Interpretability, Decision Trees, and Sequential Decision Making

Hector Kohler

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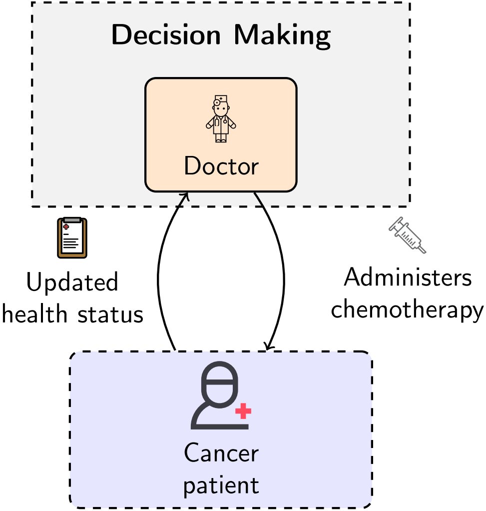
Université de Lille, CNRS, Inria, UMR CRIStAL 9189, France

November 27, 2025

Sequential decision making (SDM) and machine learning (ML)



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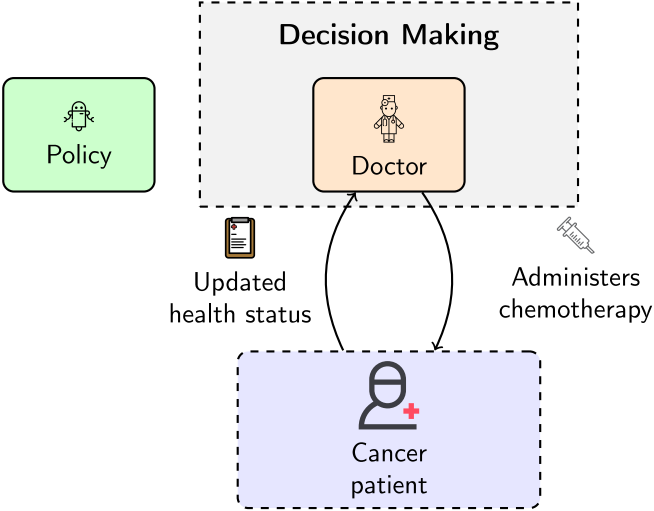

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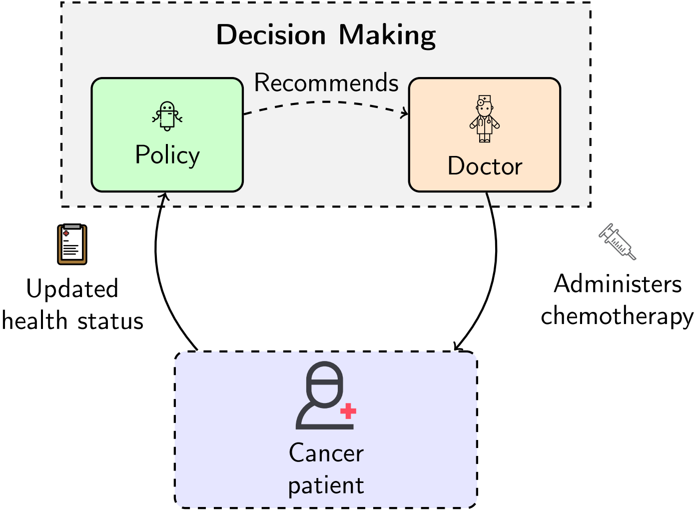

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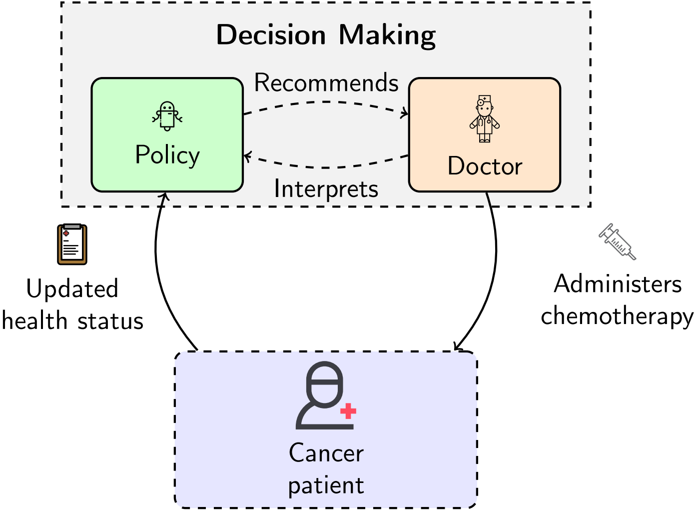

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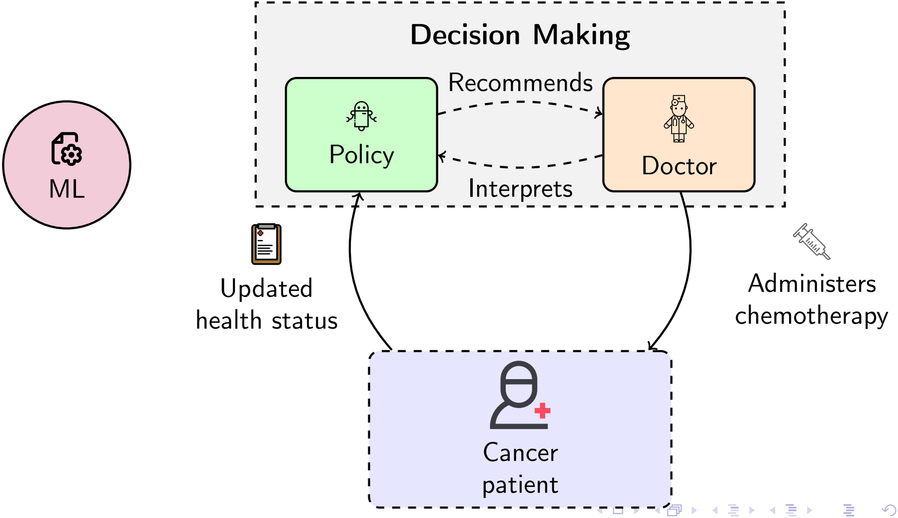

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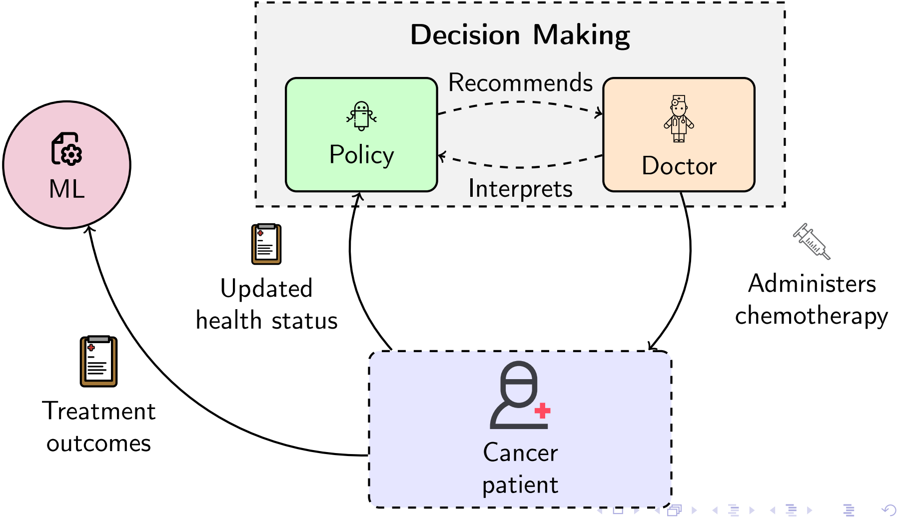

