Interpretability, Decision Trees, and Sequential Decision Making

Hector Kohler

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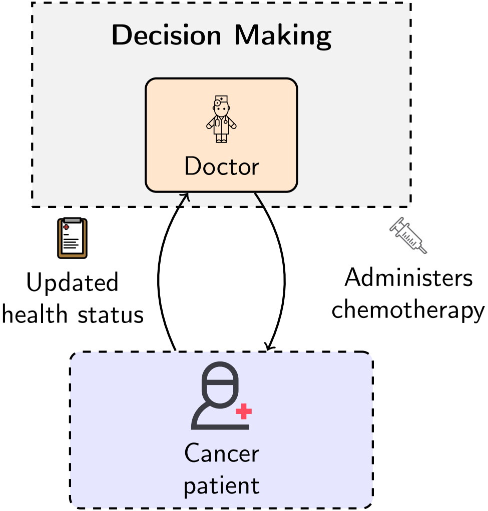
December 4, 2025

Sequential decision making (SDM)



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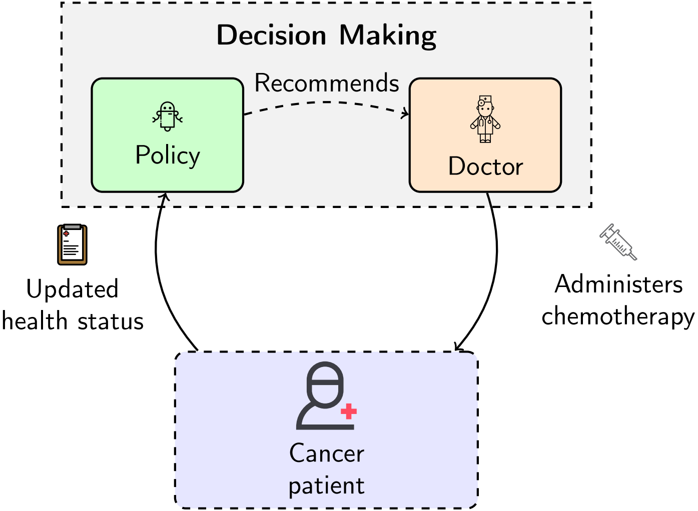
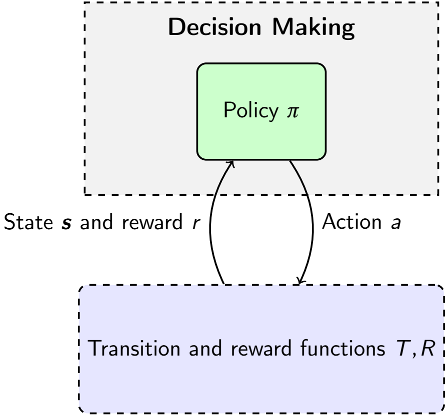


Markov decision processes (MDPs) and reinforcement learning (RL)



Markov decision processes (Puterman 1994).

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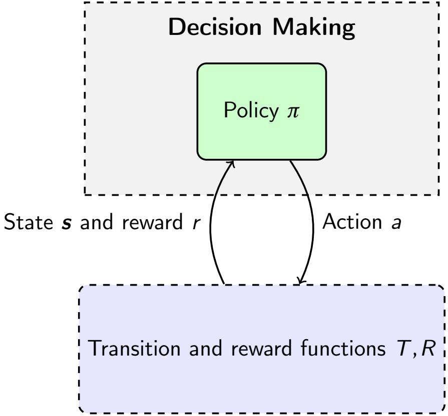

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- RL (Sutton and Barto 1998) aims to find a policy, $\pi : S \to A$ that maximizes:

$$J(\pi) = \mathop{\mathbb{E}}_{\boldsymbol{s}_t \sim T}\left[\sum_{t=0}^{\infty} \gamma^t R(s_t, \pi(s_t))\right]$$

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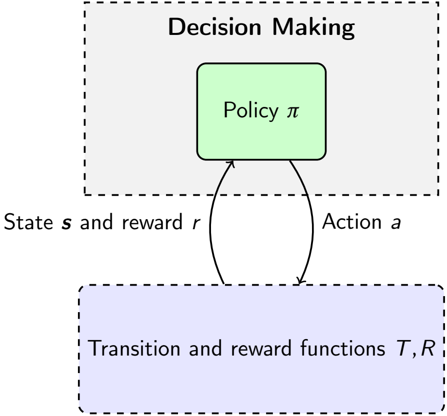

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- Lots of succesful (deep) RL algorithms (Schulman et al. 2017).

