

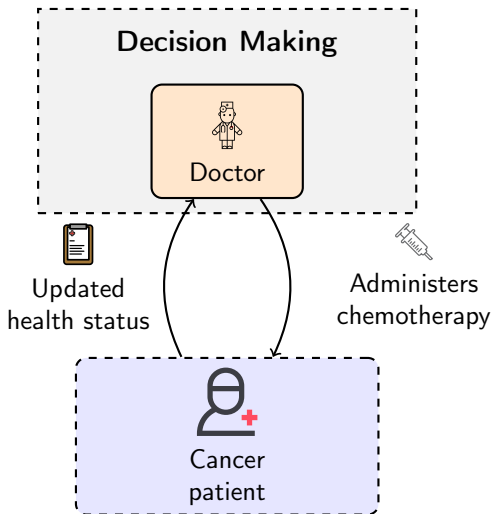
# Interpretability, Decision Trees, and Sequential Decision Making

Hector Kohler

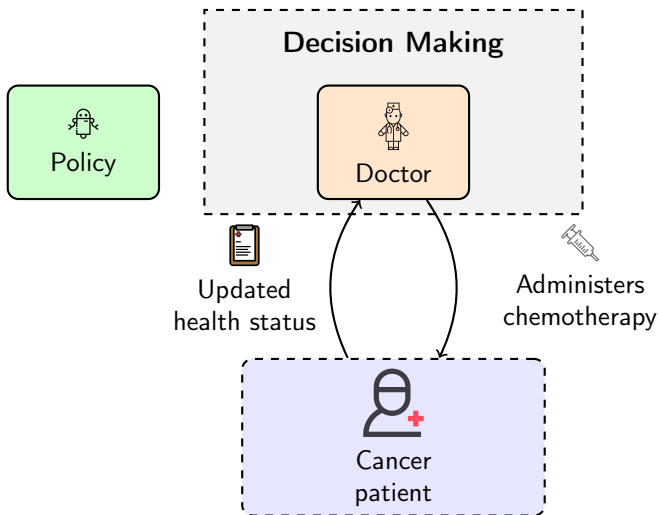
Supervised by Dr. Riad Akrou (HdR) and Prof. Philippe Preux (HdR)  
Université de Lille, CNRS, Inria, UMR CRISAL 9189, France

November 27, 2025

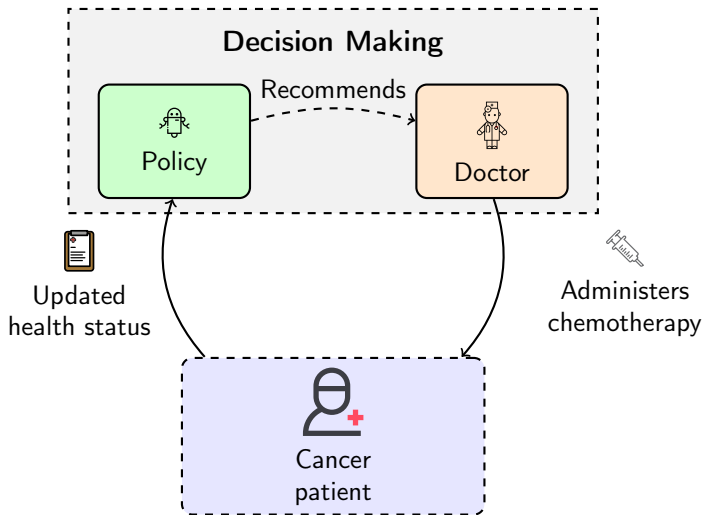
# Sequential decision making (SDM) and machine learning (ML)



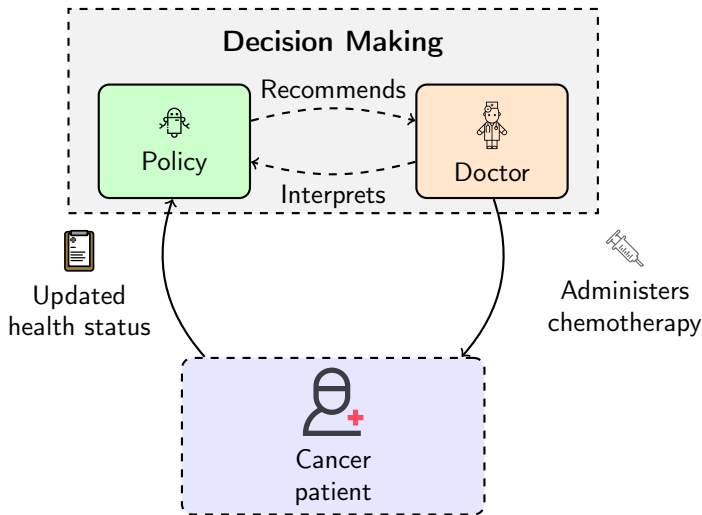
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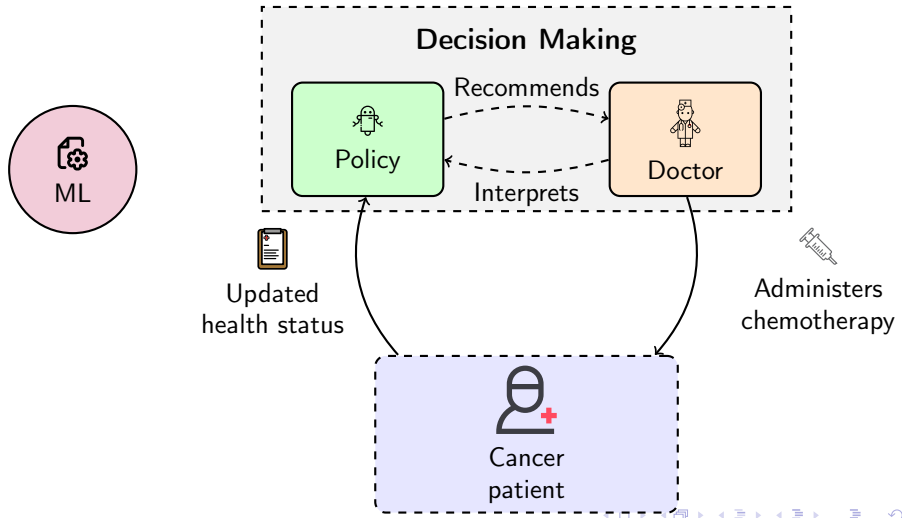
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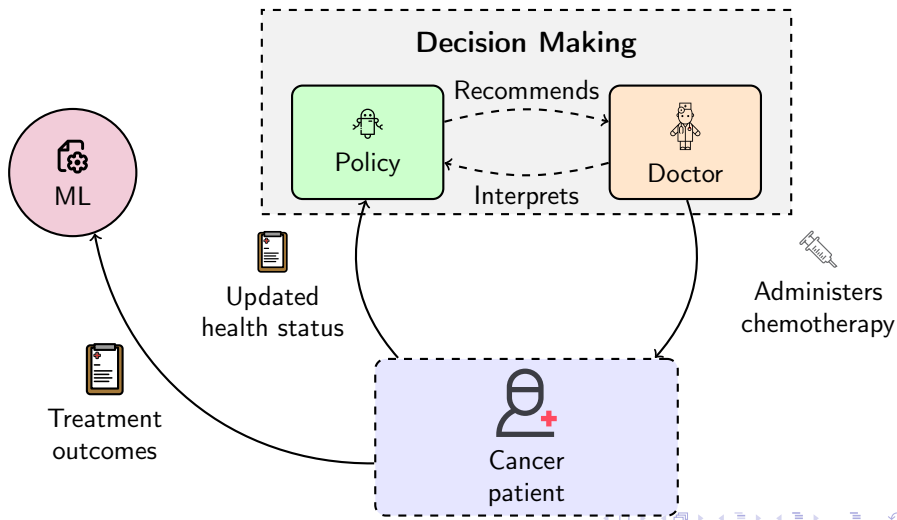
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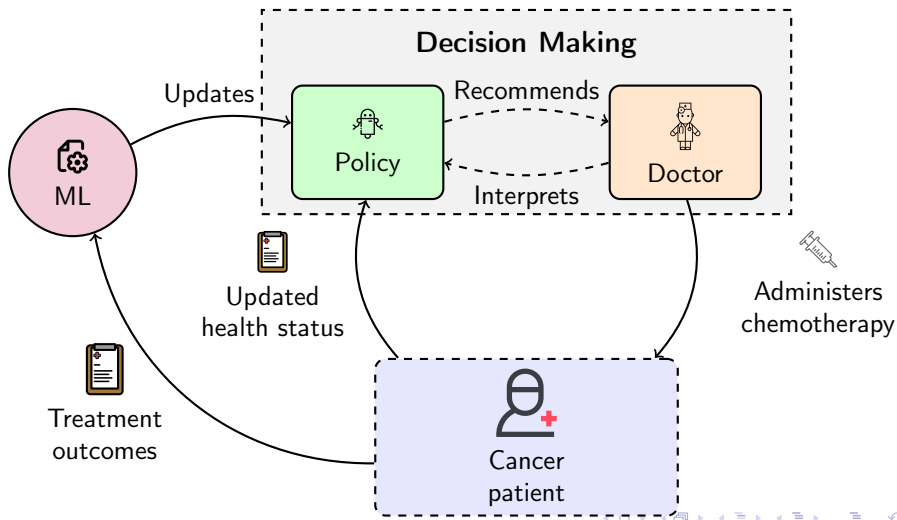
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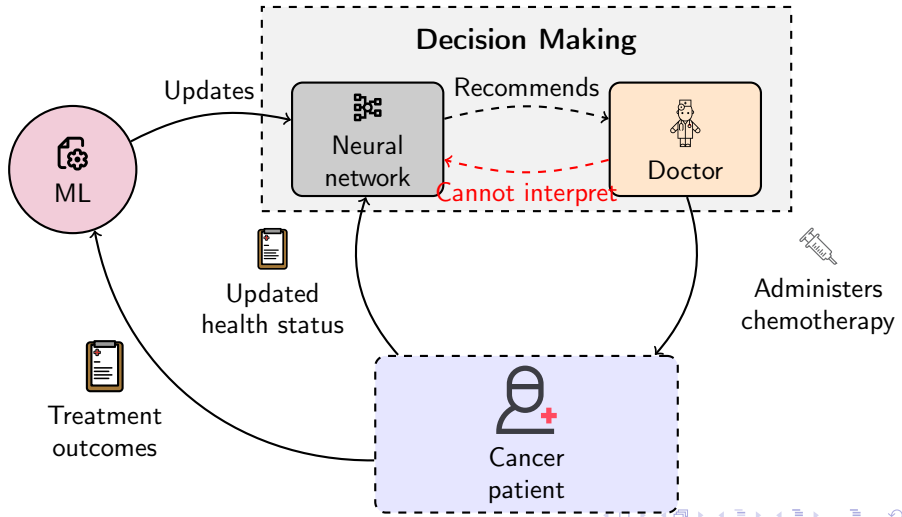


