Interpretability, Decision Trees, and Sequential Decision Making

Hector Kohler

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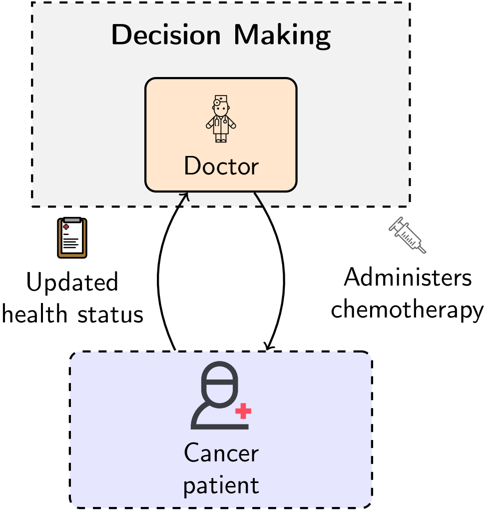
November 26, 2025

Sequential decision making (SDM) and machine learning (ML)



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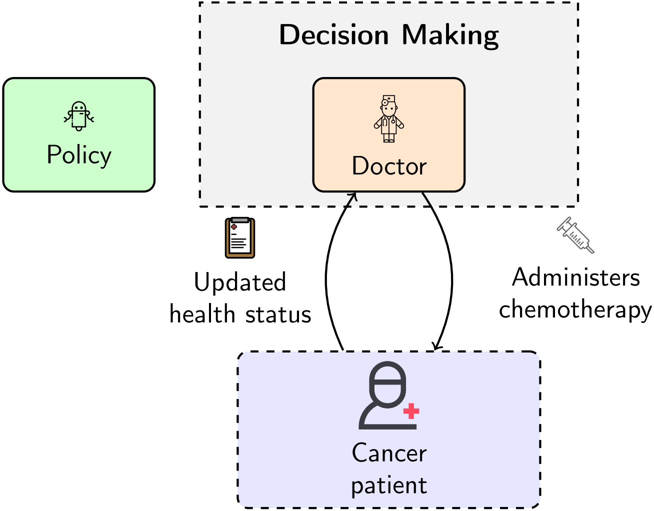

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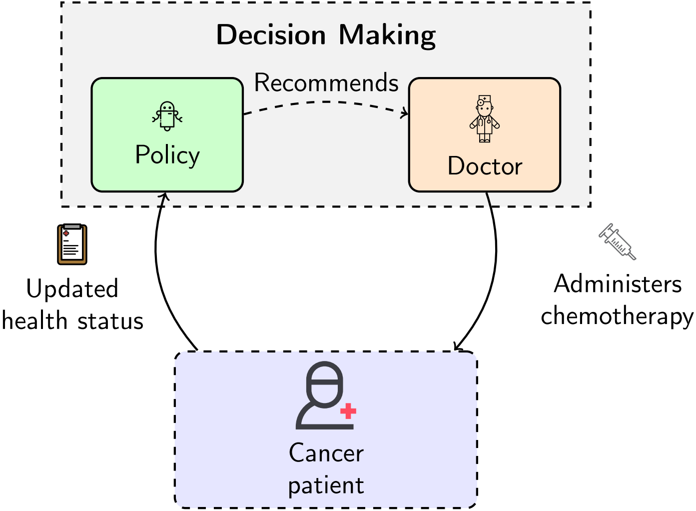

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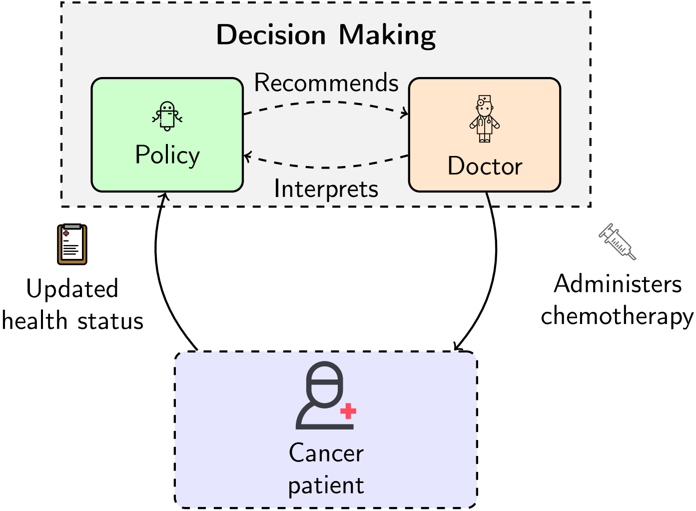

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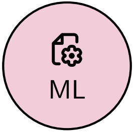

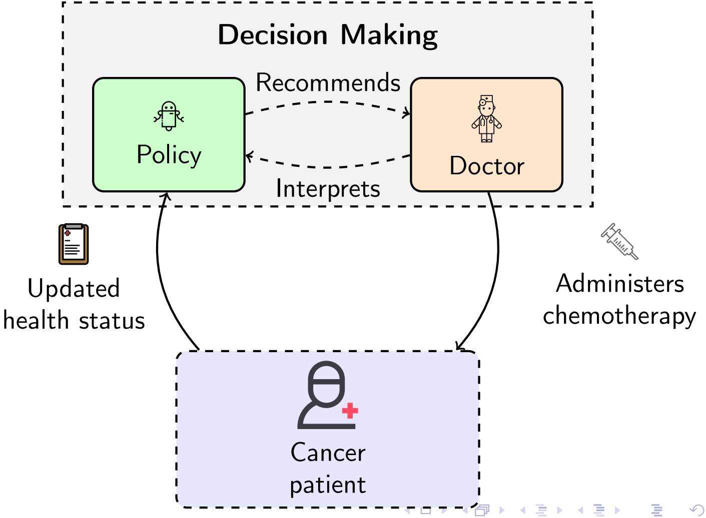

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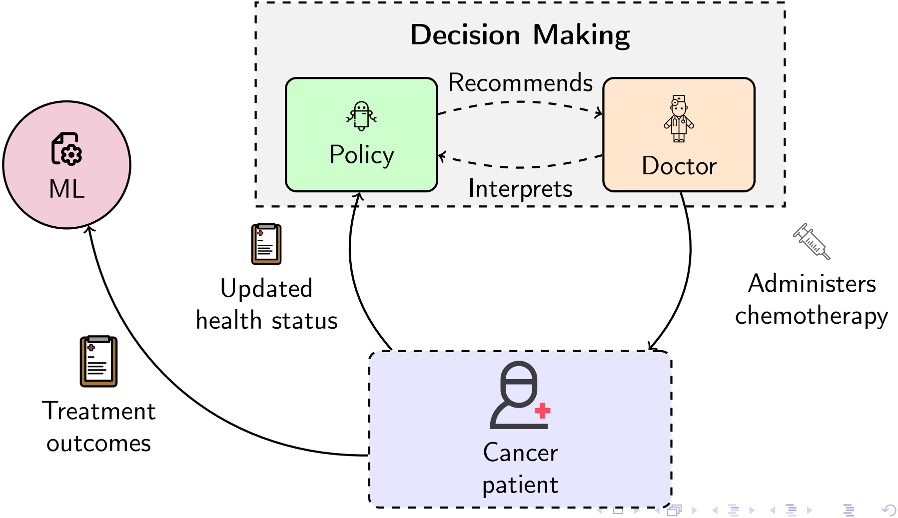

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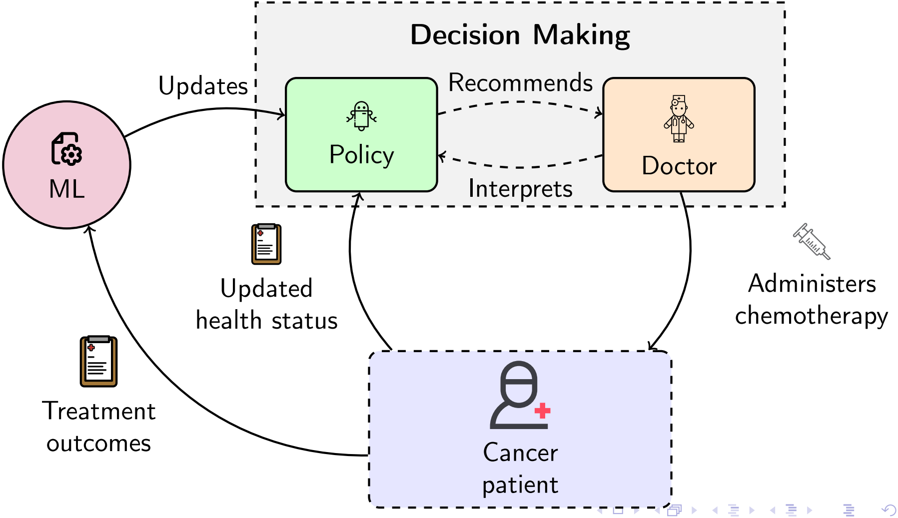

