

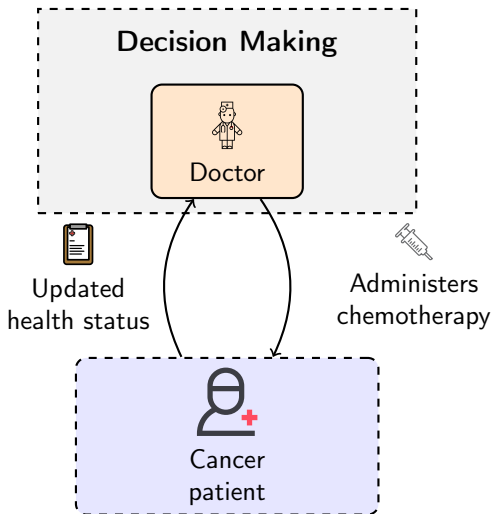
Interpretability, Decision Trees, and Sequential Decision Making

Hector Kohler

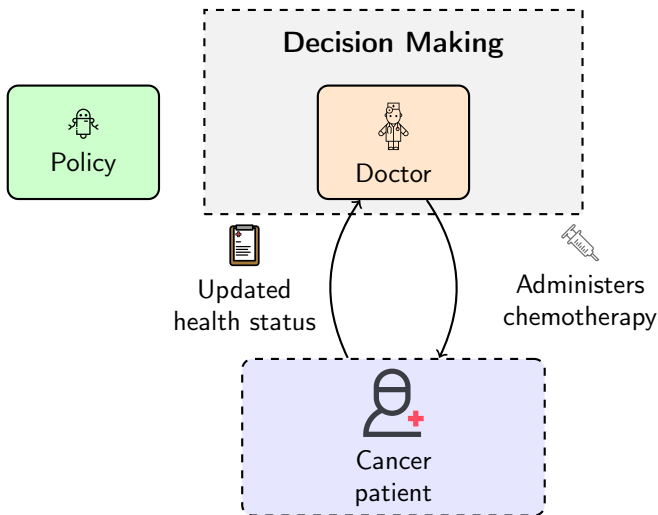
Supervised by Dr. Riad Akrou (HdR) and Prof. Philippe Preux (HdR)
Université de Lille, CNRS, Inria, UMR CRISAL 9189, France

November 25, 2025

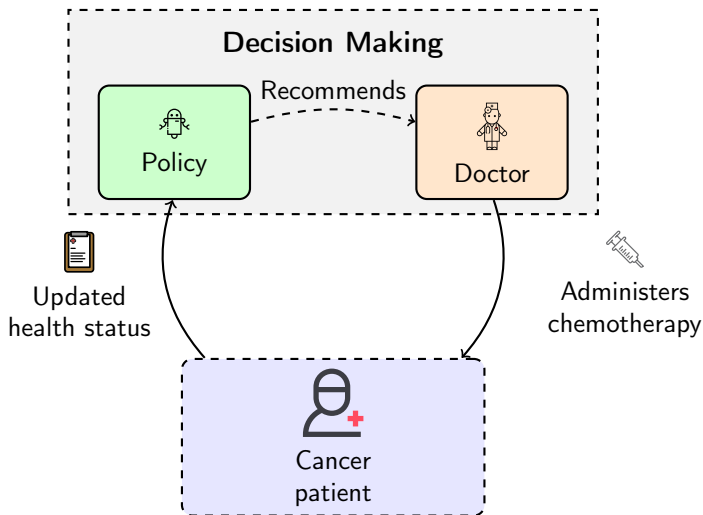
Sequential decision making (SDM) and machine learning (ML)



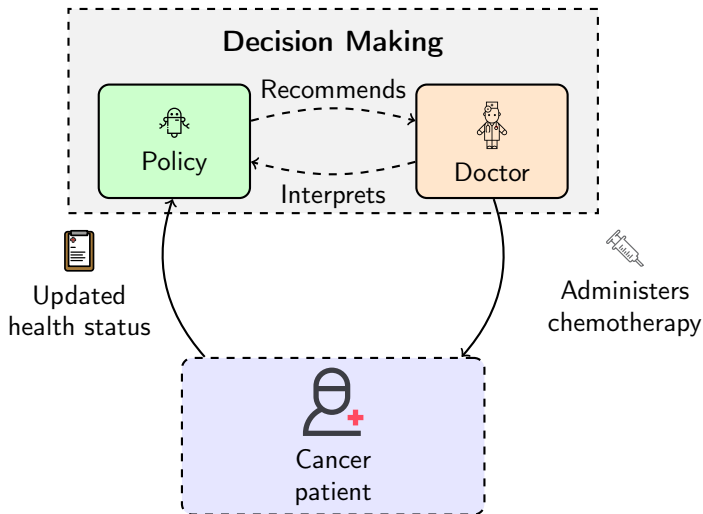
Sequential decision making (SDM) and machine learning (ML)



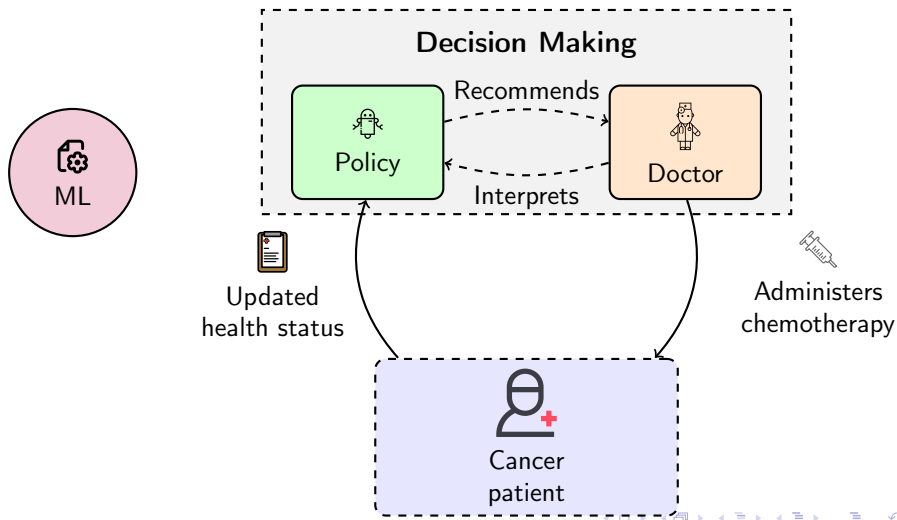
Sequential decision making (SDM) and machine learning (ML)



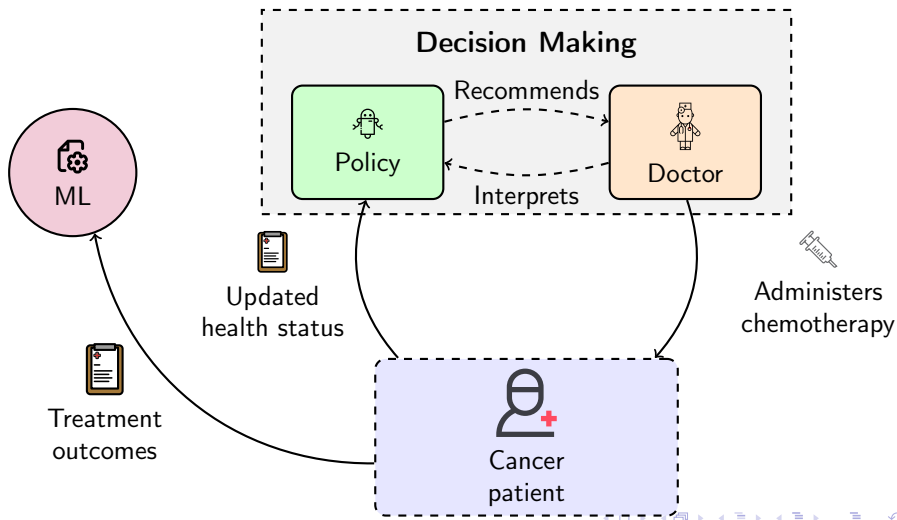
Sequential decision making (SDM) and machine learning (ML)



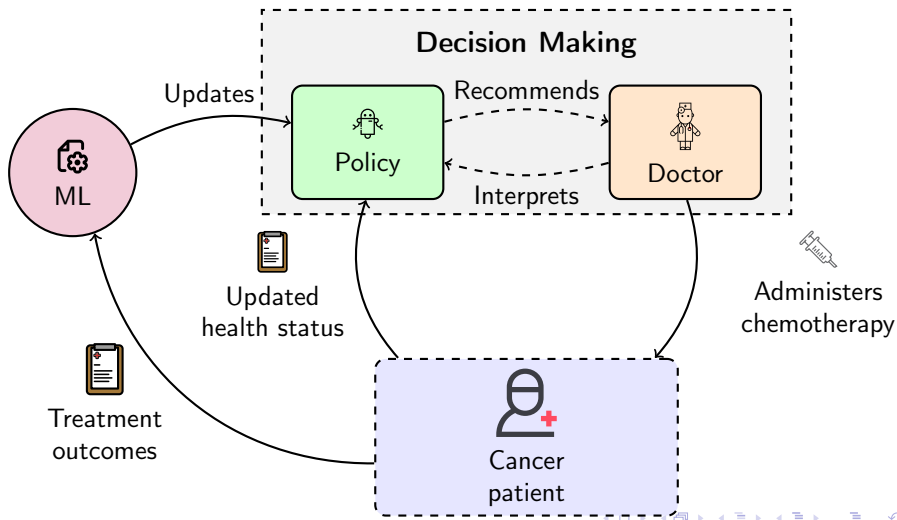
Sequential decision making (SDM) and machine learning (ML)



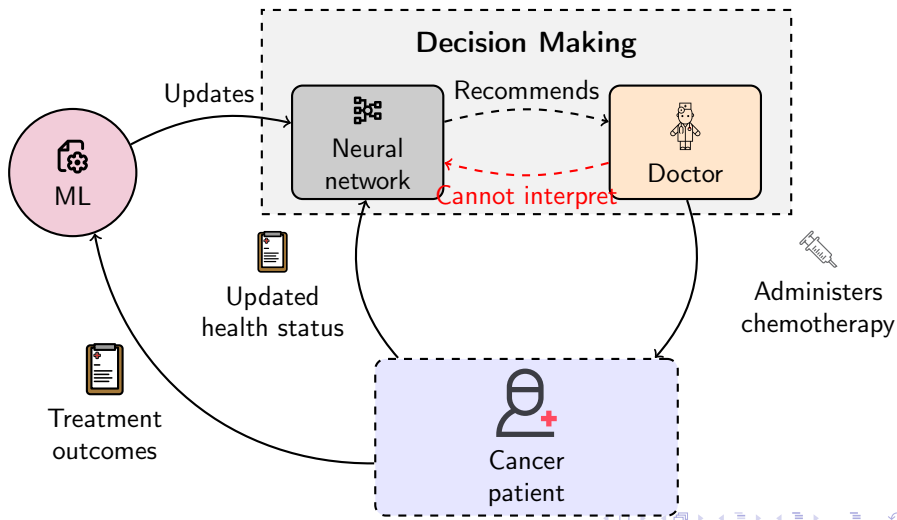
Sequential decision making (SDM) and machine learning (ML)



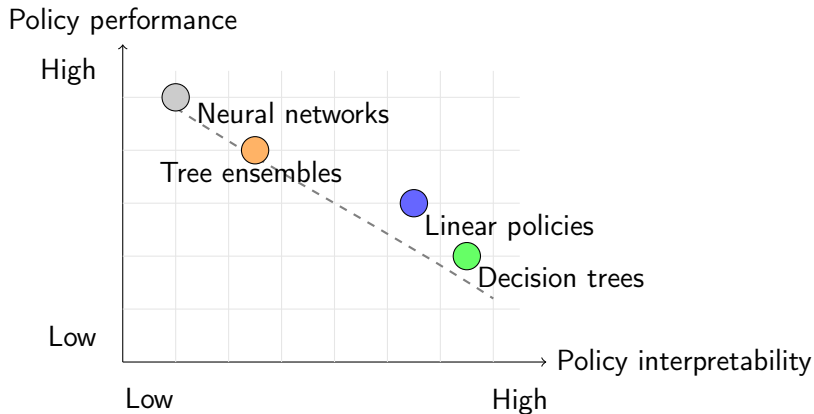
Sequential decision making (SDM) and machine learning (ML)



Sequential decision making (SDM) and machine learning (ML)

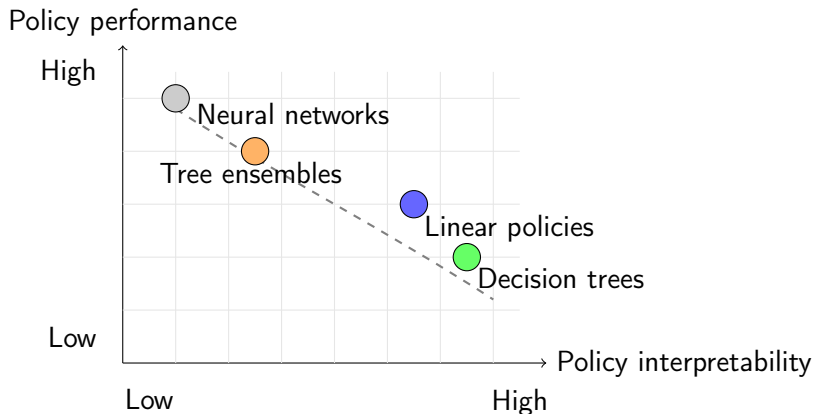


Policy interpretability



Heuristic interpretability-performance trade-offs of different policy classes.