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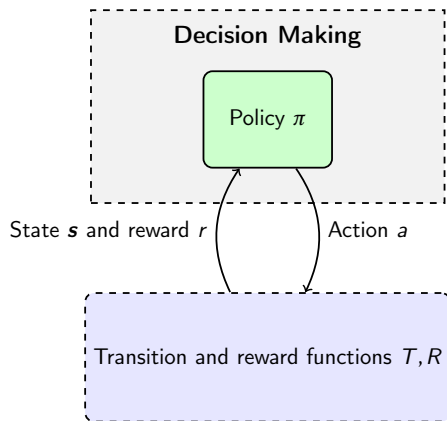




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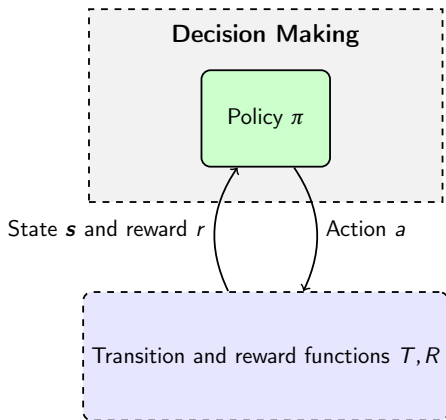


# Markov decision processes (MDPs) and reinforcement learning (RL)



Markov decision processes [Put94].

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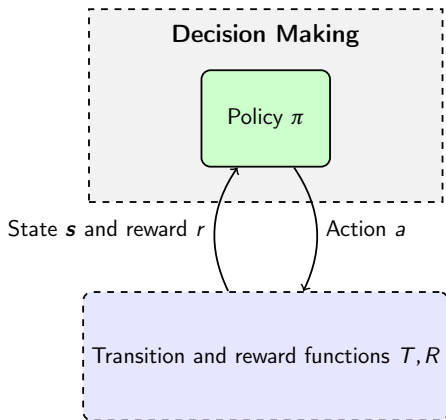


Markov decision processes [Put94].

- RL [SB98] aims to find a policy,  $\pi : S \rightarrow A$  that maximizes:

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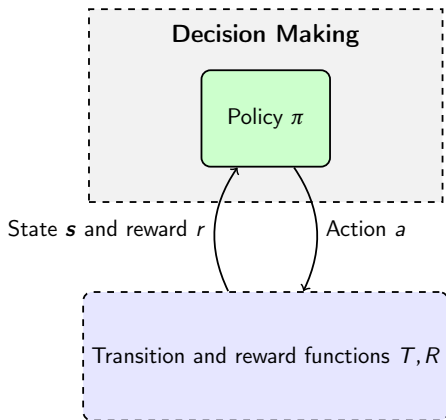
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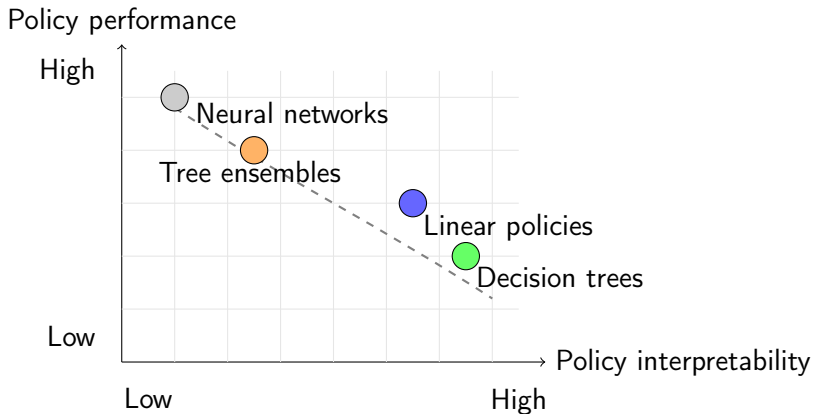
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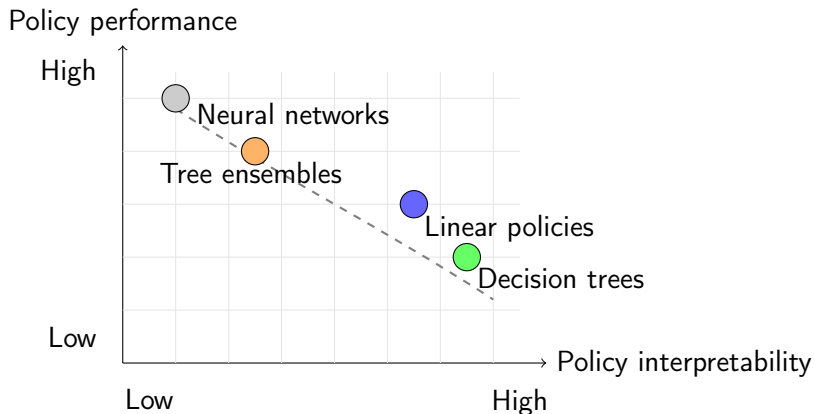
- Lots of successful RL algorithms [SB98; Mni+15; Sch+17].
- Few interpretability concerns.

# Policy interpretability



**Heuristic** interpretability-performance trade-offs of different policy classes.

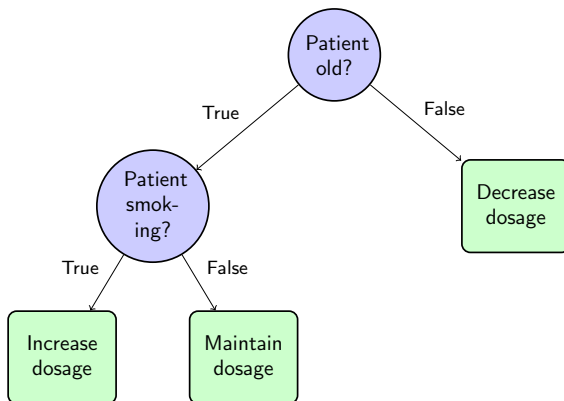
# Policy interpretability



**Heuristic** interpretability-performance trade-offs of different policy classes.

⚠ **No definition of interpretability for machine learning models!**

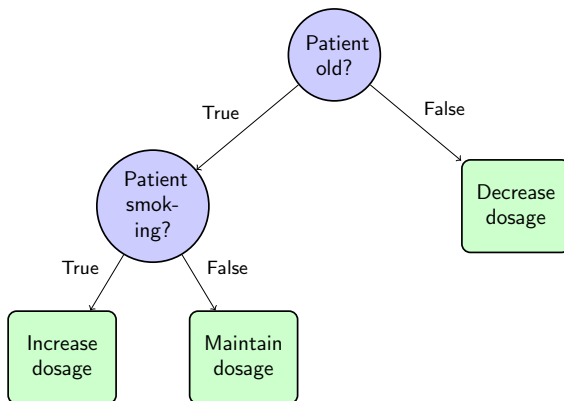
# Decision trees



A generic decision tree of depth  $D = 2$ .