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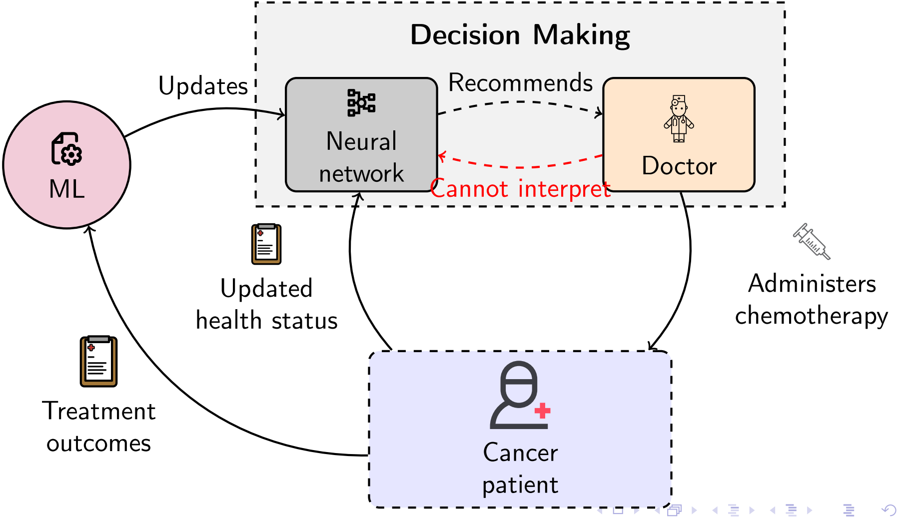
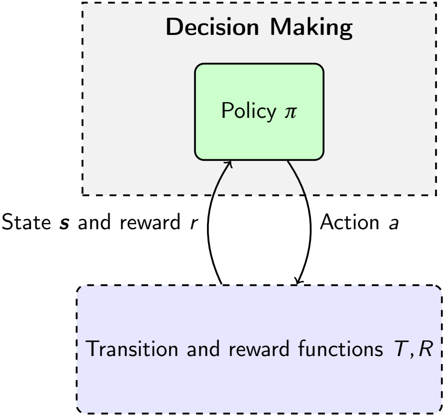


Markov decision processes (MDPs) and reinforcement learning (RL)



Markov decision processes [Put94].

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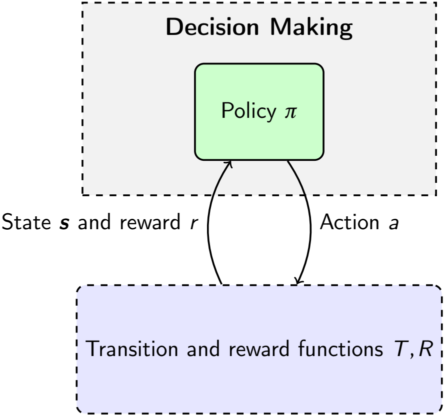

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- RL [SB98] aims to find a policy, $\pi : S \to A$ that maximizes:

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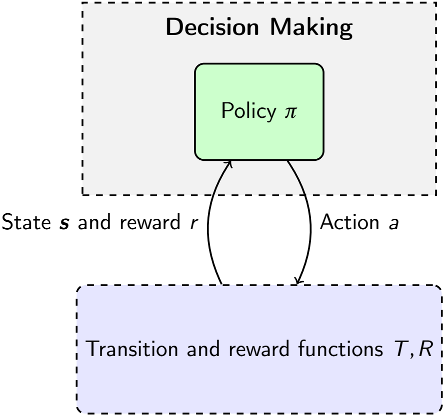

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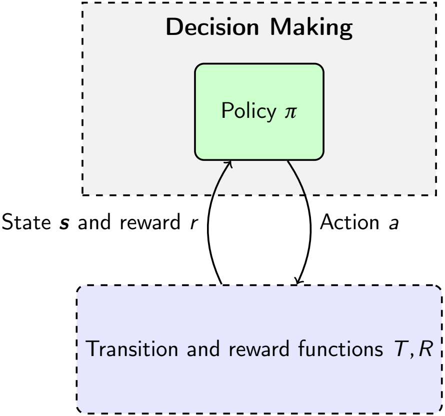

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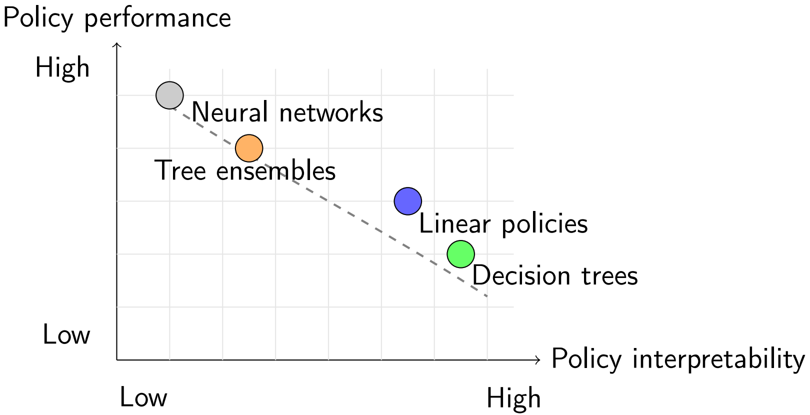
- Lots of succesful RL algorithms [SB98; Mni+15; Sch+17].
- Few interpretability concerns.

Policy interpretability



Heuristic interpretability-performance trade-offs of different policy classes.

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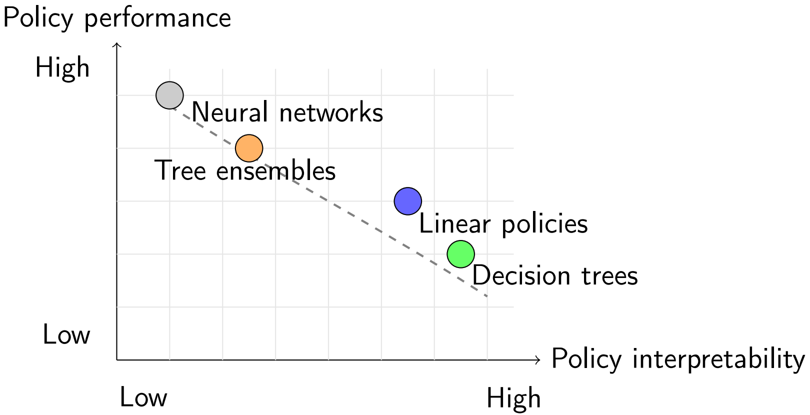

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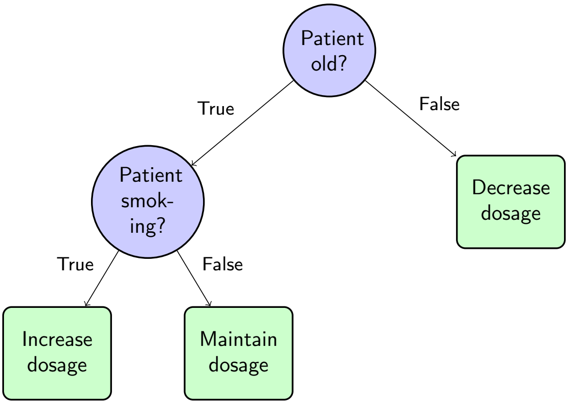
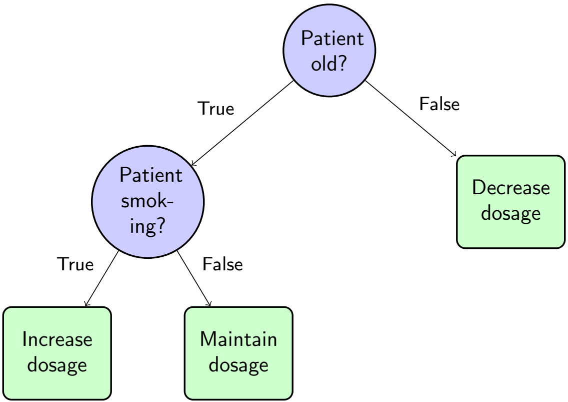
⚠ **No definition of interpretability for machine learning models!**

Decision trees



A generic decision tree of depth $D = 2$.