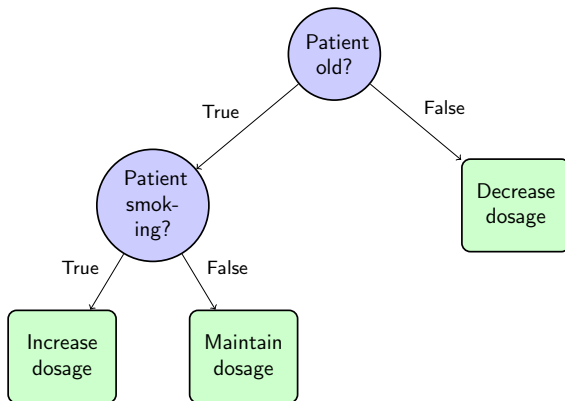
Policy interpretability



Heuristic interpretability-performance trade-offs of different policy classes.

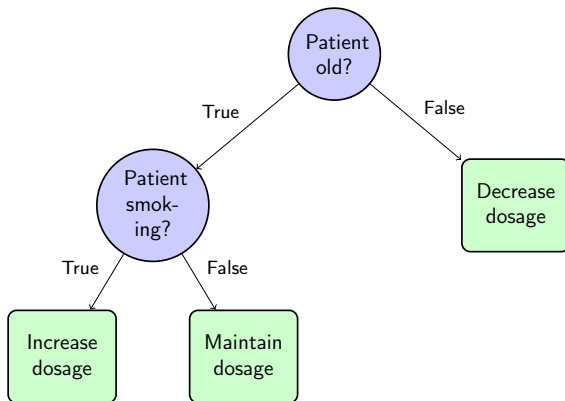
⚠ **No definition of interpretability in machine learning models!**

Decision trees



A generic decision tree of depth $D = 2$.

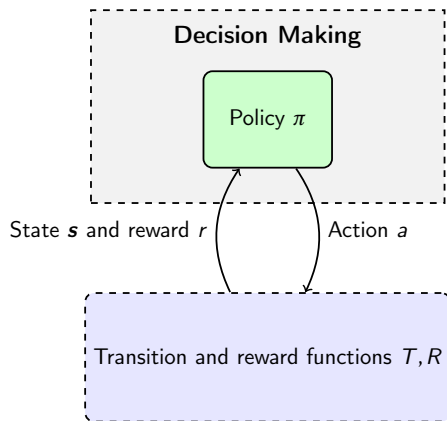
Decision trees



A generic decision tree of depth $D = 2$.

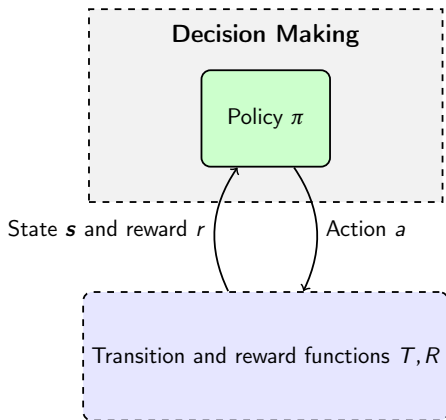
Successful algorithms for non-sequential learning: [Bre+84; BD17; Dem+22; VZ19; MMW22] ... What about SDM?

Markov decision processes (MDPs) and reinforcement learning (RL)



Markov decision processes [Put94].

Markov decision processes (MDPs) and reinforcement learning (RL)

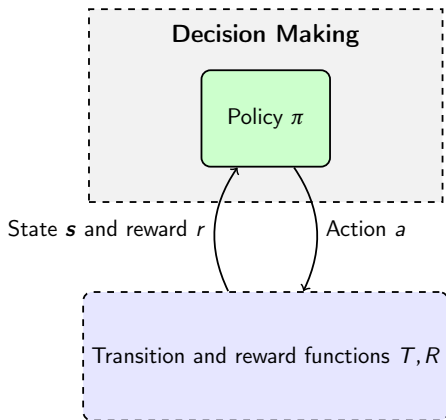


Markov decision processes [Put94].

- RL [SB98] aims to find a policy, $\pi : S \rightarrow A$ that maximizes:

$$\mathbb{E}_{s_t \sim T} \left[\sum_{t=0}^{\infty} \gamma^t R(s_t, \pi(s_t)) \right]$$

Markov decision processes (MDPs) and reinforcement learning (RL)



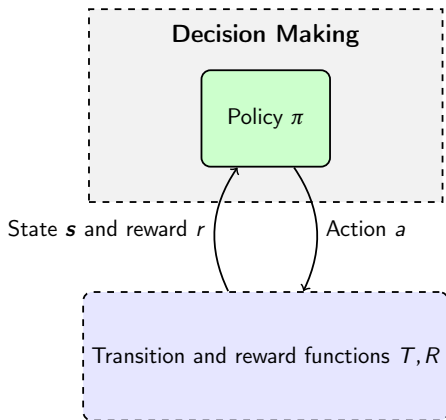
Markov decision processes [Put94].

- RL [SB98] aims to find a policy, $\pi : S \rightarrow A$ that maximizes:

$$\mathbb{E}_{s_t \sim T} \left[\sum_{t=0}^{\infty} \gamma^t R(s_t, \pi(s_t)) \right]$$

- Lots of successful RL algorithms [SB98; Mni+15; Sch+17].

Markov decision processes (MDPs) and reinforcement learning (RL)



Markov decision processes [Put94].

- RL [SB98] aims to find a policy, $\pi : S \rightarrow A$ that maximizes:

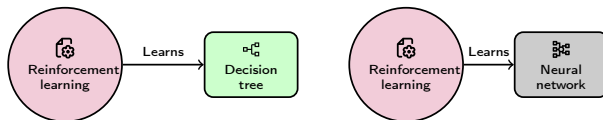
$$\mathbb{E}_{s_t \sim T} \left[\sum_{t=0}^{\infty} \gamma^t R(s_t, \pi(s_t)) \right]$$

- Lots of successful RL algorithms [SB98; Mni+15; Sch+17].
- Few interpretability concerns.

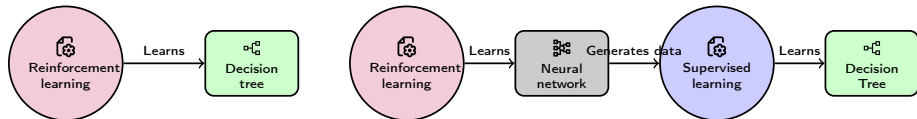
Two ways to get interpretable policies for SDM



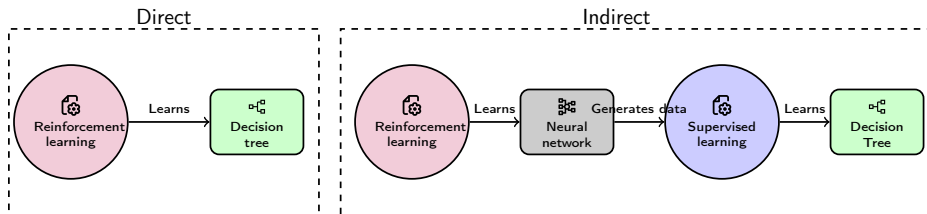
Two ways to get interpretable policies for SDM



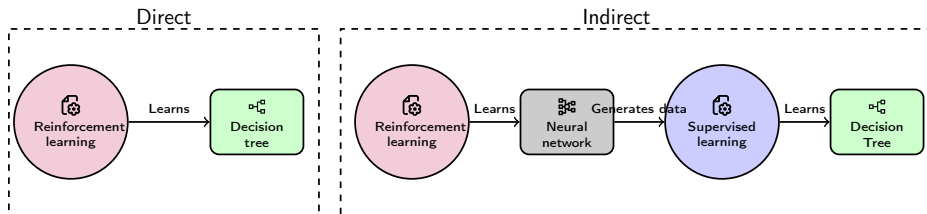
Two ways to get interpretable policies for SDM



Two ways to get interpretable policies for SDM

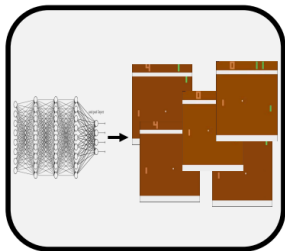


Two ways to get interpretable policies for SDM

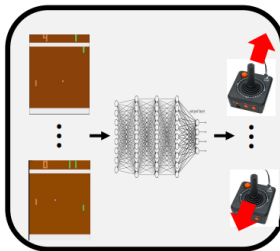


⚠ Policies obtained indirectly optimize a surrogate objective rather than a trade-off between interpretability and cumulative rewards.

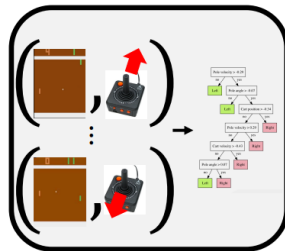
Indirect approach: imitation learning



Step 1: Use NN to generate states

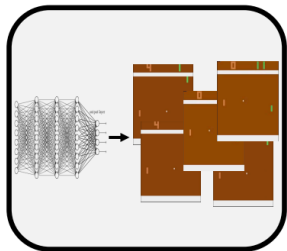


Step 2: Use NN to obtain actions

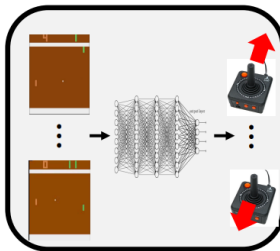


Step 3: Use supervised learning to train a decision tree

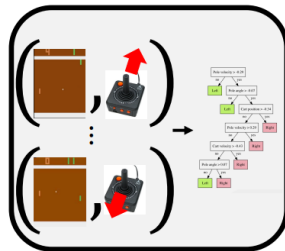
Indirect approach: imitation learning



Step 1: Use NN to generate states



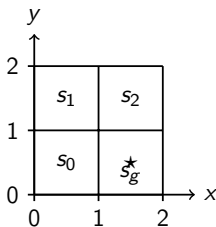
Step 2: Use NN to obtain actions



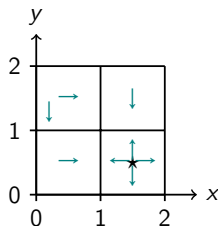
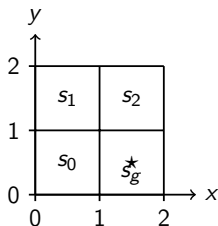
Step 3: Use supervised learning to train a decision tree

Most research focused on indirect learning of interpretable policies [RGB10; BPS18; Ver+18; Mil+24].

Grid world MDP and decision tree policies



Grid world MDP and decision tree policies

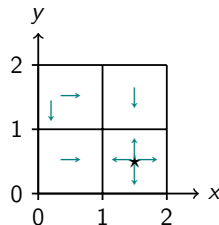
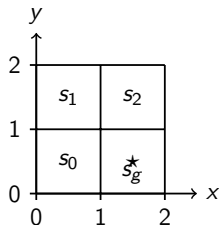


Grid world MDP and decision tree policies

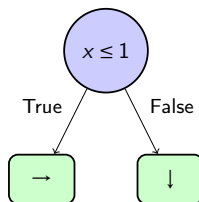


Grid world MDP and optimal actions.

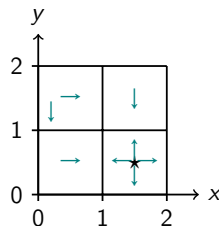
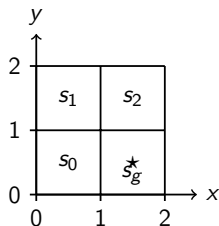
Grid world MDP and decision tree policies



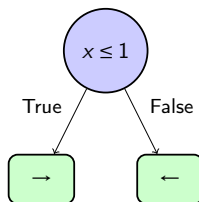
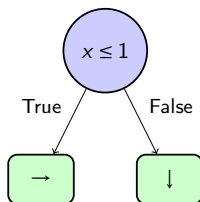
Grid world MDP and optimal actions.



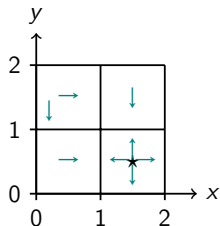
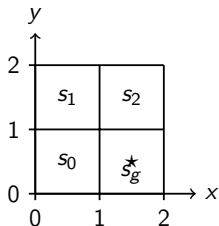
Grid world MDP and decision tree policies



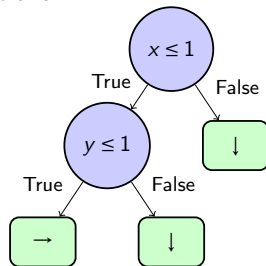
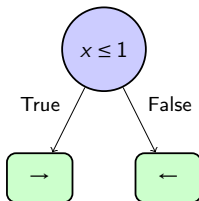
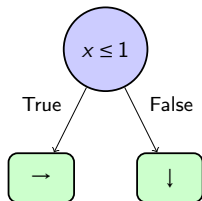
Grid world MDP and optimal actions.



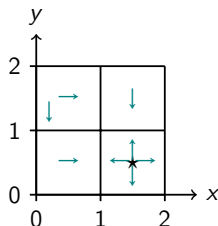
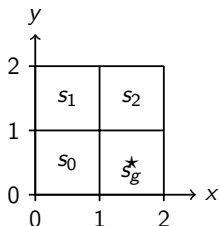
Grid world MDP and decision tree policies



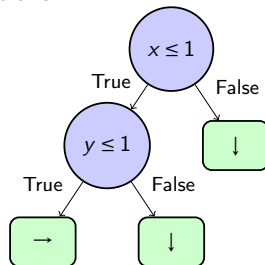
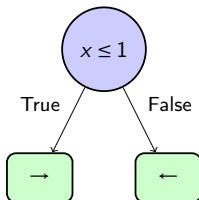
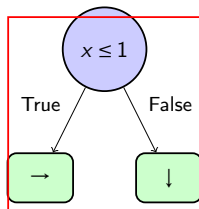
Grid world MDP and optimal actions.