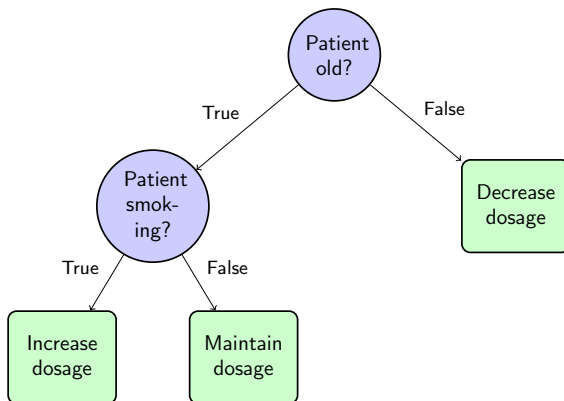
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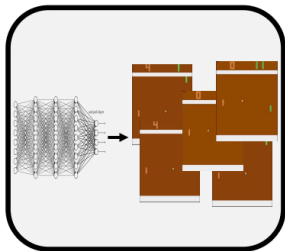


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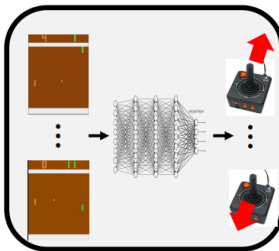
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**What about SDM?**

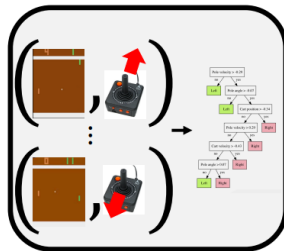
## Indirect approach: imitation learning



**Step 1:** Use NN to generate states

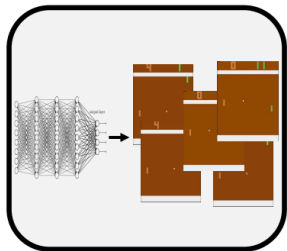


### Step 2: Use NN to obtain actions

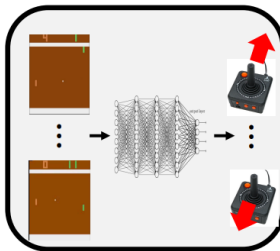


**Step 3:** Use supervised learning to train a decision tree

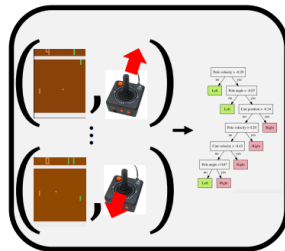
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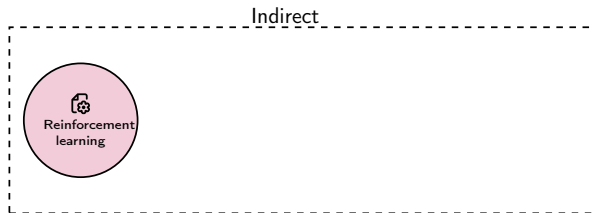
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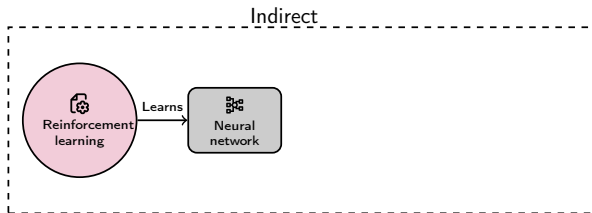
**Step 3:** Use supervised learning to train a decision tree

Most research focused on indirect learning of interpretable policies [RGB10; BPS18; Ver+18; Mil+24].

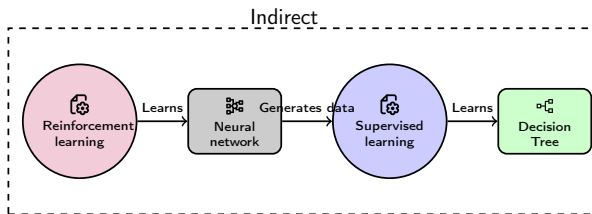
# Two ways to get interpretable policies for SDM



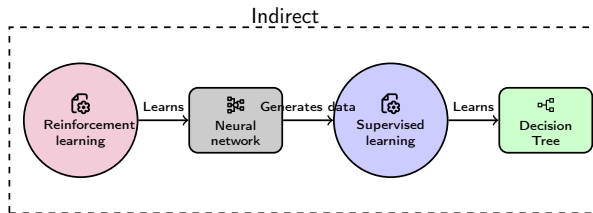
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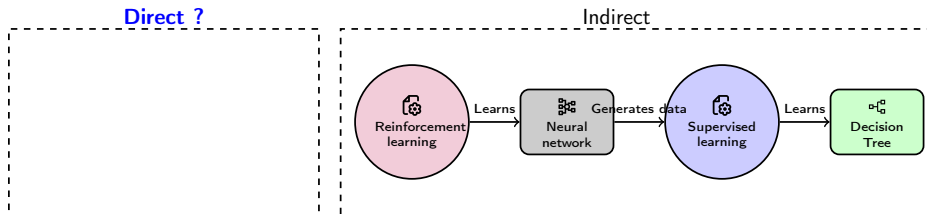
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⚠ Policies obtained indirectly optimize a surrogate objective rather than a trade-off between interpretability and cumulative rewards.

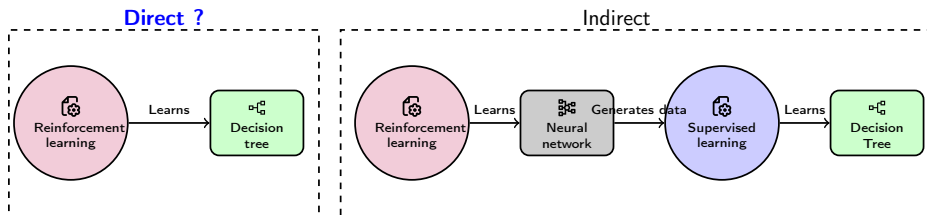


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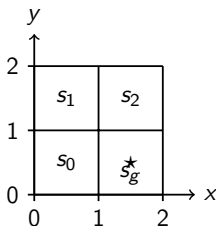
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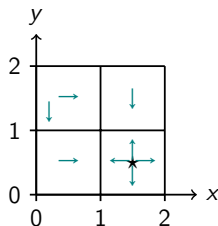
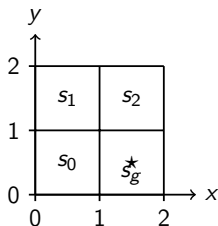
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# Grid world MDP and decision tree policies



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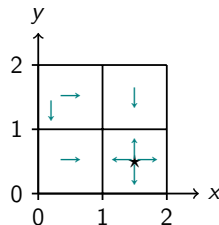
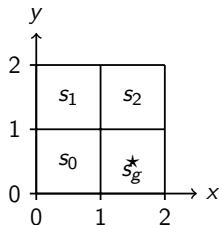


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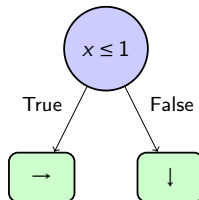


Grid world MDP and optimal actions.

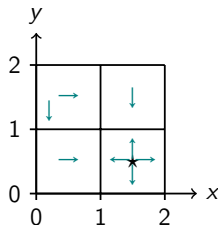
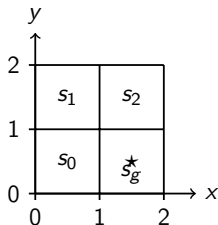
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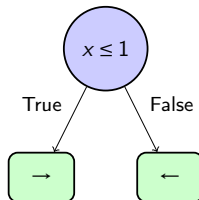
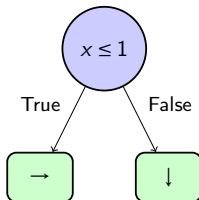
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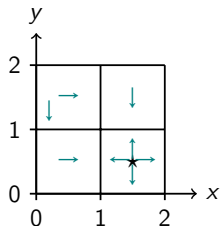
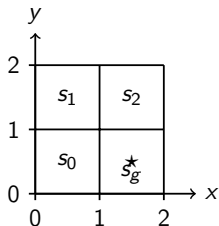
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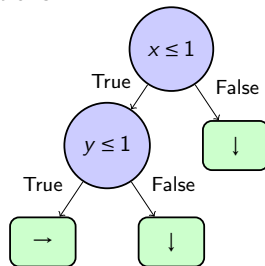
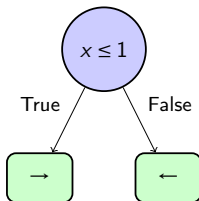
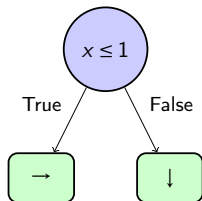
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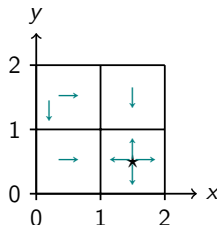
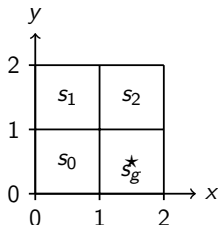
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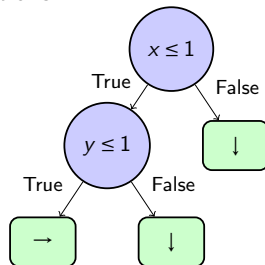
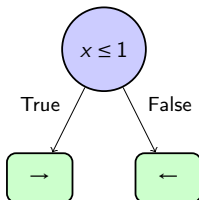
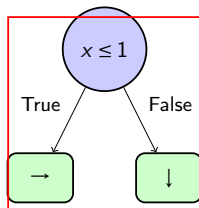
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# Grid world MDP and decision tree policies



Grid world MDP and optimal actions.



