Interpretability, Decision Trees, and Sequential Decision Making

Hector Kohler

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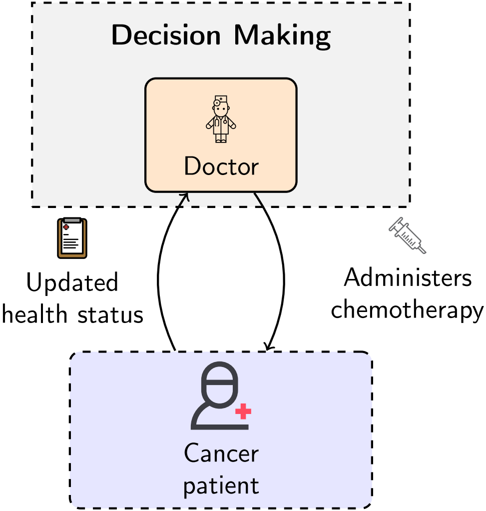
November 30, 2025

Sequential decision making (SDM)



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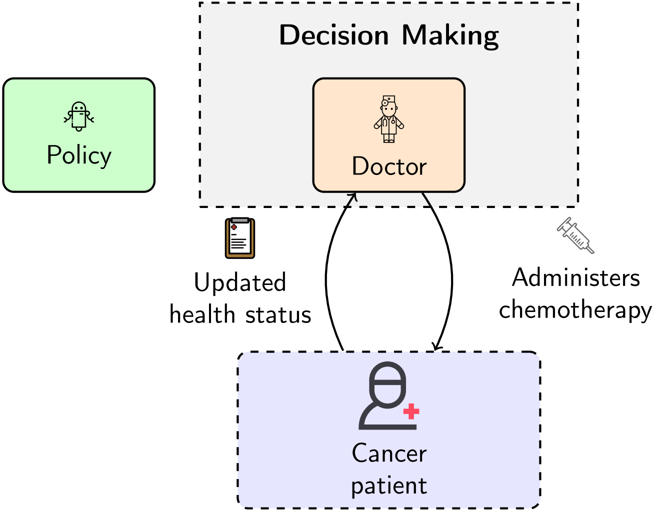

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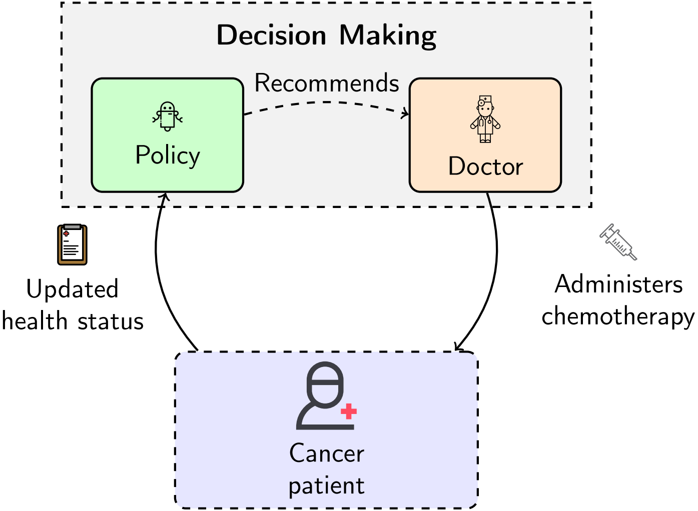
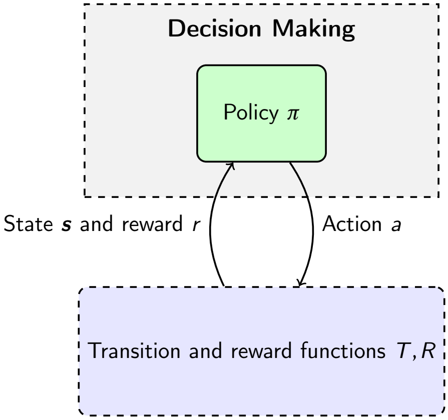


Markov decision processes (MDPs) and reinforcement learning (RL)



Markov decision processes [Put94].

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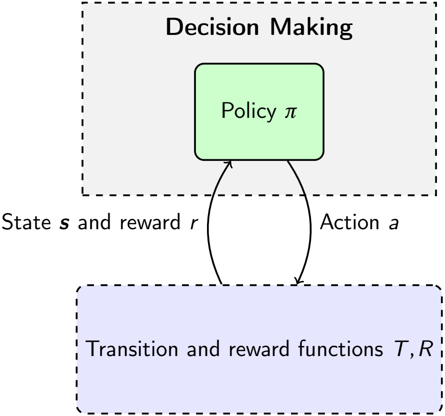

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- RL [SB98] aims to find a policy, $\pi : S \to A$ that maximizes:

$$J(\pi) = \mathop{\mathbb{E}}_{\boldsymbol{s}_t \sim T}\left[\sum_{t=0}^{\infty} \gamma^t R(s_t, \pi(s_t$$

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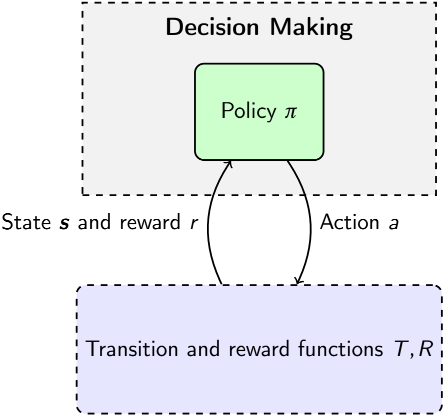

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- Lots of succesful RL algorithms [SB98; Mni+15; Sch+17].

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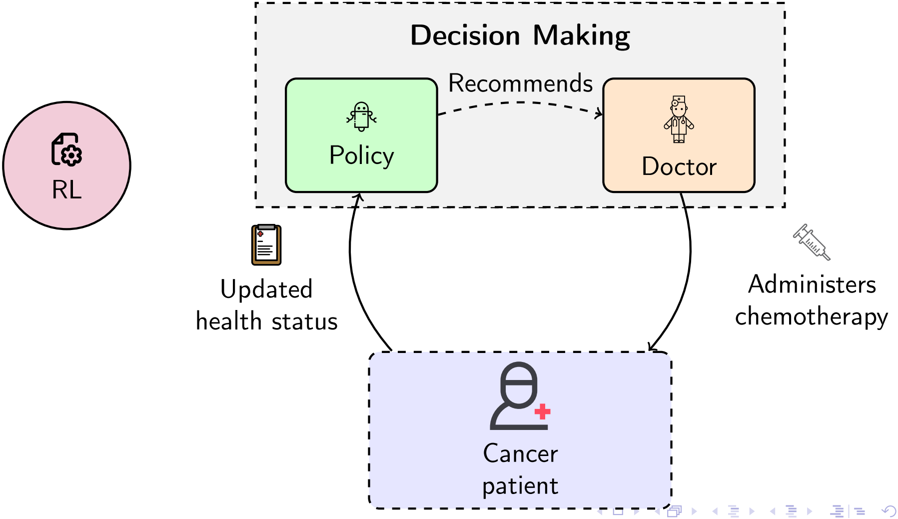

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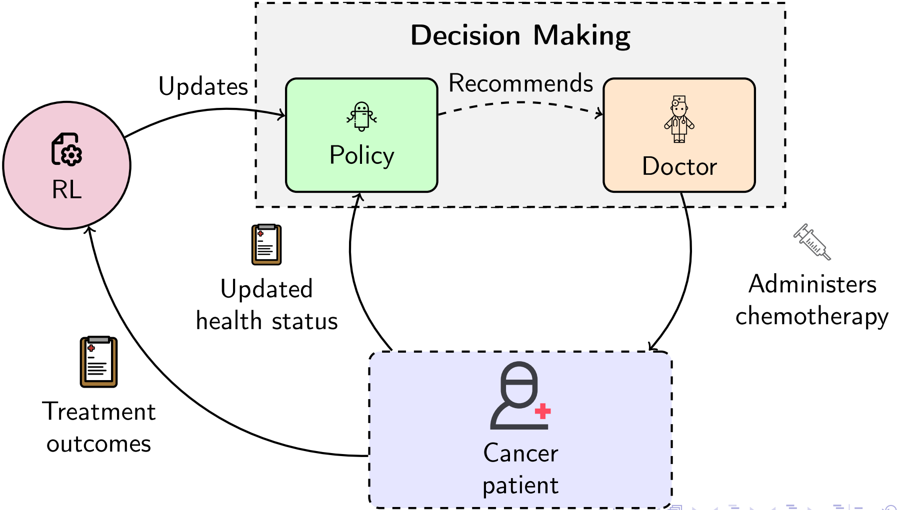

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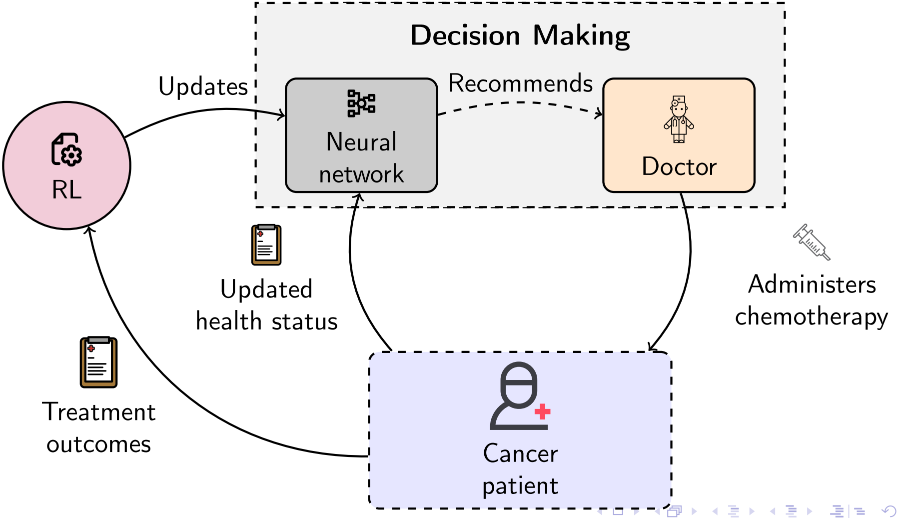