Grid world MDP and decision tree policies

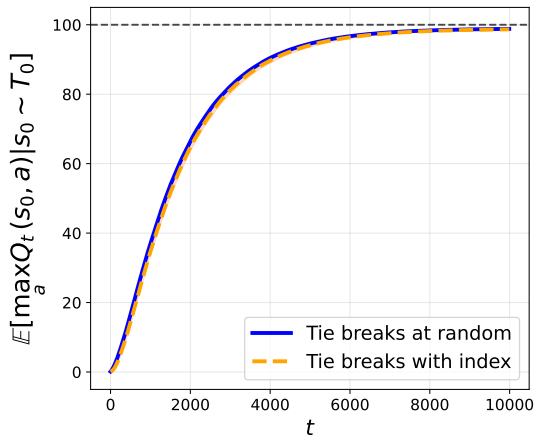


Grid world MDP and optimal actions.



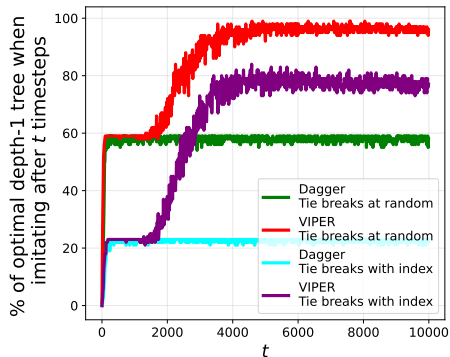
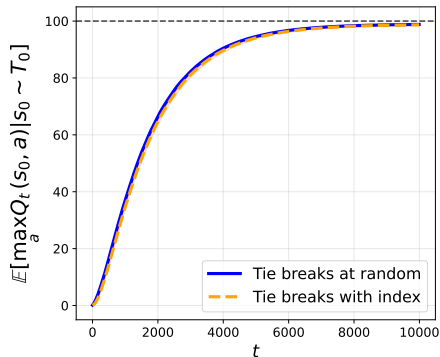
Decision tree policies with different interpretability-performance trade-offs.

Grid world MDP and decision tree policies



Sample complexity curve of Q-learning over 100 random seeds.

Grid world MDP and decision tree policies



Sample complexity curve of Q-learning over 100 random seeds and performance of indirect interpretable methods when imitating the greedy policy with a tree at different Q-learning stages.

- ① How difficult is it to directly optimize a trade-off of interpretability and performance in SDM?
- ② How to leverage sequential decision making to learn interpretable classifiers for supervised learning?
- ③ How to measure policy interpretability in sequential decision making?

- ① How difficult is it to directly optimize a trade-off of interpretability and performance in SDM?
- ② How to leverage sequential decision making to learn interpretable classifiers for supervised learning?
- ③ How to measure policy interpretability in sequential decision making?

- ① How difficult is it to directly optimize a trade-off of interpretability and performance in SDM?
- ② How to leverage sequential decision making to learn interpretable classifiers for supervised learning?
- ③ How to measure policy interpretability in sequential decision making?

- ① How difficult is it to directly optimize a trade-off of interpretability and performance in SDM?
- ② How to leverage sequential decision making to learn interpretable classifiers for supervised learning?
- ③ How to measure policy interpretability in sequential decision making?

Direct RL of decision tree policies with iterative bounding Markov decision processes

Given an MDP $\mathcal{M} \langle S, A, R, T \rangle$, an associated iterative bounding Markov decision process (IBMDP, [Top+21]) \mathcal{M}_{IB} is a tuple:

$$\langle \overbrace{S \times O}^{\text{Augmented state space}}, \underbrace{A \cup A_{info}}_{\text{Augmented action space}}, \overbrace{(R, \zeta)}^{\text{Augmented rewards}}, \underbrace{(T_{info}, T, T_0)}_{\text{Augmented transitions}} \rangle$$

Direct RL of decision tree policies with iterative bounding Markov decision processes

Given an MDP $\mathcal{M} \langle S, A, R, T \rangle$, an associated iterative bounding Markov decision process (IBMDP, [Top+21]) \mathcal{M}_{IB} is a tuple:

$$\langle \overbrace{S \times O}^{\text{Augmented state space}}, \underbrace{A \cup A_{info}}_{\text{Augmented action space}}, \overbrace{(R, \zeta)}^{\text{Augmented rewards}}, \underbrace{(T_{info}, T, T_0)}_{\text{Augmented transitions}} \rangle$$

IBMDPs promises

Direct RL of decision tree policies with iterative bounding Markov decision processes

Given an MDP $\mathcal{M} \langle S, A, R, T \rangle$, an associated iterative bounding Markov decision process (IBMDP, [Top+21]) \mathcal{M}_{IB} is a tuple:

$$\langle \overbrace{S \times O}^{\text{Augmented state space}}, \underbrace{A \cup A_{info}}_{\text{Augmented action space}}, \overbrace{(R, \zeta)}^{\text{Augmented rewards}}, \underbrace{(T_{info}, T, T_0)}_{\text{Augmented transitions}} \rangle$$

IBMDPs promises

- No need to design new algorithm: we can use deep RL.

Direct RL of decision tree policies with iterative bounding Markov decision processes

Given an MDP $\mathcal{M} \langle S, A, R, T \rangle$, an associated iterative bounding Markov decision process (IBMDP, [Top+21]) \mathcal{M}_{IB} is a tuple:

$$\langle \overbrace{S \times O}^{\text{Augmented state space}}, \underbrace{A \cup A_{info}}_{\text{Augmented action space}}, \overbrace{(R, \zeta)}^{\text{Augmented rewards}}, \underbrace{(T_{info}, T, T_0)}_{\text{Augmented transitions}} \rangle$$

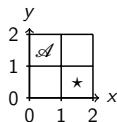
IBMDPs promises

- No need to design new algorithm: we can use deep RL.
- IBMDP rewards trade-off naturally interpretability and performances.