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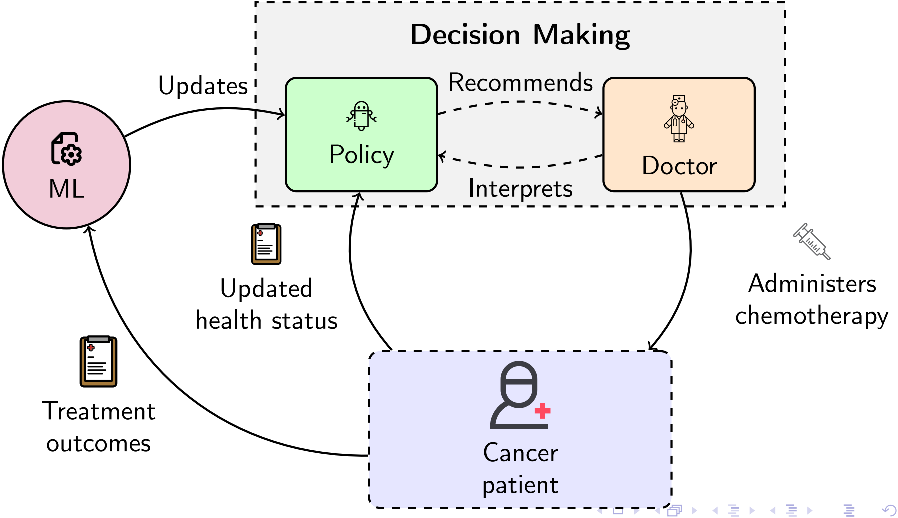

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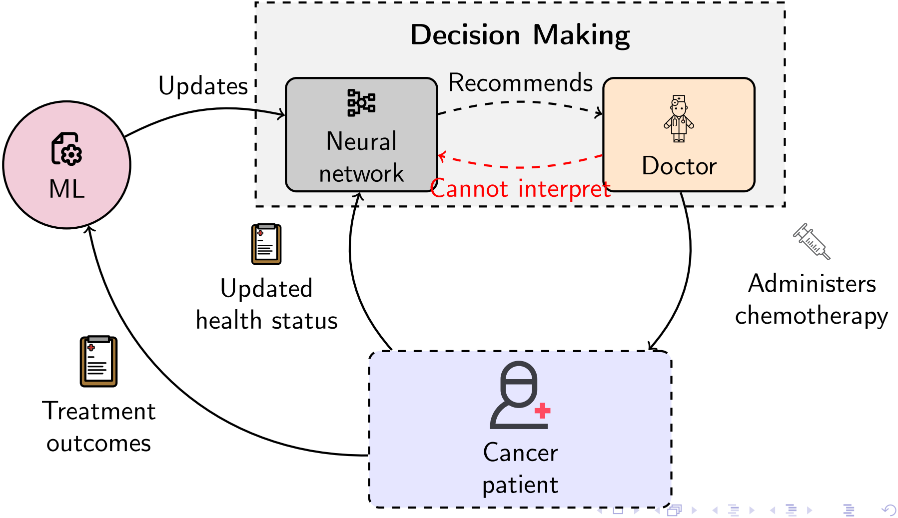
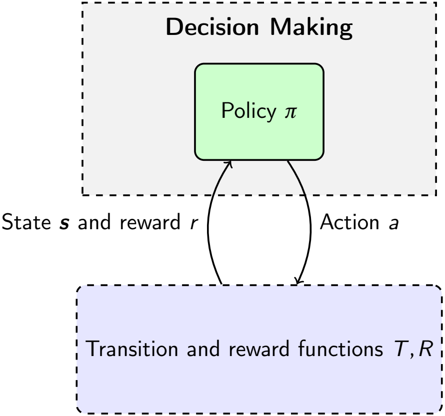


Markov decision processes (MDPs) and reinforcement learning (RL)



Markov decision processes [Put94].

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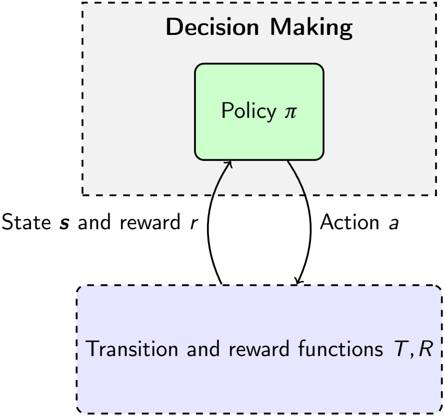

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- RL [SB98] aims to find a policy, $\pi : S \to A$ that maximizes:

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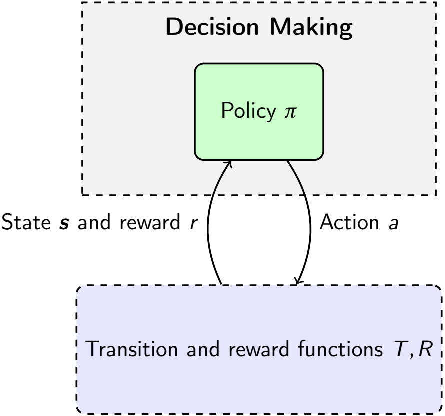

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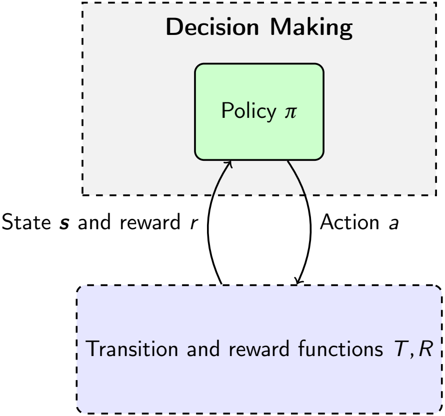

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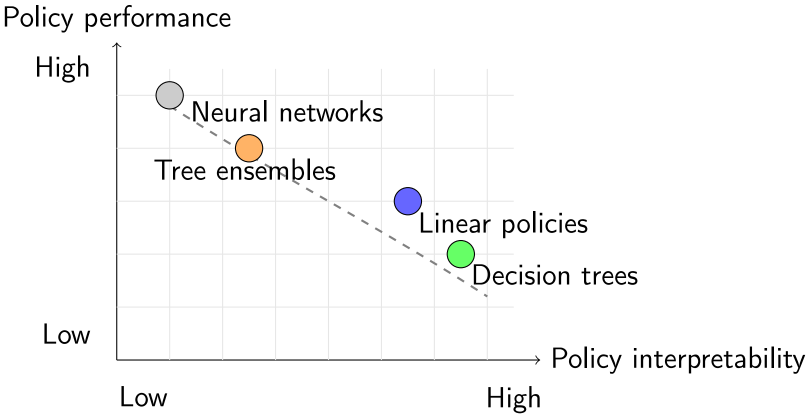
- Lots of succesful RL algorithms [SB98; Mni+15; Sch+17].
- Few interpretability concerns.

Policy interpretability



Heuristic interpretability-performance trade-offs of different policy classes.