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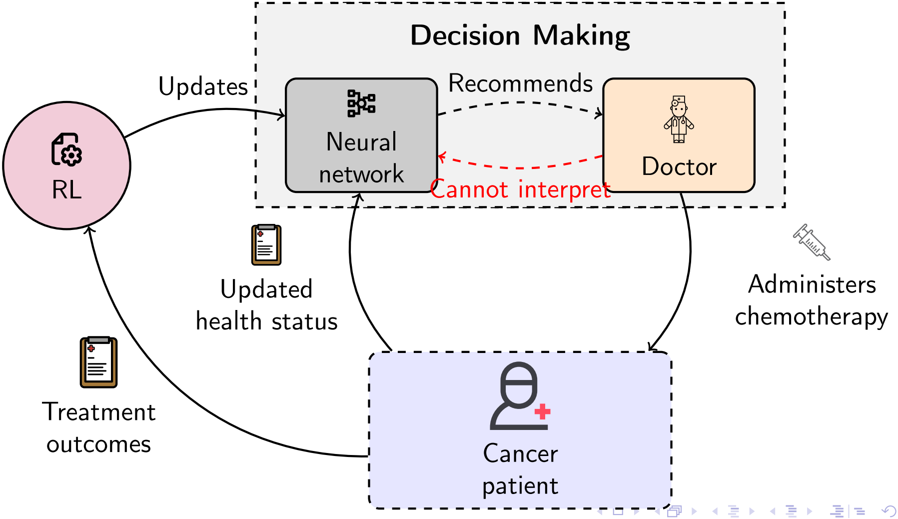

Interpretability?

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Local vs. global interpretability [Mil+24; Gla+24].

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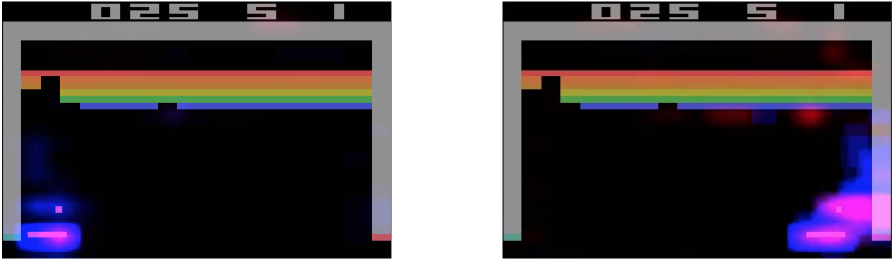

Saliency maps [Gre+18].

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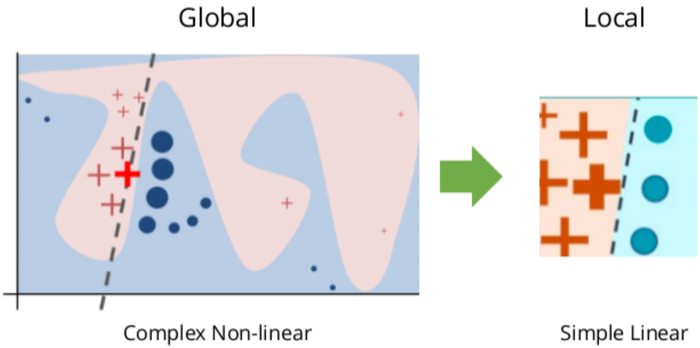

Local interpretable model-agnostic explanations (LIME) [RSG16].

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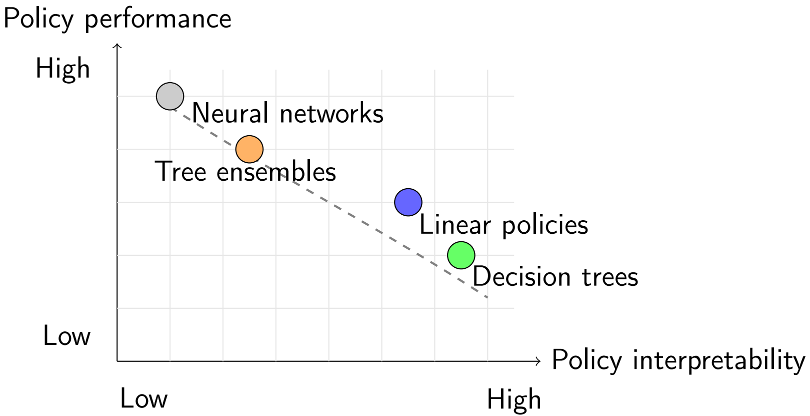

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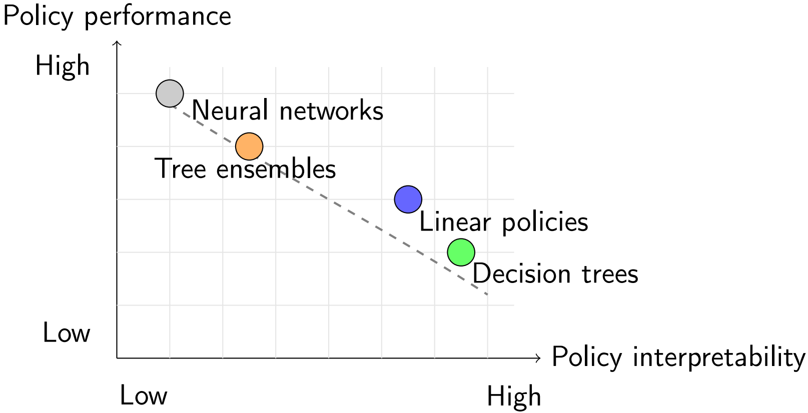

Global interpretability → works for all states.

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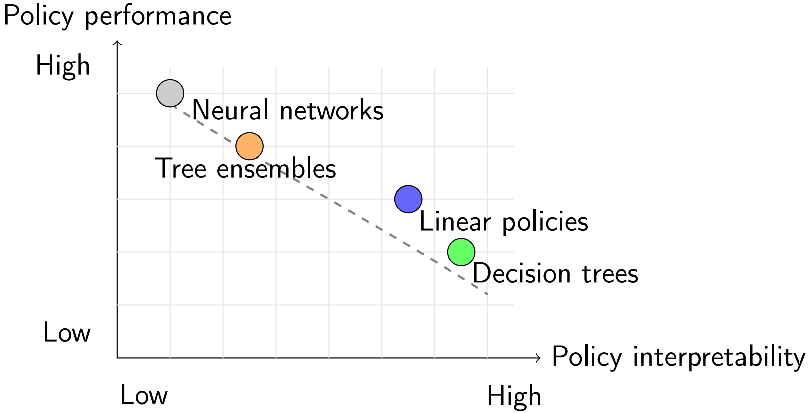
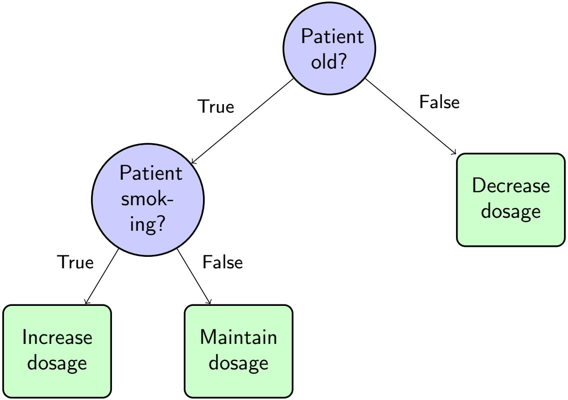


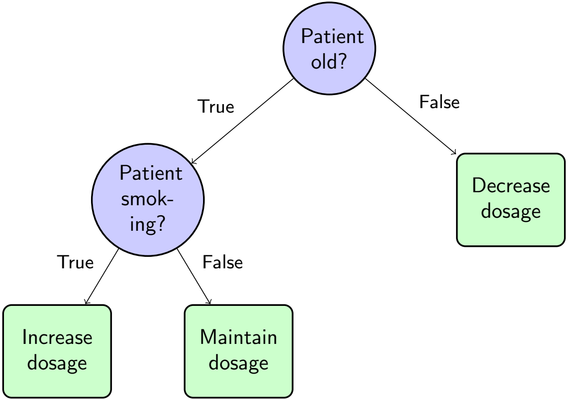
⚠**Multiple definitions** **[Lip18; DK17; Bar+20]**.

Decision trees



A generic decision tree of depth $D = 2$.

Decision trees



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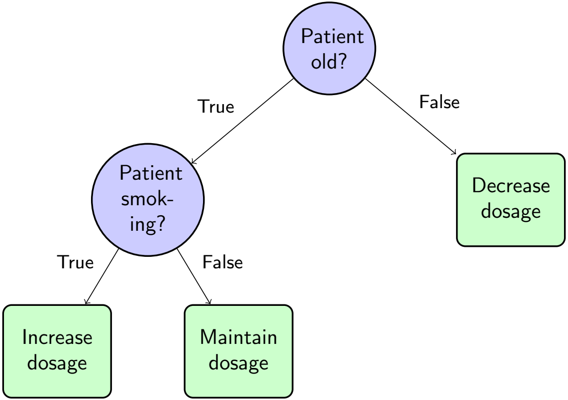

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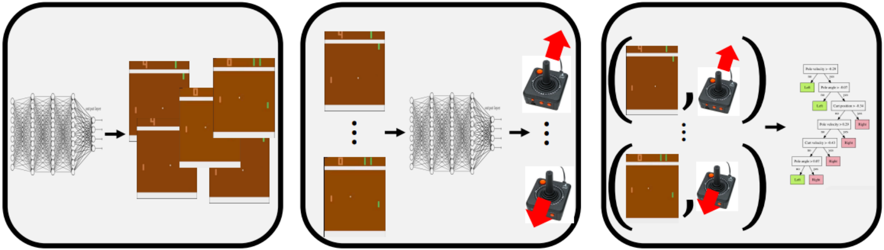
What about SDM?