Decision tree policies with different interpretability-performance trade-offs.

# Direct RL of decision tree policies with iterative bounding

## Markov decision processes

# Direct RL of decision tree policies with iterative bounding Markov decision processes

Given an MDP  $\mathcal{M} \langle S, A, R, T \rangle$ , an associated iterative bounding Markov decision process (IBMDP, [Top+21])  $\mathcal{M}_{IB}$  is an MDP:

$$\langle \overbrace{S \times O}^{\text{Augmented state space}}, \underbrace{A \cup A_{info}}_{\text{Augmented action space}}, \overbrace{(R, \zeta)}^{\text{Augmented rewards}}, \underbrace{(T_{info}, T)}_{\text{Augmented transitions}} \rangle$$

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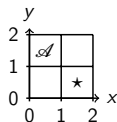
## IBMDPs promises

- No need to design new algorithm: we can use RL.
- IBMDP rewards trade-off naturally interpretability and performances.

# Grid world IBMDP example

$$t = 0$$

$$\mathbf{s}_t = (0.5, 1.5)$$



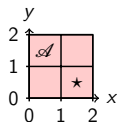


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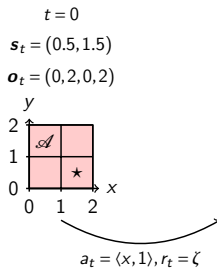
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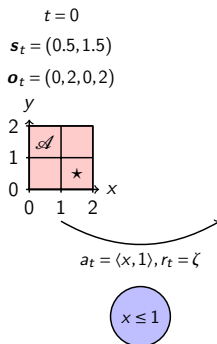
$$\mathbf{o}_t = (0, 2, 0, 2)$$



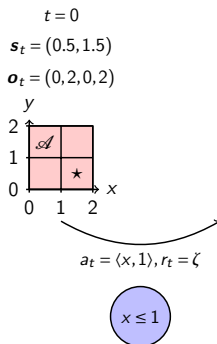
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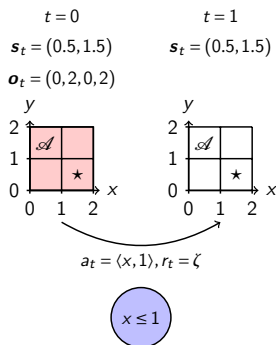
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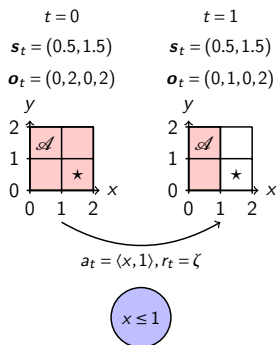
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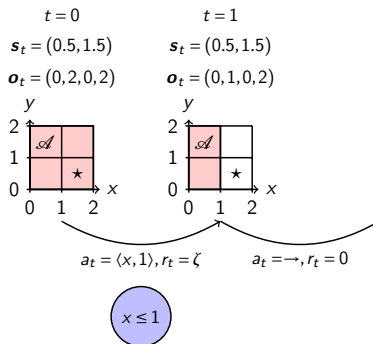
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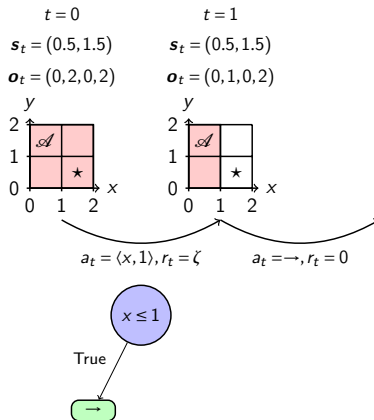
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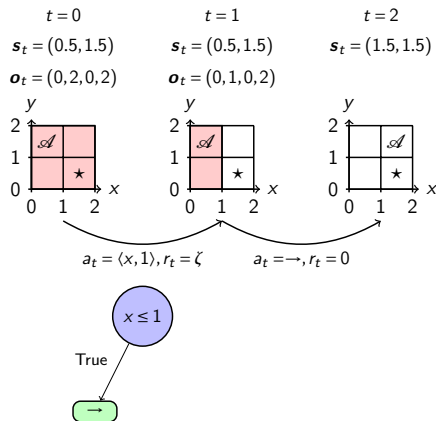


# Grid world IBMDP example

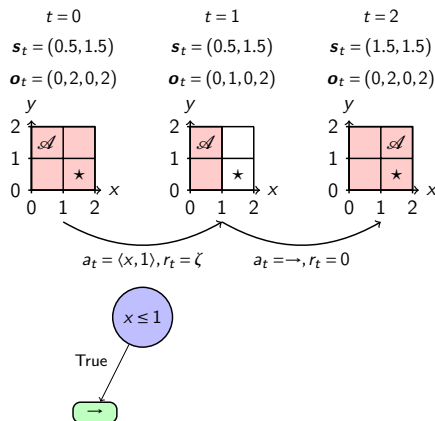




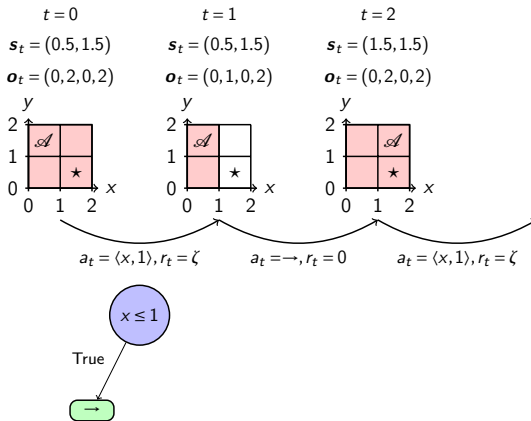
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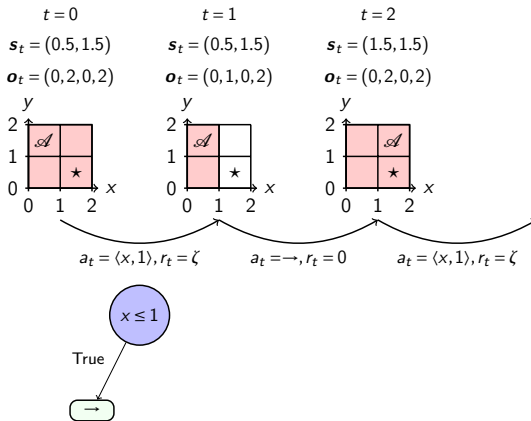
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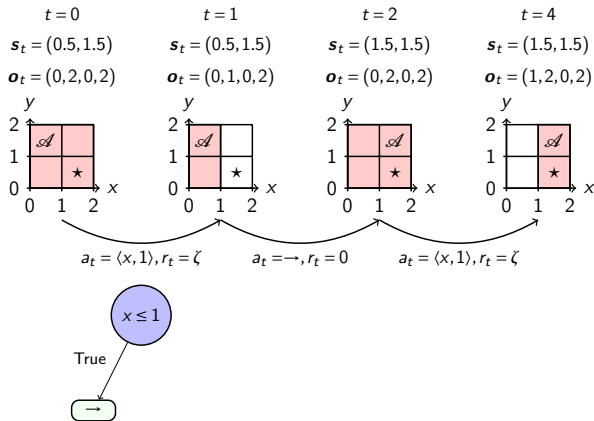
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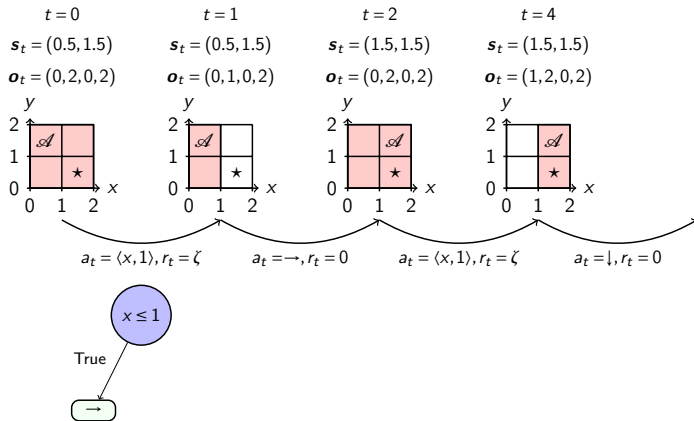
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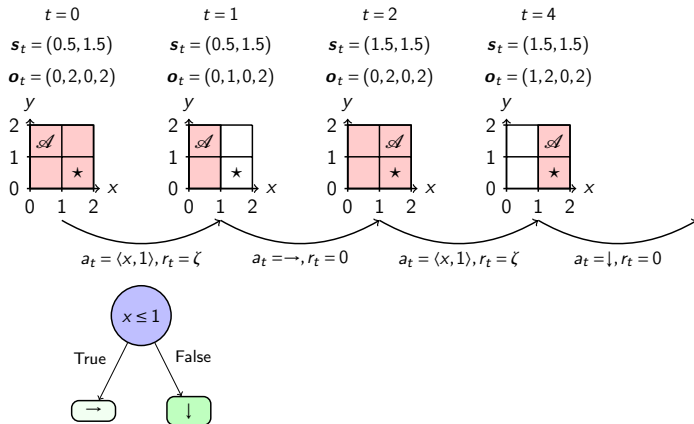
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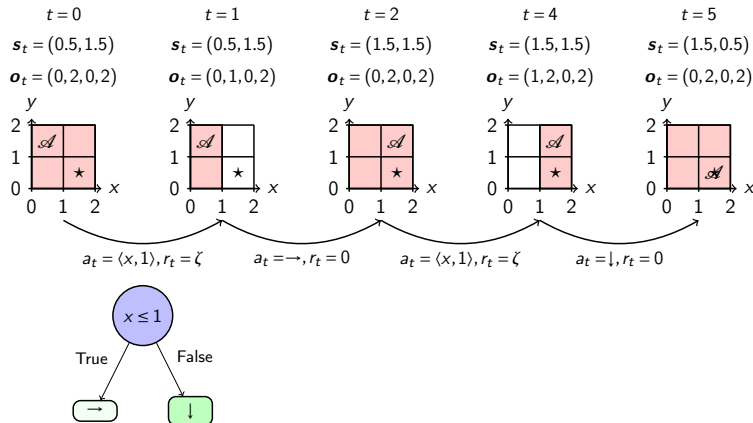
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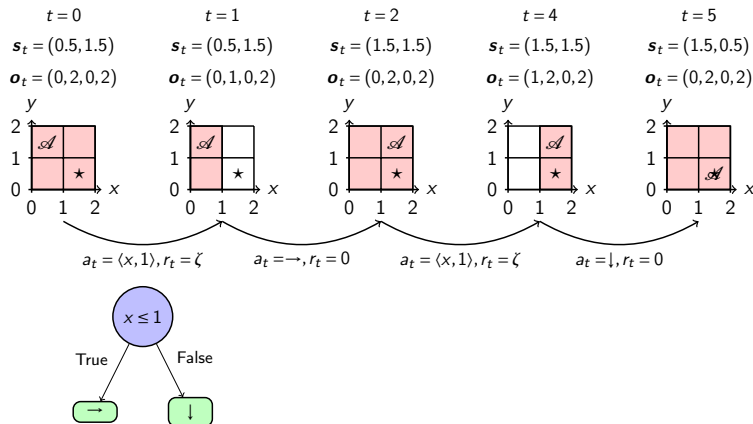