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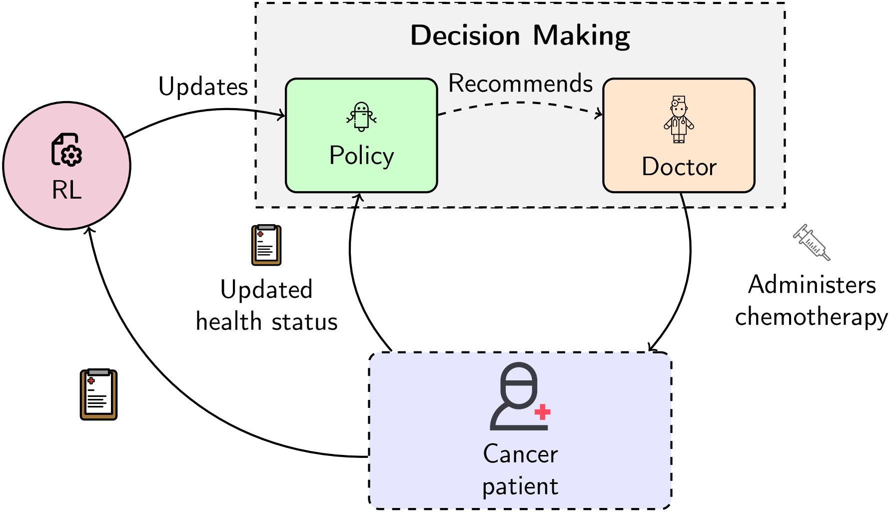

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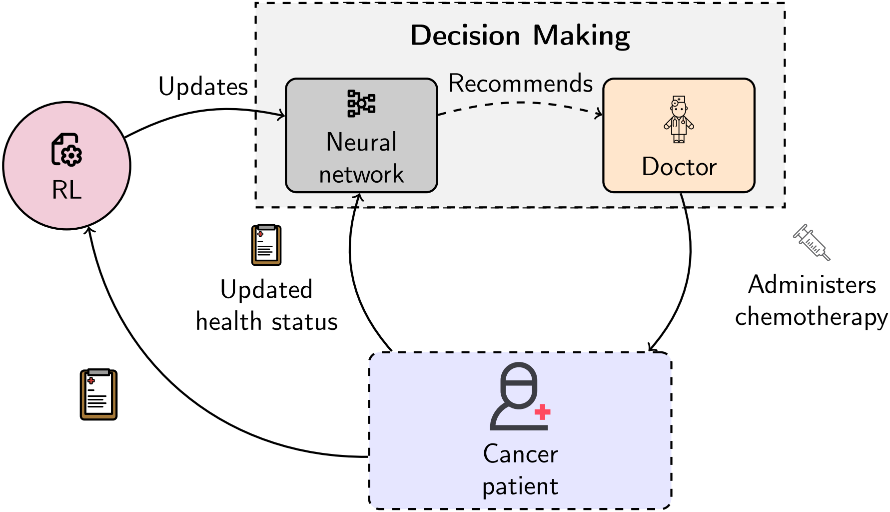

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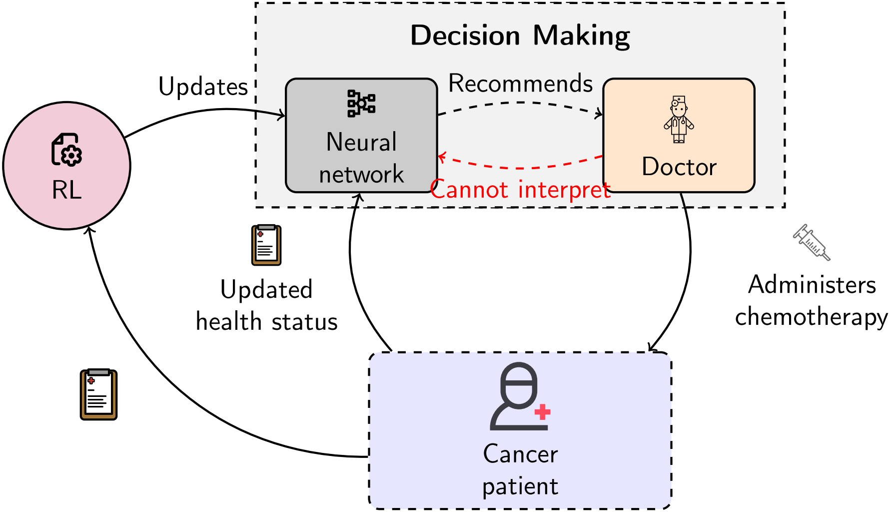

Interpretability in SDM (Glanois et al. 2024).

⚠**No concensus on definition (Lipton 2018)**.

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Saliency maps of different MDP states (Greydanus et al. 2018).

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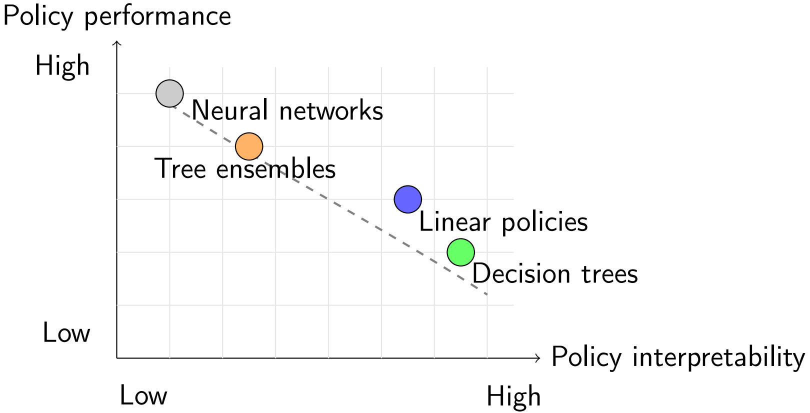
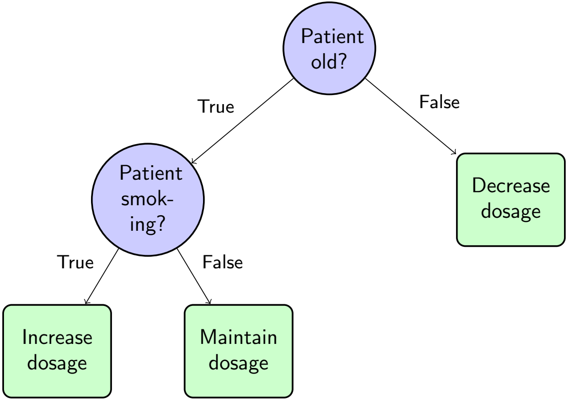


