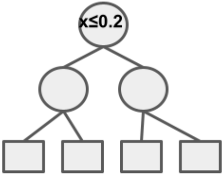

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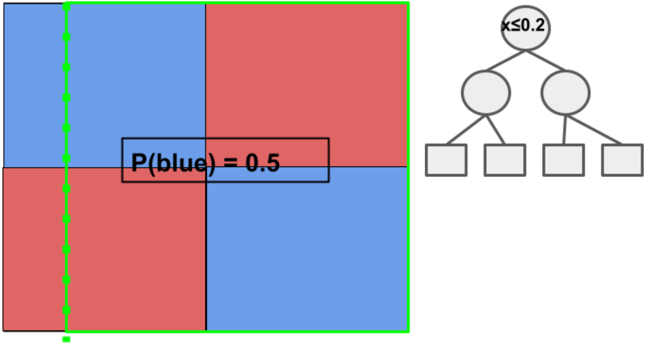

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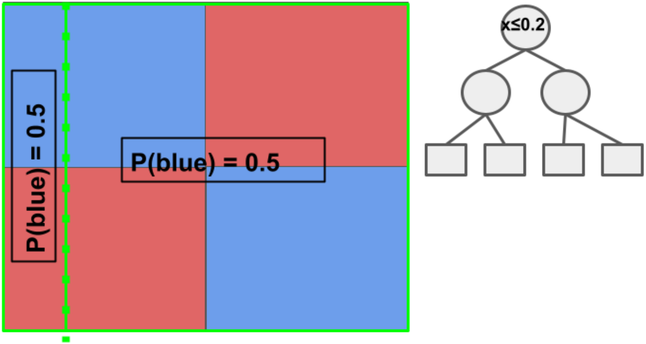

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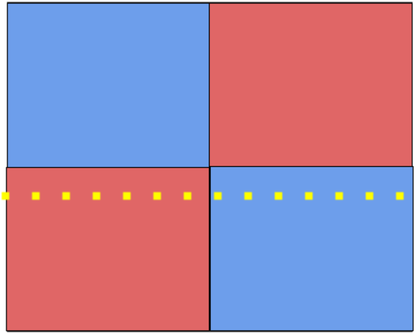

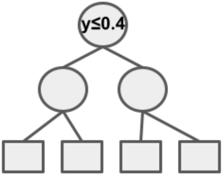

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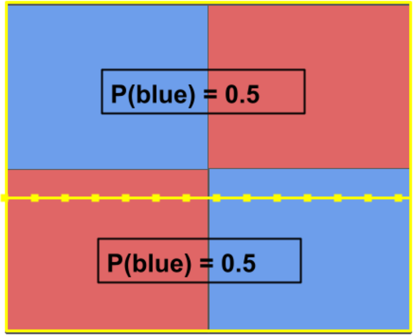


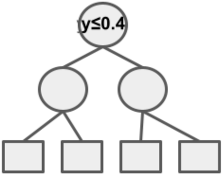

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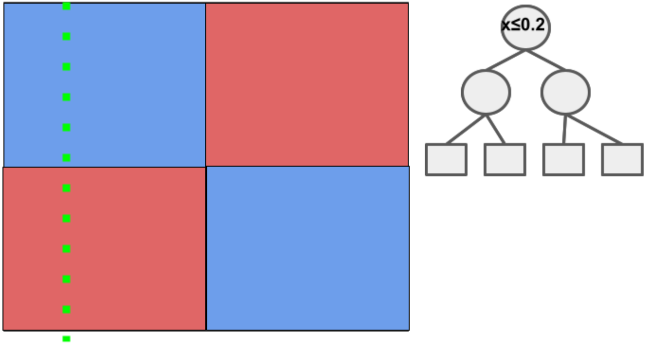