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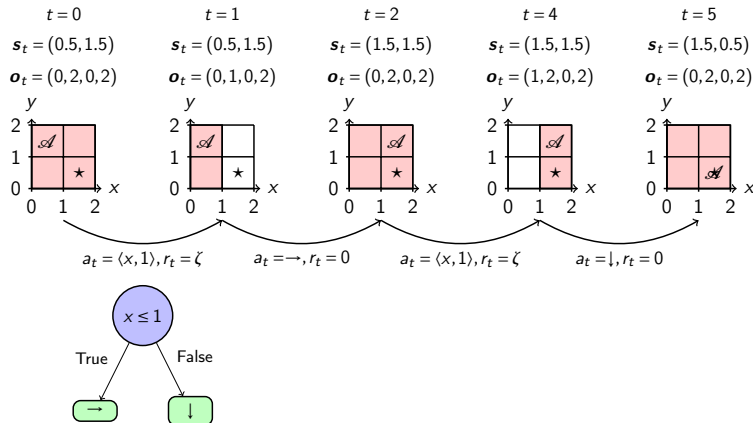




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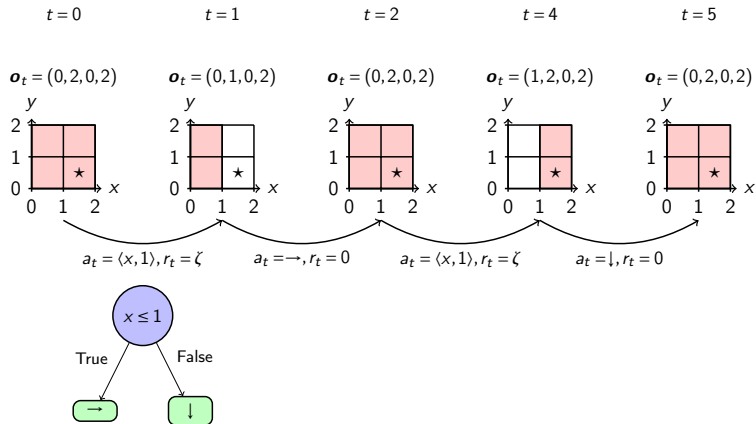


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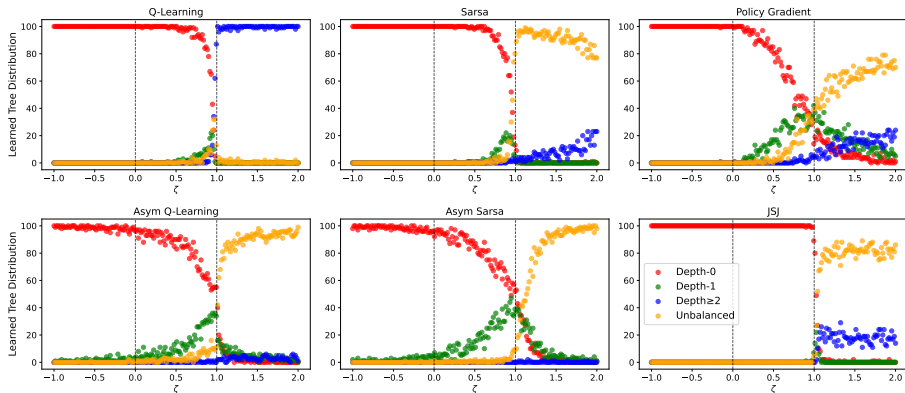
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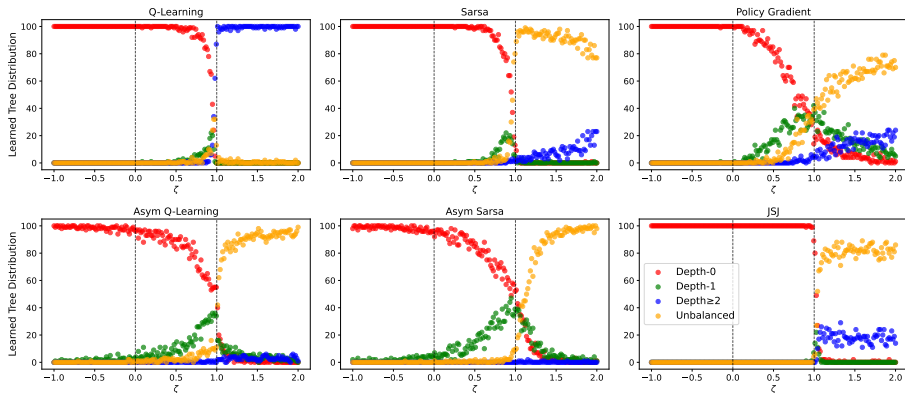
## Hard problem

Finding the best **deterministic** and **partially observable** policy is NP-hard [Lit94]!

# Result: RL cannot retrieve optimal depth-1 trees for the grid world MDP



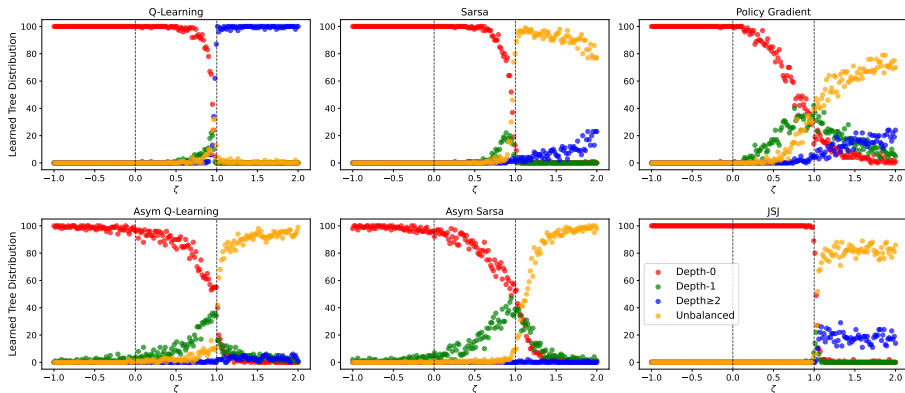
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Distributions of tree policies learned with (asymmetric) RL algorithms [SB98; SJJ94; LS98; BA22; BDA22] as a function of the interpretability reward  $\zeta$ .