Grid world IBMDP example

$$t = 0$$

$$\mathbf{s}_t = (0.5, 1.5)$$

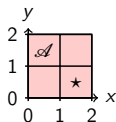


Grid world IBMDP example

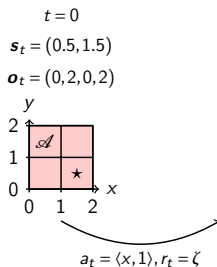
$$t = 0$$

$$\mathbf{s}_t = (0.5, 1.5)$$

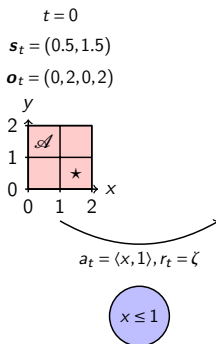
$$\mathbf{o}_t = (0, 2, 0, 2)$$



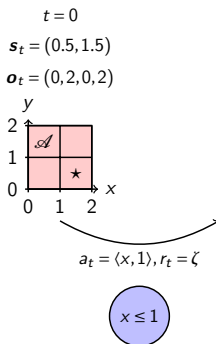
Grid world IBMDP example



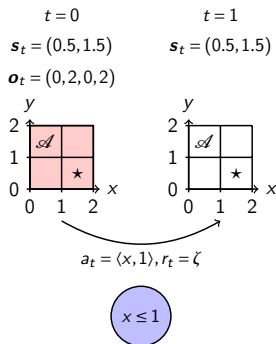
Grid world IBMDP example



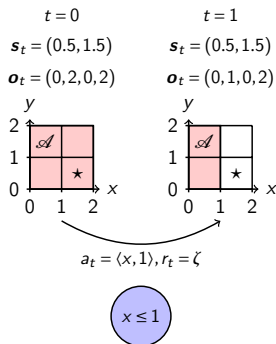
Grid world IBMDP example



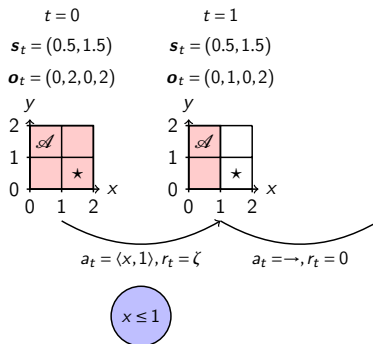
Grid world IBMDP example



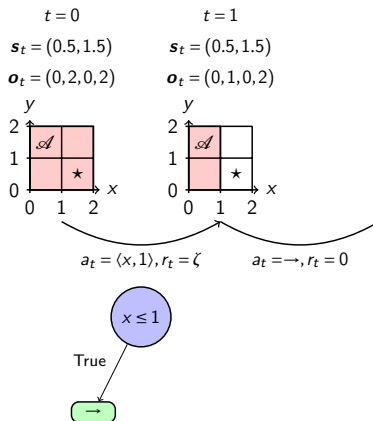
Grid world IBMDP example



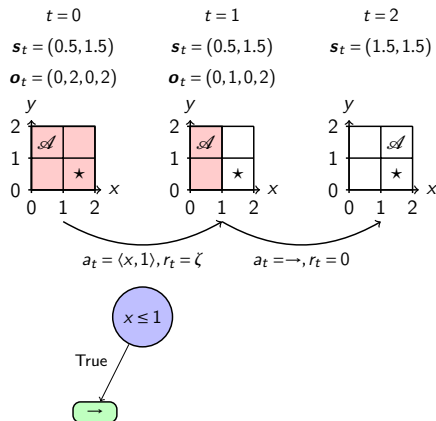
Grid world IBMDP example



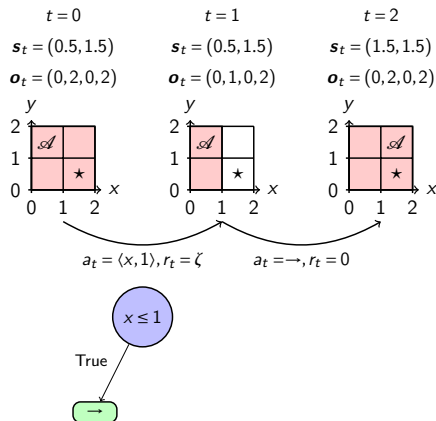
Grid world IBMDP example



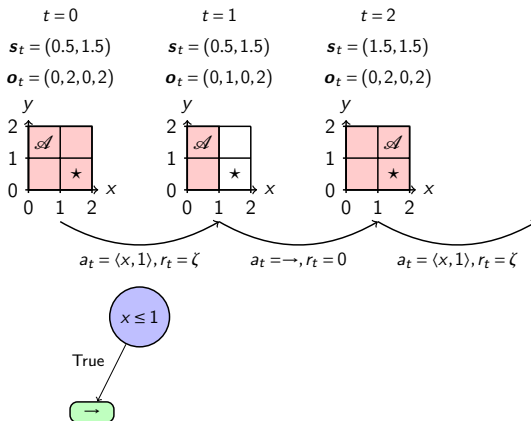
Grid world IBMDP example



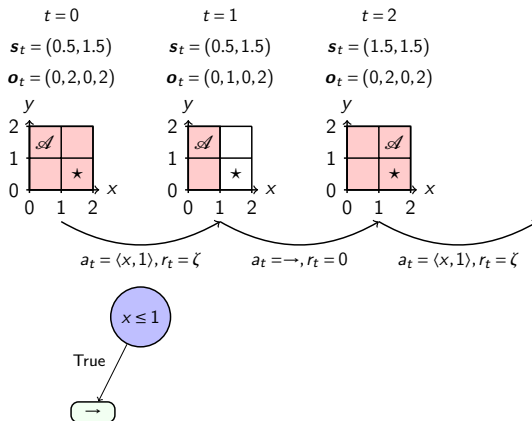
Grid world IBMDP example



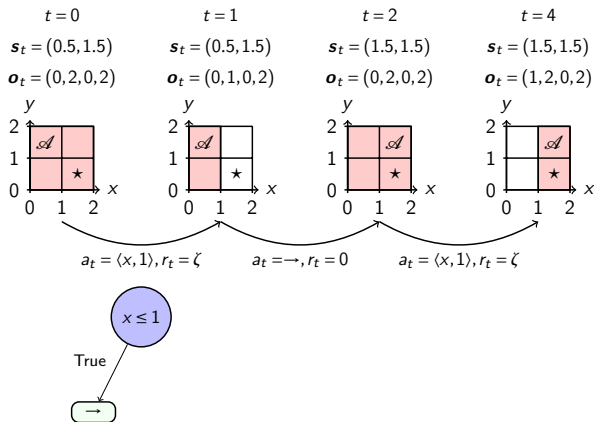
Grid world IBMDP example



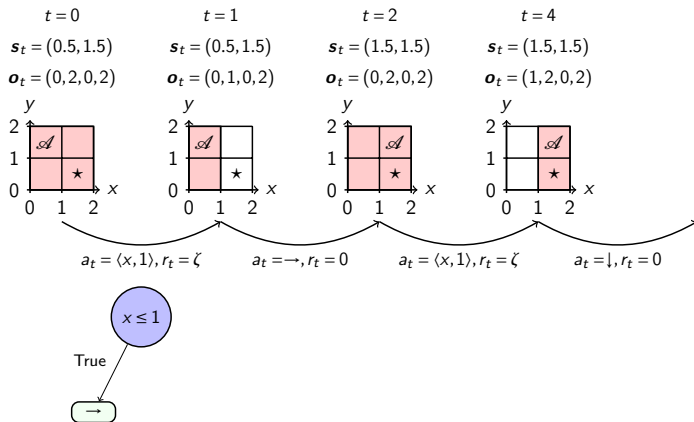
Grid world IBMDP example



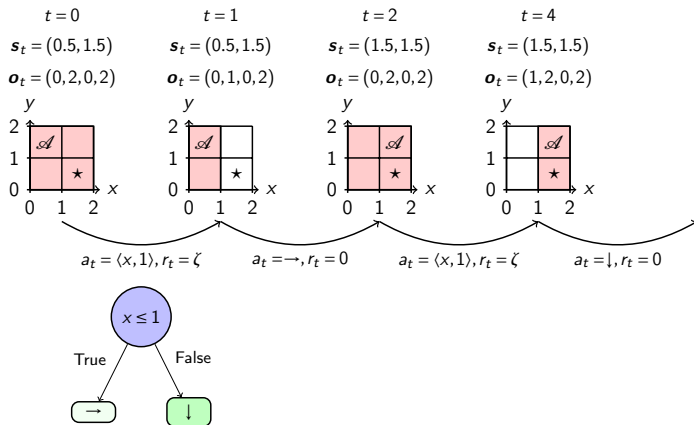
Grid world IBMDP example



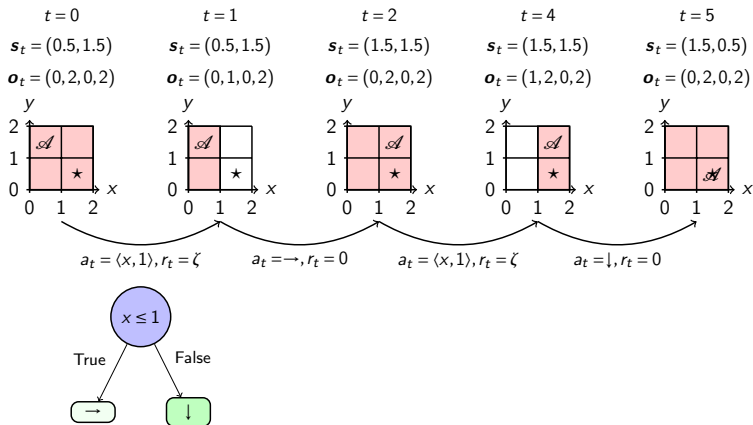
Grid world IBMDP example



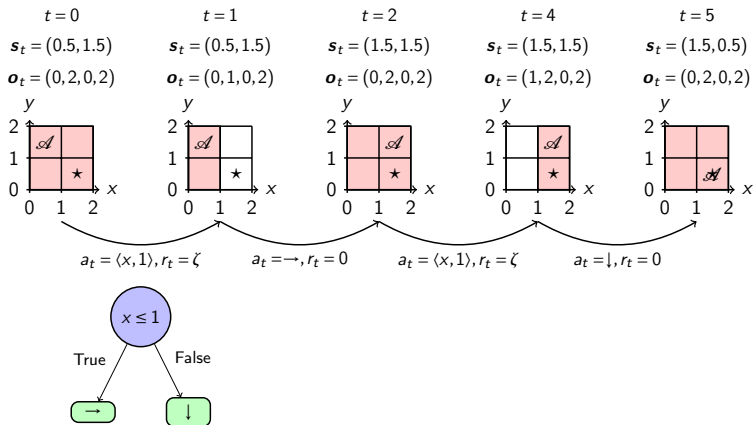
Grid world IBMDP example



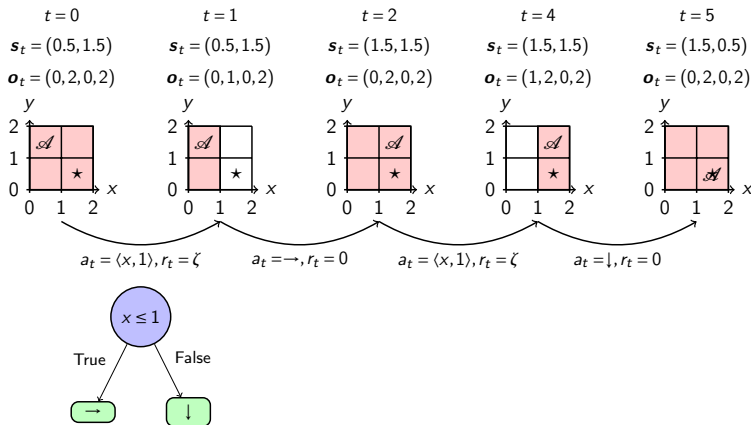
Grid world IBMDP example



Grid world IBMDP example

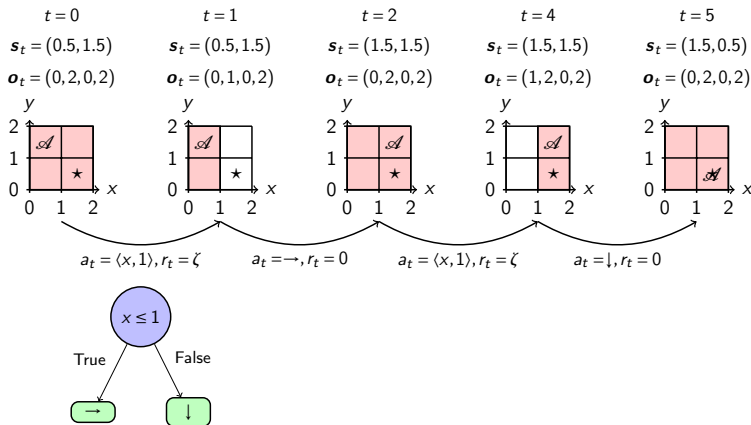


Grid world IBMDP example



- **Deterministic** and **partially observable** policies \Rightarrow decision trees.

Grid world IBMDP example



- **Deterministic** and **partially observable** policies \Rightarrow decision trees.
- $\triangle!$ Finding the best **deterministic** and **partially observable** policy is NP-hard [Lit94]!

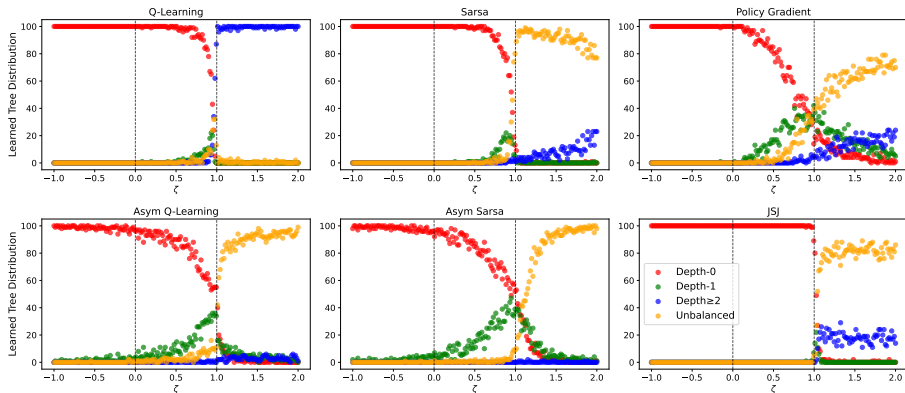
Q: Can we use reinforcement learning to directly optimize trade-offs of performance and interpretability in SDM?

Q: Can we use reinforcement learning to directly optimize trade-offs of performance and interpretability in SDM?

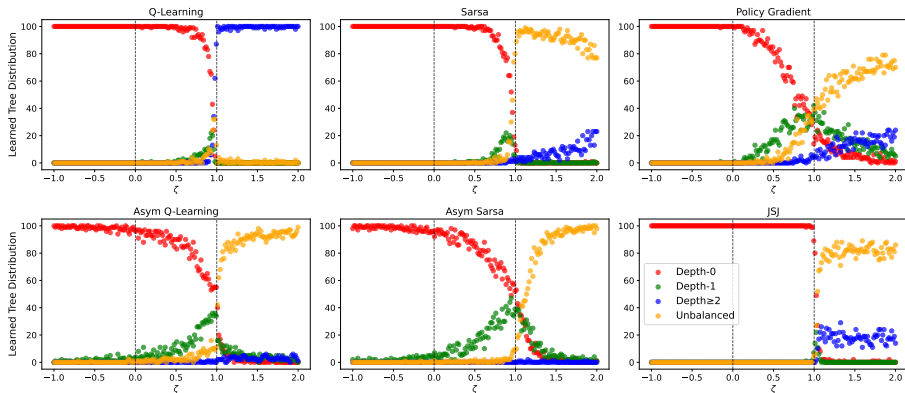
\approx

*Q: How does RL perform for optimizing **deterministic** and **partially observable** policies in IBMDPs?*

Result: RL cannot retrieve optimal depth-1 trees for the grid world MDP

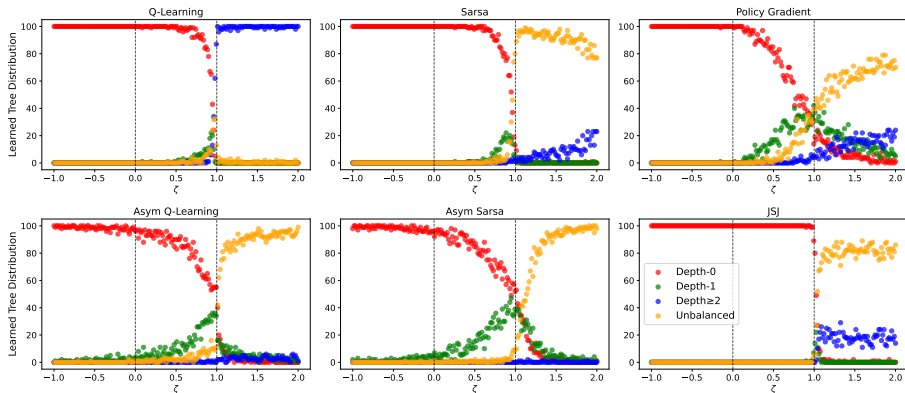


Result: RL cannot retrieve optimal depth-1 trees for the grid world MDP



Distributions of tree policies learned with (asymmetric) RL algorithms [SB98; SJJ94; LS98; BA22; BDA22] as a function of the interpretability reward ζ .

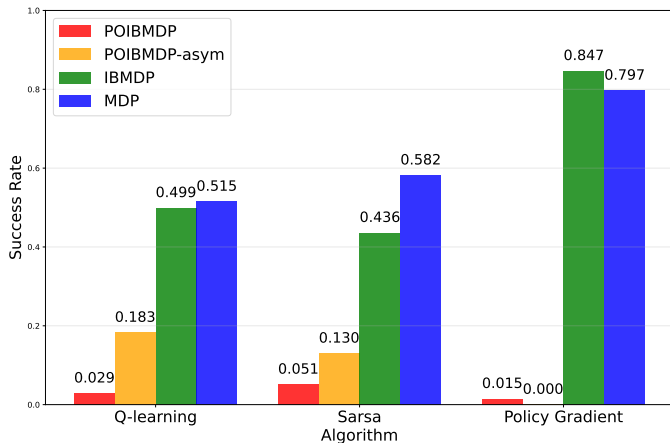
Result: RL cannot retrieve optimal depth-1 trees for the grid world MDP



Distributions of tree policies learned with (asymmetric) RL algorithms [SB98; SJJ94; LS98; BA22; BDA22] as a function of the interpretability reward ζ .

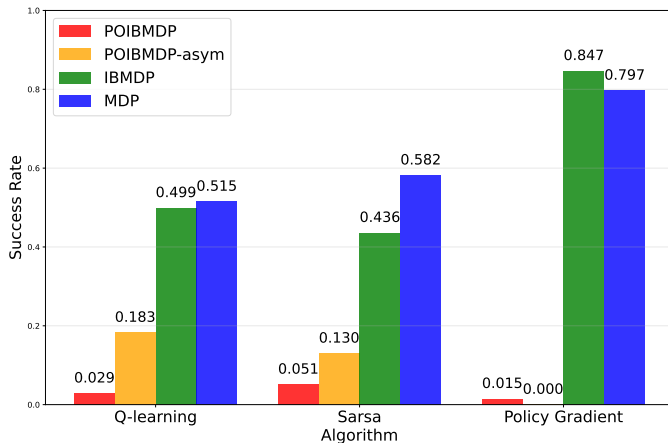
Are the poor performances due to partial observability?

Result: for similar problems, RL struggles more when there is partial observability



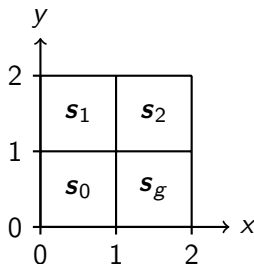
Success rates when learning deterministic partially observable or Markovian policies in the same IBMDP.