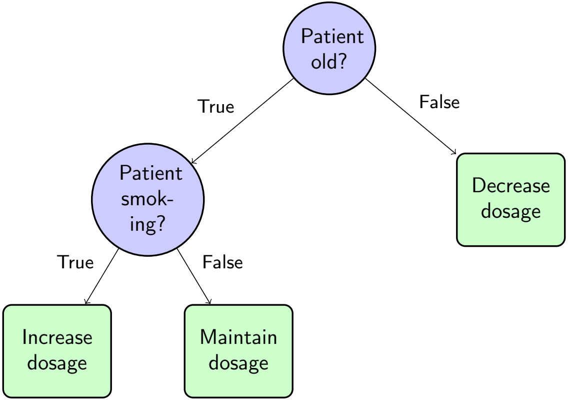
Global interpretation.

Decision trees



A generic decision tree of depth $D = 2$.

Decision trees



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Successful algorithms for classification/regression (Breiman et al. 1984).

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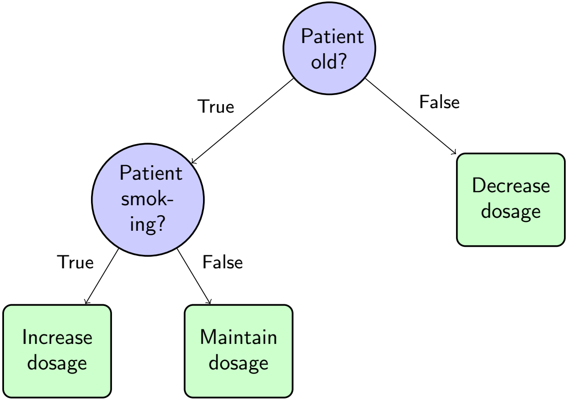

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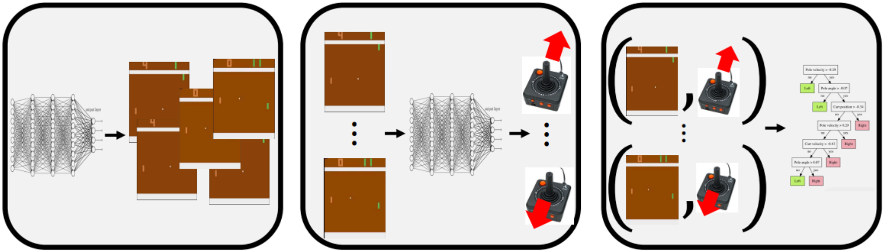
What about SDM?

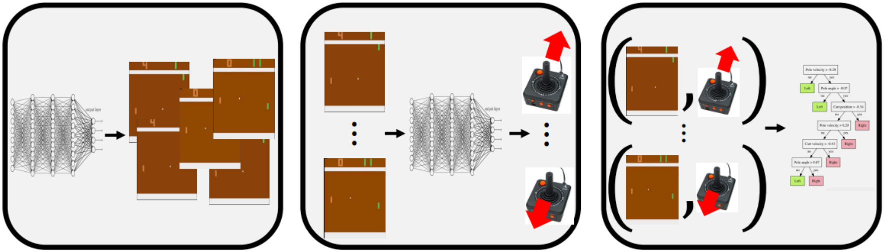
Imitation learning



Step 1: Use NN to generate states

Step 2: Use NN to obtain actions

Step 3: Use supervised learning to train a decision tree



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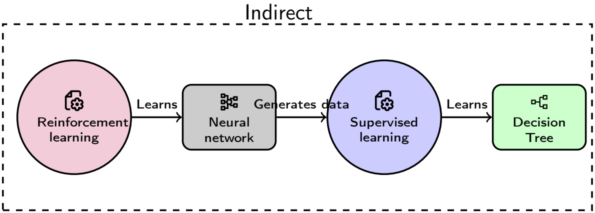
Most research focused on indirect learning of interpretable policies (Bastani, Pu, and Solar-Lezama 2018).