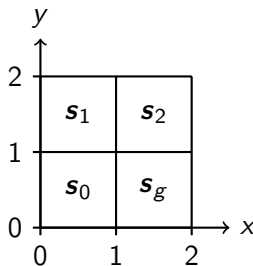
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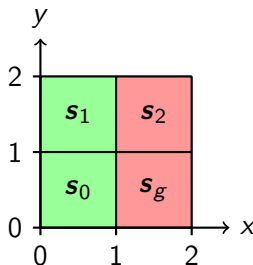
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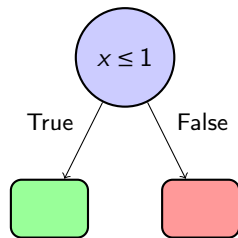
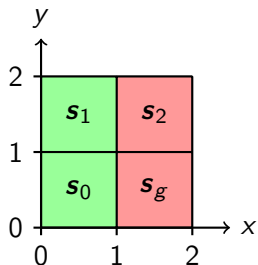


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Classification MDP and the unique optimal depth-1 tree.

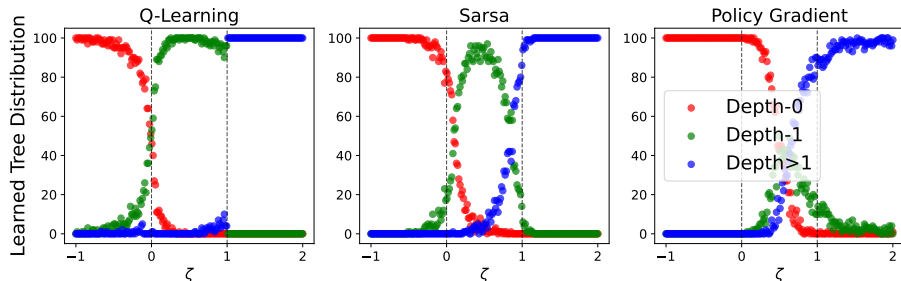
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Classification MDP and the unique optimal depth-1 tree.

**Deterministic partially observable policies for classification IBMDPs ( $\Leftrightarrow$  decision tree policies) are in fact Markovian.**

# Result: RL can retrieve optimal depth-1 trees for the toy classification MDP



Distributions of tree policies learned with various RL algorithms.

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- $N$  data points  $\{\mathbf{x}_i, y_i\}$ . Each  $\mathbf{x}_i$  is described by  $p$  features and has a label  $y_i \in \mathcal{Y}$ . We want to find a tree of depth at most  $D$   $T \in \mathcal{T}_D$  that minimizes:

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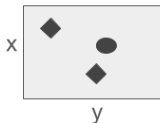
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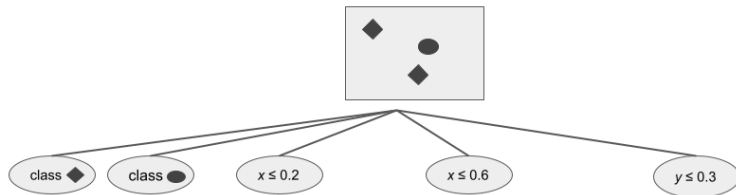
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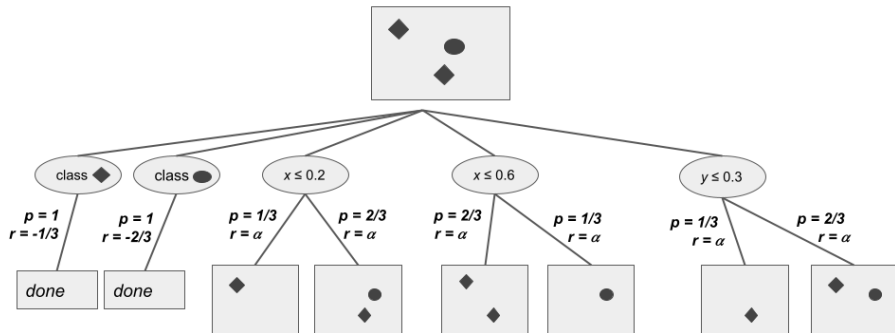
Example of decision tree induction as an MDP.

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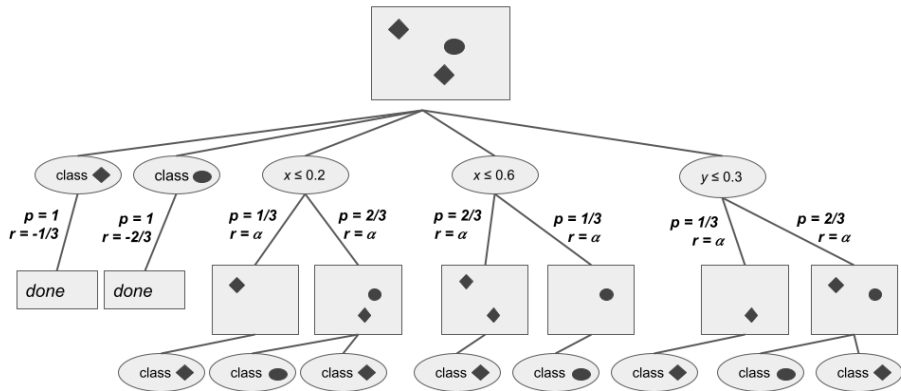
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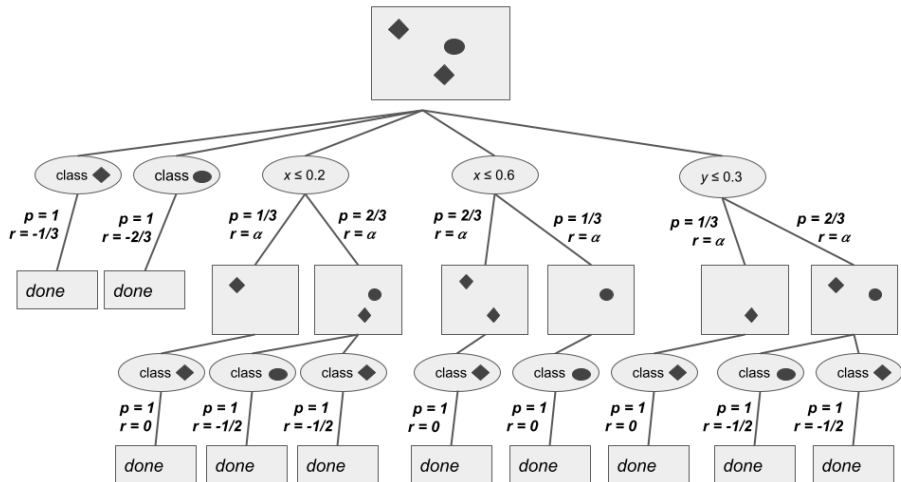
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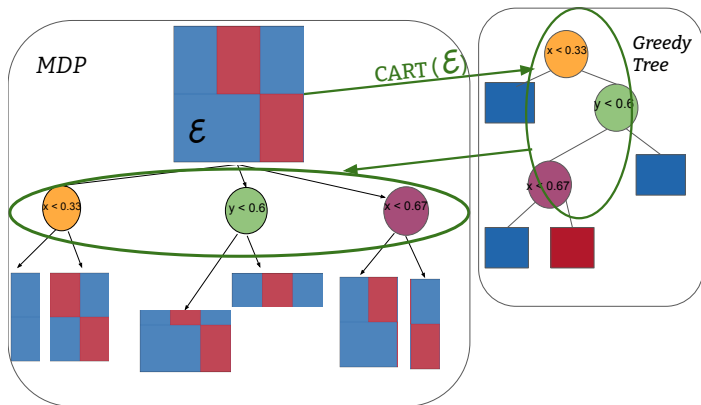
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**How to choose the  $B$  candidate actions/splits?**

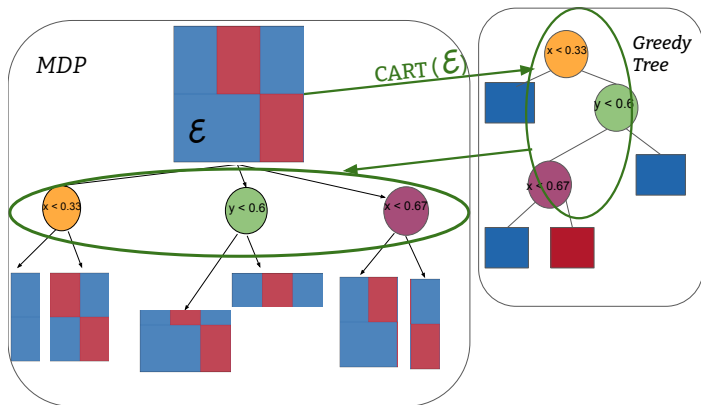
# Dynamic Programming Decision Trees (DPDT)<sup>1</sup>



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DPDT was presented at the 31st ACM SIGKDD conference.