Result: for similar problems, RL struggles more when there is partial observability

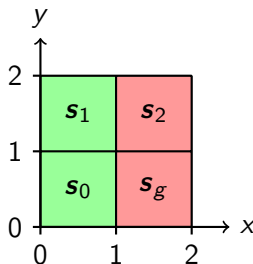


Success rates when learning deterministic partially observable or Markovian policies in the same IBMDP. **Is it all for nothing?**

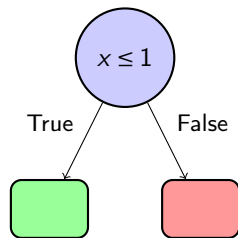
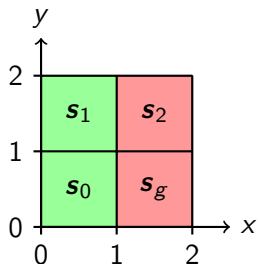
Interesting sub-class of MDPs: classification MDPs



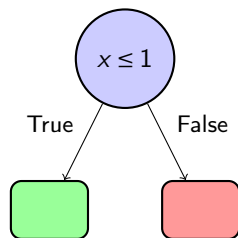
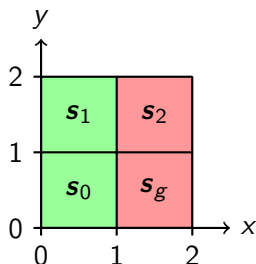
Interesting sub-class of MDPs: classification MDPs



Interesting sub-class of MDPs: classification MDPs

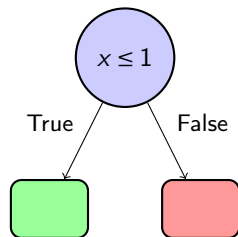
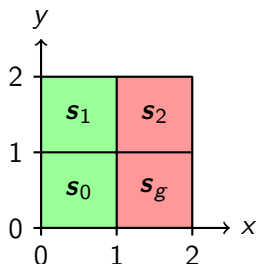


Interesting sub-class of MDPs: classification MDPs



Classification MDP and the unique optimal depth-1 tree.

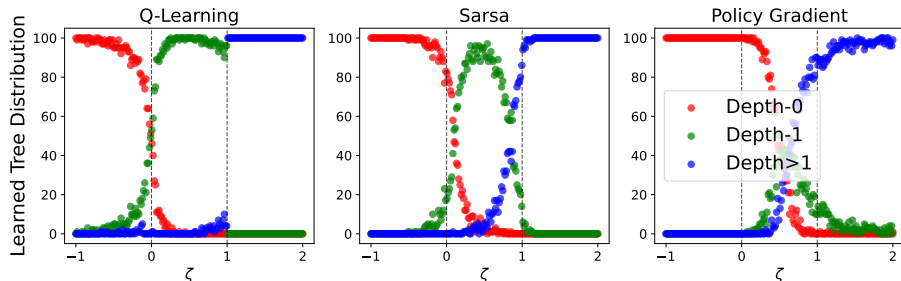
Interesting sub-class of MDPs: classification MDPs



Classification MDP and the unique optimal depth-1 tree.

⚠ Deterministic partially observable policies for classification IBMDPs (\Leftrightarrow decision tree policies) are in fact Markovian.

Result: RL can retrieve optimal depth-1 trees for the toy classification MDPs



Distributions of tree policies learned with various RL algorithms.

Perspectives for direct RL of decision tree policies.

Perspectives for direct RL of decision tree policies.

- It seems that interpretability for SDM problems can be difficult to achieve because of **partial observability**.

Perspectives for direct RL of decision tree policies.

- It seems that interpretability for SDM problems can be difficult to achieve because of **partial observability**.
- Should we focus on indirect approaches? Hybrid approaches [Wu+20]?

Perspectives for direct RL of decision tree policies.

- It seems that interpretability for SDM problems can be difficult to achieve because of **partial observability**.
- Should we focus on indirect approaches? Hybrid approaches [Wu+20]?
- Fixing the policy tree structure a priori (parametric trees, [Mar+25])?

Perspectives for direct RL of decision tree policies.

- It seems that interpretability for SDM problems can be difficult to achieve because of **partial observability**.
- Should we focus on indirect approaches? Hybrid approaches [Wu+20]?
- Fixing the policy tree structure a priori (parametric trees, [Mar+25])?
- Can other policies (programs, oblique trees, algebraic expressions. . .) be directly optimized with RL in IBMDPs?

Perspectives for direct RL of decision tree policies.

- It seems that interpretability for SDM problems can be difficult to achieve because of **partial observability**.
- Should we focus on indirect approaches? Hybrid approaches [Wu+20]?
- Fixing the policy tree structure a priori (parametric trees, [Mar+25])?
- Can other policies (programs, oblique trees, algebraic expressions. . .) be directly optimized with RL in IBMDPs?
- Design algorithms that learn deterministic partially observable policies [LBE25; LEM25]?

Perspectives for direct RL of decision tree policies.

- It seems that interpretability for SDM problems can be difficult to achieve because of **partial observability**.
- Should we focus on indirect approaches? Hybrid approaches [Wu+20]?
- Fixing the policy tree structure a priori (parametric trees, [Mar+25])?
- Can other policies (programs, oblique trees, algebraic expressions. . .) be directly optimized with RL in IBMDPs?
- Design algorithms that learn deterministic partially observable policies [LBE25; LEM25]?

RL works in classification MDPs

Perspectives for direct RL of decision tree policies.

- It seems that interpretability for SDM problems can be difficult to achieve because of **partial observability**.
- Should we focus on indirect approaches? Hybrid approaches [Wu+20]?
- Fixing the policy tree structure a priori (parametric trees, [Mar+25])?
- Can other policies (programs, oblique trees, algebraic expressions. . .) be directly optimized with RL in IBMDPs?
- Design algorithms that learn deterministic partially observable policies [LBE25; LEM25]?

RL works in classification MDPs

Q: Can we leverage SDM to design new decision tree induction algorithms for the supervised learning setting?

Perspectives for direct RL of decision tree policies.

- It seems that interpretability for SDM problems can be difficult to achieve because of **partial observability**.
- Should we focus on indirect approaches? Hybrid approaches [Wu+20]?
- Fixing the policy tree structure a priori (parametric trees, [Mar+25])?
- Can other policies (programs, oblique trees, algebraic expressions. . .) be directly optimized with RL in IBMDPs?
- Design algorithms that learn deterministic partially observable policies [LBE25; LEM25]?

RL works in classification MDPs

Q: Can we leverage SDM to design new decision tree induction algorithms for the supervised learning setting? A: Yes!

Decision trees in supervised learning

Decision trees in supervised learning

- N data points. Each x_i is described by p features and has a label $y_i \in \mathcal{Y}$.

$$\mathcal{L}(T) = \frac{1}{N} \sum_{i=1}^N \ell(y_i, f(x_i)) + \alpha C(T)$$

Decision trees in supervised learning

- N data points. Each x_i is described by p features and has a label $y_i \in \mathcal{Y}$.

$$\mathcal{L}(T) = \frac{1}{N} \sum_{i=1}^N \ell(y_i, f(x_i)) + \alpha C(T)$$

- Trees **interpretable** and **competitive with neural nets** [GOV22].

Decision trees in supervised learning

- N data points. Each x_i is described by p features and has a label $y_i \in \mathcal{Y}$.

$$\mathcal{L}(T) = \frac{1}{N} \sum_{i=1}^N \ell(y_i, f(x_i)) + \alpha C(T)$$

- Trees **interpretable** and **competitive with neural nets** [GOV22].
- Greedy algorithms **sub-optimal accuracy**, but $O(2^D)$ operations [Bre+84; Qui86; Qui93] .

Decision trees in supervised learning

- N data points. Each x_i is described by p features and has a label $y_i \in \mathcal{Y}$.

$$\mathcal{L}(T) = \frac{1}{N} \sum_{i=1}^N \ell(y_i, f(x_i)) + \alpha C(T)$$

- Trees **interpretable** and **competitive with neural nets** [GOV22].
- Greedy algorithms **sub-optimal accuracy**, but $O(2^D)$ operations [Bre+84; Qui86; Qui93] .
- Optimal algorithms, **optimal accuracy**, but $O((2Np)^D)$ operations (NP-hard) [BD17; Dem+22; LWD23; CRB24; HR76].

Decision trees in supervised learning

- N data points. Each x_i is described by p features and has a label $y_i \in \mathcal{Y}$.

$$\mathcal{L}(T) = \frac{1}{N} \sum_{i=1}^N \ell(y_i, f(x_i)) + \alpha C(T)$$

- Trees **interpretable** and **competitive with neural nets** [GOV22].
- Greedy algorithms **sub-optimal accuracy**, but $O(2^D)$ operations [Bre+84; Qui86; Qui93] .
- Optimal algorithms, **optimal accuracy**, but $O((2Np)^D)$ operations (NP-hard) [BD17; Dem+22; LWD23; CRB24; HR76].
- In between optimal and greedy?

Decision tree induction as solving MDPs

Intuition

The induction of a decision tree is made of a sequence of decisions: at each node, we must decide whether it is better to split (a subset of) \mathcal{E} , or to create a leaf node.

Intuition

The induction of a decision tree is made of a sequence of decisions: at each node, we must decide whether it is better to split (a subset of) \mathcal{E} , or to create a leaf node.

- S : data subsets.

Intuition

The induction of a decision tree is made of a sequence of decisions: at each node, we must decide whether it is better to split (a subset of) \mathcal{E} , or to create a leaf node.

- S: data subsets.
- A: test or leaf nodes that can be added to the tree.

Intuition

The induction of a decision tree is made of a sequence of decisions: at each node, we must decide whether it is better to split (a subset of) \mathcal{E} , or to create a leaf node.

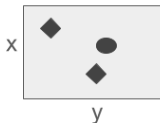
- S: data subsets.
- A: test or leaf nodes that can be added to the tree.
- R: penalty or accuracies.

Intuition

The induction of a decision tree is made of a sequence of decisions: at each node, we must decide whether it is better to split (a subset of) \mathcal{E} , or to create a leaf node.

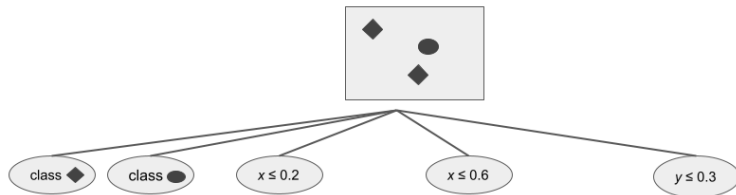
- S: data subsets.
- A: test or leaf nodes that can be added to the tree.
- R: penalty or accuracies.
- T: node traversals.

Decision tree induction as solving MDPs



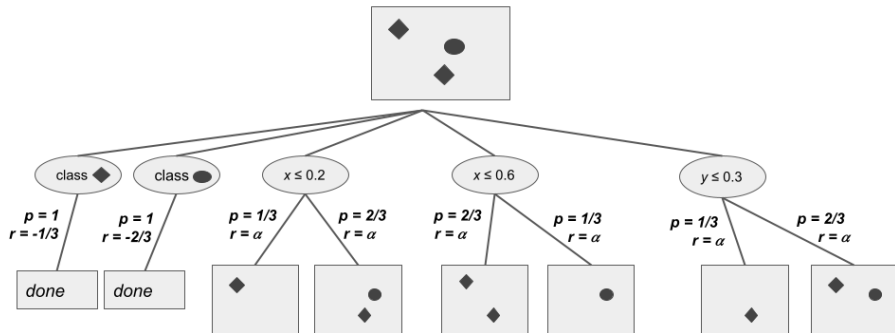
Example of decision tree induction as an MDP.

Decision tree induction as solving MDPs



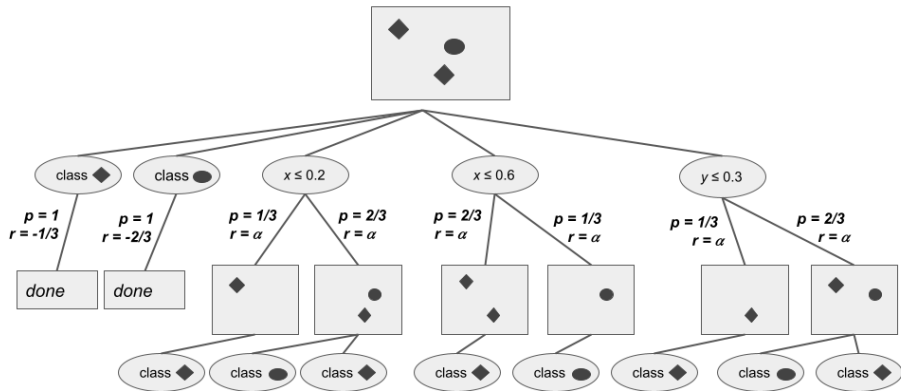
Example of decision tree induction as an MDP.

Decision tree induction as solving MDPs



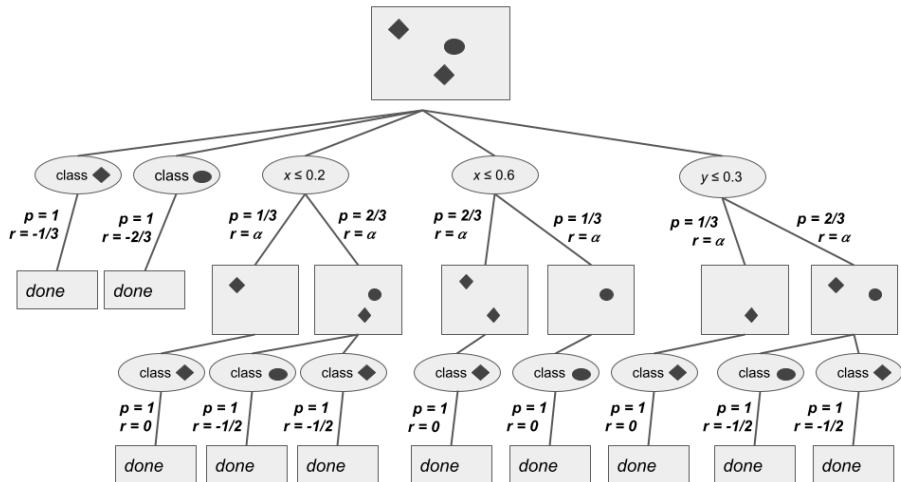
Example of decision tree induction as an MDP.