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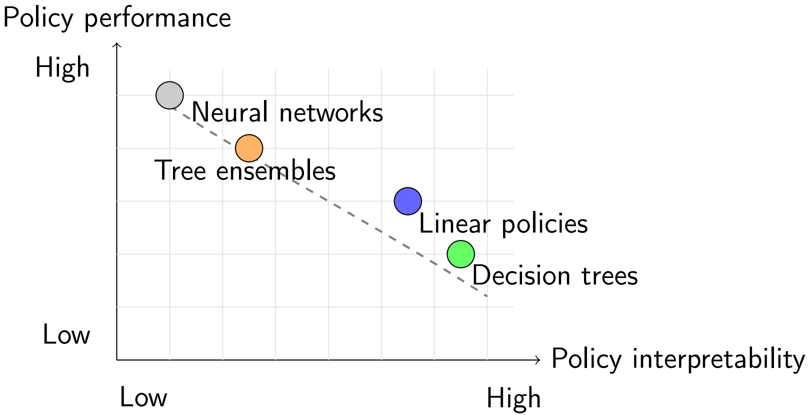

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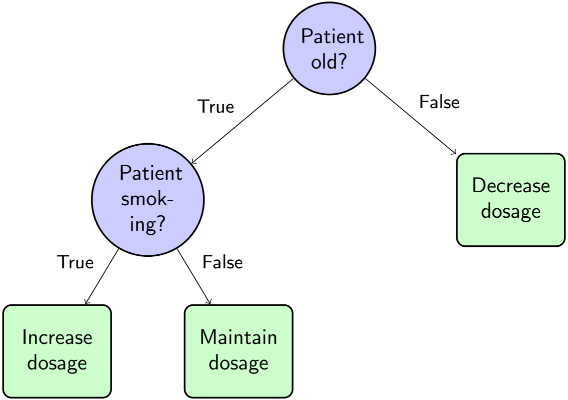
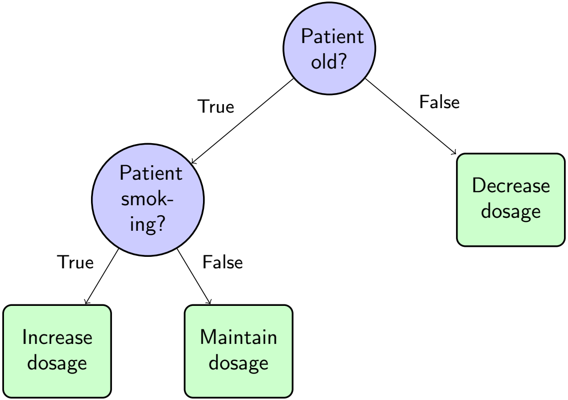
⚠ **No definition of interpretability for machine learning models!**

Decision trees



A generic decision tree of depth $D = 2$.

Decision trees



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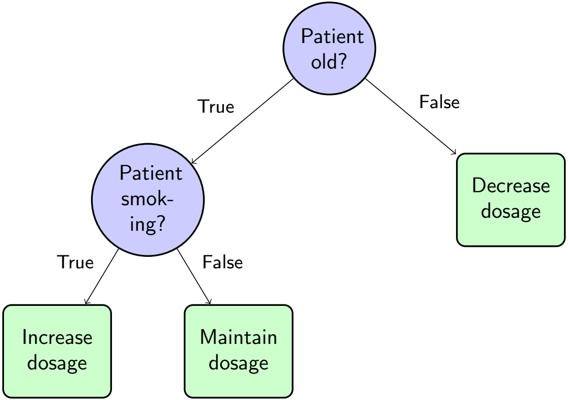

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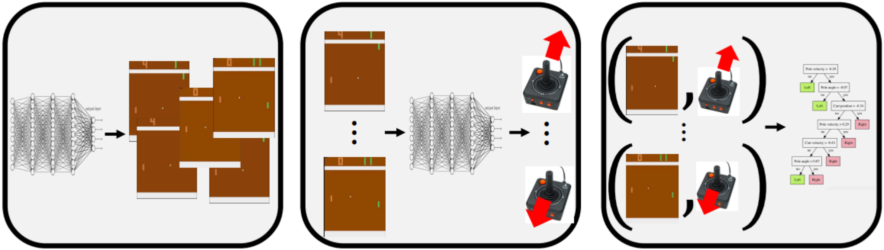
What about SDM?

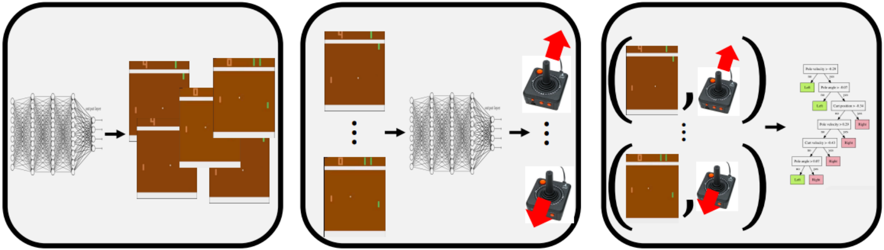
Indirect approach: imitation learning



Step 1: Use NN to generate states

Step 2: Use NN to obtain actions

Step 3: Use supervised learning to train a decision tree



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Most research focused on indirect learning of interpretable policies [RGB10; BPS18; Ver+18; Mil+24].

Two ways to get interpretable policies for SDM

Indirect

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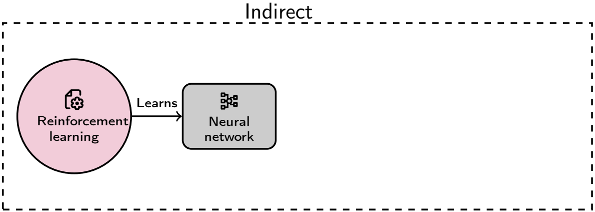

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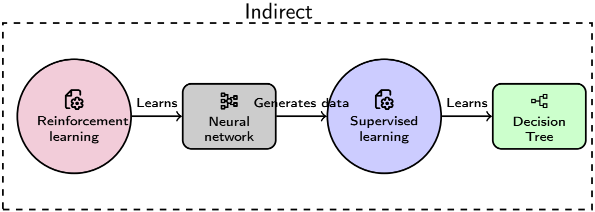

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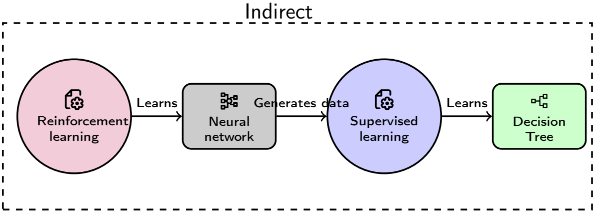

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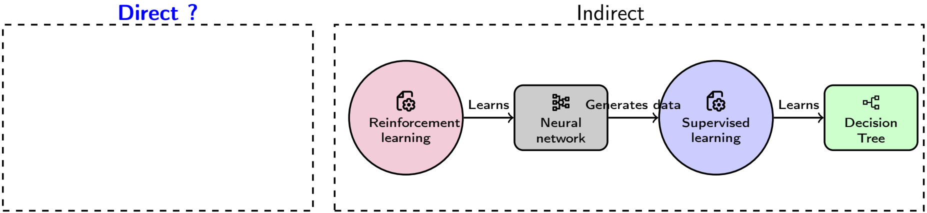

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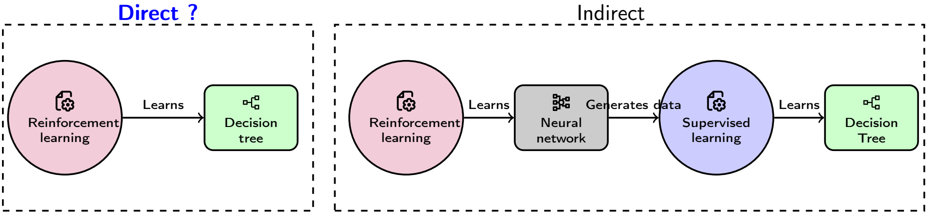

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Contributions

1. Can we directly optimize a trade-off of interpretability and performance in SDM?
2. Can we leverage SDM to learn interpretable classifiers for supervised learning?
3. How to measure policy interpretability in SDM?

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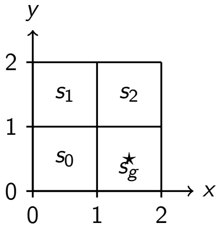

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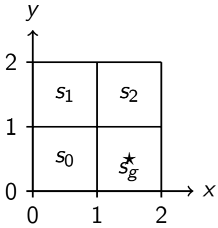

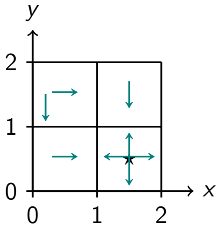

Grid world MDP and decision tree policies

Grid world MDP and optimal actions.