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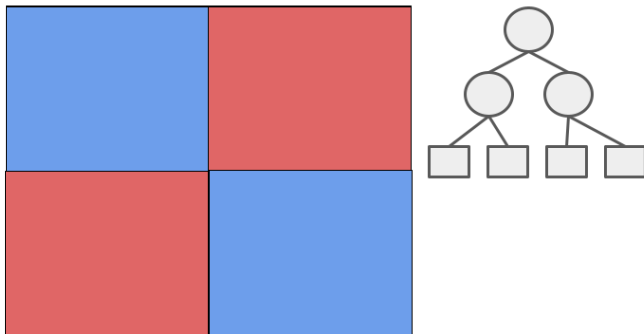
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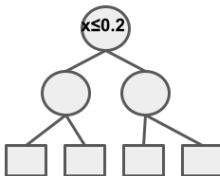
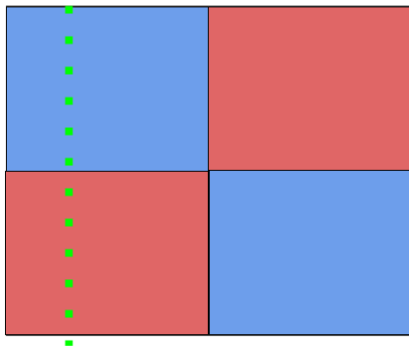
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*There exist a depth budget  $D$  and a dataset for which DPDT trees are strictly better than greedy trees.*

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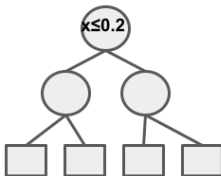
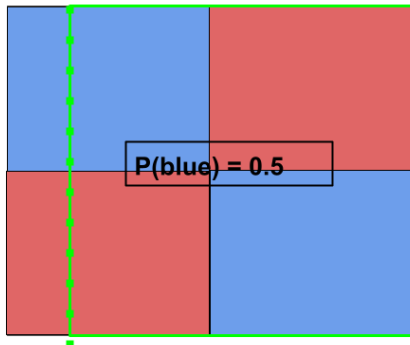


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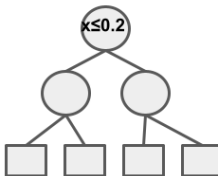
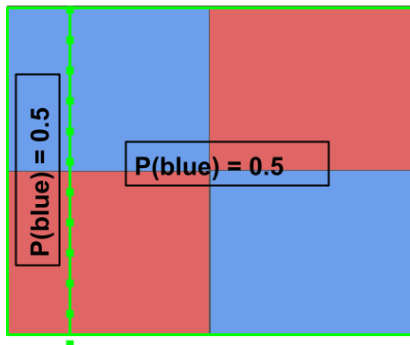




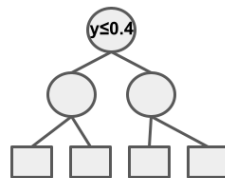
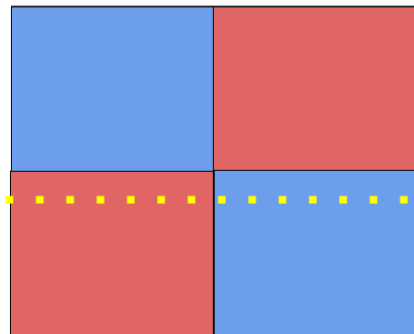
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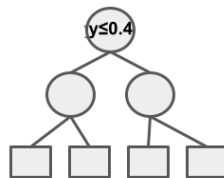
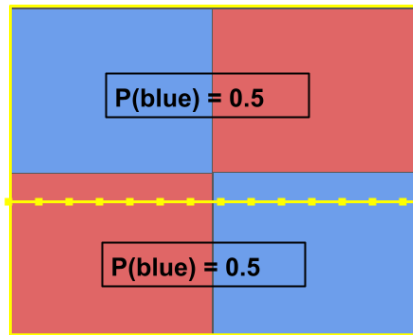
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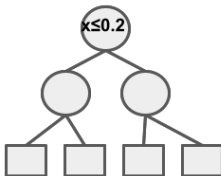
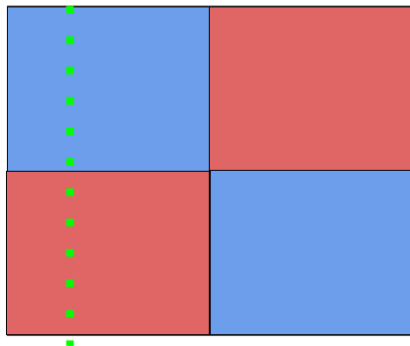
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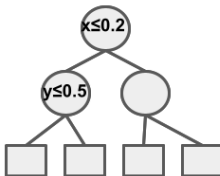
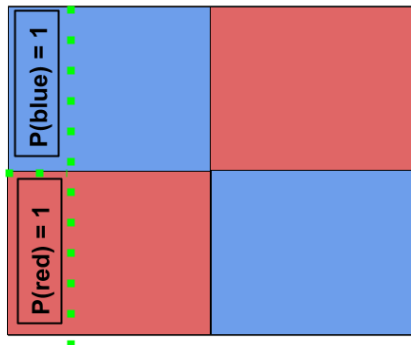
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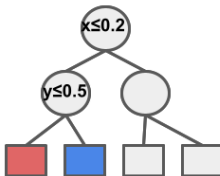
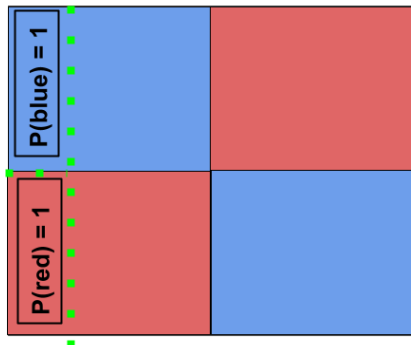
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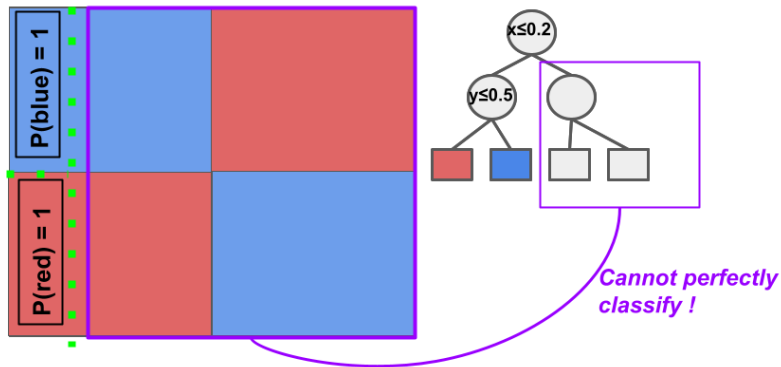
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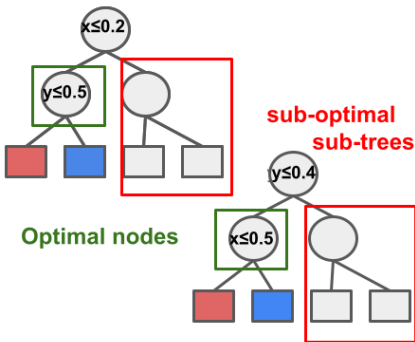
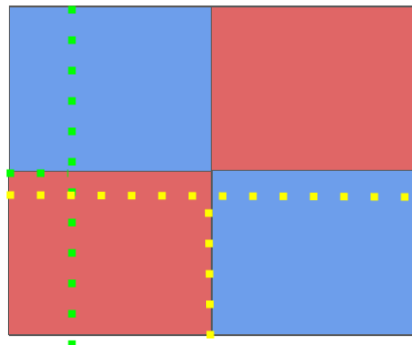


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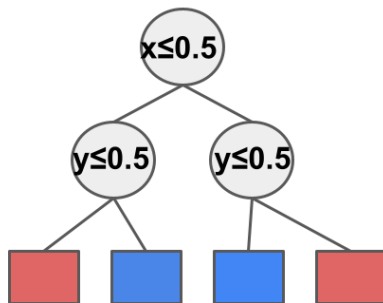
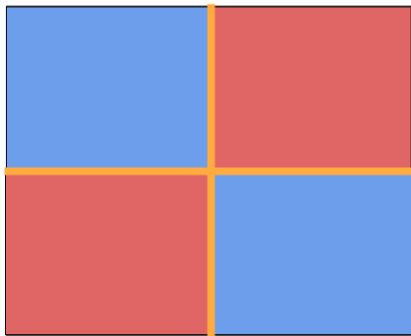




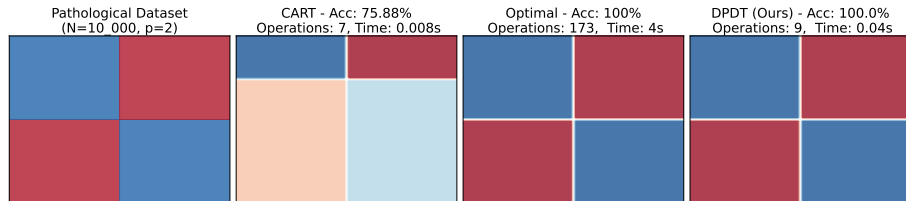
# DPDT trees can be strictly better than greedy trees



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# Fast like greedy trees, accurate like optimal trees



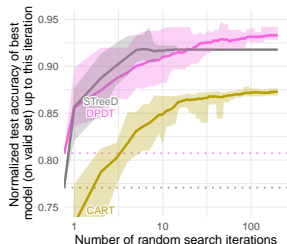
Comparison of greedy, optimal, and DPDT depth-2 trees on the checkersboard dataset.