Decision trees



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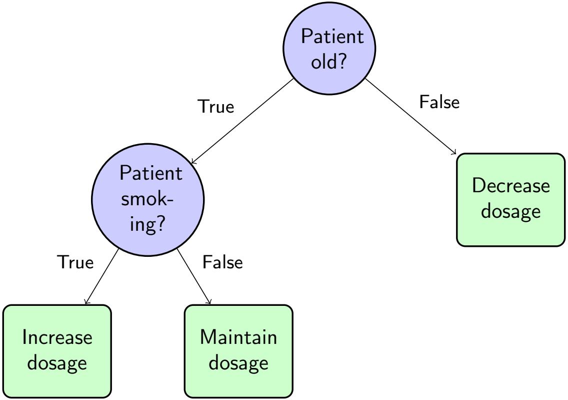

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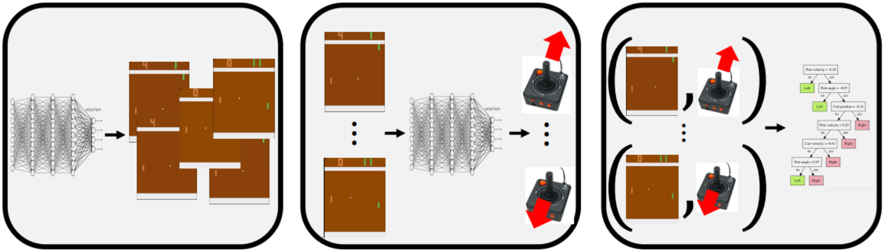
What about SDM?

Indirect approach: imitation learning



Step 1: Use NN to generate states

Step 2: Use NN to obtain actions

Step 3: Use supervised learning to train a decision tree

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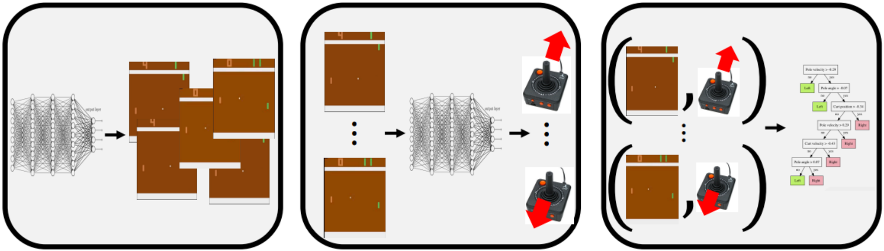

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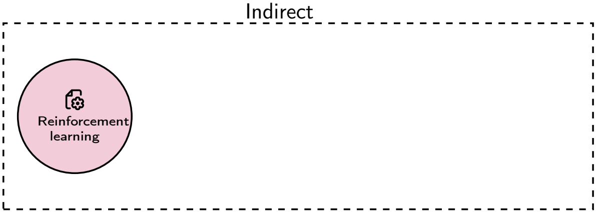
Most research focused on indirect learning of interpretable policies [RGB10; BPS18; Ver+18; Mil+24].

Two ways to get interpretable policies for SDM



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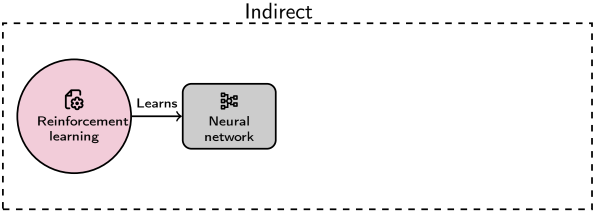

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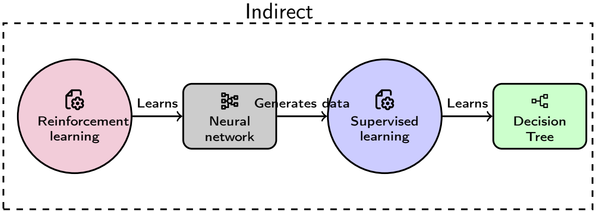

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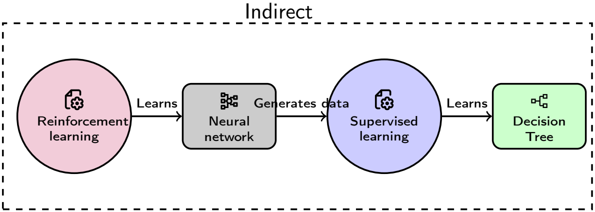

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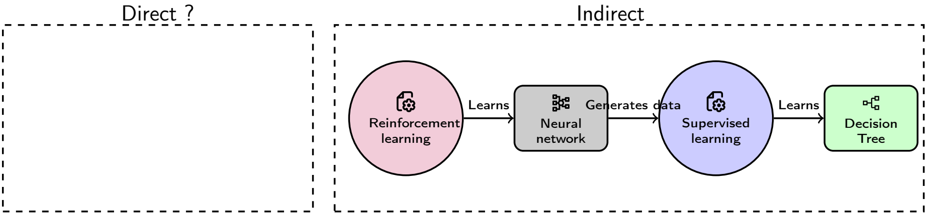

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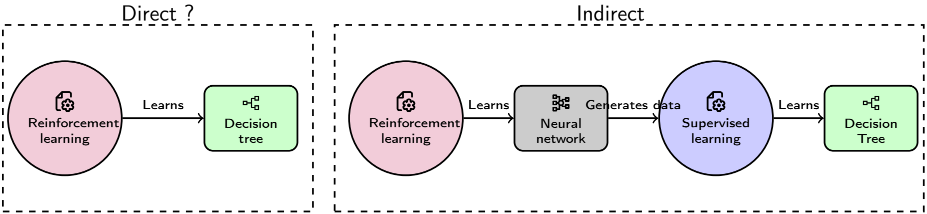

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Contributions

1. How difficult is it to directly optimize a trade-off of interpretability and performance in SDM?
2. How to leverage DM to learn interpretable classifiers for supervised learning?
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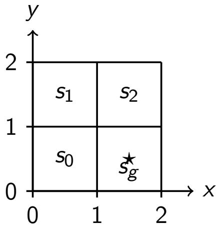

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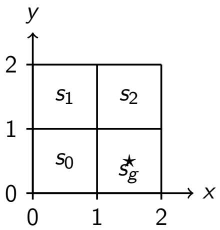

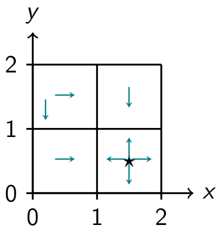

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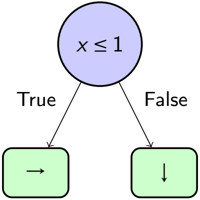

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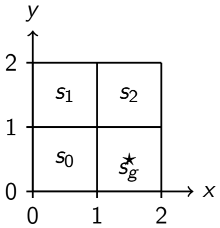

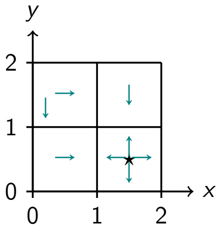

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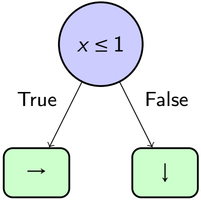

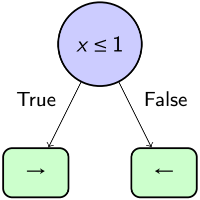

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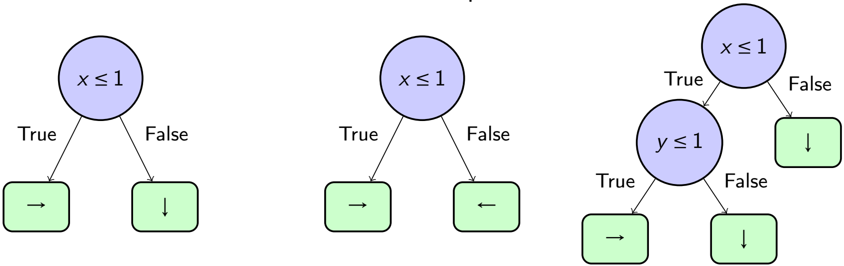

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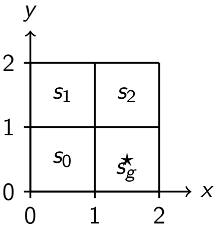

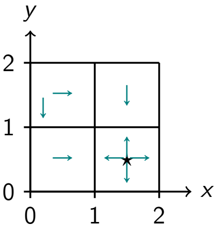

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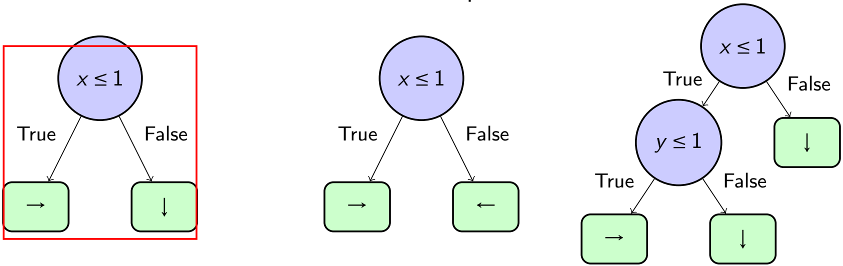

Decision tree policies with different interpretability-performance trade-offs.

Direct RL of decision tree policies with iterative bounding Markov decision processes

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Given an MDP $\mathcal{M}$ $\langle S, A, R, T\rangle$, an associated iterative bounding Markov decision process (IBMDP, [Top+21]) $\mathcal{M}_{IB}$ is an MDP:

$$\langle \overbrace{S \times O}^{\text{Augmented state space}}, \underbrace{A \cup A_{info}}_{\text{Augmented action space}}, \overbrace{(R, \zeta)}^{\text{Augmented rewards}}, \underbrace{(T_{info}, T)}_{\text{Augmented transitions}} \rangle$$

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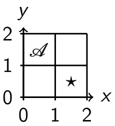
IBMDPs promises

- No need to design new algorithm: we can use RL.
- IBMDP rewards trade-off naturally interpretability and performances.