Two ways to get interpretable policies for SDM (Glanois et al. 2024)



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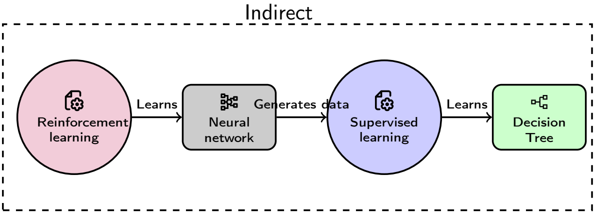

⚠**Policies obtained indirectly optimize a surrogate objective rather than an MDP cumulative rewards.**

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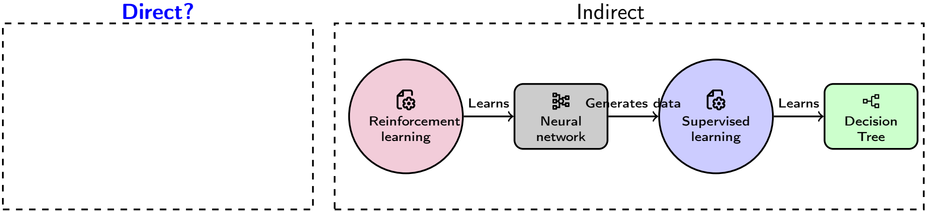

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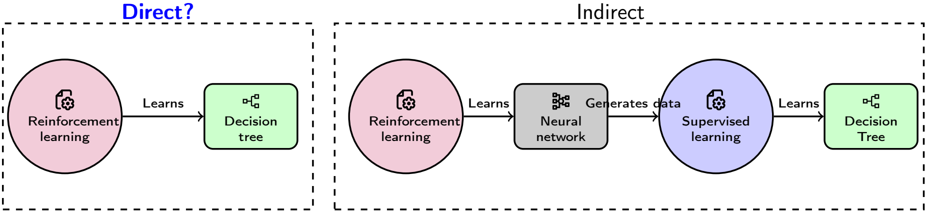

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Contributions

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2. Can we leverage SDM to learn decision trees for classification/regression?
3. How to measure policy interpretability in SDM?

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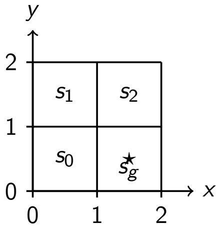

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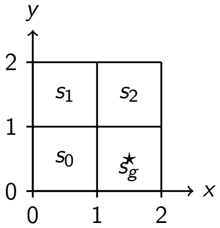

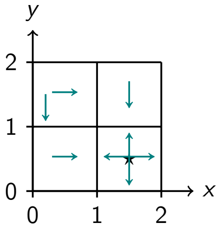

Grid world MDP and decision tree policies

Grid world MDP and optimal actions.

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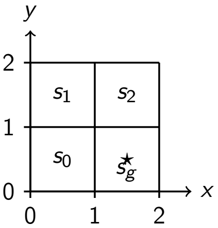

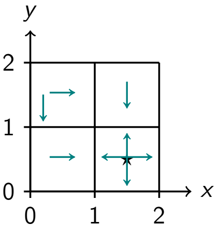

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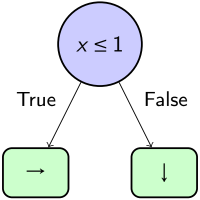

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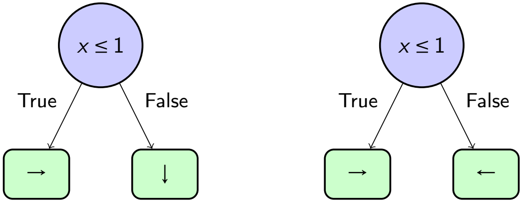

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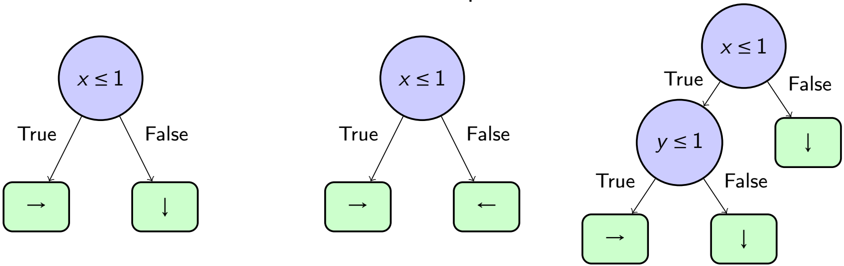

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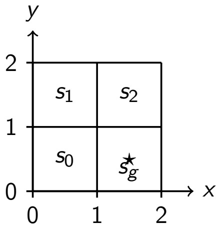

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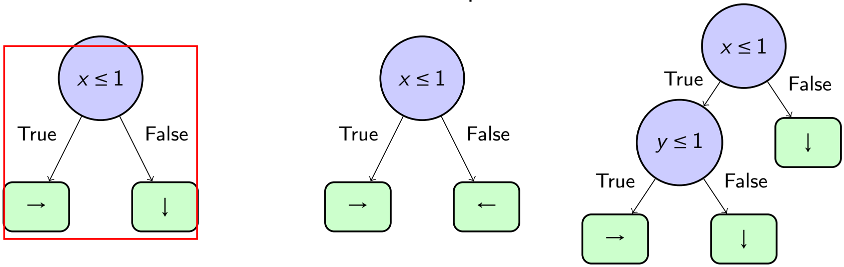
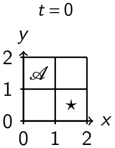


Decision tree policies with different interpretability-performance trade-offs.