# Comparing tree accuracy to complexity

Train accuracy and operation count when learning depth-3 decision trees.

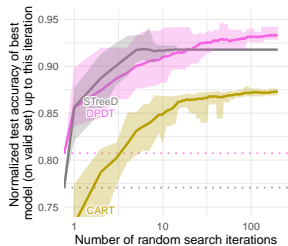
|         |        |    | Accuracy         |                |               |              | Operations       |                |               |              |
|---------|--------|----|------------------|----------------|---------------|--------------|------------------|----------------|---------------|--------------|
| Dataset | N      | p  | Opt<br>Quant-BnB | Greedy<br>CART | DPDT<br>light | DPDT<br>full | Opt<br>Quant-BnB | Greedy<br>CART | DPDT<br>light | DPDT<br>full |
| room    | 8103   | 16 | 0.992            | 0.968          | 0.991         | 0.992        | $10^6$           | 15             | 286           | 16100        |
| bean    | 10888  | 16 | 0.871            | 0.777          | 0.812         | 0.853        | $5 \cdot 10^6$   | 15             | 295           | 25900        |
| eeg     | 11984  | 14 | 0.708            | 0.666          | 0.689         | 0.706        | $2 \cdot 10^6$   | 13             | 289           | 26000        |
| avila   | 10430  | 10 | 0.585            | 0.532          | 0.574         | 0.585        | $3 \cdot 10^7$   | 9              | 268           | 24700        |
| magic   | 15216  | 10 | 0.831            | 0.801          | 0.822         | 0.828        | $6 \cdot 10^6$   | 15             | 298           | 28000        |
| htru    | 14318  | 8  | 0.981            | 0.979          | 0.979         | 0.980        | $6 \cdot 10^7$   | 15             | 295           | 25300        |
| occup.  | 8143   | 5  | 0.994            | 0.989          | 0.991         | 0.994        | $7 \cdot 10^5$   | 13             | 280           | 16300        |
| skin    | 196045 | 3  | 0.969            | 0.966          | 0.966         | 0.966        | $7 \cdot 10^4$   | 15             | 301           | 23300        |
| fault   | 1552   | 27 | 0.682            | 0.553          | 0.672         | 0.674        | $9 \cdot 10^8$   | 13             | 295           | 24200        |
| segment | 1848   | 18 | 0.887            | 0.574          | 0.812         | 0.879        | $2 \cdot 10^6$   | 7              | 220           | 16300        |
| page    | 4378   | 10 | 0.971            | 0.964          | 0.970         | 0.970        | $10^7$           | 15             | 298           | 22400        |
| bidding | 5056   | 9  | 0.993            | 0.981          | 0.985         | 0.993        | $3 \cdot 10^5$   | 13             | 256           | 9360         |
| raisin  | 720    | 7  | 0.894            | 0.869          | 0.879         | 0.886        | $4 \cdot 10^6$   | 15             | 295           | 20900        |
| rice    | 3048   | 7  | 0.938            | 0.933          | 0.934         | 0.937        | $2 \cdot 10^7$   | 15             | 298           | 25500        |
| wilt    | 4339   | 5  | 0.996            | 0.993          | 0.994         | 0.995        | $3 \cdot 10^5$   | 13             | 274           | 11300        |
| bank    | 1097   | 4  | 0.983            | 0.933          | 0.971         | 0.980        | $6 \cdot 10^4$   | 13             | 271           | 7990         |

# DPDT trees generalization

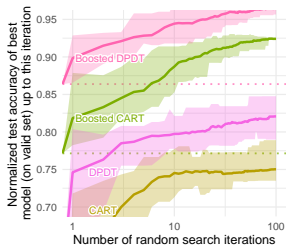


DPDT depth-5 trees vs.  
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# DPDT trees generalization

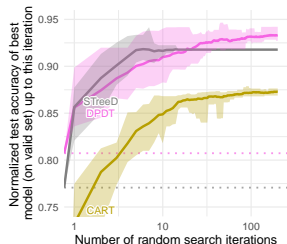


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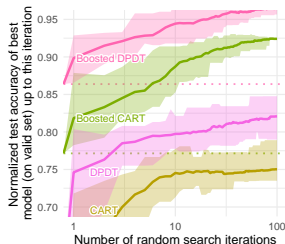


Boosted DPDT vs. Boosted  
CART

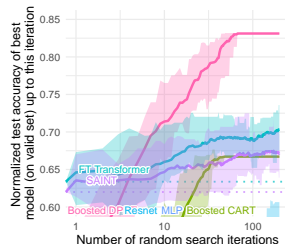
# DPDT trees generalization



DPDT depth-5 trees vs. other depth-5 trees



Boosted DPDT vs. Boosted CART



Boosted DPDT vs. other classifiers





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**A: It depends.**

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[Luo+24]

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# We propose policy unfolding

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```
# Decision tree for Mountain Car
def play(x):
    if x[1] <= -0.2597:
        if x[1] <= -0.6378:
            return 0
        else:
            if x[0] <= -1.0021:
                return 2
            else:
                return 0
    else:
        if x[1] <= -0.0508:
            if x[0] <= 0.2979:
                if x[0] <= 0.0453:
                    return 2
                else:
                    if x[1] <=
-0.2156:
                        return 0
                    else:
                        return 2
            else:
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```

```
# Small ReLU MLP for Pendulum
def play(x):
    h_layer_0_0 = 1.238*x[0]+0.971*x[1]
                +0.430*x[2]+0.933
    h_layer_0_0 = max(0, h_layer_0_0)
    h_layer_0_1 = -1.221*x[0]+1.001
                *x[1]-0.423*x[2]
                +0.475
    h_layer_0_1 = max(0, h_layer_0_1)
    h_layer_1_0 = -0.109*h_layer_0_0
                -0.377*h_layer_0_1
                +1.694
    h_layer_1_0 = max(0, h_layer_1_0)
    h_layer_1_1 = -3.024*h_layer_0_0
                -1.421*h_layer_0_1
                +1.530
    h_layer_1_1 = max(0, h_layer_1_1)
    h_layer_2_0 = -1.790*h_layer_1_0
                +2.840*h_layer_1_1
                +0.658
    y_0 = h_layer_2_0
    return [y_0]
```

# Empirical validation

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- 2 What kind of results we can obtain using our proposed methodology?

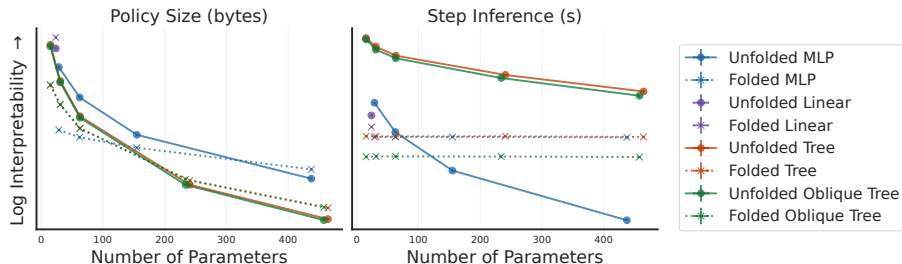
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## Setup

We imitate  $\sim 40000$  expert policies from `stable-baselines3` using various policy classes/nb parameters on various environments.

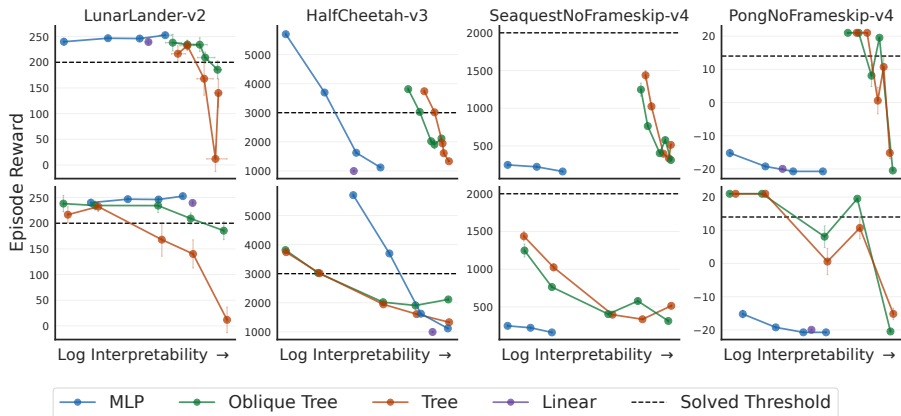


# Result: unfolding policies is necessary to respect consensus



Aggregated policies interpretability on classic control environments

# Result: there is no dominating policy class for all environments



Interpretability-Performance trade-offs. Top row, interpretability is measured with step inference times. Bottom row, the interpretability is measured with policy size.



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- Tree-like policy classes can have good inductive bias (e.g. Atari).
- What about (very) big models?
- Can we use our policy programs as low level skills (hierarchical RL)?

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## My hope

Motivate interpretability by finding a real-world problem where interpretability is *really* necessary [Nag+24].

# Broader perspectives

- **Deep learning:** Can we design deep learning layers that take datasets and output candidate splits?



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- **Teaching:** Can we use unfolded policies (and interpretability) for teaching?

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