Grid world IBMDP example

$t = 0$

$\boldsymbol{s}_t = (0.5, 1.5)$



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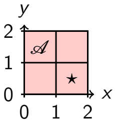
$$\boldsymbol{o}_t = (0, 2, 0, 2)$$



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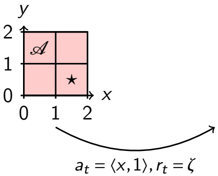

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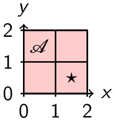

$a_t = \langle x, 1 \rangle, r_t = \zeta$

$x \leq 1$

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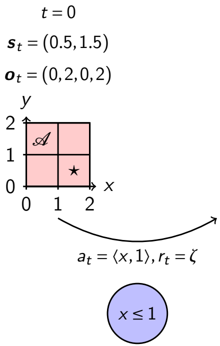

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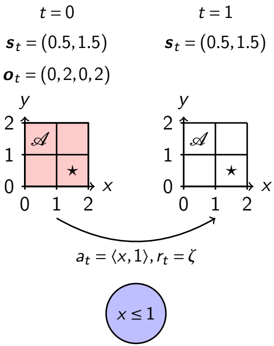

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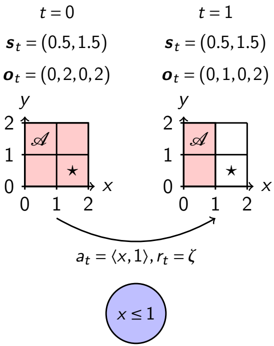

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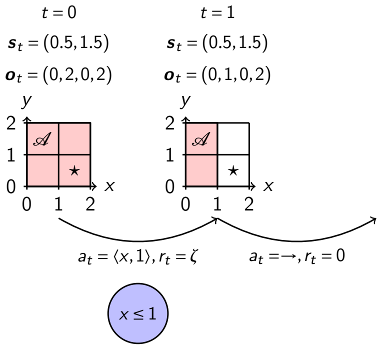

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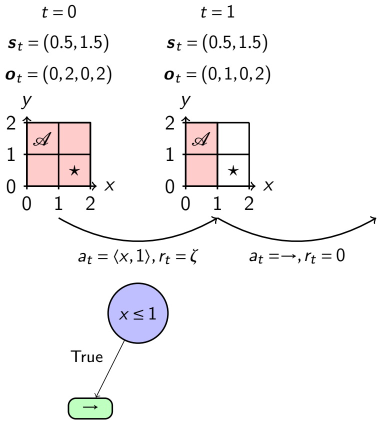

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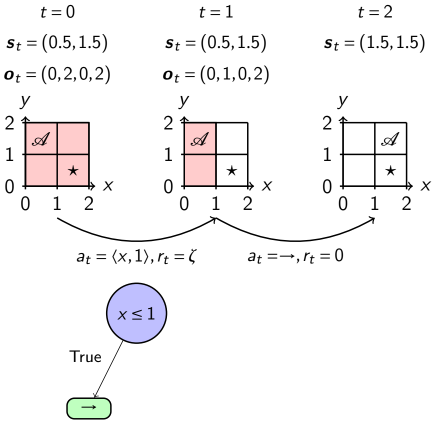

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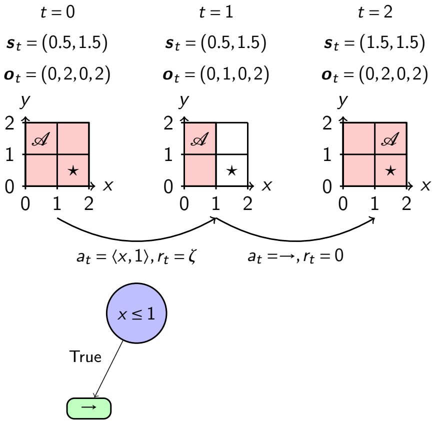

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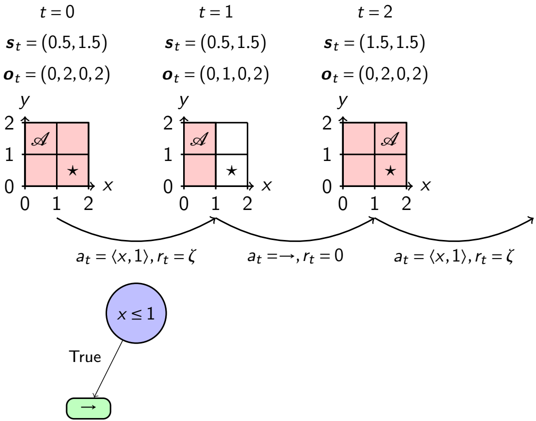

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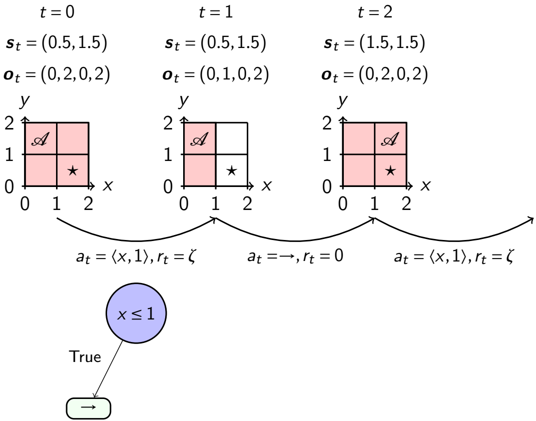

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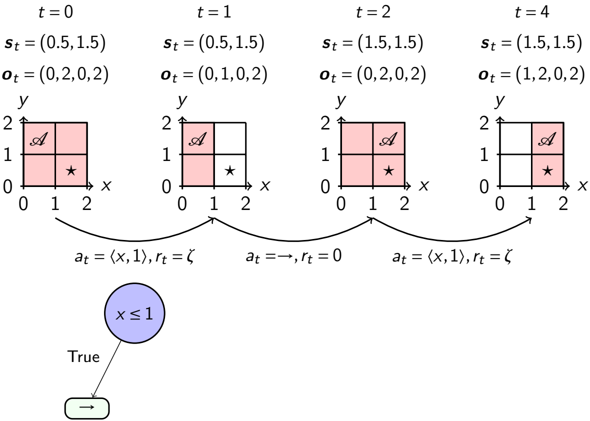

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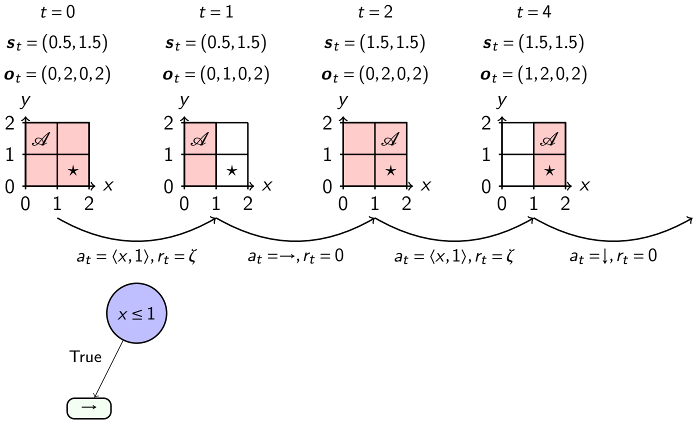

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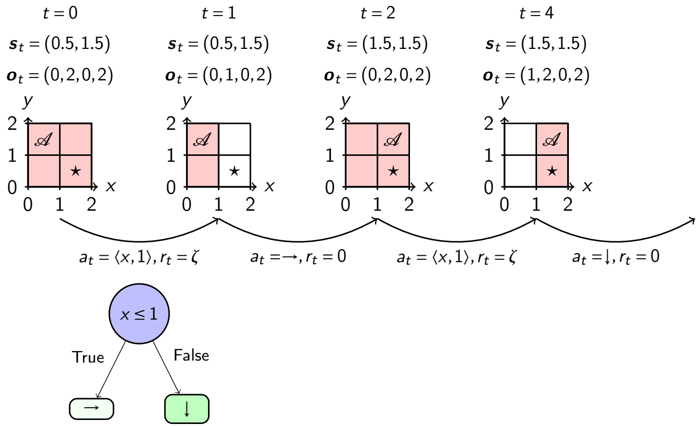