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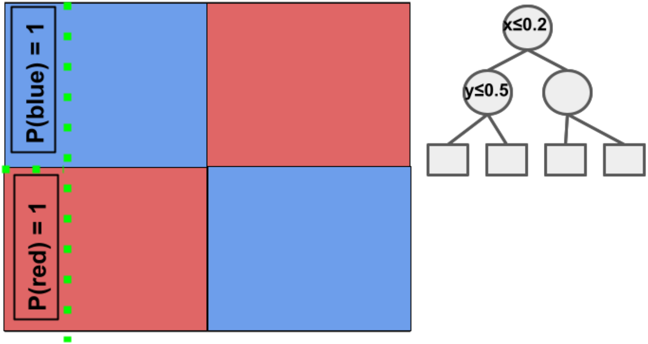

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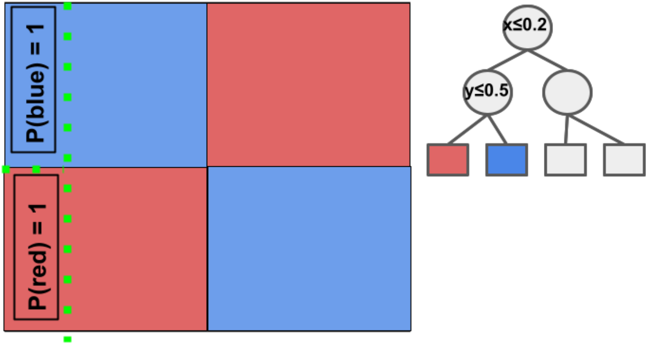

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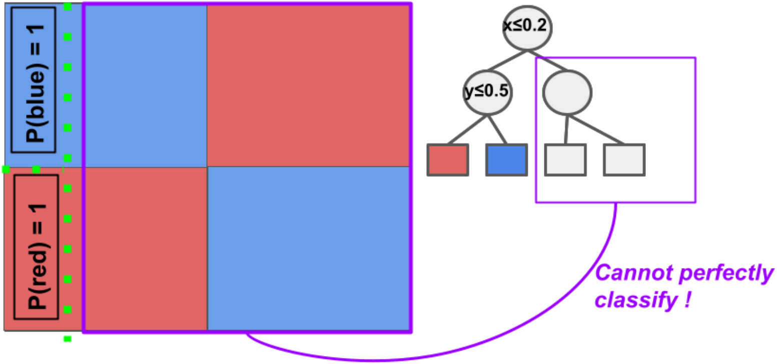

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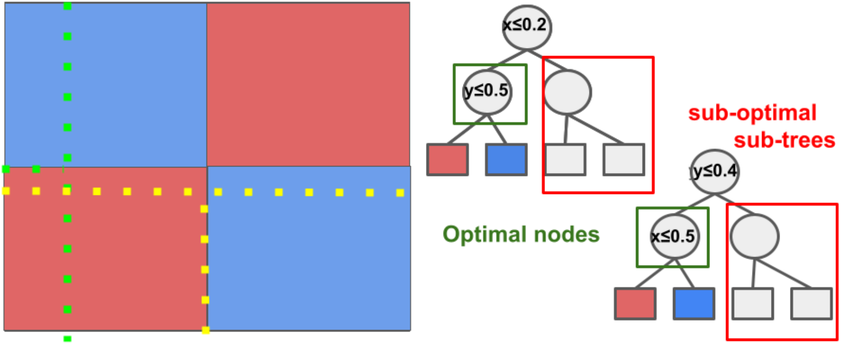