Decision tree induction as solving MDPs



Example of decision tree induction as an MDP.

Decision tree induction as solving MDPs



Example of decision tree induction as an MDP.

Controlling the time complexity of decision tree induction

Controlling the time complexity of decision tree induction

- Greedy algorithms consider only one candidate action in each state which is the test that minimizes some impurity criterion

Controlling the time complexity of decision tree induction

- Greedy algorithms consider only one candidate action in each state which is the test that minimizes some impurity criterion
→ **MDP state space size is $O(2^D)$.**

Controlling the time complexity of decision tree induction

- Greedy algorithms consider only one candidate action in each state which is the test that minimizes some impurity criterion
→ **MDP state space size is $O(2^D)$.**
- Optimal algorithms consider all possible actions in each state

Controlling the time complexity of decision tree induction

- Greedy algorithms consider only one candidate action in each state which is the test that minimizes some impurity criterion
→ **MDP state space size is $O(2^D)$.**
- Optimal algorithms consider all possible actions in each state
→ **MDP state space size is $O((2Np)^D)$.**

Controlling the time complexity of decision tree induction

- Greedy algorithms consider only one candidate action in each state which is the test that minimizes some impurity criterion
→ **MDP state space size is $O(2^D)$.**
- Optimal algorithms consider all possible actions in each state
→ **MDP state space size is $O((2Np)^D)$.**
- Let's choose candidate actions adaptively

Controlling the time complexity of decision tree induction

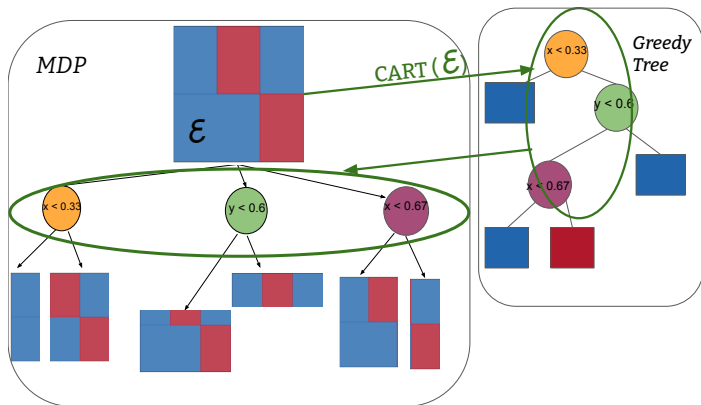
- Greedy algorithms consider only one candidate action in each state which is the test that minimizes some impurity criterion
→ **MDP state space size is $O(2^D)$.**
- Optimal algorithms consider all possible actions in each state
→ **MDP state space size is $O((2Np)^D)$.**
- Let's choose candidate actions adaptively
→ for each MDP state consider B actions: **state space size is $O((2B)^D)$.**

Controlling the time complexity of decision tree induction

- Greedy algorithms consider only one candidate action in each state which is the test that minimizes some impurity criterion
→ **MDP state space size is $O(2^D)$.**
- Optimal algorithms consider all possible actions in each state
→ **MDP state space size is $O((2Np)^D)$.**
- Let's choose candidate actions adaptively
→ for each MDP state consider B actions: **state space size is $O((2B)^D)$.**

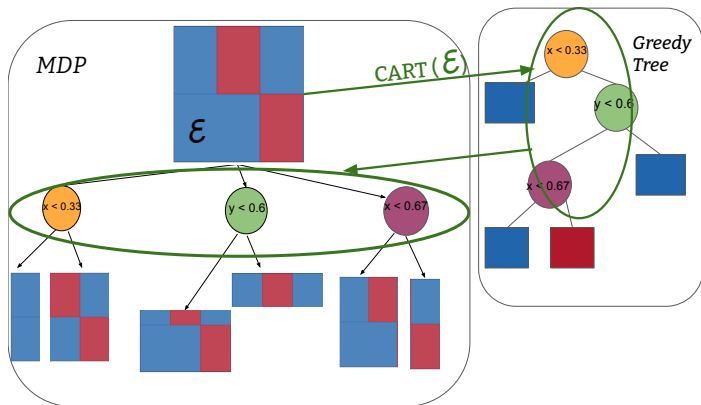
How to choose the B candidate actions/splits?

Dynamic Programming Decision Trees (DPDT)¹



¹Because states are entire datasets, we implement DPDT with a depth-first search to limit the space complexity.

Dynamic Programming Decision Trees (DPDT)¹



DPDT was presented at the 31st ACM SIGKDD conference.

¹Because states are entire datasets, we implement DPDT with a depth-first search to limit the space complexity.

Theory of generating candidate splits with CART

Theory of generating candidate splits with CART

Theorem (DPDT trees are not worse than greedy trees)

Theory of generating candidate splits with CART

Theorem (DPDT trees are not worse than greedy trees)

The greedy tree is always a solution of the MDPs we solve. Because we solve the MDPs exactly with DP, if the greedy tree is the best solution, DPDT will find it.

Theory of generating candidate splits with CART

Theorem (DPDT trees are not worse than greedy trees)

The greedy tree is always a solution of the MDPs we solve. Because we solve the MDPs exactly with DP, if the greedy tree is the best solution, DPDT will find it.

Theorem (DPDT trees can be strictly better than greedy trees)

Theory of generating candidate splits with CART

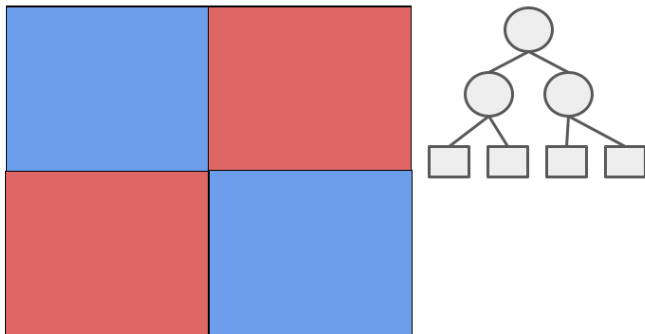
Theorem (DPDT trees are not worse than greedy trees)

The greedy tree is always a solution of the MDPs we solve. Because we solve the MDPs exactly with DP, if the greedy tree is the best solution, DPDT will find it.

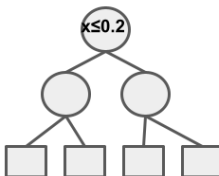
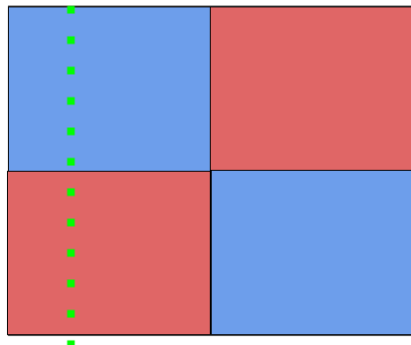
Theorem (DPDT trees can be strictly better than greedy trees)

There exist a depth budget D and a dataset for which DPDT trees are strictly better than greedy trees.

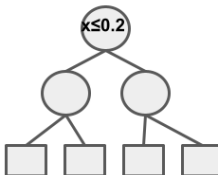
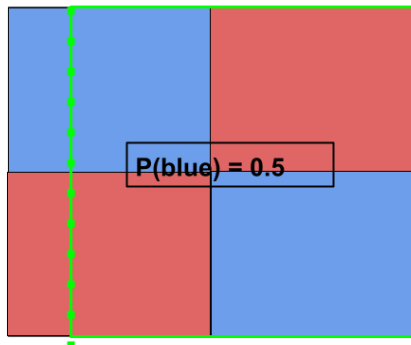
DPDT trees can be strictly better than greedy trees



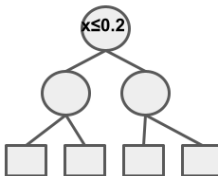
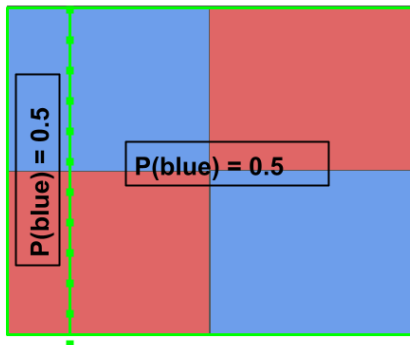
DPDT trees can be strictly better than greedy trees



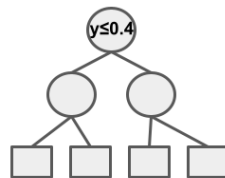
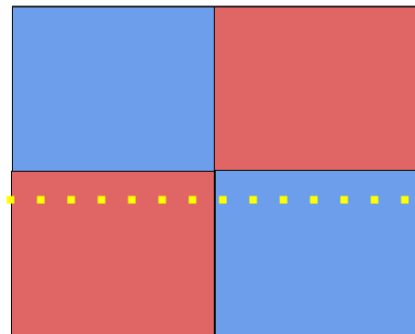
DPDT trees can be strictly better than greedy trees



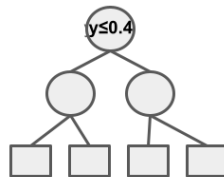
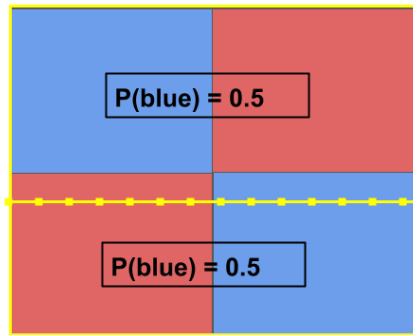
DPDT trees can be strictly better than greedy trees



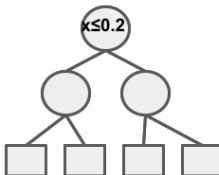
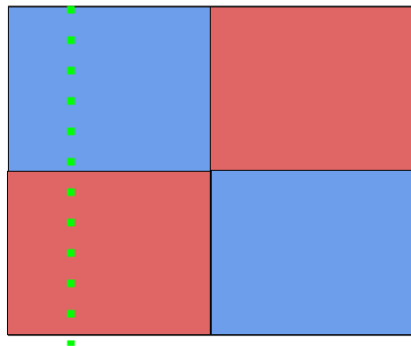
DPDT trees can be strictly better than greedy trees



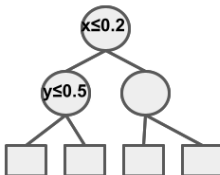
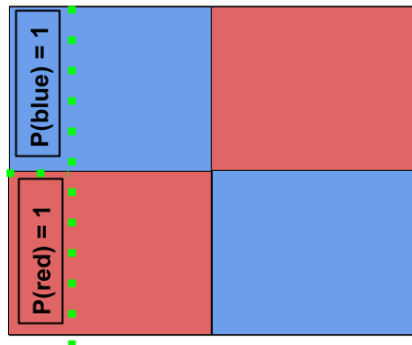
DPDT trees can be strictly better than greedy trees



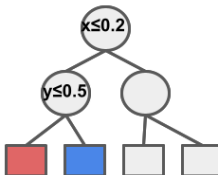
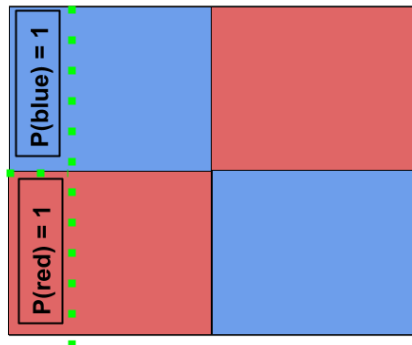
DPDT trees can be strictly better than greedy trees



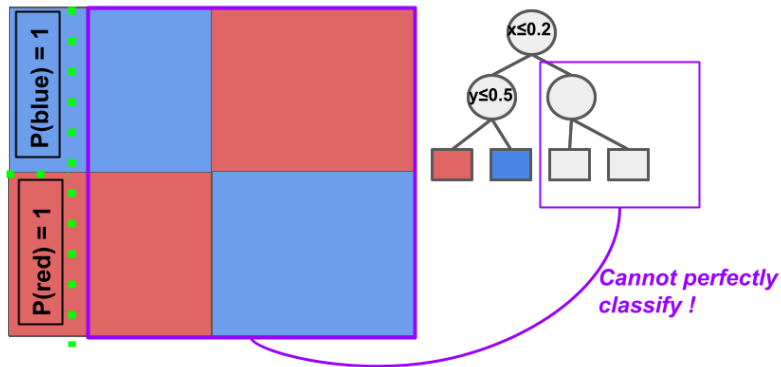
DPDT trees can be strictly better than greedy trees



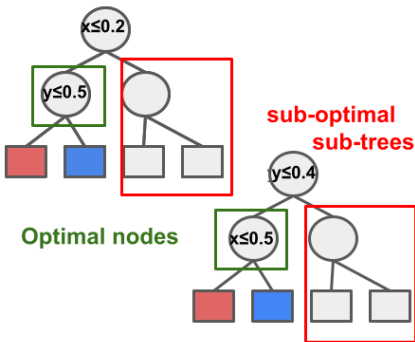
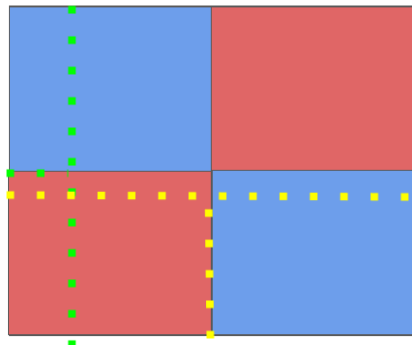
DPDT trees can be strictly better than greedy trees