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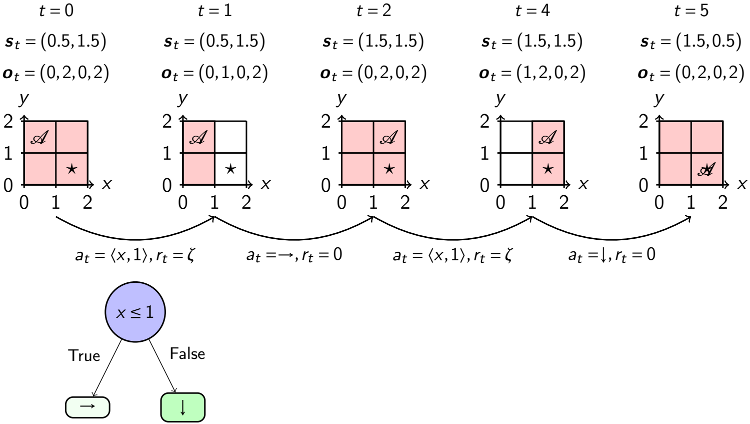

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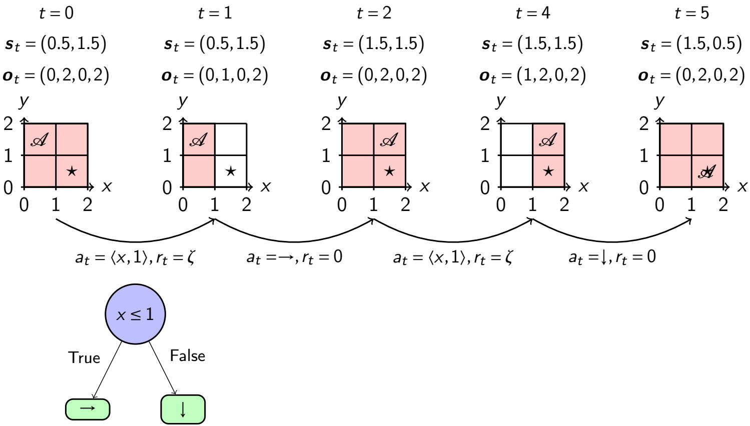

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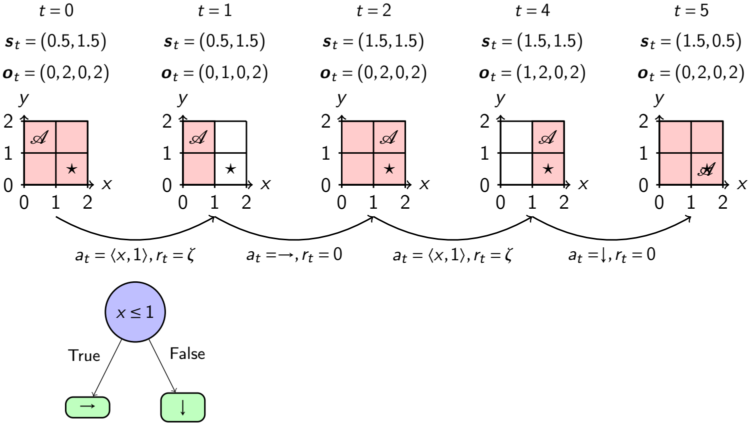

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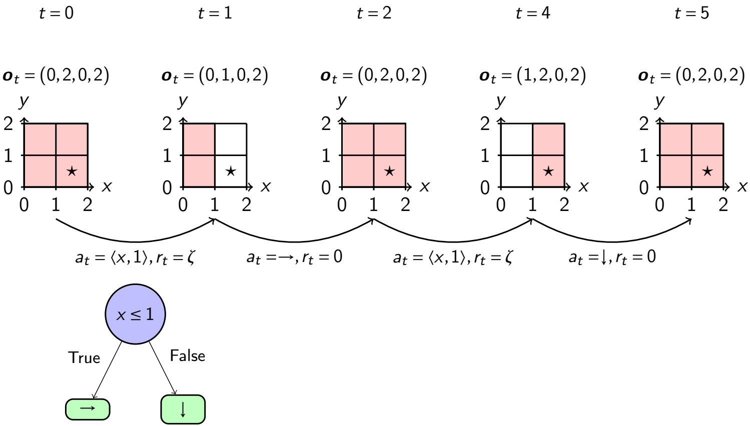

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Q: Can we use reinforcement learning to directly optimize trade-offs of performance and interpretability in SDM?

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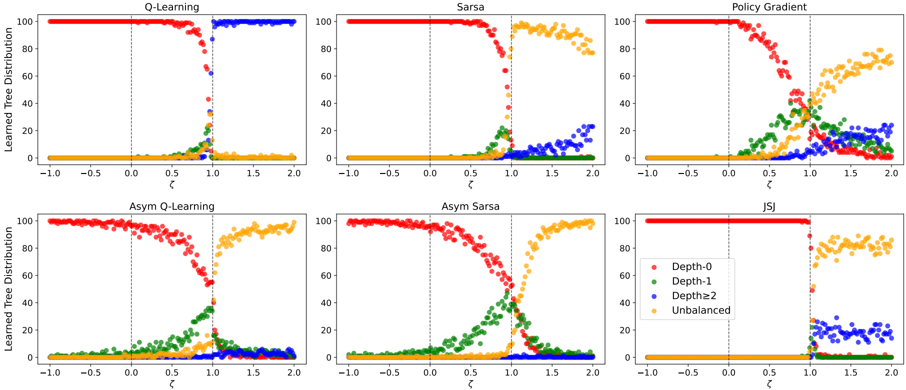
*Q: How does RL perform for optimizing **deterministic** and **partially observable** policies in IBMDPs?*

Hard problem

Finding the best **deterministic** and **partially observable** policy is NP-hard [Lit94]!

Result: RL cannot retrieve optimal depth-1 trees for the grid world MDP



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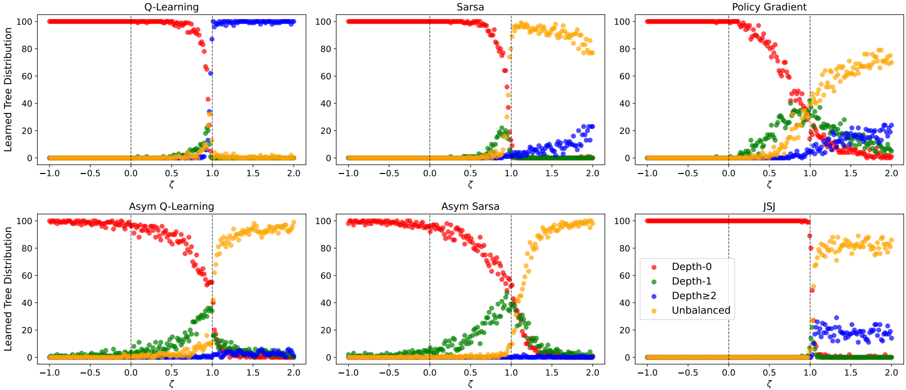

Distributions of tree policies learned with (asymmetric) RL algorithms [SB98; SJJ94; LS98; BA22; BDA22] as a function of the interpretability reward ζ.

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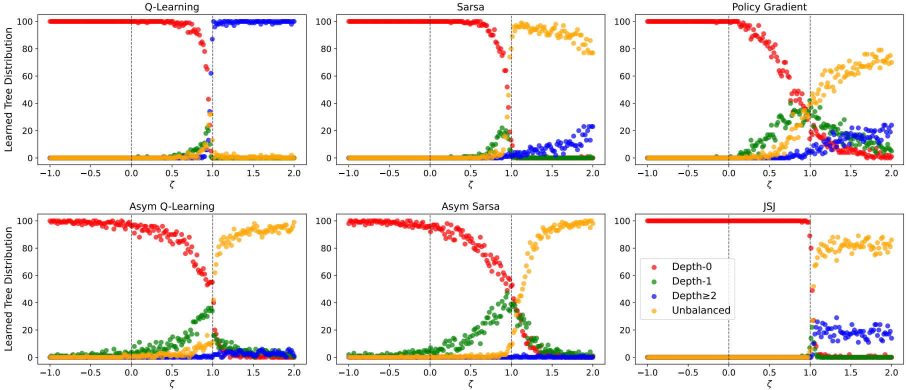

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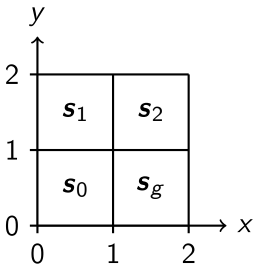
Are the poor performances due to partial observability?

Result: decision tree policies for classification MDPs are standard Markovian policies in IBMDPs



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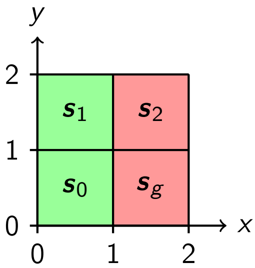

Classification MDP and the unique optimal depth-1 tree.