Iterative bounding Markov decision processes (Topin et al. 2021)



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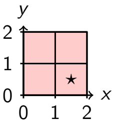
$t = 0$



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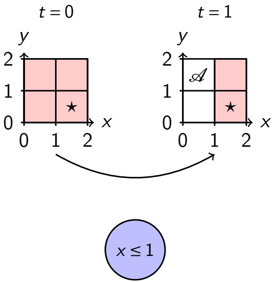

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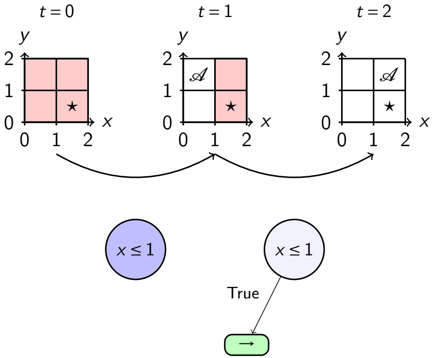

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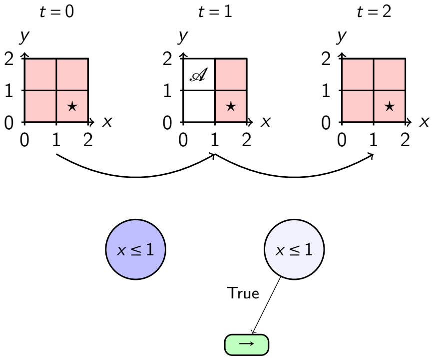

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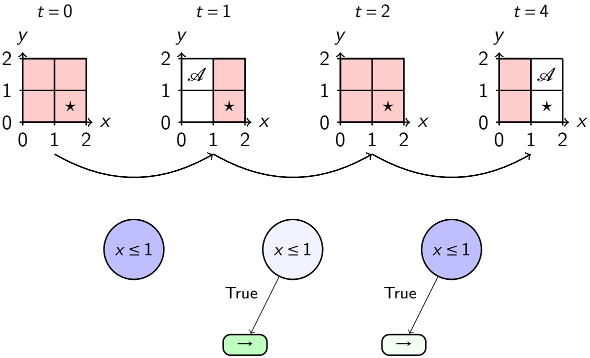

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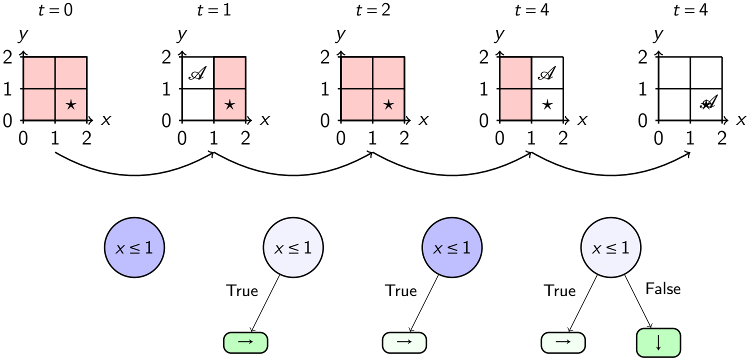

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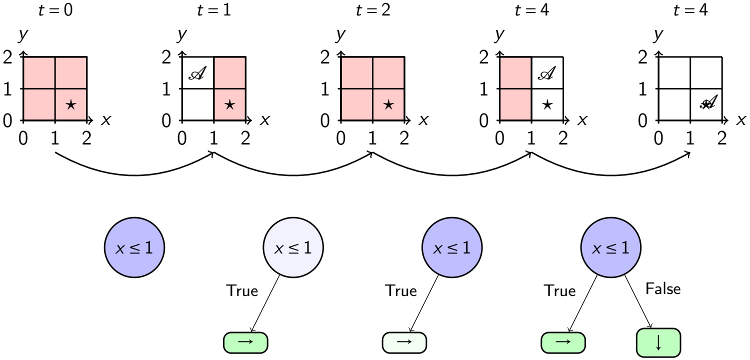

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Given an MDP $\mathscr{M}$ $\langle S, A, R, T\rangle$, an IBMDP is an MDP

$$\langle \overbrace{S \times O}^{\text{State space}}, \underbrace{A \cup A_{info}}_{\text{Action space}}, \overbrace{(R, \zeta)}^{\text{Reward function}}, \underbrace{(T_{info}, T)}_{\text{Transitions}} \rangle$$

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- ⚠**IBMDP policies $\pi_{po} : O \rightarrow A \cup A_{info}$ are decision tree policies for $\mathscr{M}$.**

RL for memoryless policies in POMDPs

RL for memoryless policies

- Finding the best deterministic and memoryless policy in a POMDP is NP-hard (Littman 1994)!
- The best memoryless policy can be stochastic (Singh, Jaakkola, and Jordan 1994).
- Value-based RL converges to sub-optimal solutions.

Asymmetric RL (Pinto et al. 2017)

- Value-based → learns $Q(o, a)$ with TD targets $U(s, a)$ (Baisero, Daley, and Amato 2022).
- Actor-critic → policy gradient on $\pi(o, a)$ using a critic $V(s)$ (Baisero and Amato 2022).
- Supposed to work better for our problem (Lambrechts, Ernst, and Mahajan 2025).