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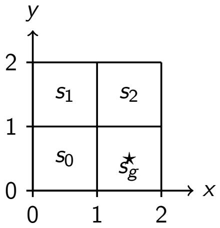

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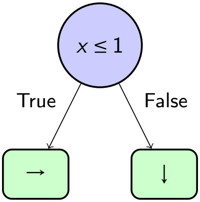

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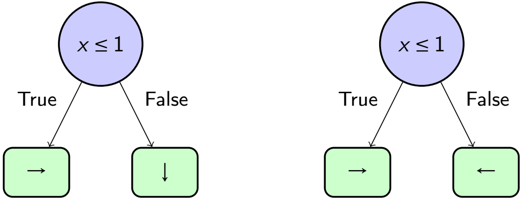

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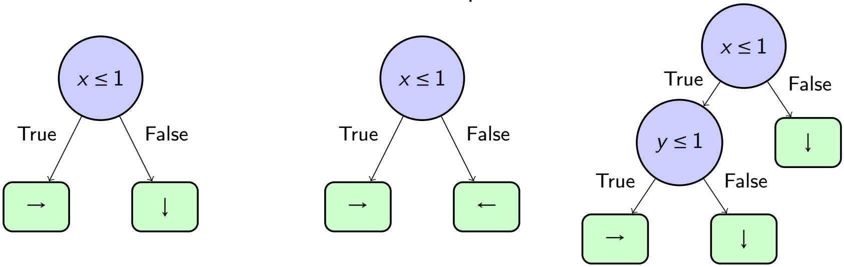

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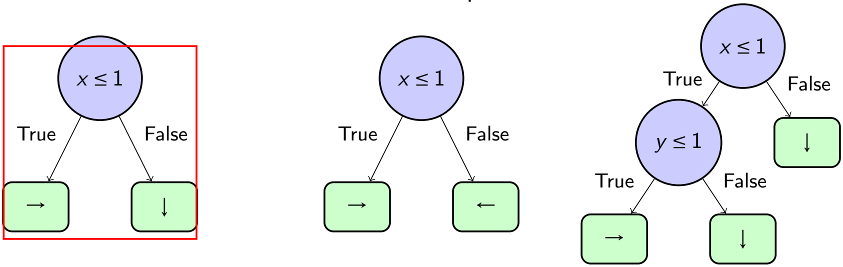

Decision tree policies with different interpretability-performance trade-offs.

Grid world MDP and decision tree policies: indirect approach

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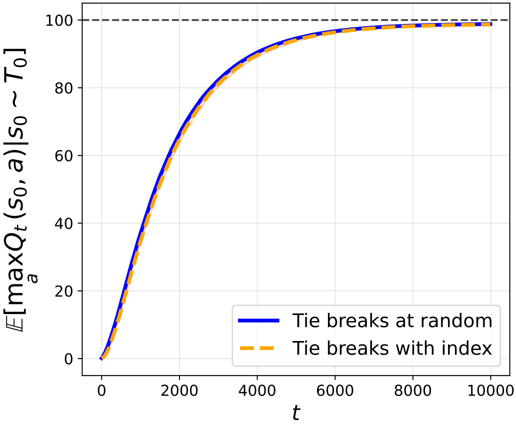

Sample complexity curve of Q-learning over 100 random seeds.

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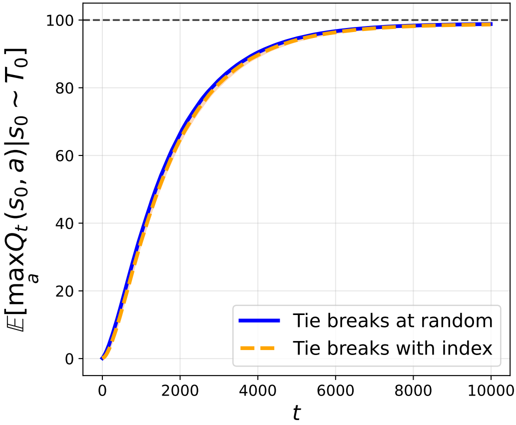

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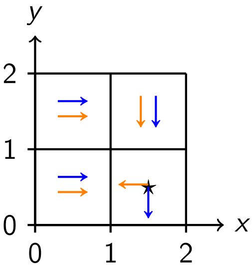
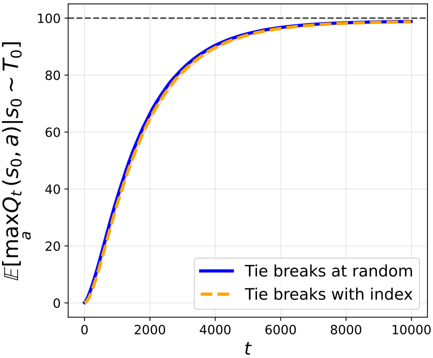
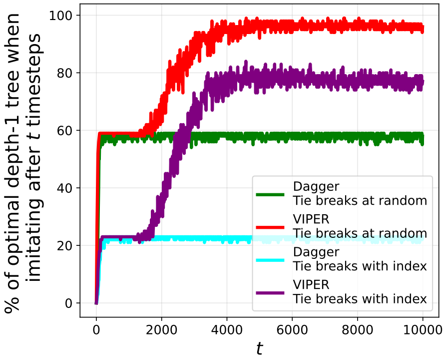


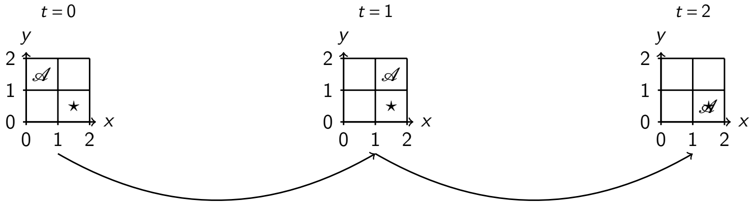
Expert policies.





Sample complexity curve of Q-learning over 100 random seeds and performance of indirect interpretable methods when imitating the greedy policy with a tree at different Q-learning stages.

Iterative bounding Markov decision processes [Top+21]



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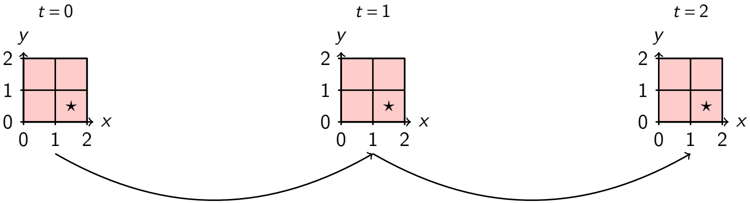

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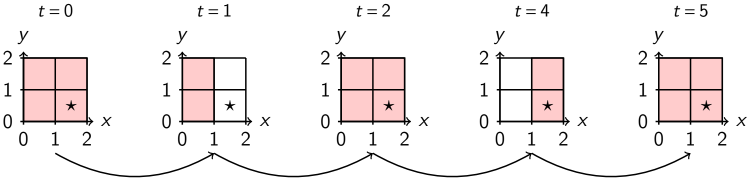

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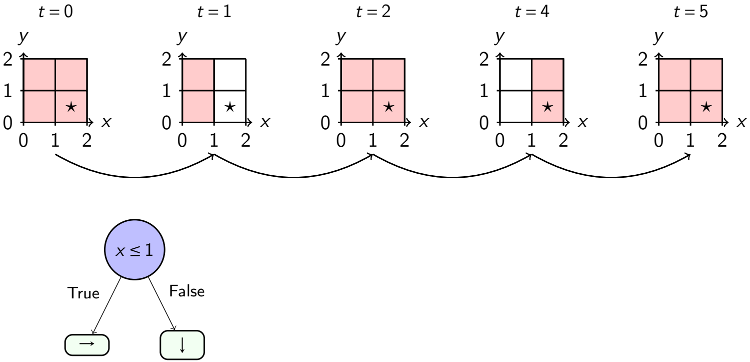