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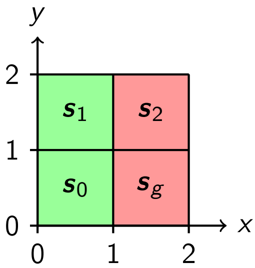
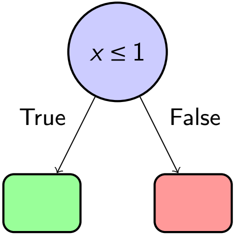
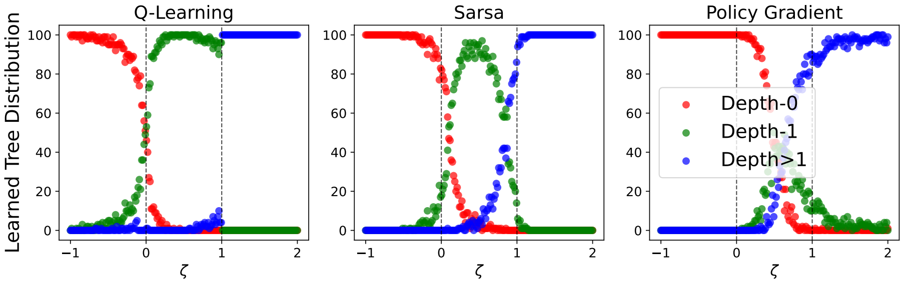




Classification MDP and the unique optimal depth-1 tree.
Deterministic partially observable policies for classification IBMDPs ($\Leftrightarrow$ decision tree policies) are in fact Markovian.

Result: RL can retrieve optimal depth-1 trees for the toy classification MDP



Distributions of tree policies learned with various RL algorithms.

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Q: Can we leverage SDM to design new decision tree induction algorithms for the supervised learning setting? **A: Yes!**

Decision trees in supervised learning

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- N data points. Each x_i is described by p features and has a label $y_i \in \mathscr{Y}$.

$$\mathscr{L}(T) = \frac{1}{N}\sum_{i=1}^{N}\ell(y_i, f(x_i)) + \alpha C(T)$$

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- In between optimal and greedy?

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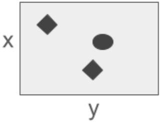

Example of decision tree induction as an MDP.

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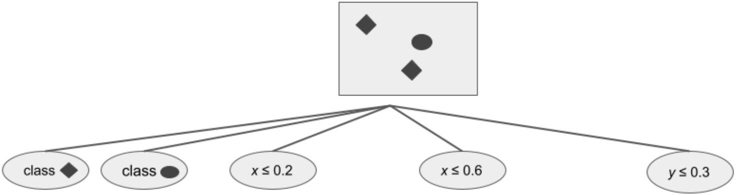

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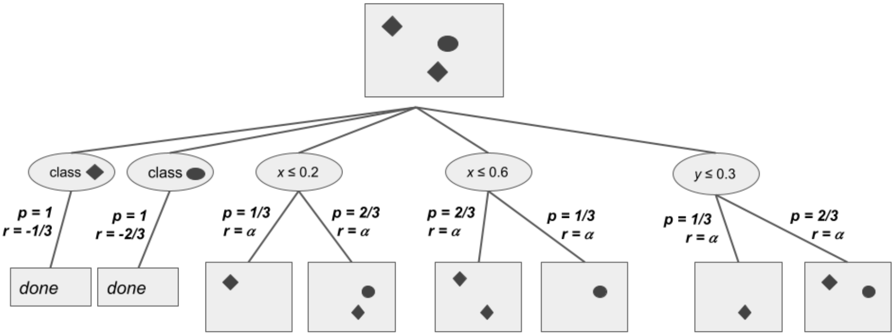

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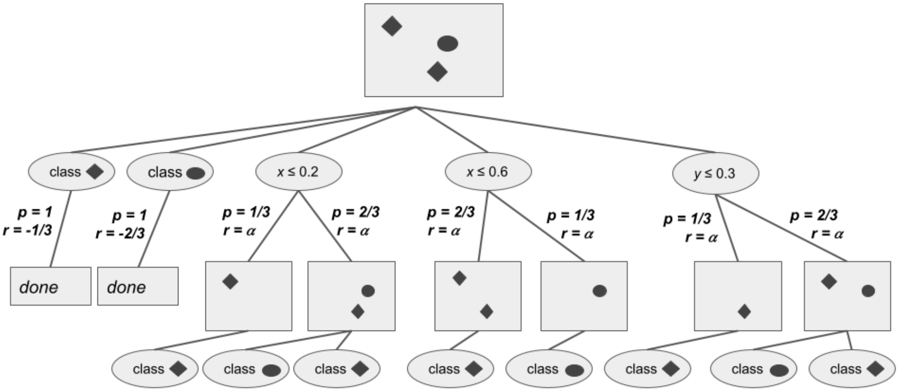

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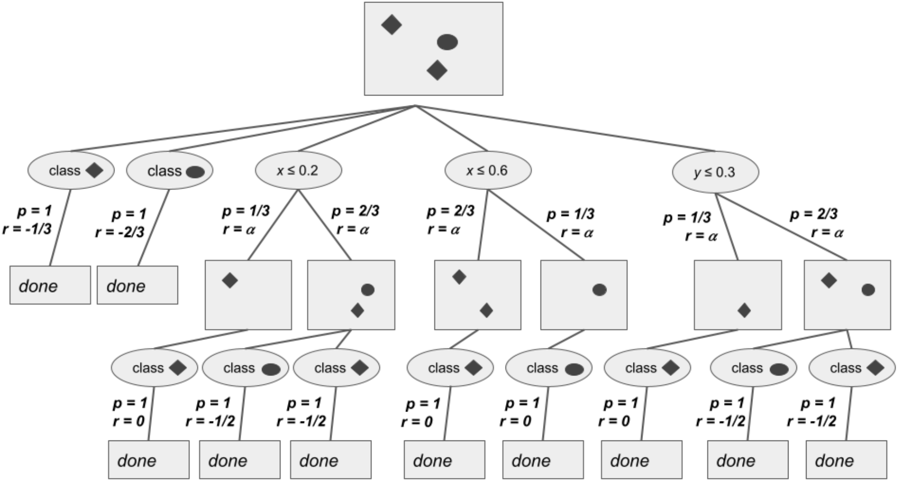

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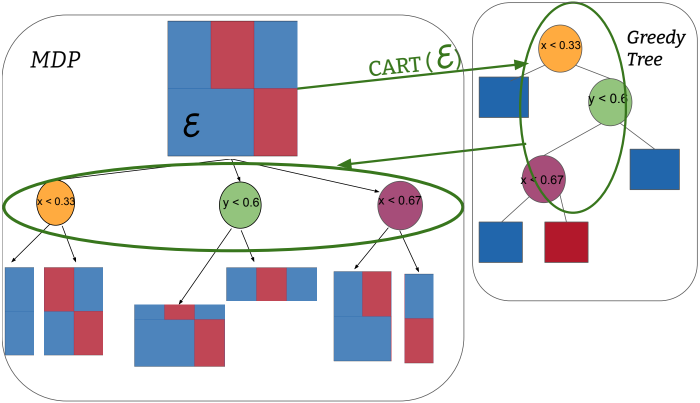
How to choose the B candidate actions/splits?

Dynamic Programming Decision Trees (DPDT)[1]



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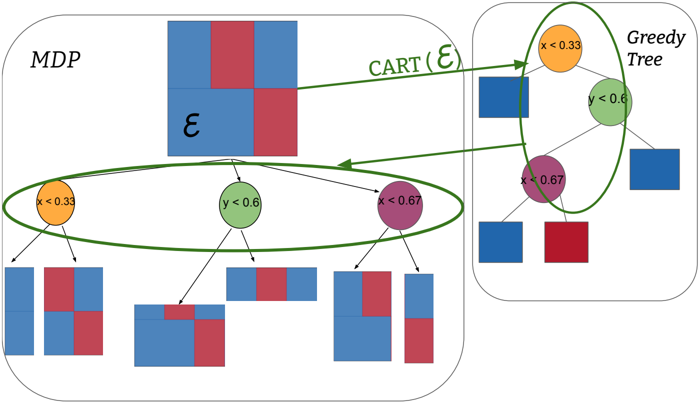

DPDT was presented at the 31st ACM SIGKDD conference.

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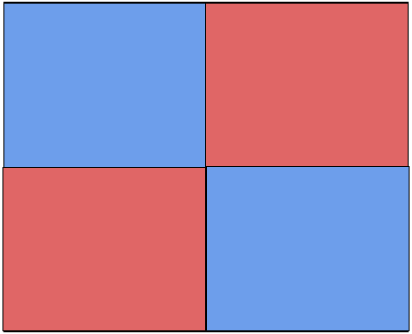
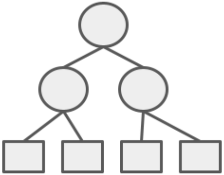
Theorem (DPDT trees can be strictly better than greedy trees)

There exist a depth budget D and a dataset for which DPDT trees are strictly better than greedy trees.