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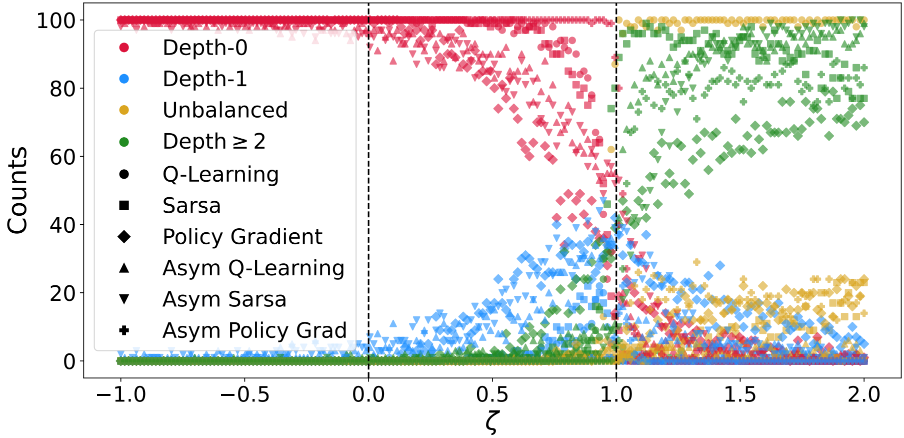
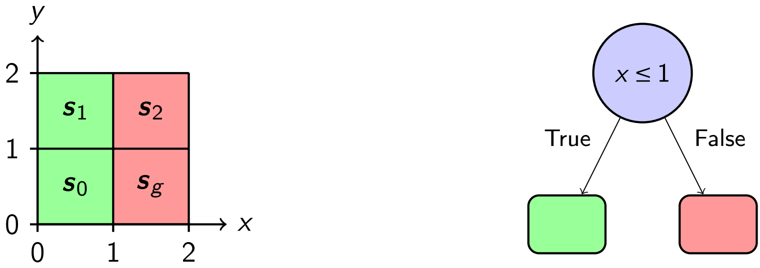
Result: RL cannot retrieve optimal depth-1 trees for the grid world MDP



Distributions of tree policies learned with (asymmetric) RL algorithms as a function of the interpretability reward ζ.

Direct RL of decision trees for classification tasks does not involve partial observability



Classification MDP and the unique optimal depth-1 tree.

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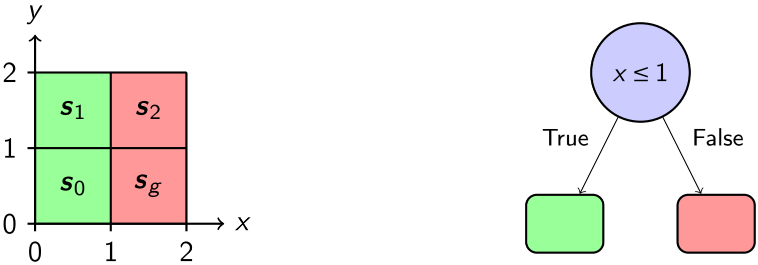
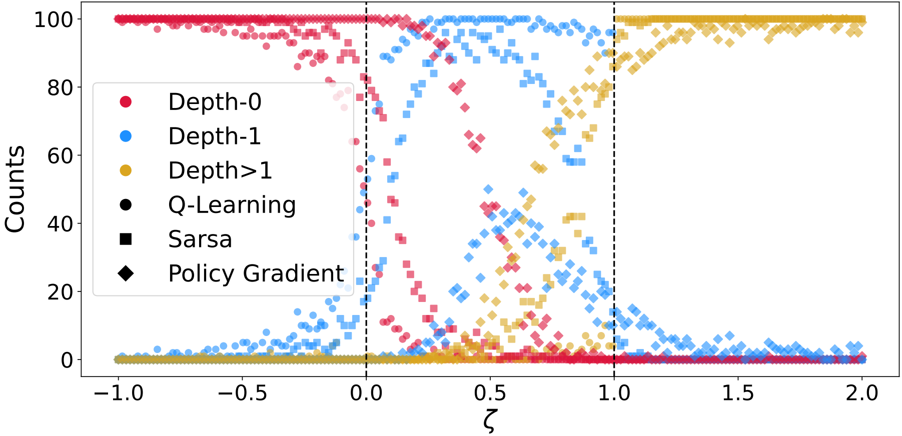


Classification MDP and the unique optimal depth-1 tree.

Partial observations are sufficient statistics of the full states in IBMDPs for classification tasks.

Result: RL can retrieve optimal depth-1 trees for the toy classification MDP



Distributions of tree policies learned with various RL algorithms.

Perspectives for direct RL of decision tree policies.

- It seems that directly learning decision tree trading off interpretability and performances in MDPs can be difficult to achieve because of **partial observability**.
- Should we focus on indirect approaches? Hybrid approaches (Wu et al. 2020)?
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RL can train good decision trees for classification MDPs

Q: Can we leverage SDM to design new decision tree induction algorithms for classification/regression?

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Decision trees in supervised learning

- N data points $\{\boldsymbol{x}_i, y_i\}$. Each $\boldsymbol{x}_i$ is described by p features and has a label $y_i \in \mathscr{Y}$. We want to find a tree of depth at most D, $T \in \mathscr{T}_D$ that minimizes:

$$\mathscr{L}_\alpha(T) = \frac{1}{N}\sum_{i=1}^{N}\ell(y_i, T(x_i)) + \alpha C(T)$$

- Trees **interpretable** and **competitive with neural nets** (Grinsztajn, Oyallon, and Varoquaux 2022).
- Greedy algorithms **sub-optimal accuracy**, but $\boldsymbol{O(2^D)}$ operations (Breiman et al. 1984) .
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Decision tree induction as solving MDPs

Intuition

The induction of a decision tree is made of a sequence of decisions: at each node, we must decide whether it is better to split (a subset of) the training data, or to create a leaf node.