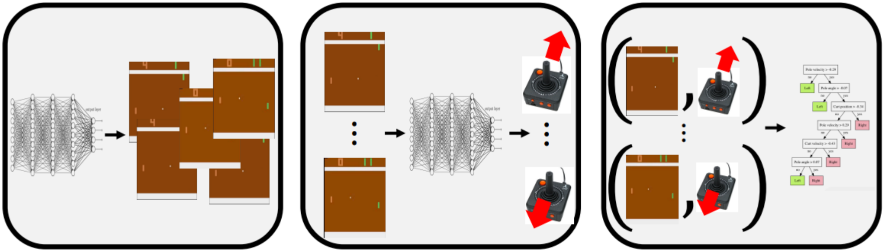
Imitation learning



Step 1: Use NN to generate states

Step 2: Use NN to obtain actions

Step 3: Use supervised learning to train a decision tree



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Most research focused on indirect learning of interpretable policies [RGB10; BPS18; Ver+18; Mil+24].

Two ways to get interpretable policies for SDM

Indirect

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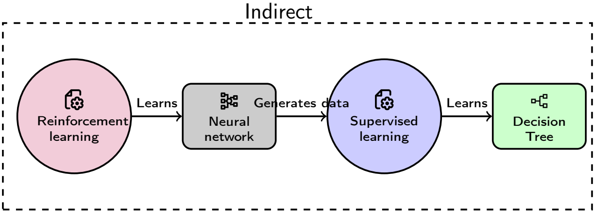

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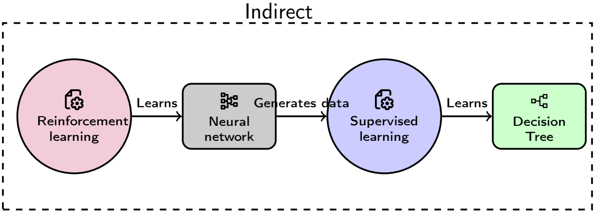

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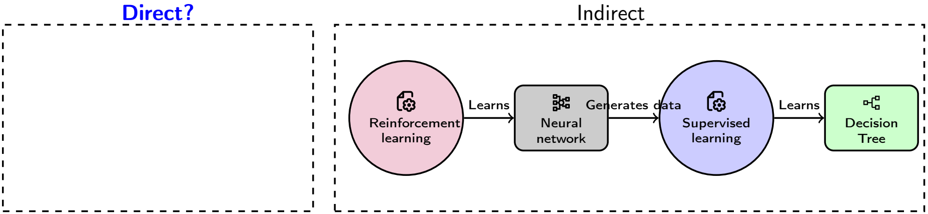

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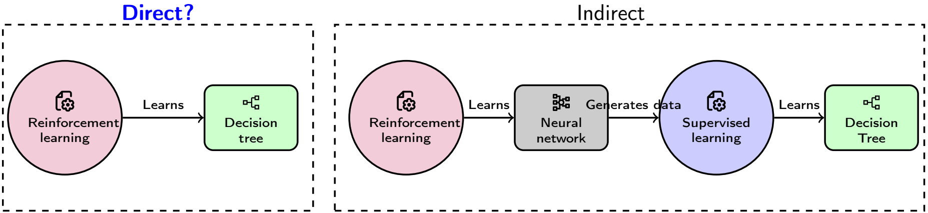

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2. Can we leverage SDM to learn decision trees for classification/regression?
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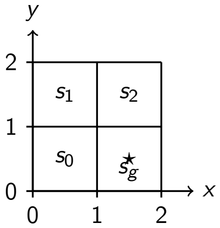

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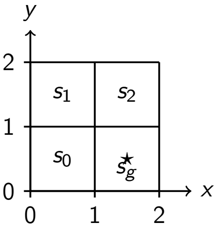

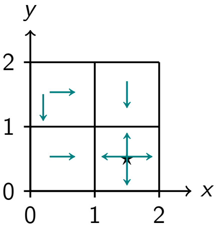

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Grid world MDP and optimal actions.

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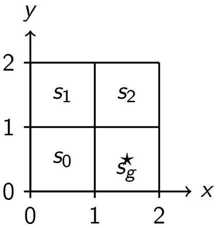

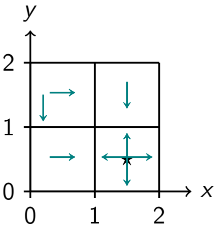

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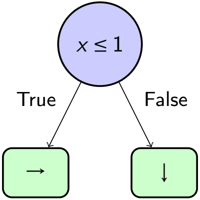

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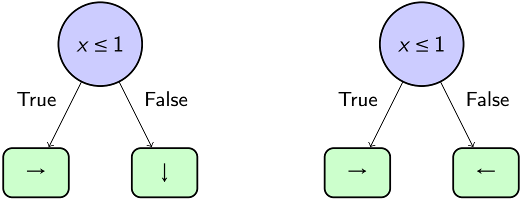

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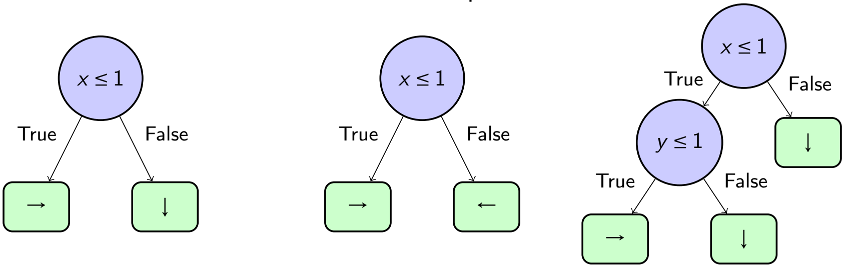

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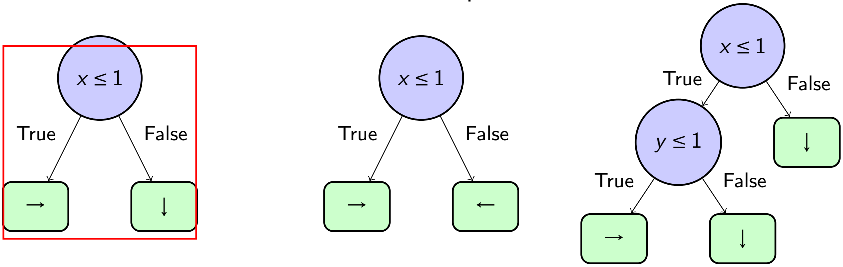
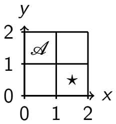


Decision tree policies with different interpretability-performance trade-offs.

Iterative bounding Markov decision processes [Top+21]

$t = 0$



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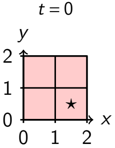

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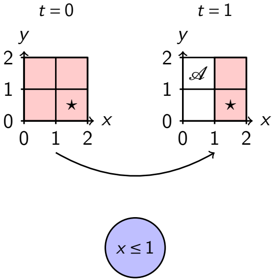

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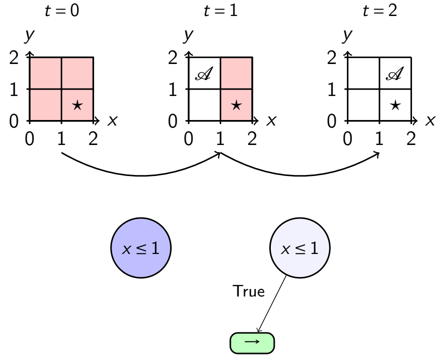

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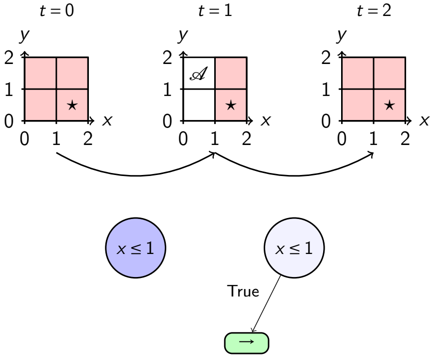

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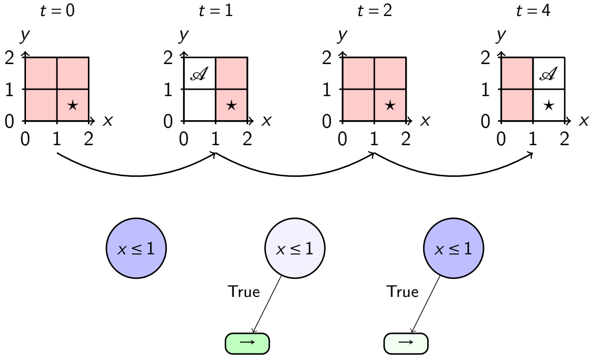

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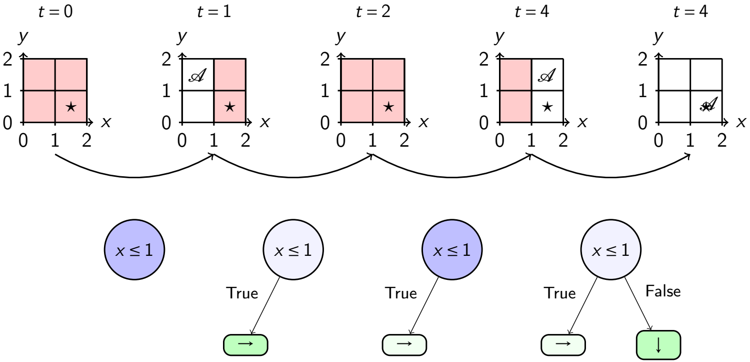

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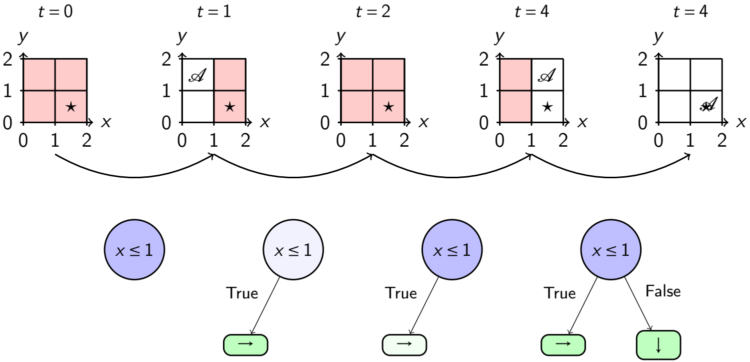

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- ⚠**IBMDP policies $\pi_{po} : O \to A \cup A_{info}$ are decision tree policies for $\mathcal{M}$.**

RL for memoryless policies in POMDPs

RL for memoryless policies

- Finding the best **deterministic** and memoryless policy in a POMDP is NP-hard [Lit94]!
- The best memoryless policy can be stochastic [SJJ94].
- Value-based RL converges to sub-optimal solutions [SJJ94; LS98].