DPDT trees can be strictly better than greedy trees





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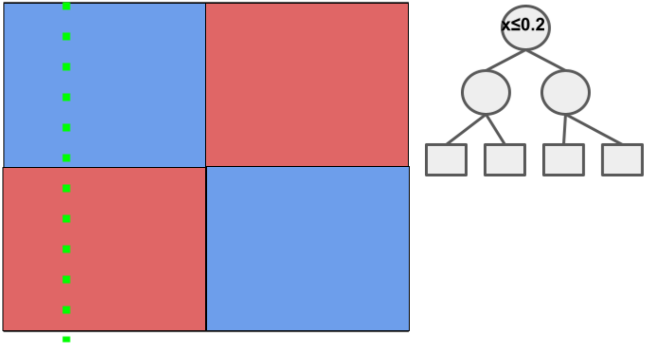

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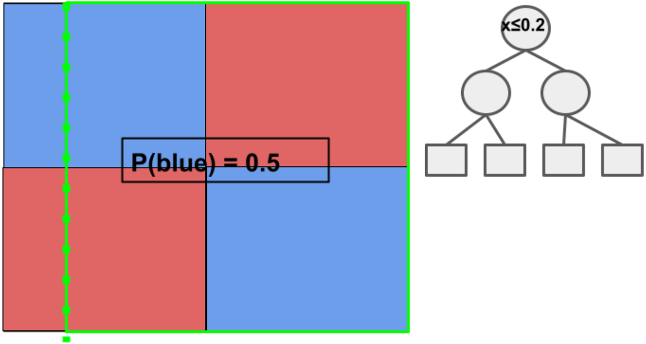

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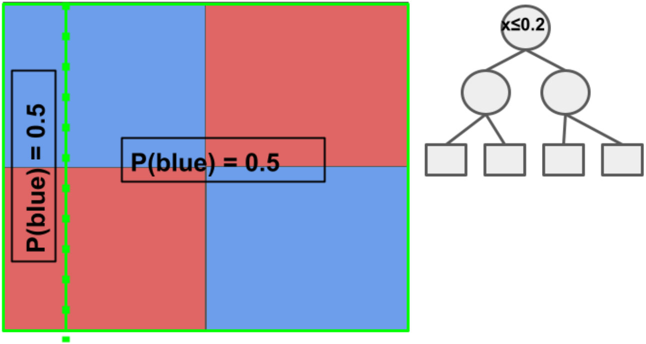

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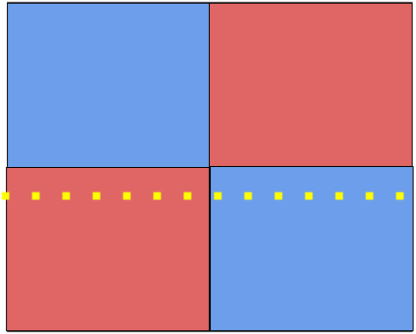

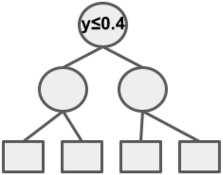

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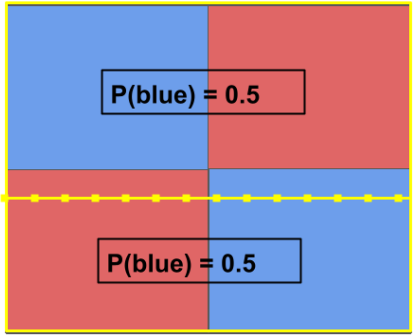

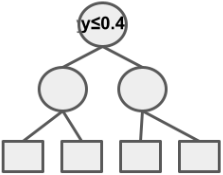

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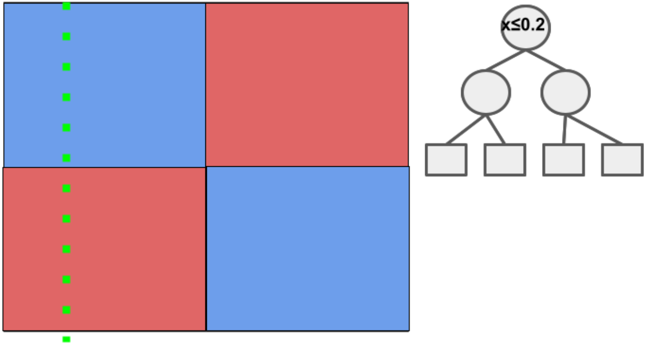

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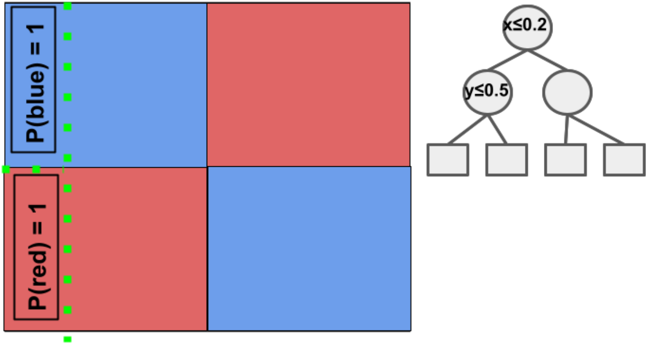

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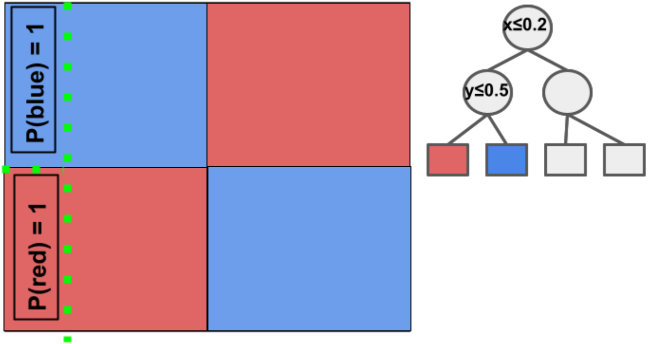

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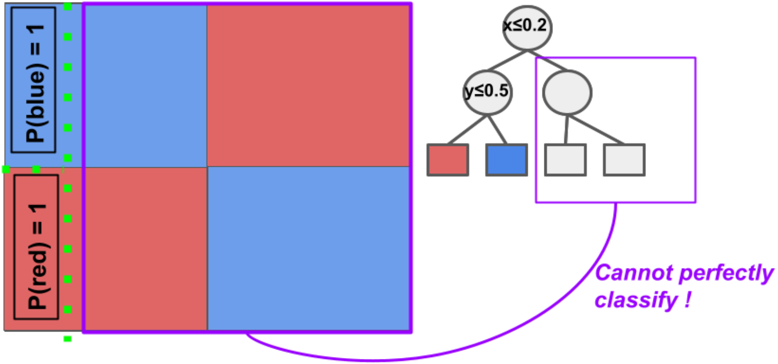

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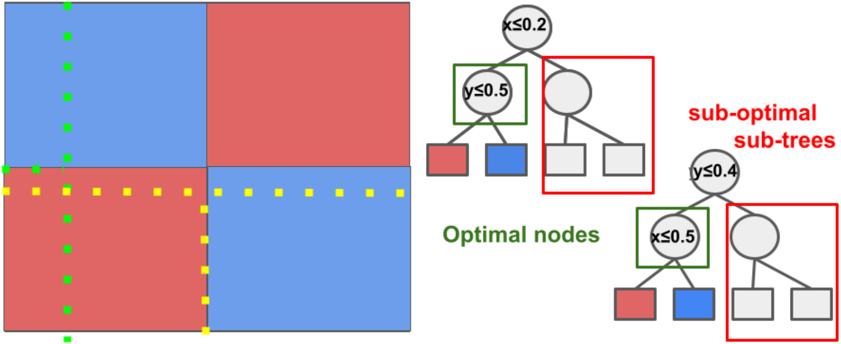

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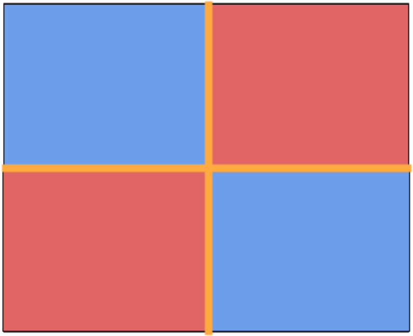
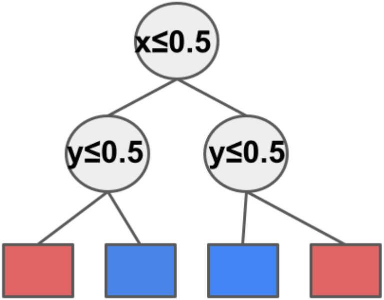
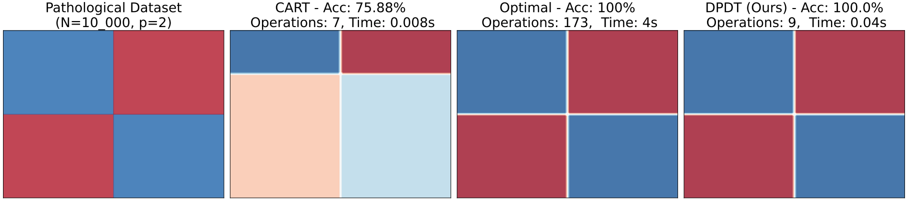




Fast like greedy trees, accurate like optimal trees



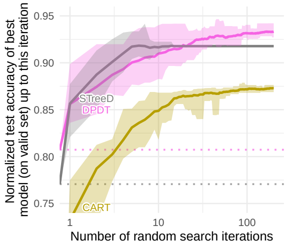
Comparison of greedy, optimal, and DPDT depth-2 trees on the checkersboard dataset.