- S: data subsets.
- A: test or leaf nodes that can be added to the tree.
- R: interpretability term $-\alpha$ and misclassifications.
- T: node traversals.

Proposition (Objective Equivalence)

Let π be a deterministic policy of the MDP. Then $J_\alpha(\pi) = -\mathscr{L}_\alpha(E(\pi, s_0))$ where E is an algorithm that extracts a decision tree from π (Topin et al. 2021).

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Controlling the time complexity of decision tree induction

- Greedy algorithms consider only one candidate action in each state which is the test that minimizes some impurity criterion
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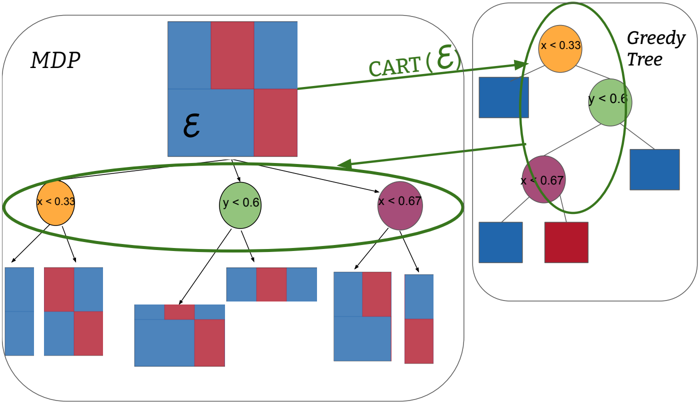

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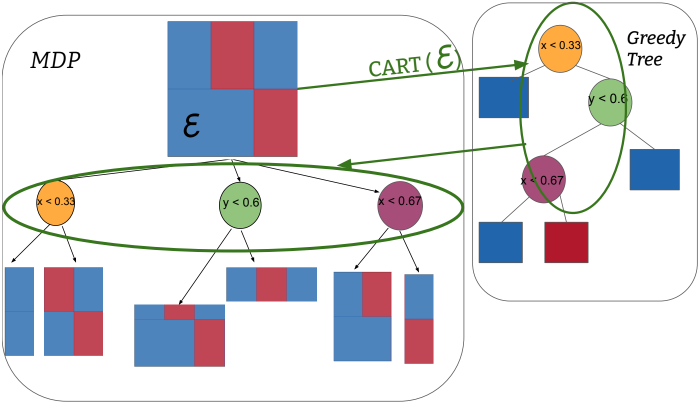

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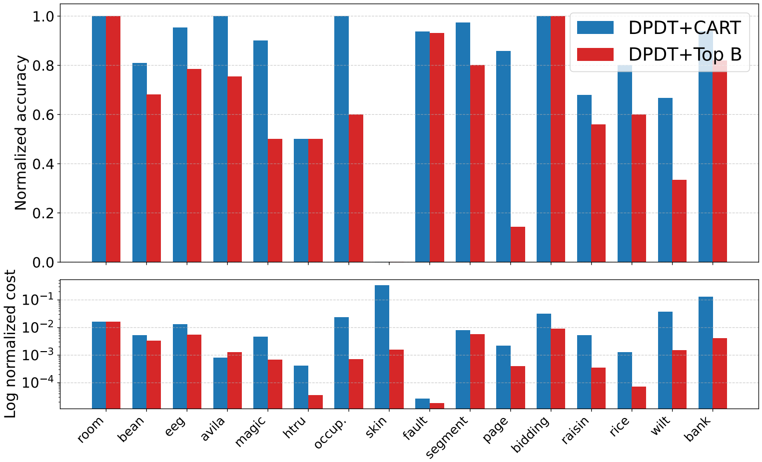
Fast like greedy trees, accurate like optimal trees



Train accuracies against cost for detph-3 trees.

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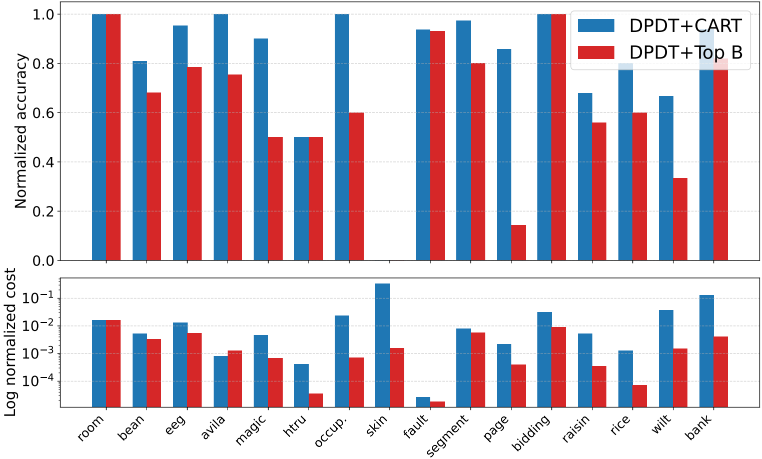