Asymmetric RL

- Access to hidden states during training but not at execution [Pin+17].
- Value-based → learns $Q(o, a)$ with TD targets $Q(s, a)$ [BDA22].
- Actor-critic[a] → policy gradient on $\pi(o, a)$ using a critic $V(s)$ [BA22].
- Supposed to work better for our problem [LEM25].

[a] Although those return stochastic policies, we can be greedy.

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[a] Although those return stochastic policies, we can be greedy.

RL for memoryless policies in POMDPs

RL for memoryless policies

- Finding the best **deterministic** and memoryless policy in a POMDP is NP-hard [Lit94]!
- The best memoryless policy can be stochastic [SJJ94].
- Value-based RL converges to sub-optimal solutions [SJJ94; LS98].

Asymmetric RL

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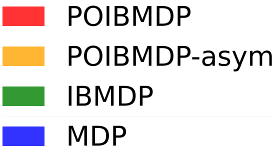
Result: for similar problems, RL struggles more when there is partial observability



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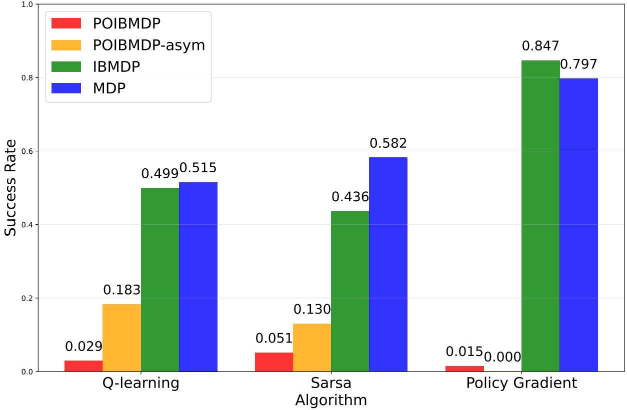

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- It seems that directly learning decision tree trading off interpretability and performances in MDPs can be difficult to achieve because of **partial observability**.
- Should we focus on indirect approaches? Hybrid approaches [Wu+20]?
- Fixing the policy tree structure a priori (parametric trees, [Mar+25])?

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Decision trees in supervised learning

- N data points $\{\boldsymbol{x}_i, y_i\}$. Each $\boldsymbol{x}_i$ is described by p features and has a label $y_i \in \mathscr{Y}$. We want to find a tree of depth at most D $T \in \mathscr{T}_D$ that minimizes:

$$\mathscr{L}_\alpha(T) = \frac{1}{N}\sum_{i=1}^{N} \ell(y_i, T(x_i)) + \alpha C(T)$$

- Trees **interpretable** and **competitive with neural nets** [GOV22].
- Greedy algorithms **sub-optimal accuracy**, but $\boldsymbol{O(2^D)}$ operations [Bre+84; Qui86; Qui93] .
- Optimal algorithms, optimal accuracy, but $\boldsymbol{O((2Np)^D)}$ operations (NP-hard) [BD17; Dem+22; LWD23; CRB24; HR76].
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Decision tree induction as solving MDPs

Intuition

The induction of a decision tree is made of a sequence of decisions: at each node, we must decide whether it is better to split (a subset of) the training data, or to create a leaf node.

- S: data subsets.
- A: test or leaf nodes that can be added to the tree.
- R: interpretability term $-\alpha$ and accuracies.
- T: node traversals.

Proposition (Objective Equivalence)

Let π be a deterministic policy of the MDP. Then $J_\alpha(\pi) = -\mathscr{L}_\alpha(E(\pi, s_0))$ where E is an algorithm that extracts a decision tree from π [Top+21].

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Controlling the time complexity of decision tree induction

- Greedy algorithms consider only one candidate action in each state which is the test that minimizes some impurity criterion
  → **MDP state space size is $O(2^D)$**.
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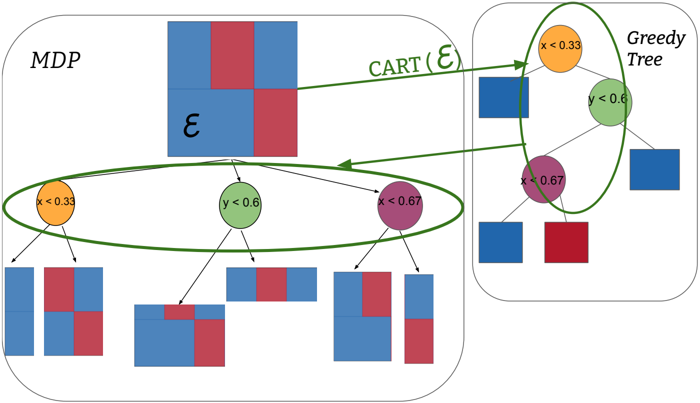

We can use greedy trees nodes as candidate actions.

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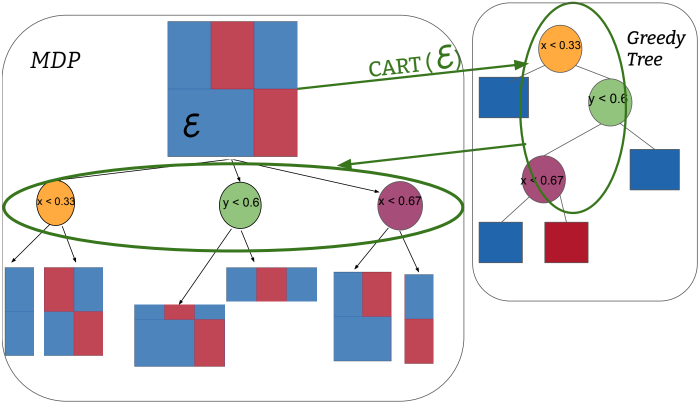

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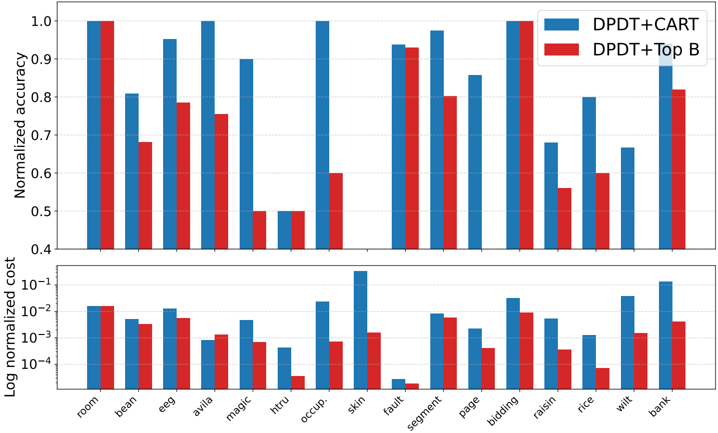
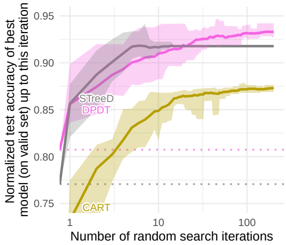
Fast like greedy trees, accurate like optimal trees



Train accuracies against cost for detph-3 trees.

Large scale evaluation of DPDT trees generalization [GOV22]



DPDT depth-5 trees vs. other detph-5 trees

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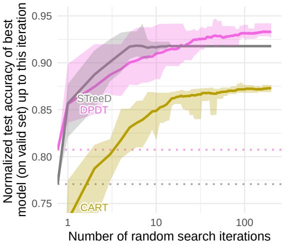

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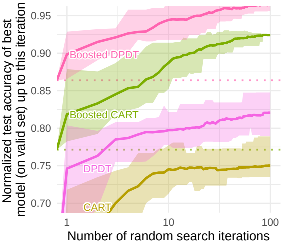

Boosted DPDT vs. Boosted CART

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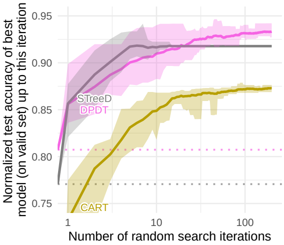

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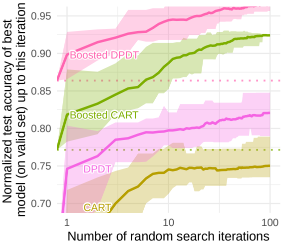