Comparing tree accuracy to complexity

Train accuracy and operation count when learning depth-3 decision trees.

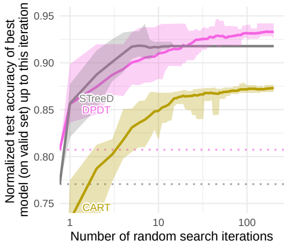
			Accuracy				Operations			
Dataset	N	p	Opt Quant-BnB	Greedy CART	DPDT light	DPDT full	Opt Quant-BnB	Greedy CART	DPDT light	DPDT full
room	8103	16	**0.992**	0.968	0.991	**0.992**	10^6	15	286	16100
bean	10888	16	**0.871**	0.777	0.812	0.853	$5 \cdot 10^6$	15	295	25900
eeg	11984	14	**0.708**	0.666	0.689	0.706	$2 \cdot 10^6$	13	289	26000
avila	10430	10	**0.585**	0.532	0.574	**0.585**	$3 \cdot 10^7$	9	268	24700
magic	15216	10	**0.831**	0.801	0.822	0.828	$6 \cdot 10^6$	15	298	28000
htru	14318	8	**0.981**	0.979	0.979	0.980	$6 \cdot 10^7$	15	295	25300
occup.	8143	5	**0.994**	0.989	0.991	**0.994**	$7 \cdot 10^5$	13	280	16300
skin	196045	3	**0.969**	0.966	0.966	0.966	$7 \cdot 10^4$	15	301	23300
fault	1552	27	**0.682**	0.553	0.672	0.674	$9 \cdot 10^8$	13	295	24200
segment	1848	18	**0.887**	0.574	0.812	0.879	$2 \cdot 10^6$	7	220	16300
page	4378	10	**0.971**	0.964	0.970	0.970	10^7	15	298	22400
bidding	5056	9	**0.993**	0.981	0.985	**0.993**	$3 \cdot 10^5$	13	256	9360
raisin	720	7	**0.894**	0.869	0.879	0.886	$4 \cdot 10^6$	15	295	20900
rice	3048	7	**0.938**	0.933	0.934	0.937	$2 \cdot 10^7$	15	298	25500
wilt	4339	5	**0.996**	0.993	0.994	0.995	$3 \cdot 10^5$	13	274	11300
bank	1097	4	**0.983**	0.933	0.971	0.980	$6 \cdot 10^4$	13	271	7990

DPDT trees generalization



DPDT depth-5 trees vs. other detph-5 trees

DPDT trees generalization



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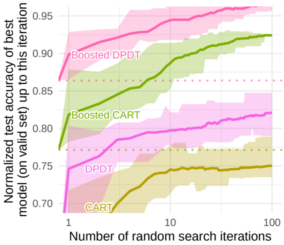
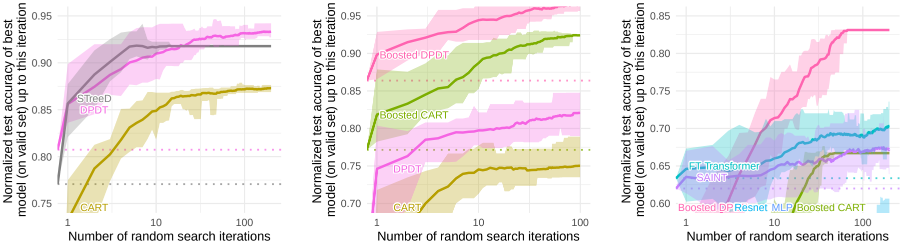


Boosted DPDT vs. Boosted CART

DPDT trees generalization



DPDT depth-5 trees vs. other detph-5 trees

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Boosted DPDT vs. other classifiers

Perspectives

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Q: Are decision trees really the most interpretable model?
A: It depends.

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- Time to formally verify a policy decreases with interpretability [Bar+20].

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- Different implementations (matrix operations vs fully sequentially) ...

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```python
# Decision tree for Mountain Car
def play(x):
    if x[1] <= -0.2597:
        if x[1] <= -0.6378:
            return 0
        else:
            if x[0] <= -1.0021:
                return 2
            else:
                return 0
    else:
        if x[1] <= -0.0508:
            if x[0] <= 0.2979:
                if x[0] <= 0.0453:
                    return 2
                else:
                    if x[1] <=
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                    else:
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            else:
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```

```
# Small ReLU MLP for Pendulum
def play(x):
    h_layer_0_0 = 1.238*x[0]+0.971*x
      [1]
                  +0.430*x[2]+0.933
    h_layer_0_0 = max(0, h_layer_0_0
     )
    h_layer_0_1 = -1.221*x[0]+1.001
                  *x[1]-0.423*x[2]
                  +0.475
    h_layer_0_1 = max(0, h_layer_0_1
     )
    h_layer_1_0 = -0.109*h_layer_0_0
                  -0.377*h_layer_0_1
                  +1.694
    h_layer_1_0 = max(0, h_layer_1_0
     )
    h_layer_1_1 = -3.024*h_layer_0_0
                  -1.421*h_layer_0_1
                  +1.530
    h_layer_1_1 = max(0, h_layer_1_1
     )

    h_layer_2_0 = -1.790*h_layer_1_0
                  +2.840*h_layer_1_1
                  +0.658
    y_0 = h_layer_2_0
    return [y_0]
```

Empirical validation

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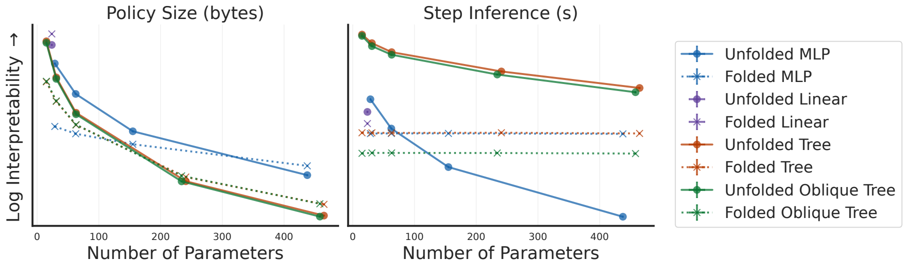
Setup

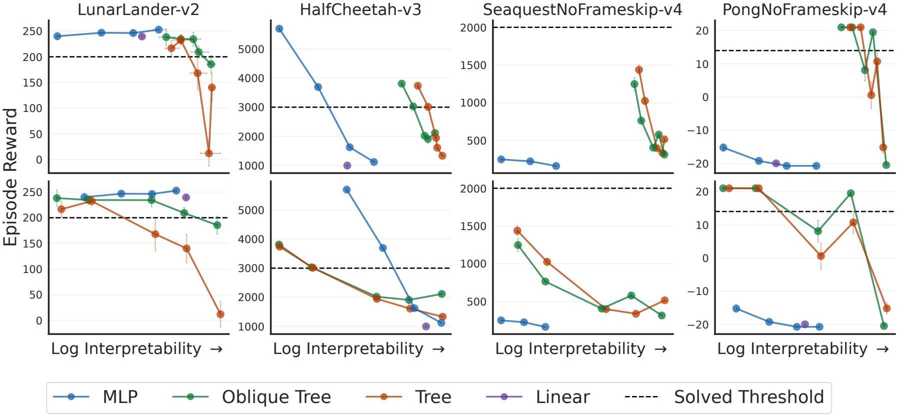
We imitate ~ 40000 expert policies from `stable-baselines3` using various policy classes/nb parameters on various environments.

Result: unfolding policies is necessary to respect consensus



Aggregated policies interpretability on classic control environments

Result: there is no dominating policy class for all environments



Interpretability-Performance trade-offs. Top row, interpretability is measured with step inference times. Bottom row, the interpretability is measured with policy size.

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- Tree-like policy classes can have good inductive bias (e.g. Atari).
- What about (very) big models?
- Can we use our policy programs as low level skills (hierarchical RL)?

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My hope

Motivate interpretability by finding a real-world problem where interpretability is *really* necessary [Nag+24].

Broader perspectives

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- **Deep learning:** Can we design deep learning layers that take datasets and output candidate splits?

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- **Combinatorial optimization:** Can we formulate other combinatorial/NP-hard problems as MDPs and design other DPDT-like algorithms?
- **Human-computer interaction:** Can we do large scale human study of the ~40K programs interpertability?

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