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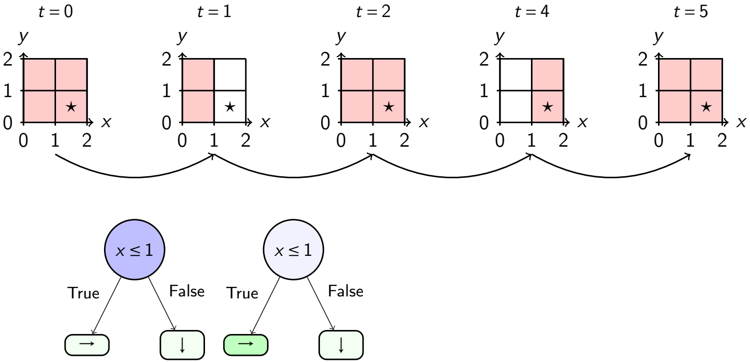

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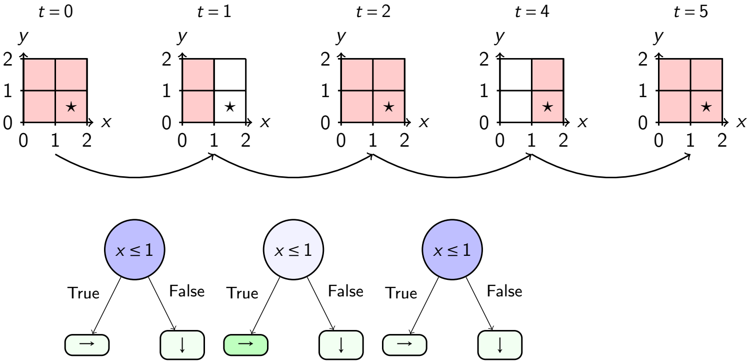

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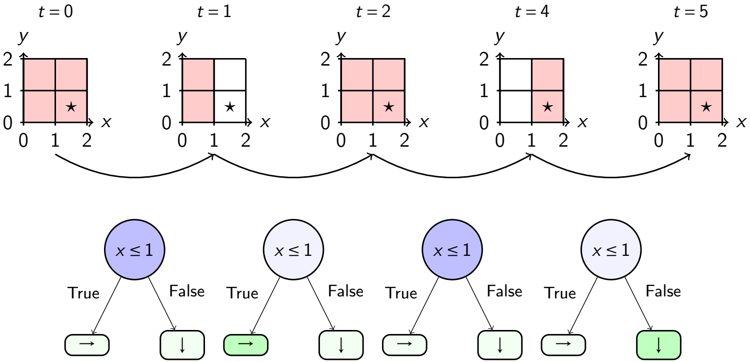

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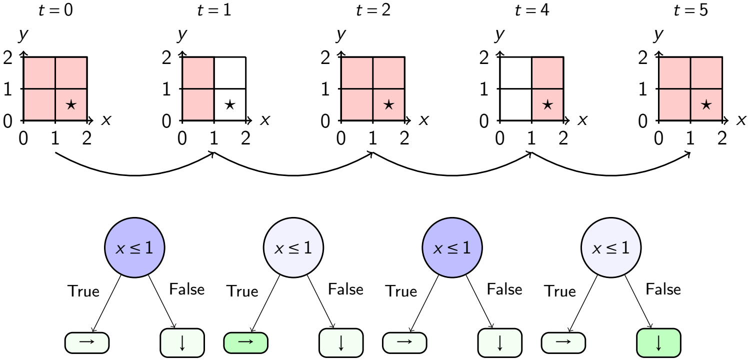

⚠ **To learn decision tree policies for an MDP we need to hide and allow agents to query information about state features.**

Deterministic partially observable policies in IBMDPs

IBMDPs promises

- No need to design new algorithm: we can use RL.
- IBMDP rewards trade-off naturally interpretability and performances.

RL for partially observable policies

- Finding the best **deterministic** and **partially observable** policy is NP-hard [Lit94]!
- The best partially observable policy can be stochastic [SJJ94].
- Value-based RL converges to sub-optimal solutions [SJJ94].

Asymmetric RL

- Access to hidden states during training but not at execution [Pin+17].
- Value-based → learns $Q(o, a)$ with TD targets $Q(s, a)$ [BDA22].
- Actor-critic[a] → policy gradient on $\pi(o, a)$ using a critic $V(s)$ [BA22].
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- The best partially observable policy can be stochastic [SJJ94].
- Value-based RL converges to sub-optimal solutions [SJJ94].

Asymmetric RL

- Access to hidden states during training but not at execution [Pin+17].
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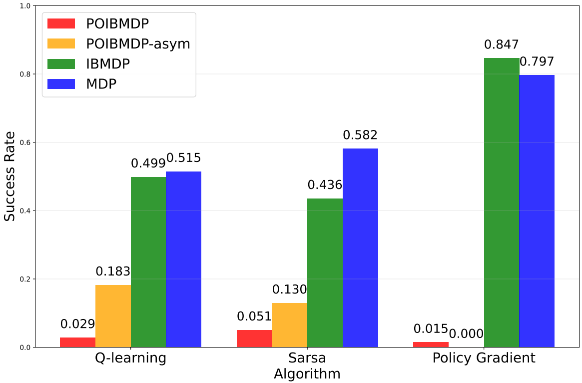
Result: for similar problems, RL struggles more when there is partial observability



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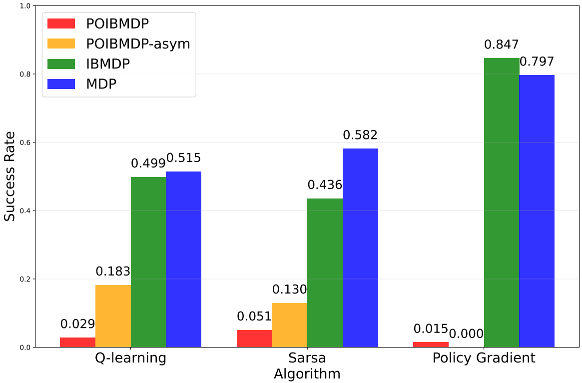

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- It seems that interpretability for SDM problems can be difficult to achieve because of **partial observability**.
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- N data points $\{\boldsymbol{x}_i, y_i\}$. Each $\boldsymbol{x}_i$ is described by p features and has a label $y_i \in \mathcal{Y}$. We want to find a tree of depth at most D $T \in \mathcal{T}_D$ that minimizes:

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- Trees **interpretable** and **competitive with neural nets** [GOV22].
- Greedy algorithms **sub-optimal accuracy**, but $\boldsymbol{O(2^D)}$ operations [Bre+84; Qui86; Qui93] .
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Decision tree induction as solving MDPs

Intuition

The induction of a decision tree is made of a sequence of decisions: at each node, we must decide whether it is better to split (a subset of) the training data, or to create a leaf node.