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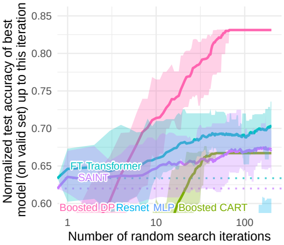
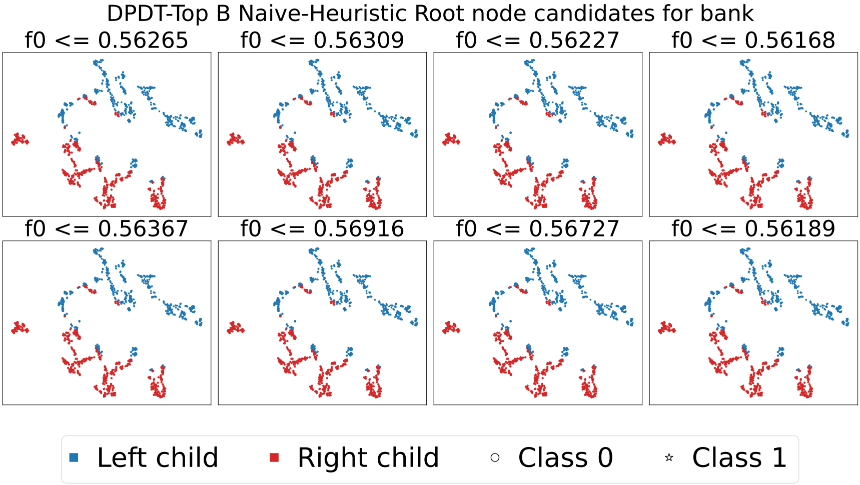


Boosted DPDT vs. other classifiers

CART generates more diverse splits than Top B



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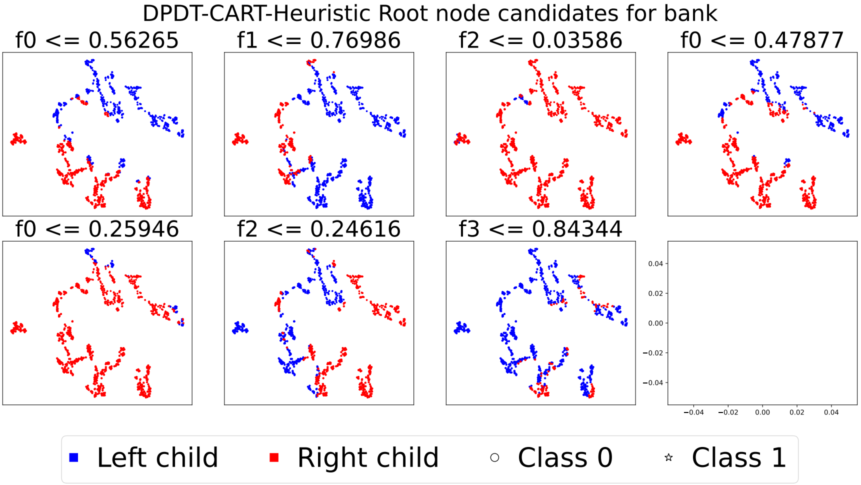