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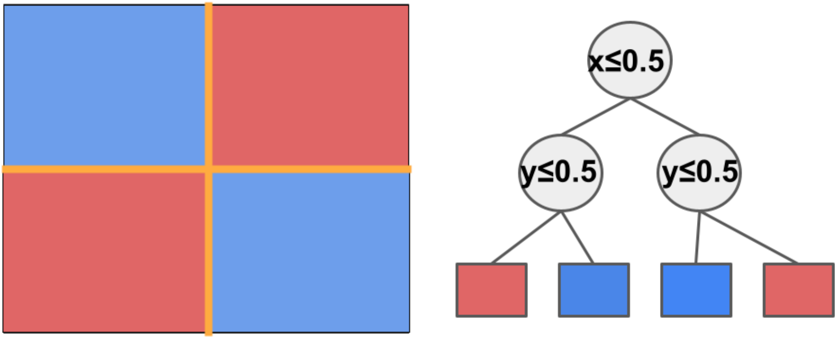
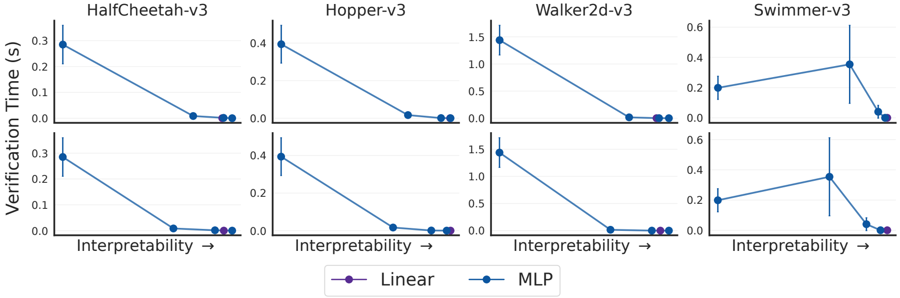
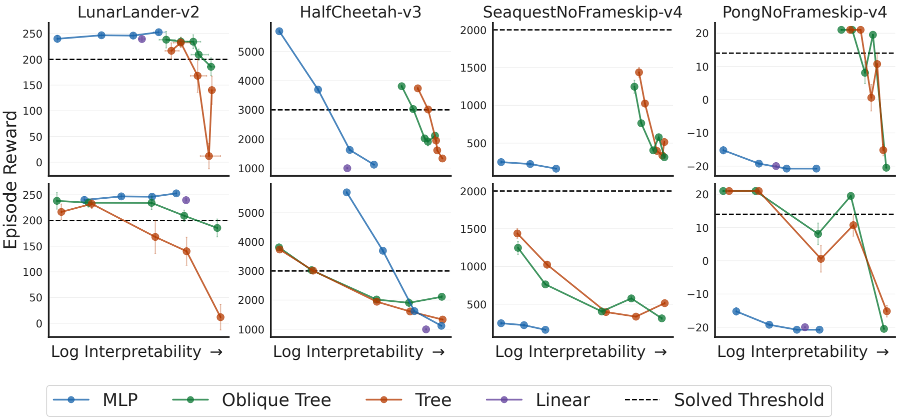


# Result: verification time does scale with step inference time




Verification time as a function of policy interpretability. Top row, interpretability is measured with step inference times. Bottom row, the interpretability is measured with policy size.

# Result: there is no dominating policy class for all environments




Interpretability-Performance trade-offs. Top row, interpretability is measured with step inference times. Bottom row, the interpretability is measured with policy size.

# We propose policy unfolding

```python
# Decision tree for Mountain Car
def play(x):
    if x[1] <= -0.2597:
        if x[1] <= -0.6378:
            return 0
        else:
            if x[0] <= -1.0021:
                return 2
            else:
                return 0
    else:
        if x[1] <= -0.0508:
            if x[0] <= 0.2979:
                if x[0] <= 0.0453:
                    return 2
                else:
                    if x[1] <=
   -0.2156:
                        return 0
                    else:
                        return 2
            else:
                return 0
        else:
            return 2
```

```python
# Small ReLU MLP for Pendulum
def play(x):
    h_layer_0_0 = 1.238*x[0]+0.971*x
     [1]
                  +0.430*x[2]+0.933
    h_layer_0_0 = max(0, h_layer_0_0
     )
    h_layer_0_1 = -1.221*x[0]+1.001
                  *x[1]-0.423*x[2]
                  +0.475
    h_layer_0_1 = max(0, h_layer_0_1
     )
    h_layer_1_0 = -0.109*h_layer_0_0
                  -0.377*h_layer_0_1
                  +1.694
    h_layer_1_0 = max(0, h_layer_1_0
     )
    h_layer_1_1 = -3.024*h_layer_0_0
                  -1.421*h_layer_0_1
                  +1.530
    h_layer_1_1 = max(0, h_layer_1_1
     )

    h_layer_2_0 = -1.790*h_layer_1_0
                  +2.840*h_layer_1_1
                  +0.658
    y_0 = h_layer_2_0
    return [y_0]
```