- S: data subsets.
- A: test or leaf nodes that can be added to the tree.
- R: penalty or accuracies.
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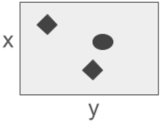

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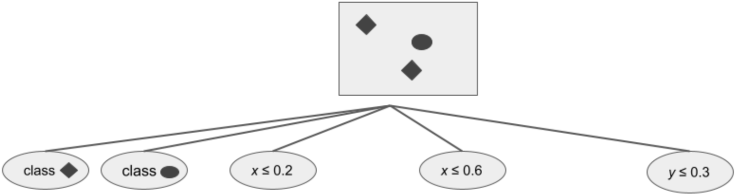

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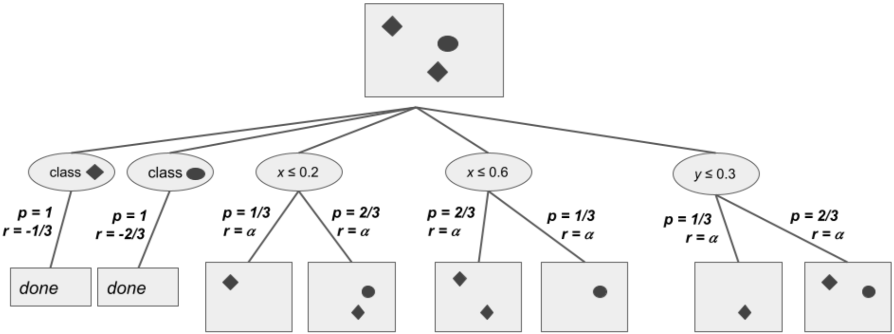

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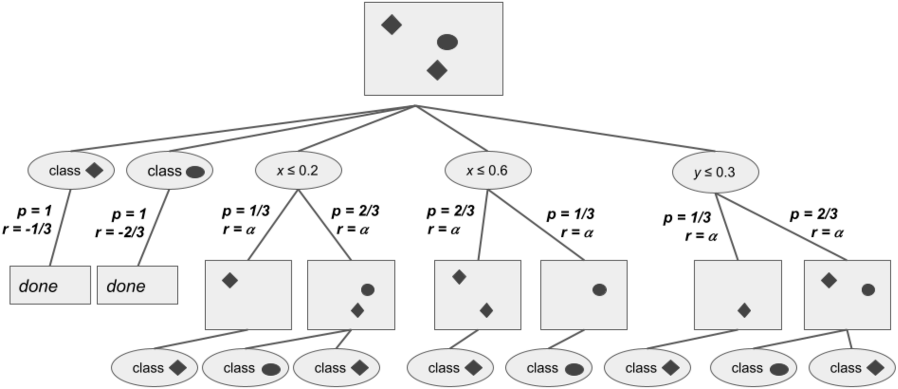

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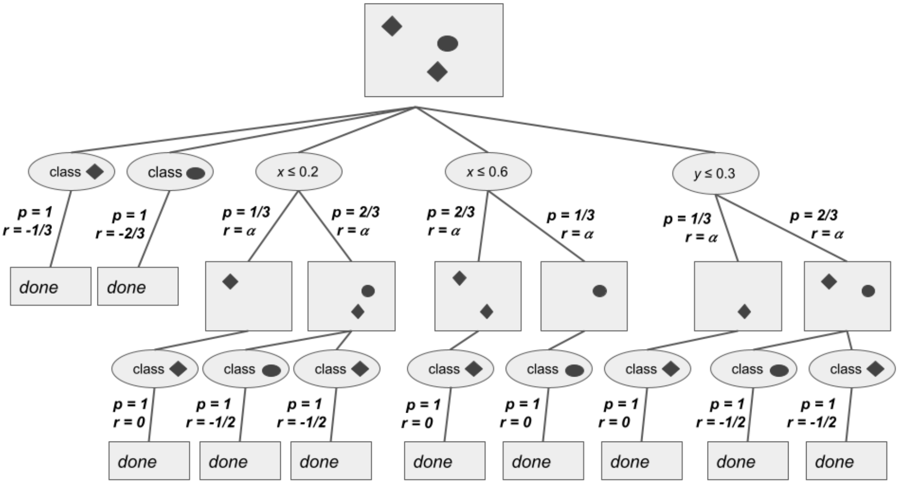

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Controlling the time complexity of decision tree induction

- Greedy algorithms consider only one candidate action in each state which is the test that minimizes some impurity criterion
  → **MDP state space size is $O(2^D)$**.
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- **Dynamic Programming Decision Trees (DPDT)**: Let's choose candidate actions adaptively
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How to choose the B candidate actions/splits?

Top-B greedy splits [Bla+23], quantiles, random...

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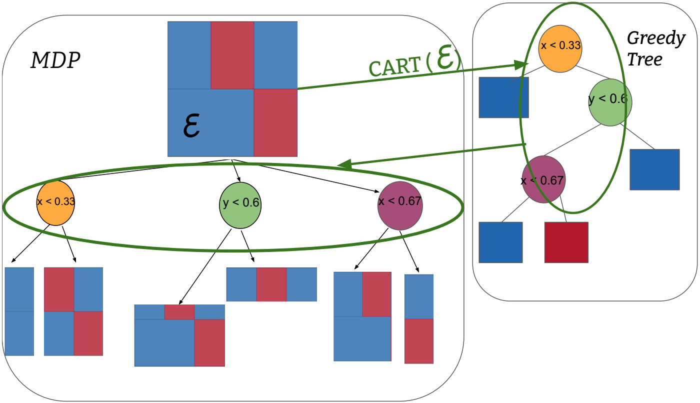

We can use greedy trees nodes as candidate actions.

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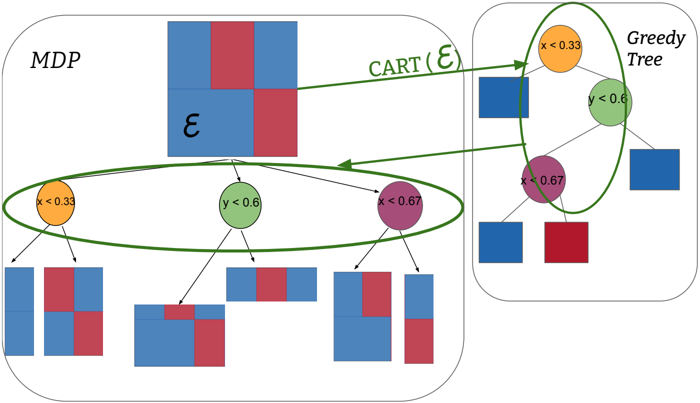

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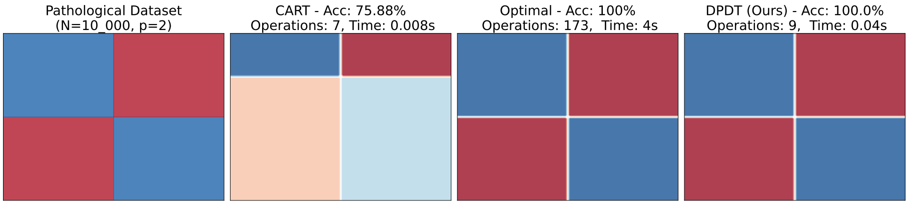
Fast like greedy trees, accurate like optimal trees



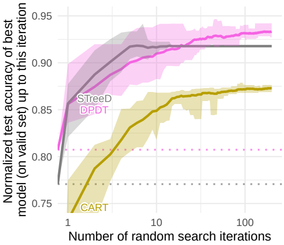
Comparison of greedy, optimal, and DPDT depth-2 trees on the checkersboard dataset.

Fast like greedy trees, accurate like optimal trees

Comparison of accuracies and operations for depth-3 trees.