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- Different hardwares (CPUs vs GPUs).
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- Less parameters mean more interpretability [Fre14; Lav99].
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We propose policy unfolding

```
# Decision tree for Mountain Car
def play(x):
    if x[1] <= -0.2597:
        if x[1] <= -0.6378:
            return 0
        else:
            if x[0] <= -1.0021:
                return 2
            else:
                return 0
    else:
        if x[1] <= -0.0508:
            if x[0] <= 0.2979:
                if x[0] <= 0.0453:
                    return 2
                else:
                    if x[1] <=
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# Small ReLU MLP for Pendulum
def play(x):
    h_layer_0_0 = 1.238*x[0]+0.971*x
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    h_layer_0_0 = max(0, h_layer_0_0
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    h_layer_0_1 = -1.221*x[0]+1.001
                  *x[1]-0.423*x[2]
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    h_layer_0_1 = max(0, h_layer_0_1
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    h_layer_1_0 = -0.109*h_layer_0_0
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    h_layer_1_1 = max(0, h_layer_1_1
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    h_layer_2_0 = -1.790*h_layer_1_0
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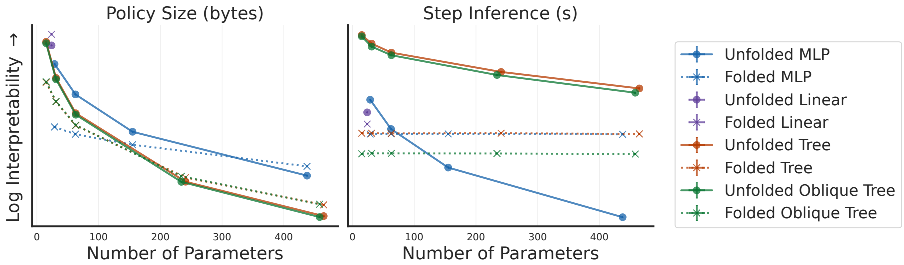

Aggregated policies interpretability on classic control environments

Perspectives

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  → Focus on indirect approaches and/or on POMDP research first?
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[BA22] Andrea Baisero and Christopher Amato. “Unbiased Asymmetric Reinforcement Learning under Partial Observability”. In: *Proceedings of the 21st International Conference on Autonomous Agents and Multiagent Systems*. AAMAS ’22. Virtual Event, New Zealand: International Foundation for Autonomous Agents and Multiagent Systems, 2022, pp. 44–52. ISBN: 9781450392136.

[Bar+20] Pablo Barceló et al. “Model interpretability through the lens of computational complexity”. In: *Advances in neural information processing systems* (2020).

[BD17] Dimitris Bertsimas and Jack Dunn. “Optimal classification trees”. In: *Machine Learning* 106 (2017), pp. 1039–1082.

[BDA22] Andrea Baisero, Brett Daley, and Christopher Amato. “Asymmetric DQN for partially observable reinforcement learning”. In: *Proceedings of the Thirty-Eighth Conference on Uncertainty in Artificial Intelligence*. Ed. by James Cussens and Kun Zhang. Vol. 180. Proceedings of Machine Learning

Research. PMLR, Jan. 2022, pp. 107–117. URL: https://proceedings.mlr.press/v180/baisero22a.html.

[Bla+23] Guy Blanc et al. "Harnessing the power of choices in decision tree learning". In: *Advances in Neural Information Processing Systems* 36 (2023), pp. 80220–80232.

[BPS18] Osbert Bastani, Yewen Pu, and Armando Solar-Lezama. "Verifiable Reinforcement Learning via Policy Extraction". In: (2018).

[Bre+84] L Breiman et al. *Classification and Regression Trees*. Wadsworth, 1984.

[CRB24] Ayman Chaouki, Jesse Read, and Albert Bifet. "Branches: A Fast Dynamic Programming and Branch & Bound algorithm for Optimal Decision Trees". In: (2024). arXiv: 2406.02175 [cs.LG]. URL: https://arxiv.org/abs/2406.02175.

[Dem+22] Emir Demirovic et al. "MurTree: Optimal Decision Trees via Dynamic Programming and Search". In: *Journal of Machine Learning Research* 23.26 (2022), pp. 1–47. URL: http://jmlr.org/papers/v23/20-520.html.

[DK17] Finale Doshi-Velez and Been Kim. "Towards A Rigorous Science of Interpretable Machine Learning". In: (2017). arXiv: 1702.08608 [stat.ML]. URL: https://arxiv.org/abs/1702.08608.

[Fre14] Alex A. Freitas. "Comprehensible classification models: a position paper". In: *SIGKDD Explor. Newsl.* 15.1 (Mar. 2014), pp. 1–10. ISSN: 1931-0145. DOI: 10.1145/2594473.2594475. URL: https://doi.org/10.1145/2594473.2594475.

[Gla+24] Claire Glanois et al. "A survey on interpretable reinforcement learning". In: *Machine Learning* (2024), pp. 1–44.

[GOV22] Léo Grinsztajn, Edouard Oyallon, and Gaël Varoquaux. "Why do tree-based models still outperform deep learning on typical tabular data?" In: *Advances in neural information processing systems* 35 (2022), pp. 507–520.

[Gre+18] Sam Greydanus et al. *Visualizing and Understanding Atari Agents*. 2018.

[HR76] Laurent Hyafil and Ronald L. Rivest. "Constructing optimal binary decision trees is NP-complete". In: *Information Processing Letters* 5.1 (1976), pp. 15–17. ISSN: 0020-0190. DOI: https://doi.org/10.1016/0020-0190(76)90095-8. URL: https://www.sciencedirect.com/science/article/pii/0020019076900958.

[Lav99] Nada Lavrač. "Selected techniques for data mining in medicine". In: *Artificial Intelligence in Medicine* 16.1 (1999). Data Mining Techniques and Applications in Medicine, pp. 3–23. ISSN: 0933-3657. DOI: https://doi.org/10.1016/S0933-3657(98)00062-1. URL: https://www.sciencedirect.com/science/article/pii/S0933365798000621.

[LEM25] Gaspard Lambrechts, Damien Ernst, and Aditya Mahajan. "A Theoretical Justification for Asymmetric Actor-Critic algorithms". In: *Forty-second International Conference on Machine Learning*. 2025. URL: https://openreview.net/forum?id=F1yANMCnAn.