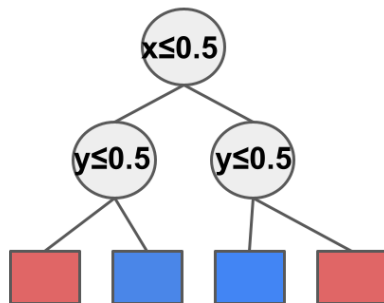
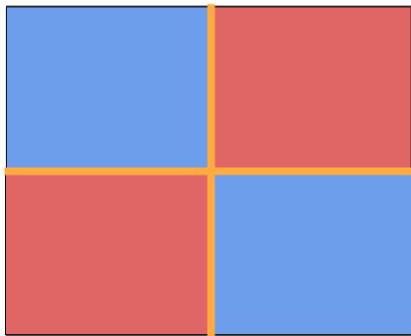
DPDT trees can be strictly better than greedy trees



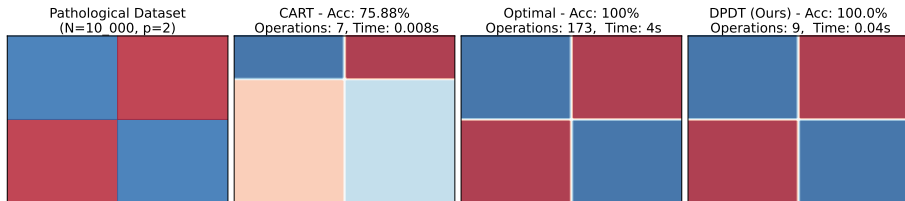
DPDT trees can be strictly better than greedy trees



DPDT trees can be strictly better than greedy trees



Fast like greedy trees, accurate like optimal trees



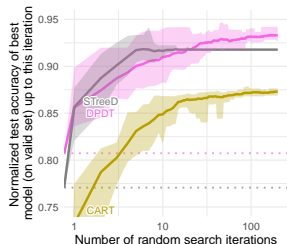
Comparison of greedy, optimal, and DPDT depth-2 trees on the checkersboard dataset.

Comparing tree accuracy to complexity

Train accuracy and operation count when learning depth-3 decision trees.

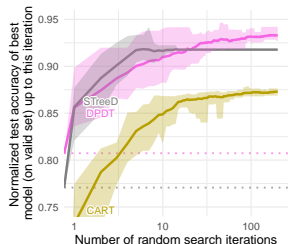
Dataset	N	p	Accuracy				Operations			
			Opt Quant-BnB	Greedy CART	DPDT light	DPDT full	Opt Quant-BnB	Greedy CART	DPDT light	DPDT full
room	8103	16	0.992	0.968	0.991	0.992	10^6	15	286	16100
bean	10888	16	0.871	0.777	0.812	0.853	$5 \cdot 10^6$	15	295	25900
eeg	11984	14	0.708	0.666	0.689	0.706	$2 \cdot 10^6$	13	289	26000
avila	10430	10	0.585	0.532	0.574	0.585	$3 \cdot 10^7$	9	268	24700
magic	15216	10	0.831	0.801	0.822	0.828	$6 \cdot 10^6$	15	298	28000
htru	14318	8	0.981	0.979	0.979	0.980	$6 \cdot 10^7$	15	295	25300
occup.	8143	5	0.994	0.989	0.991	0.994	$7 \cdot 10^5$	13	280	16300
skin	196045	3	0.969	0.966	0.966	0.966	$7 \cdot 10^4$	15	301	23300
fault	1552	27	0.682	0.553	0.672	0.674	$9 \cdot 10^8$	13	295	24200
segment	1848	18	0.887	0.574	0.812	0.879	$2 \cdot 10^6$	7	220	16300
page	4378	10	0.971	0.964	0.970	0.970	10^7	15	298	22400
bidding	5056	9	0.993	0.981	0.985	0.993	$3 \cdot 10^5$	13	256	9360
raisin	720	7	0.894	0.869	0.879	0.886	$4 \cdot 10^6$	15	295	20900
rice	3048	7	0.938	0.933	0.934	0.937	$2 \cdot 10^7$	15	298	25500
wilt	4339	5	0.996	0.993	0.994	0.995	$3 \cdot 10^5$	13	274	11300
bank	1097	4	0.983	0.933	0.971	0.980	$6 \cdot 10^4$	13	271	7990

DPDT trees generalization

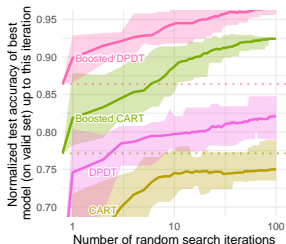


DPDT depth-5 trees vs.
other depth-5 trees

DPDT trees generalization

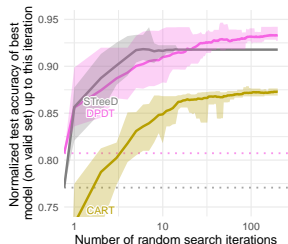


DPDT depth-5 trees vs.
other detph-5 trees

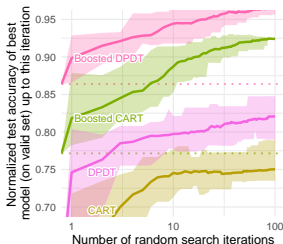


Boosted DPDT vs. Boosted
CART

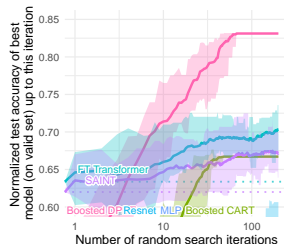
DPDT trees generalization



DPDT depth-5 trees vs. other detph-5 trees



Boosted DPDT vs. Boosted CART



Boosted DPDT vs. other classifiers

- New SOTA decision tree induction with dynamic programming in MDPs.

- New SOTA decision tree induction with dynamic programming in MDPs.
- What about using DPDT for indirect decision tree policy learning for SDM?

- New SOTA decision tree induction with dynamic programming in MDPs.
- What about using DPDT for indirect decision tree policy learning for SDM?
- What performances could we reach with an industry-grade implementation of XGboost+DPDT?

- New SOTA decision tree induction with dynamic programming in MDPs.
- What about using DPDT for indirect decision tree policy learning for SDM?
- What performances could we reach with an industry-grade implementation of XGboost+DPDT?

Let us take a step back

Q: Are decision trees really the most interpretable model?

- New SOTA decision tree induction with dynamic programming in MDPs.
- What about using DPDT for indirect decision tree policy learning for SDM?
- What performances could we reach with an industry-grade implementation of XGboost+DPDT?

Let us take a step back

Q: Are decision trees really the most interpretable model?

A: It depends.

How to measure policy interpretability?

How to measure policy interpretability?

Challenges [Gla+24; Lip18; DK17]

How to measure policy interpretability?

Challenges [Gla+24; Lip18; DK17]

- No definition of interpretability.

How to measure policy interpretability?

Challenges [Gla+24; Lip18; DK17]

- No definition of interpretability.
- Measuring might require humans.

How to measure policy interpretability?

Challenges [Gla+24; Lip18; DK17]

- No definition of interpretability.
- Measuring might require humans.

The notion of *simulatability* [Lip18]

How to measure policy interpretability?

Challenges [Gla+24; Lip18; DK17]

- No definition of interpretability.
- Measuring might require humans.

The notion of *simulatability* [Lip18]

- Interpretability \simeq how long for human to make the same computations.

How to measure policy interpretability?

Challenges [Gla+24; Lip18; DK17]

- No definition of interpretability.
- Measuring might require humans.

The notion of *simulatability* [Lip18]

- Interpretability \simeq how long for human to make the same computations.
- Interpretability \simeq how much effort for a human to read through the entire policy.

How to measure policy interpretability?

Challenges [Gla+24; Lip18; DK17]

- No definition of interpretability.
- Measuring might require humans.

The notion of *simulatability* [Lip18]

- Interpretability \simeq how long for human to make the same computations.
- Interpretability \simeq how much effort for a human to read through the entire policy.
- Less parameters mean more interpretability [Fre14; Lav99].

How to measure policy interpretability?

Challenges [Gla+24; Lip18; DK17]

- No definition of interpretability.
- Measuring might require humans.

The notion of *simulatability* [Lip18]

- Interpretability \simeq how long for human to make the same computations.
- Interpretability \simeq how much effort for a human to read through the entire policy.
- Less parameters mean more interpretability [Fre14; Lav99].
- Time to formally verify a policy decreases with interpretability [Bar+20].

A methodology to measure policy interpretability without humans

A methodology to measure policy interpretability without humans

Simulatability [Lip18]

A methodology to measure policy interpretability without humans

Simulatability [Lip18]

- 1 How long it takes for human to make the same computations given an input \simeq policy inference time.

A methodology to measure policy interpretability without humans

Simulatability [Lip18]

- 1 How long it takes for human to make the same computations given an input \simeq policy inference time.
- 2 How much effort it would take a human to read through the entire policy once \simeq policy size in memory.

A methodology to measure policy interpretability without humans

Simulatability [Lip18]

- 1 How long it takes for human to make the same computations given an input \simeq policy inference time.
- 2 How much effort it would take a human to read through the entire policy once \simeq policy size in memory.

Not that simple in practice [Luo+24]

A methodology to measure policy interpretability without humans

Simulatability [Lip18]

- 1 How long it takes for human to make the same computations given an input \simeq policy inference time.
- 2 How much effort it would take a human to read through the entire policy once \simeq policy size in memory.

Not that simple in practice [Luo+24]

- Different hardwares (CPUs vs GPUs).

A methodology to measure policy interpretability without humans

Simulatability [Lip18]

- 1 How long it takes for human to make the same computations given an input \simeq policy inference time.
- 2 How much effort it would take a human to read through the entire policy once \simeq policy size in memory.

Not that simple in practice [Luo+24]

- Different hardwares (CPUs vs GPUs).
- Different implementations (matrix operations vs fully sequentially) ...

We propose policy unfolding

We propose policy unfolding

```
# Decision tree for Mountain Car
def play(x):
    if x[1] <= -0.2597:
        if x[1] <= -0.6378:
            return 0
        else:
            if x[0] <= -1.0021:
                return 2
            else:
                return 0
    else:
        if x[1] <= -0.0508:
            if x[0] <= 0.2979:
                if x[0] <= 0.0453:
                    return 2
                else:
                    if x[1] <=
-0.2156:
                        return 0
                    else:
                        return 2
            else:
                return 0
        else:
            return 2
```

We propose policy unfolding

```
# Decision tree for Mountain Car
def play(x):
    if x[1] <= -0.2597:
        if x[1] <= -0.6378:
            return 0
        else:
            if x[0] <= -1.0021:
                return 2
            else:
                return 0
    else:
        if x[1] <= -0.0508:
            if x[0] <= 0.2979:
                if x[0] <= 0.0453:
                    return 2
                else:
                    if x[1] <=
-0.2156:
                        return 0
                    else:
                        return 2
            else:
                return 0
        else:
            return 2
```

```
# Small ReLU MLP for Pendulum
def play(x):
    h_layer_0_0 = 1.238*x[0]+0.971*x
    [1]
    +0.430*x[2]+0.933
    h_layer_0_0 = max(0, h_layer_0_0
    )
    h_layer_0_1 = -1.221*x[0]+1.001
    *x[1]-0.423*x[2]
    +0.475
    h_layer_0_1 = max(0, h_layer_0_1
    )
    h_layer_1_0 = -0.109*h_layer_0_0
    -0.377*h_layer_0_1
    +1.694
    h_layer_1_0 = max(0, h_layer_1_0
    )
    h_layer_1_1 = -3.024*h_layer_0_0
    -1.421*h_layer_0_1
    +1.530
    h_layer_1_1 = max(0, h_layer_1_1
    )
    h_layer_2_0 = -1.790*h_layer_1_0
    +2.840*h_layer_1_1
    +0.658
    y_0 = h_layer_2_0
    return [y_0]
```

Empirical validation

- 1 Is policy unfolding necessary?

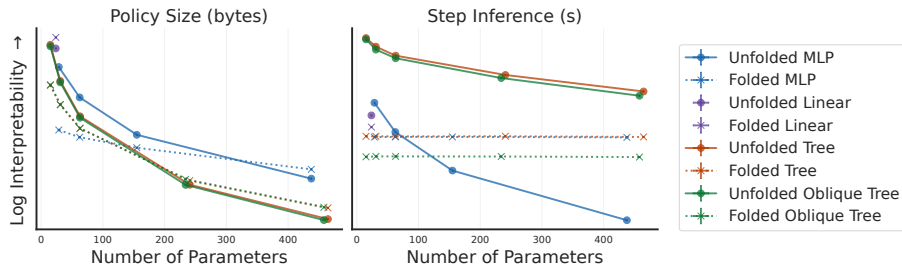
- 1 Is policy unfolding necessary?
- 2 What kind of results we can obtain using our proposed methodology?

- 1 Is policy unfolding necessary?
- 2 What kind of results we can obtain using our proposed methodology?