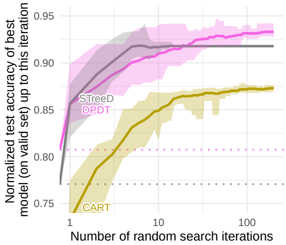
	Accuracy						Operations					
Dataset	Opt	Greedy	DPDT CART$^-$	DPDT CART$^+$	DPDT TopB$^-$	DPDT TopB$^+$	Opt	Greedy	DPDT CART$^-$	DPDT CART$^+$	DPDT TopB$^-$	DPDT TopB$^+$
room	**0.992**	0.968	0.991	**0.992**	0.990	**0.992**	10^6	15	286	16100	111	16100
bean	**0.871**	0.777	0.812	0.853	0.804	0.841	$5 \cdot 10^6$	15	295	25900	112	16800
eeg	**0.708**	0.666	0.689	0.706	0.684	0.699	$2 \cdot 10^6$	13	289	26000	95	11000
avila	**0.585**	0.532	0.574	**0.585**	0.563	0.572	$3 \cdot 10^7$	9	268	24700	60	38900
magic	**0.831**	0.801	0.822	0.828	0.807	0.816	$6 \cdot 10^6$	15	298	28000	70	4190
htru	**0.981**	0.979	0.979	0.980	0.979	0.980	$6 \cdot 10^7$	15	295	25300	55	2180
occup.	**0.994**	0.989	0.991	**0.994**	0.990	0.992	$7 \cdot 10^5$	13	280	16300	33	510
skin	**0.969**	0.966	0.966	0.966	0.966	0.966	$7 \cdot 10^4$	15	301	23300	20	126
fault	**0.682**	0.553	0.672	0.674	0.672	0.673	$9 \cdot 10^8$	13	295	24200	111	16800
segment	**0.887**	0.574	0.812	0.879	0.786	0.825	$2 \cdot 10^6$	7	220	16300	68	11400
page	**0.971**	0.964	0.970	0.970	0.964	0.965	10^7	15	298	22400	701	4050
bidding	**0.993**	0.981	0.985	**0.993**	0.985	**0.993**	$3 \cdot 10^5$	13	256	9360	58	2700
raisin	**0.894**	0.869	0.879	0.886	0.875	0.883	$4 \cdot 10^6$	15	295	20900	48	1440
rice	**0.938**	0.933	0.934	0.937	0.933	0.936	$2 \cdot 10^7$	15	298	25500	49	1470
wilt	**0.996**	0.993	0.994	0.995	0.994	0.994	$3 \cdot 10^5$	13	274	11300	33	465
bank	**0.983**	0.933	0.971	0.980	0.951	0.974	$6 \cdot 10^4$	13	271	7990	26	256

DPDT trees generalization



DPDT depth-5 trees vs. other detph-5 trees

DPDT trees generalization



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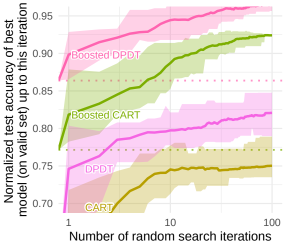

Boosted DPDT vs. Boosted CART

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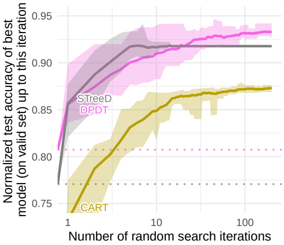

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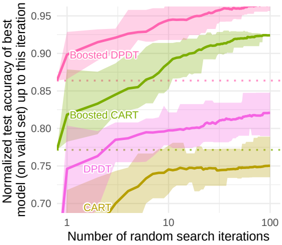

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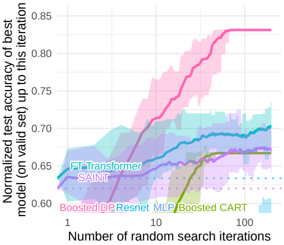

Boosted DPDT vs. other classifiers

Why generating candidate splits with CART?

Theorem (DPDT trees are not worse than greedy trees)

The greedy tree is always a solution of the MDPs we solve. Because we solve the MDPs exactly with DP, if the greedy tree is the best solution, DPDT will find it.

Theorem (DPDT trees can be strictly better than greedy trees)

There exist a depth budget D and a dataset for which DPDT trees are strictly better than greedy trees.

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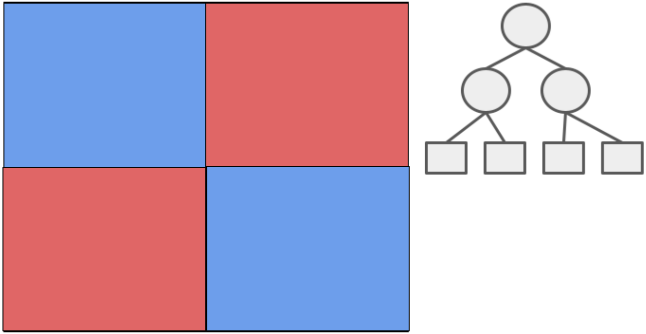

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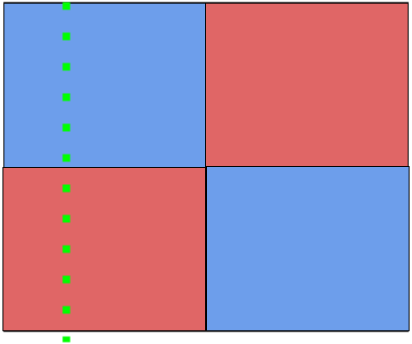

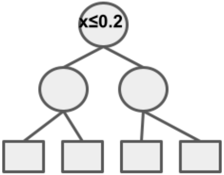

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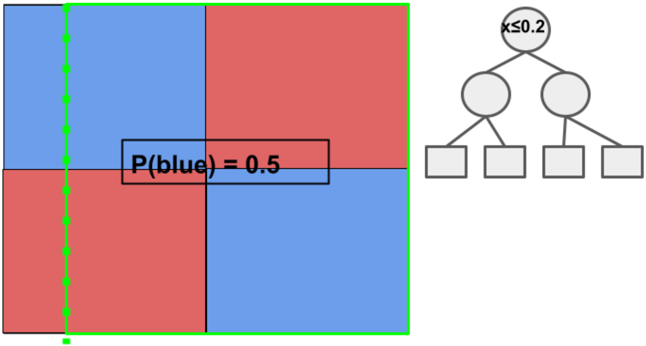

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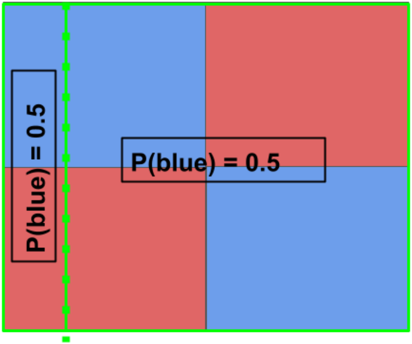

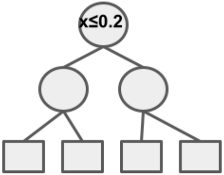

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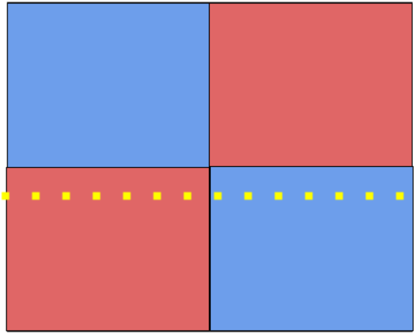

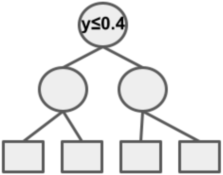

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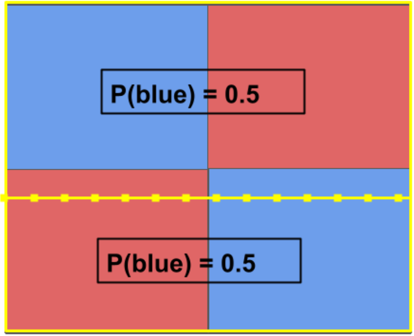

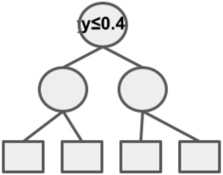

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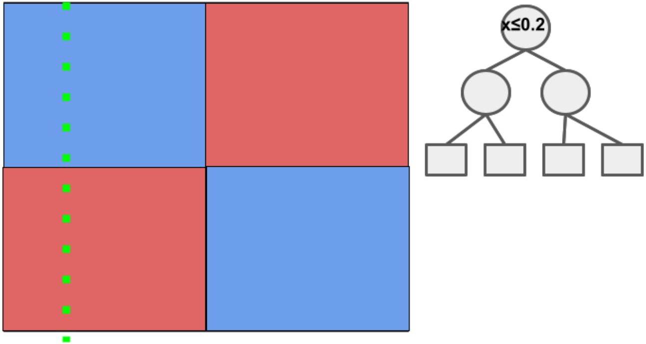