[Lip18] Zachary C. Lipton. "The Mythos of Model Interpretability: In machine learning, the concept of interpretability is both important and slippery.". In: *Queue* 16.3 (2018), pp. 31–57.

[Lit94] Michael L. Littman. "Memoryless policies: theoretical limitations and practical results". In: *Proceedings of the Third International Conference on Simulation of Adaptive Behavior: From Animals to Animats 3: From Animals to Animats 3*. SAB94. Brighton, United Kingdom: MIT Press, 1994, pp. 238–245. ISBN: 0262531224.

[LS98] John Loch and Satinder P. Singh. "Using Eligibility Traces to Find the Best Memoryless Policy in Partially Observable Markov Decision Processes". In: *Proceedings of the Fifteenth International Conference on Machine Learning*. ICML '98. San Francisco, CA, USA: Morgan Kaufmann Publishers Inc., 1998, pp. 323–331. ISBN: 1558605568.

[Luo+24] Lirui Luo et al. "End-to-End Neuro-Symbolic Reinforcement Learning with Textual Explanations". In: *International Conference on Machine Learning (ICML)* (2024).

[LWD23] Jacobus van der Linden, Mathijs de Weerdt, and Emir Demirović. "Necessary and Sufficient Conditions for Optimal Decision Trees using Dynamic Programming". In: *Advances in Neural Information Processing Systems* 36 (2023). Ed. by A. Oh et al., pp. 9173–9212.

[Mar+25] Sascha Marton et al. "Mitigating Information Loss in Tree-Based Reinforcement Learning via Direct Optimization". In: (2025). URL: https://openreview.net/forum?id=qpXctF2aLZ.

[Mil+24] Stephanie Milani et al. "Explainable Reinforcement Learning: A Survey and Comparative Review". In: *ACM Comput. Surv.* 56.7 (Apr. 2024). ISSN: 0360-0300. DOI: 10.1145/3616864. URL: https://doi.org/10.1145/3616864.

[MMW22] Rahul Mazumder, Xiang Meng, and Haoyue Wang. "Quant-BnB: A Scalable Branch-and-Bound Method for Optimal Decision Trees with Continuous Features". In: *Proceedings of the 39th International Conference on Machine Learning*. Proceedings of Machine Learning Research 162

(17–23 Jul 2022). Ed. by Kamalika Chaudhuri et al., pp. 15255–15277. URL: https://proceedings.mlr.press/v162/mazumder22a.html.

[Mni+15] Volodymyr Mnih et al. "Human-level control through deep reinforcement learning". In: *nature* 518.7540 (2015), pp. 529–533.

[Nag+24] Myura Nagendran et al. "Eye tracking insights into physician behaviour with safe and unsafe explainable AI recommendations". In: *NPJ Digital Medicine* 7.1 (2024), p. 202.

[Pin+17] Lerrel Pinto et al. *Asymmetric Actor Critic for Image-Based Robot Learning*. 2017. arXiv: 1710.06542 [cs.RO]. URL: https://arxiv.org/abs/1710.06542.

[Put94] Martin L. Puterman. *Markov Decision Processes: Discrete Stochastic Dynamic Programming*. John Wiley & Sons, 1994.

[Qui86] J. R. Quinlan. "Induction of Decision Trees". In: *Mach. Learn.* 1.1 (1986), pp. 81–106.

[Qui93] J Ross Quinlan. “C4. 5: Programs for machine learning”. In: *Morgan Kaufmann google schola* 2 (1993), pp. 203–228.

[RGB10] Stéphane Ross, Geoffrey J. Gordon, and J. Andrew Bagnell. “A Reduction of Imitation Learning and Structured Prediction to No-Regret Online Learning”. In: (2010).

[RSG16] Marco Tulio Ribeiro, Sameer Singh, and Carlos Guestrin. “"Why Should I Trust You?": Explaining the Predictions of Any Classifier”. In: KDD ’16 (2016), pp. 1135–1144. DOI: 10.1145/2939672.2939778. URL: https://doi.org/10.1145/2939672.2939778.

[SB98] Richard S. Sutton and Andrew G. Barto. *Reinforcement Learning: An Introduction*. Cambridge, MA: The MIT Press, 1998.

[Sch+17] John Schulman et al. “Proximal policy optimization algorithms”. In: *arXiv preprint arXiv:1707.06347* (2017).

[SJJ94] Satinder P. Singh, Tommi S. Jaakkola, and Michael I. Jordan. “Learning without state-estimation in partially observable Markovian decision processes”. In: *Proceedings of the Eleventh*

International Conference on International Conference on Machine Learning. ICML'94. New Brunswick, NJ, USA: Morgan Kaufmann Publishers Inc., 1994, pp. 284–292. ISBN: 1558603352.

[Top+21] Nicholay Topin et al. "Iterative bounding mdps: Learning interpretable policies via non-interpretable methods". In: *Proceedings of the AAAI Conference on Artificial Intelligence* 35 (2021), pp. 9923–9931.

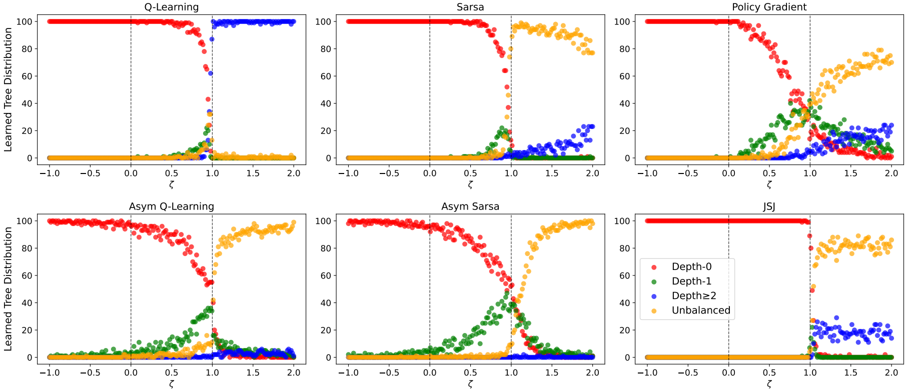
[Ver+18] Abhinav Verma et al. "Programmatically interpretable reinforcement learning". In: (2018), pp. 5045–5054.

[VZ19] Sicco Verwer and Yingqian Zhang. "Learning optimal classification trees using a binary linear program formulation". In: *Proceedings of the AAAI conference on artificial intelligence* 33 (2019), pp. 1625–1632.

[Wu+20] Mike Wu et al. "Regional Tree Regularization for Interpretability in Deep Neural Networks". In: 34 (Apr. 2020), pp. 6413–6421. DOI: 10.1609/aaai.v34i04.6112. URL:

https://ojs.aaai.org/index.php/AAAI/article/view/6112.

Result: RL cannot retrieve optimal depth-1 trees for the grid world MDP



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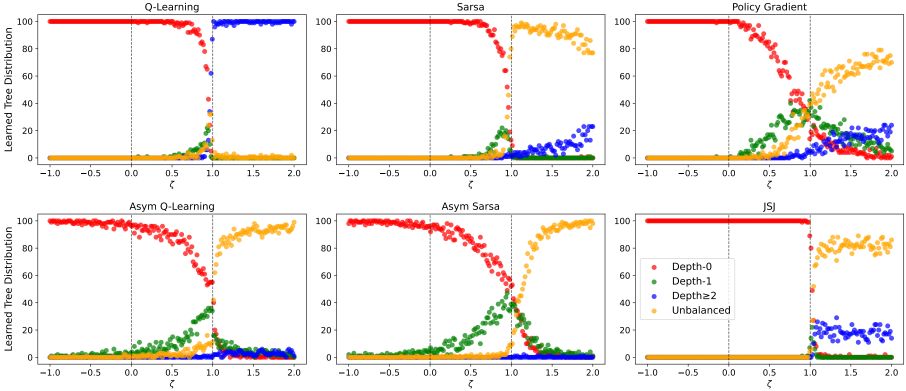

Distributions of tree policies learned with (asymmetric) RL algorithms [SB98; SJJ94; LS98; BA22; BDA22] as a function of the interpretability reward ζ.

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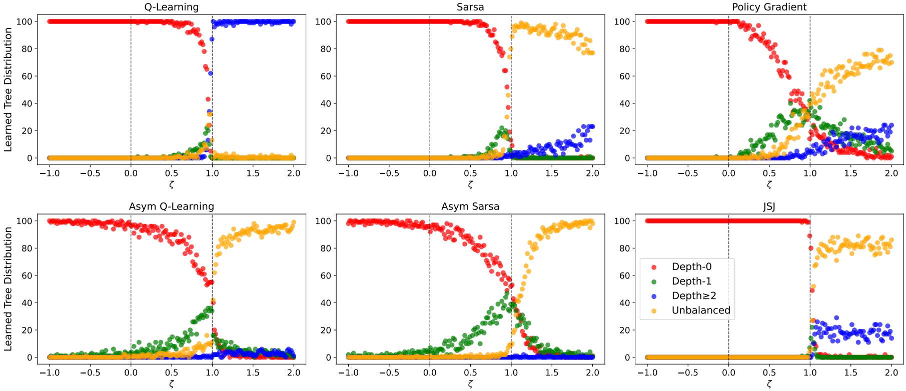

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Are the poor performances due to partial observability?