Setup

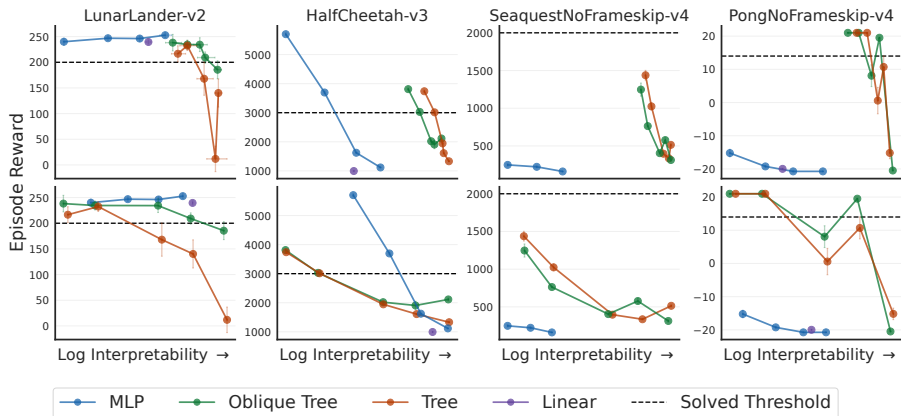
We imitate ~ 40000 expert policies from `stable-baselines3` using various policy classes/nb parameters on various environments.

Result: unfolding policies is necessary to respect consensus



Aggregated policies interpretability on classic control environments

Result: there is no dominating policy class for all environments



Interpretability-Performance trade-offs. Top row, interpretability is measured with step inference times. Bottom row, the interpretability is measured with policy size.

- Beliefs such as "trees are more interpretable than neural networks" should be used with caution.

- Beliefs such as "trees are more interpretable than neural networks" should be used with caution.
- Tree-like policy classes can have good inductive bias (e.g. Atari).

- Beliefs such as "trees are more interpretable than neural networks" should be used with caution.
- Tree-like policy classes can have good inductive bias (e.g. Atari).
- What about (very) big models?

- Beliefs such as "trees are more interpretable than neural networks" should be used with caution.
- Tree-like policy classes can have good inductive bias (e.g. Atari).
- What about (very) big models?
- Can we use our policy programs as low level skills (hierarchical RL)?

Conclusion: interpretable SDM is a difficult research topic

Conclusion: interpretable SDM is a difficult research topic

- Technical challenges: **partial observability in SDM, NP-hardness.**

Conclusion: interpretable SDM is a difficult research topic

- Technical challenges: **partial observability in SDM, NP-hardness**.
→ Focus on indirect approaches and/or on POMDP research first.

Conclusion: interpretable SDM is a difficult research topic

- Technical challenges: **partial observability in SDM, NP-hardness**.
→ Focus on indirect approaches and/or on POMDP research first.
- Fundamental challenges: **no definition**.

Conclusion: interpretable SDM is a difficult research topic

- Technical challenges: **partial observability in SDM, NP-hardness**.
→ Focus on indirect approaches and/or on POMDP research first.
- Fundamental challenges: **no definition**.
→ Discuss with the community (InterpPol workshop).

Conclusion: interpretable SDM is a difficult research topic

- Technical challenges: **partial observability in SDM, NP-hardness**.
→ Focus on indirect approaches and/or on POMDP research first.
- Fundamental challenges: **no definition**.
→ Discuss with the community (InterpPol workshop).
- **Decision trees offer good inductive bias for SDM in games or tabular data.**

Conclusion: interpretable SDM is a difficult research topic

- Technical challenges: **partial observability in SDM, NP-hardness**.
→ Focus on indirect approaches and/or on POMDP research first.
- Fundamental challenges: **no definition**.
→ Discuss with the community (InterpPol workshop).
- **Decision trees offer good inductive bias for SDM in games or tabular data.**

My hope

Motivate interpretability by finding a real-world problem where interpretability is *really* necessary [Nag+24].

Broader perspectives

- **Deep learning:** Can we design deep learning layers that take datasets and output candidate splits?

- **Deep learning:** Can we design deep learning layers that take datasets and output candidate splits?
- **Combinatorial optimization:** Can we formulate other combinatorial/NP-hard problems as MDPs and design other DPDT-like algorithms?

- **Deep learning:** Can we design deep learning layers that take datasets and output candidate splits?
- **Combinatorial optimization:** Can we formulate other combinatorial/NP-hard problems as MDPs and design other DPDT-like algorithms?
- **Human-computer interaction:** Can we do large scale human study of the $\sim 40K$ programs interpretability?

- [BA22] Andrea Baisero and Christopher Amato. “Unbiased Asymmetric Reinforcement Learning under Partial Observability”. In: *Proceedings of the 21st International Conference on Autonomous Agents and Multiagent Systems*. AAMAS '22. Virtual Event, New Zealand: International Foundation for Autonomous Agents and Multiagent Systems, 2022, pp. 44–52. ISBN: 9781450392136.
- [Bar+20] Pablo Barceló et al. “Model interpretability through the lens of computational complexity”. In: *Advances in neural information processing systems* (2020).
- [BD17] Dimitris Bertsimas and Jack Dunn. “Optimal classification trees”. In: *Machine Learning* 106 (2017), pp. 1039–1082.
- [BDA22] Andrea Baisero, Brett Daley, and Christopher Amato. “Asymmetric DQN for partially observable reinforcement learning”. In: *Proceedings of the Thirty-Eighth Conference on Uncertainty in Artificial Intelligence*. Ed. by James Cussens and Kun Zhang. Vol. 180. Proceedings of Machine Learning

Research. PMLR, Jan. 2022, pp. 107–117. URL: <https://proceedings.mlr.press/v180/baisero22a.html>.

- [BPS18] Osbert Bastani, Yewen Pu, and Armando Solar-Lezama. “Verifiable Reinforcement Learning via Policy Extraction”. In: (2018).
- [Bre+84] L Breiman et al. *Classification and Regression Trees*. Wadsworth, 1984.
- [CRB24] Ayman Chaouki, Jesse Read, and Albert Bifet. “Branches: A Fast Dynamic Programming and Branch & Bound algorithm for Optimal Decision Trees”. In: (2024). arXiv: 2406.02175 [cs.LG]. URL: <https://arxiv.org/abs/2406.02175>.
- [Dem+22] Emir Demirovic et al. “MurTree: Optimal Decision Trees via Dynamic Programming and Search”. In: *Journal of Machine Learning Research* 23.26 (2022), pp. 1–47. URL: <http://jmlr.org/papers/v23/20-520.html>.
- [DK17] Finale Doshi-Velez and Been Kim. “Towards A Rigorous Science of Interpretable Machine Learning”. In: (2017). arXiv:

1702.08608 [stat.ML]. URL:
<https://arxiv.org/abs/1702.08608>.

- [Fre14] Alex A. Freitas. “Comprehensible classification models: a position paper”. In: *SIGKDD Explor. Newsl.* 15.1 (Mar. 2014), pp. 1–10. ISSN: 1931-0145. DOI: 10.1145/2594473.2594475. URL: <https://doi.org/10.1145/2594473.2594475>.
- [Gla+24] Claire Glanois et al. “A survey on interpretable reinforcement learning”. In: *Machine Learning* (2024), pp. 1–44.
- [GOV22] Léo Grinsztajn, Edouard Oyallon, and Gaël Varoquaux. “Why do tree-based models still outperform deep learning on typical tabular data?” In: *Advances in neural information processing systems* 35 (2022), pp. 507–520.
- [HR76] Laurent Hyafil and Ronald L. Rivest. “Constructing optimal binary decision trees is NP-complete”. In: *Information Processing Letters* 5.1 (1976), pp. 15–17. ISSN: 0020-0190. DOI: [https://doi.org/10.1016/0020-0190\(76\)90095-8](https://doi.org/10.1016/0020-0190(76)90095-8).

URL: <https://www.sciencedirect.com/science/article/pii/0020019076900958>.

- [Lav99] Nada Lavrač. “Selected techniques for data mining in medicine”. In: *Artificial Intelligence in Medicine* 16.1 (1999). Data Mining Techniques and Applications in Medicine, pp. 3–23. ISSN: 0933-3657. DOI: [https://doi.org/10.1016/S0933-3657\(98\)00062-1](https://doi.org/10.1016/S0933-3657(98)00062-1). URL: <https://www.sciencedirect.com/science/article/pii/S0933365798000621>.
- [LBE25] Gaspard Lambrechts, Adrien Bolland, and Damien Ernst. “Informed POMDP: Leveraging Additional Information in Model-Based RL”. In: *Reinforcement Learning Journal* 2 (2025), pp. 763–784.
- [LEM25] Gaspard Lambrechts, Damien Ernst, and Aditya Mahajan. “A Theoretical Justification for Asymmetric Actor-Critic algorithms”. In: *Forty-second International Conference on Machine Learning*. 2025. URL: <https://openreview.net/forum?id=F1yANMCnAn>.

- [Lip18] Zachary C. Lipton. “The Mythos of Model Interpretability: In machine learning, the concept of interpretability is both important and slippery.”. In: *Queue* 16.3 (2018), pp. 31–57.
- [Lit94] Michael L. Littman. “Memoryless policies: theoretical limitations and practical results”. In: *Proceedings of the Third International Conference on Simulation of Adaptive Behavior: From Animals to Animats 3: From Animals to Animats 3*. SAB94. Brighton, United Kingdom: MIT Press, 1994, pp. 238–245. ISBN: 0262531224.
- [LS98] John Loch and Satinder P. Singh. “Using Eligibility Traces to Find the Best Memoryless Policy in Partially Observable Markov Decision Processes”. In: *Proceedings of the Fifteenth International Conference on Machine Learning*. ICML '98. San Francisco, CA, USA: Morgan Kaufmann Publishers Inc., 1998, pp. 323–331. ISBN: 1558605568.
- [Luo+24] Lirui Luo et al. “End-to-End Neuro-Symbolic Reinforcement Learning with Textual Explanations”. In: *International Conference on Machine Learning (ICML)* (2024).

- [LWD23] Jacobus van der Linden, Mathijs de Weerd, and Emir Demirović. “Necessary and Sufficient Conditions for Optimal Decision Trees using Dynamic Programming”. In: *Advances in Neural Information Processing Systems* 36 (2023). Ed. by A. Oh et al., pp. 9173–9212.
- [Mar+25] Sascha Marton et al. “Mitigating Information Loss in Tree-Based Reinforcement Learning via Direct Optimization”. In: (2025). URL: <https://openreview.net/forum?id=qpXctF2aLZ>.
- [Mil+24] Stephanie Milani et al. “Explainable Reinforcement Learning: A Survey and Comparative Review”. In: *ACM Comput. Surv.* 56.7 (Apr. 2024). ISSN: 0360-0300. DOI: 10.1145/3616864. URL: <https://doi.org/10.1145/3616864>.
- [MMW22] Rahul Mazumder, Xiang Meng, and Haoyue Wang. “Quant-BnB: A Scalable Branch-and-Bound Method for Optimal Decision Trees with Continuous Features”. In: *Proceedings of the 39th International Conference on Machine Learning*. Proceedings of Machine Learning Research 162

(17–23 Jul 2022). Ed. by Kamalika Chaudhuri et al., pp. 15255–15277. URL: <https://proceedings.mlr.press/v162/mazumder22a.html>.

- [Mni+15] Volodymyr Mnih et al. “Human-level control through deep reinforcement learning”. In: *nature* 518.7540 (2015), pp. 529–533.
- [Nag+24] Myura Nagendran et al. “Eye tracking insights into physician behaviour with safe and unsafe explainable AI recommendations”. In: *NPJ Digital Medicine* 7.1 (2024), p. 202.
- [Put94] Martin L. Puterman. *Markov Decision Processes: Discrete Stochastic Dynamic Programming*. John Wiley & Sons, 1994.
- [Qui86] J. R. Quinlan. “Induction of Decision Trees”. In: *Mach. Learn.* 1.1 (1986), pp. 81–106.
- [Qui93] J Ross Quinlan. “C4. 5: Programs for machine learning”. In: *Morgan Kaufmann google schola* 2 (1993), pp. 203–228.

- [RGB10] Stéphane Ross, Geoffrey J. Gordon, and J. Andrew Bagnell. “A Reduction of Imitation Learning and Structured Prediction to No-Regret Online Learning”. In: (2010).
- [SB98] Richard S. Sutton and Andrew G. Barto. *Reinforcement Learning: An Introduction*. Cambridge, MA: The MIT Press, 1998.
- [Sch+17] John Schulman et al. “Proximal policy optimization algorithms”. In: *arXiv preprint arXiv:1707.06347* (2017).
- [SJJ94] Satinder P. Singh, Tommi S. Jaakkola, and Michael I. Jordan. “Learning without state-estimation in partially observable Markovian decision processes”. In: *Proceedings of the Eleventh International Conference on International Conference on Machine Learning*. ICML’94. New Brunswick, NJ, USA: Morgan Kaufmann Publishers Inc., 1994, pp. 284–292. ISBN: 1558603352.
- [Top+21] Nicholay Topin et al. “Iterative bounding mdps: Learning interpretable policies via non-interpretable methods”. In:

Proceedings of the AAAI Conference on Artificial Intelligence
35 (2021), pp. 9923–9931.

- [Ver+18] Abhinav Verma et al. “Programmatically interpretable reinforcement learning”. In: (2018), pp. 5045–5054.
- [VZ19] Sicco Verwer and Yingqian Zhang. “Learning optimal classification trees using a binary linear program formulation”. In: *Proceedings of the AAAI conference on artificial intelligence* 33 (2019), pp. 1625–1632.
- [Wu+20] Mike Wu et al. “Regional Tree Regularization for Interpretability in Deep Neural Networks”. In: 34 (Apr. 2020), pp. 6413–6421. DOI: 10.1609/aaai.v34i04.6112. URL: <https://ojs.aaai.org/index.php/AAAI/article/view/6112>.