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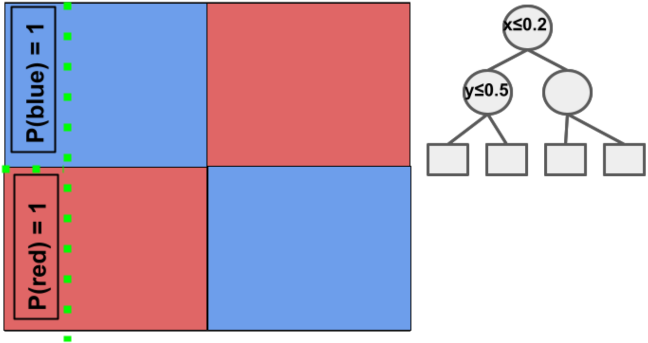

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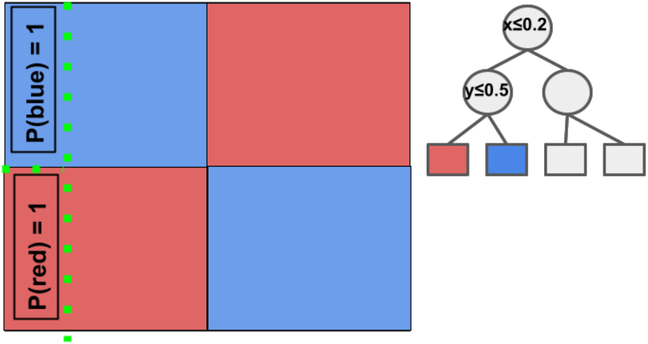

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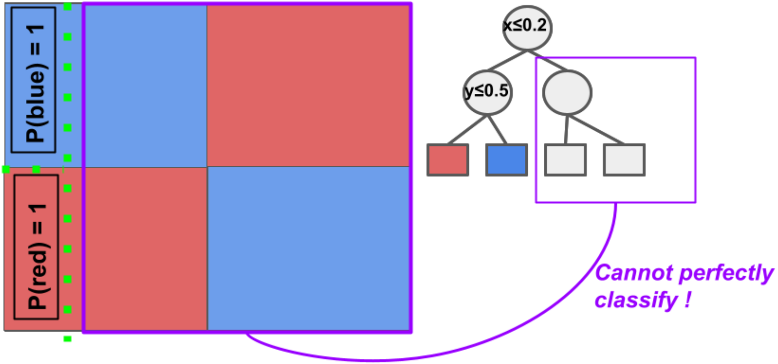

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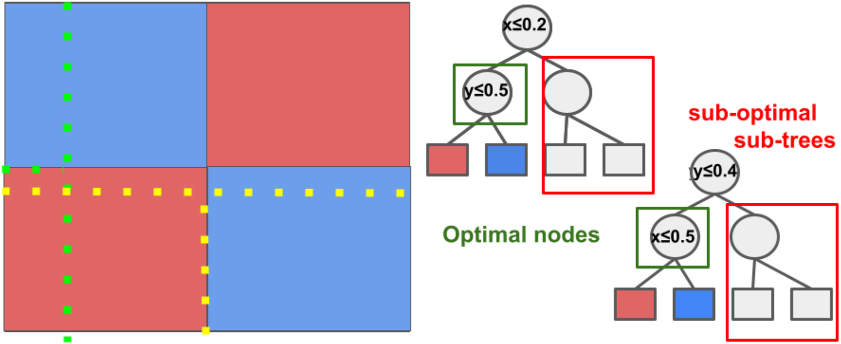

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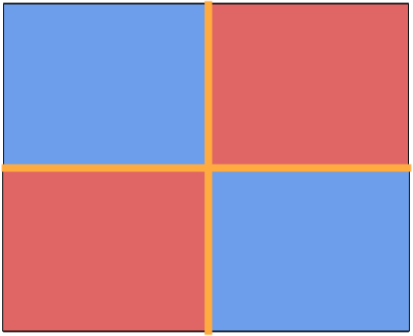

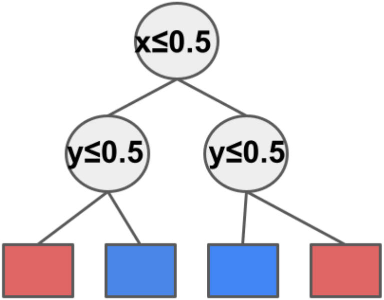

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We propose policy unfolding

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# Decision tree for Mountain Car
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        if x[1] <= -0.6378:
            return 0
        else:
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2. What kind of results we can obtain using our proposed methodology?

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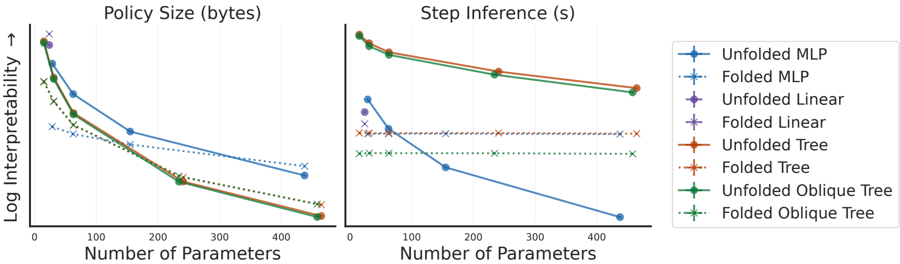
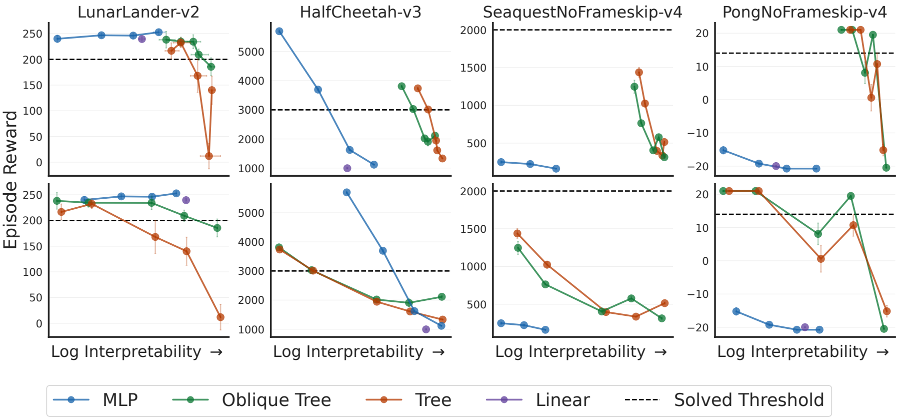
Result: unfolding policies is necessary to respect consensus



Aggregated policies interpretability on classic control environments

Result: there is no dominating policy class for all environments



Interpretability-Performance trade-offs. Top row, interpretability is measured with step inference times. Bottom row, the interpretability is measured with policy size.

Perspectives

- Beliefs such as "trees are more interpretable than neural networks" should be used with caution.
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- What about (very) big models?
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Conclusion: interpretable SDM is a difficult research topic

- Technical challenges: **partial observability in SDM, NP-hardness**.
  → Focus on indirect approaches and/or on POMDP research first.
- Fundamental challenges: **no definition**.
  → Discuss with the community (InterpPol workshop).
- **Decision trees offer good inductive bias for SDM in games or tabular data**.

My hope

Motivate interpretability by finding a real-world problem where interpretability is *really* necessary [Nag+24].

Conclusion: interpretable SDM is a difficult research topic

- Technical challenges: **partial observability in SDM, NP-hardness**.
  → Focus on indirect approaches and/or on POMDP research first.
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