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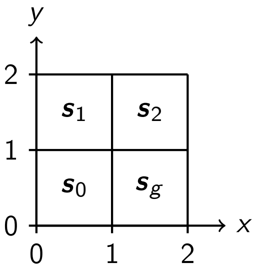

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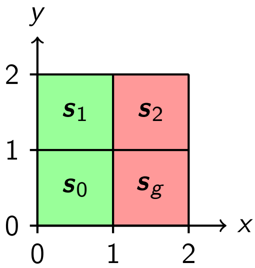

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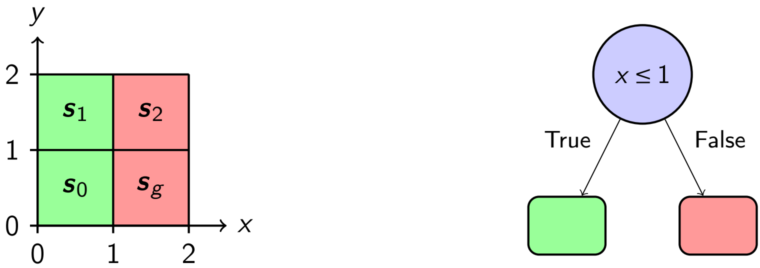

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Deterministic memoryless policies for classification IBMDPs ($\Leftrightarrow$ decision tree policies) are in fact Markovian.

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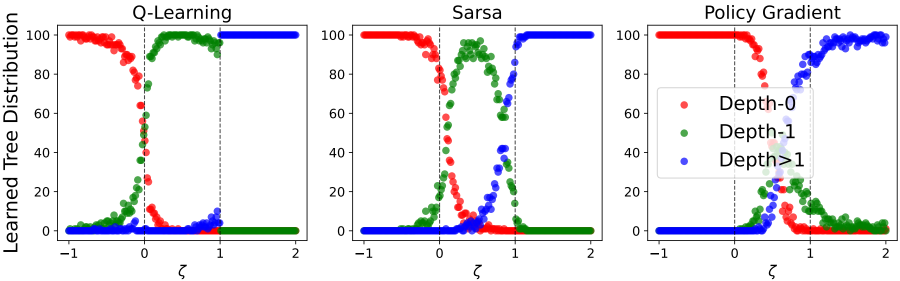
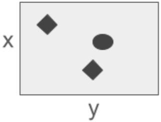


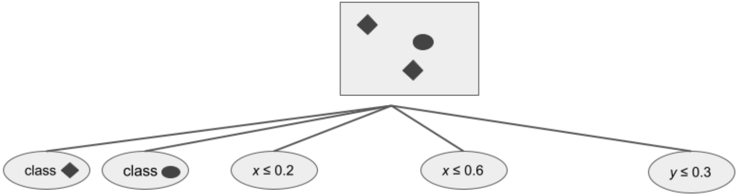
Distributions of tree policies learned with various RL algorithms.

Decision tree induction as solving MDPs



Example of decision tree induction as an MDP.

Decision tree induction as solving MDPs



Example of decision tree induction as an MDP.

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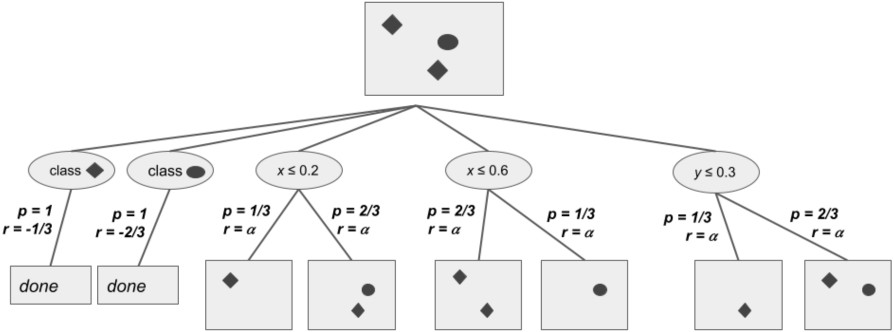

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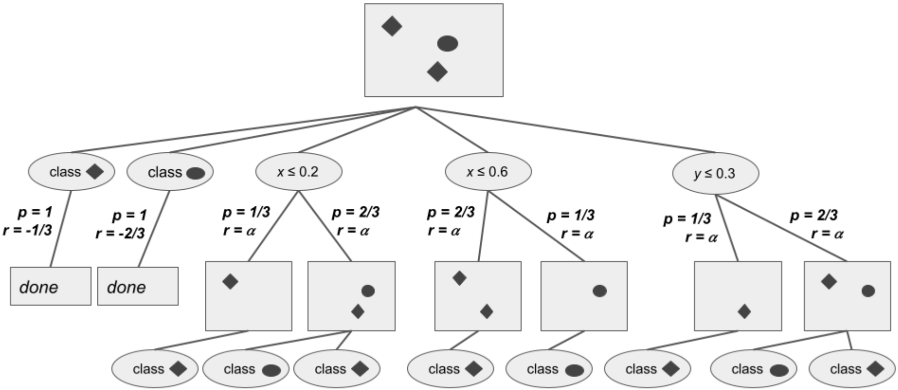

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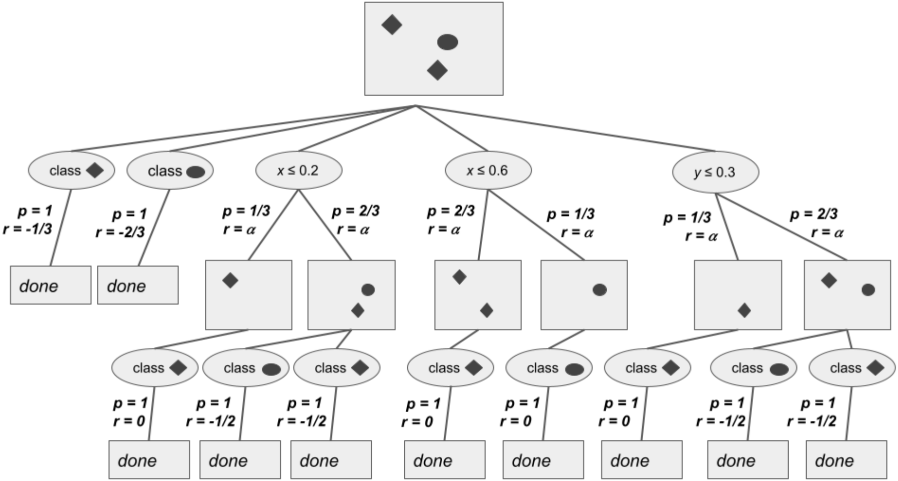
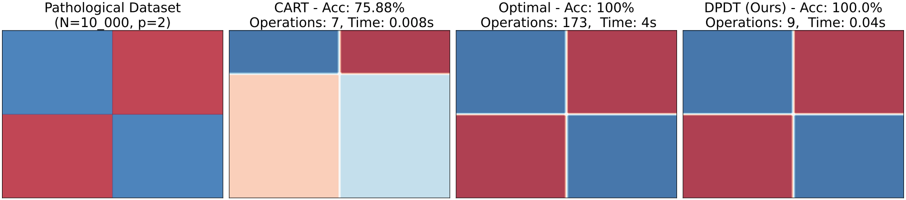


Example of decision tree induction as an MDP.

Fast like greedy trees, accurate like optimal trees



Comparison of greedy, optimal, and DPDT depth-2 trees on the checkersboard dataset.

Fast like greedy trees, accurate like optimal trees

Comparison of accuracies and operations for depth-3 trees.

	Accuracy						**Operations**					
Dataset	Opt	Greedy	DPDT				Opt	Greedy	DPDT			
			CART$^-$	CART$^+$	TopB$^-$	TopB$^+$			CART$^-$	CART$^+$	TopB$^-$	TopB$^+$
room	**0.992**	0.968	0.991	**0.992**	0.990	**0.992**	10^6	15	286	16100	111	16100
bean	**0.871**	0.777	0.812	0.853	0.804	0.841	$5 \cdot 10^6$	15	295	25900	112	16800
eeg	**0.708**	0.666	0.689	0.706	0.684	0.699	$2 \cdot 10^6$	13	289	26000	95	11000
avila	**0.585**	0.532	0.574	**0.585**	0.563	0.572	$3 \cdot 10^7$	9	268	24700	60	38900
magic	**0.831**	0.801	0.822	0.828	0.807	0.816	$6 \cdot 10^6$	15	298	28000	70	4190
htru	**0.981**	0.979	0.979	0.980	0.979	0.980	$6 \cdot 10^7$	15	295	25300	55	2180
occup.	**0.994**	0.989	0.991	**0.994**	0.990	0.992	$7 \cdot 10^5$	13	280	16300	33	510
skin	**0.969**	0.966	0.966	0.966	0.966	0.966	$7 \cdot 10^4$	15	301	23300	20	126
fault	**0.682**	0.553	0.672	0.674	0.672	0.673	$9 \cdot 10^8$	13	295	24200	111	16800
segment	**0.887**	0.574	0.812	0.879	0.786	0.825	$2 \cdot 10^6$	7	220	16300	68	11400
page	**0.971**	0.964	0.970	0.970	0.964	0.965	10^7	15	298	22400	701	4050
bidding	**0.993**	0.981	0.985	**0.993**	0.985	**0.993**	$3 \cdot 10^5$	13	256	9360	58	2700
raisin	**0.894**	0.869	0.879	0.886	0.875	0.883	$4 \cdot 10^6$	15	295	20900	48	1440
rice	**0.938**	0.933	0.934	0.937	0.933	0.936	$2 \cdot 10^7$	15	298	25500	49	1470
wilt	**0.996**	0.993	0.994	0.995	0.994	0.994	$3 \cdot 10^5$	13	274	11300	33	465
bank	**0.983**	0.933	0.971	0.980	0.951	0.974	$6 \cdot 10^4$	13	271	7990	26	256

Why generating candidate splits with CART?

Theorem (DPDT trees are not worse than greedy trees)

The greedy tree is always a solution of the MDPs we solve. Because we solve the MDPs exactly with DP, if the greedy tree is the best solution, DPDT will find it.

Theorem (DPDT trees can be strictly better than greedy trees)

There exist a depth budget D and a dataset for which DPDT trees are strictly better than greedy trees.[a]

[a] cf. checkersboard dataset.

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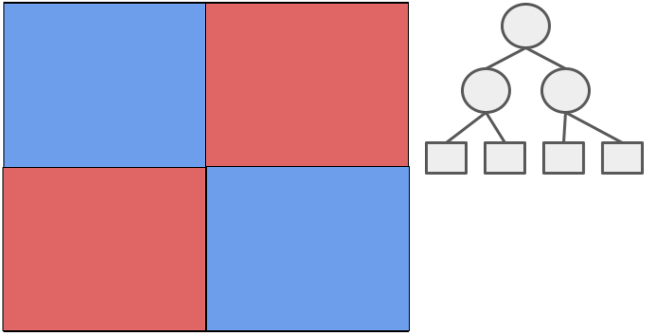

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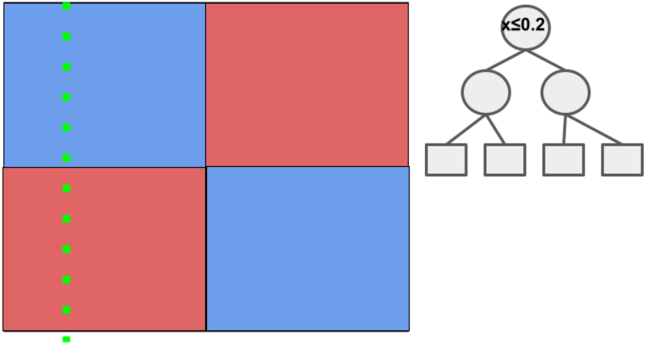

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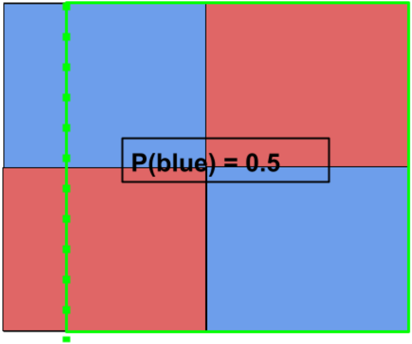

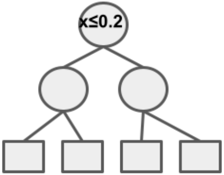

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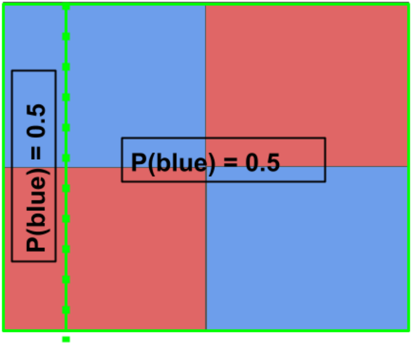

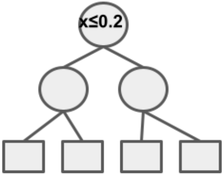

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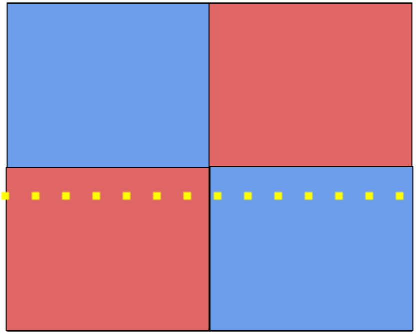
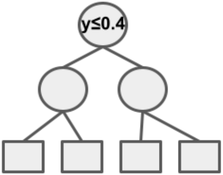
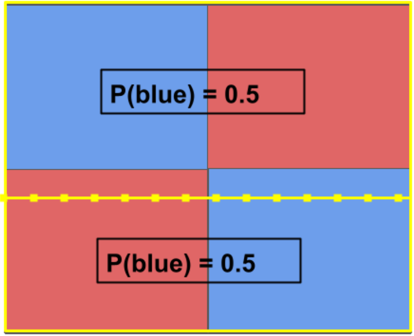
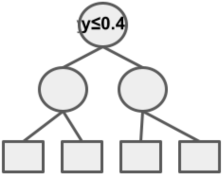


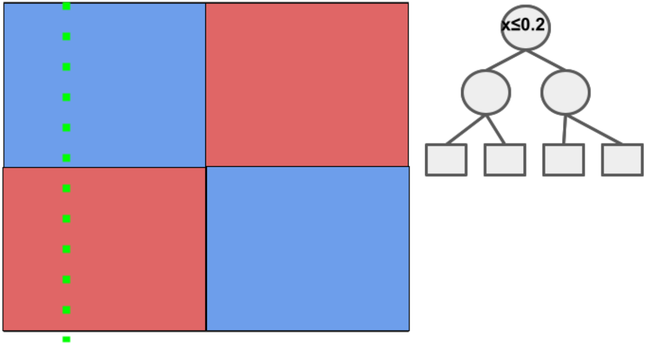


DPDT trees can be strictly better than greedy trees





DPDT trees can be strictly better than greedy trees



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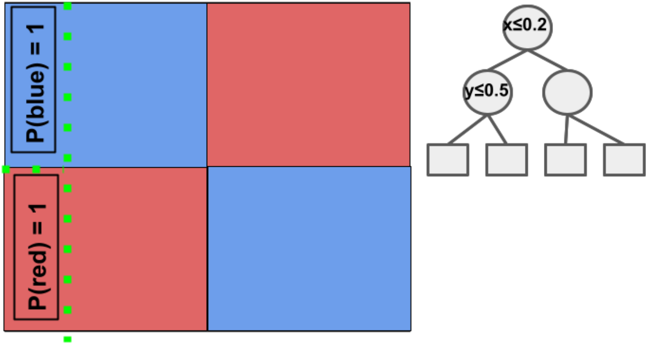

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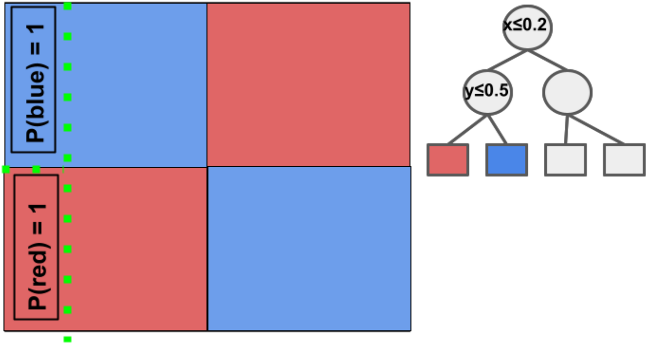

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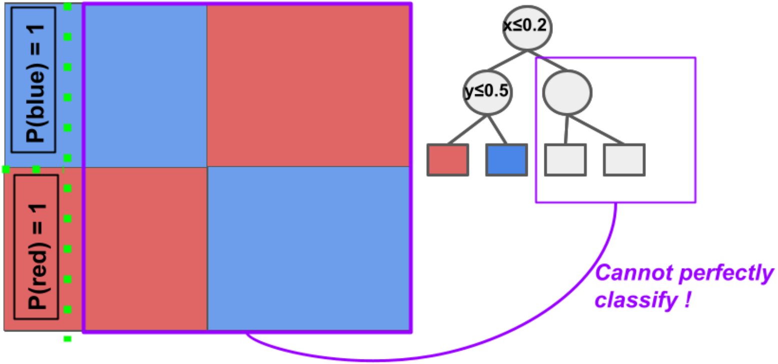

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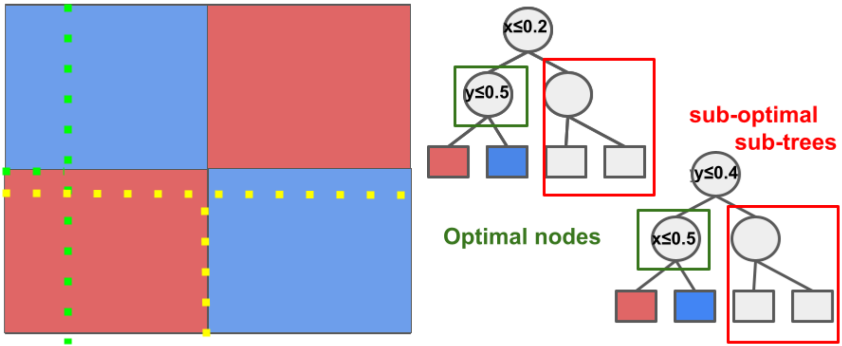

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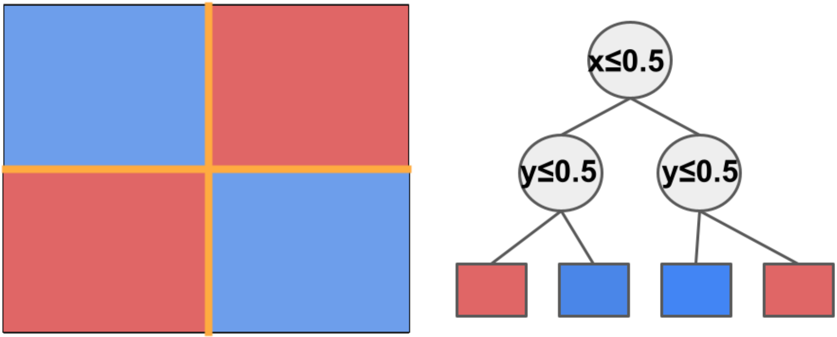
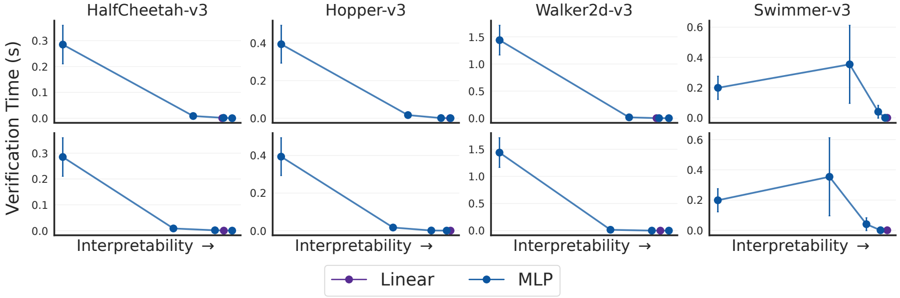
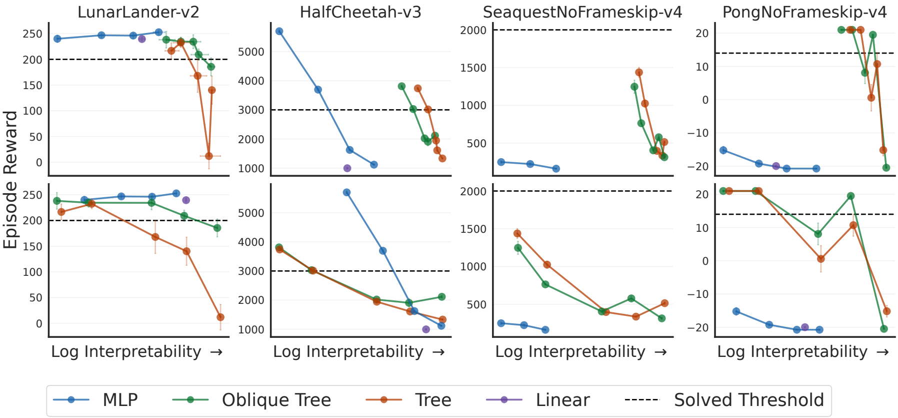


Result: verification time does scale with step inference time



Verification time as a function of policy interpretability. Top row, interpretability is measured with step inference times. Bottom row, the interpretability is measured with policy size.

Result: there is no dominating policy class for all environments



Interpretability-Performance trade-offs. Top row, interpretability is measured with step inference times. Bottom row, the interpretability is measured with policy size.