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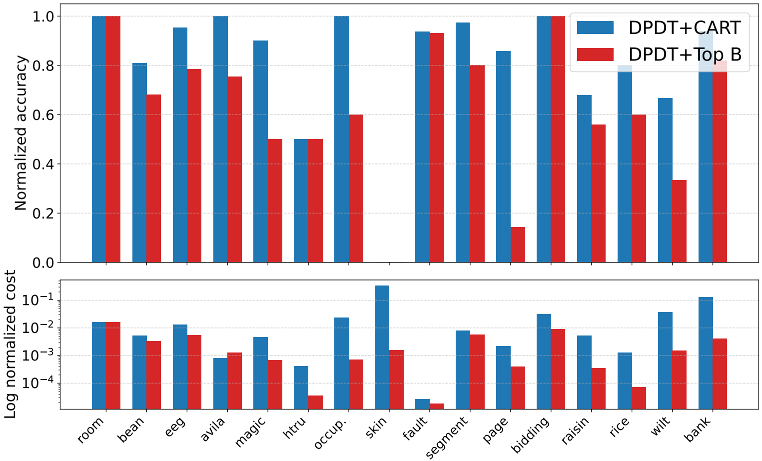

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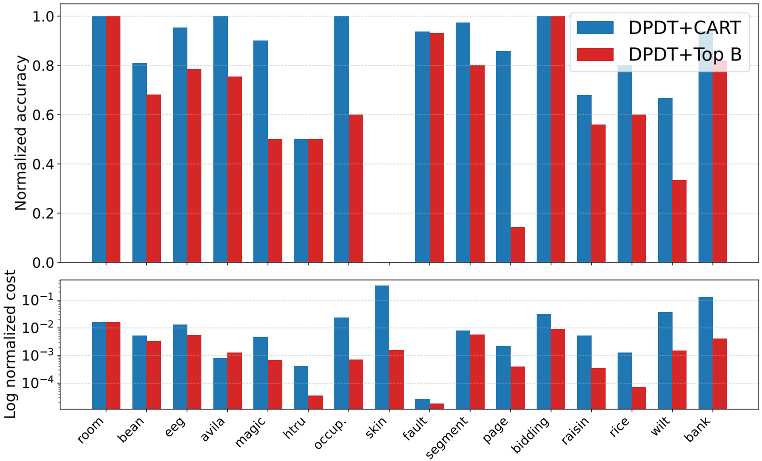

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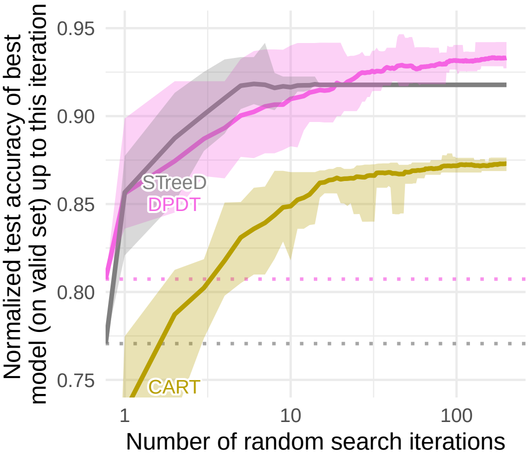

DPDT depth-5 trees vs. other detph-5 trees

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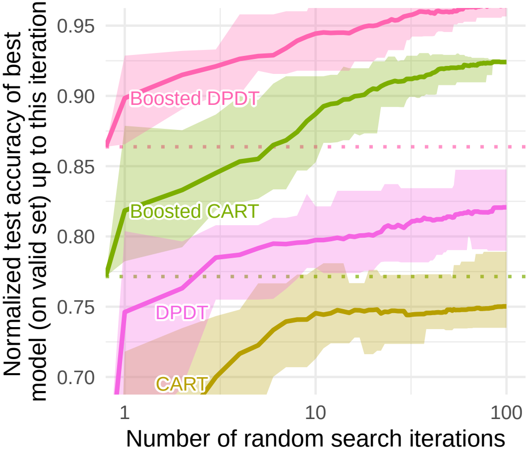

Boosted DPDT vs. Boosted CART

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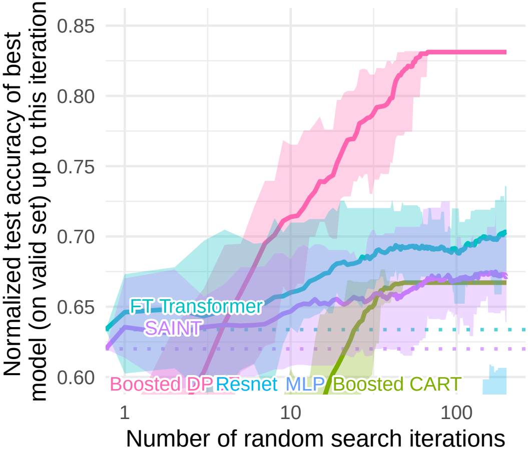

Boosted DPDT vs. other classifiers

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Let us take a step back

Q: Are decision trees really the most interpretable model?

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We propose policy unfolding

```
# Small ReLU MLP for Pendulum
def play(x):
    h_layer_0_0 = 1.68 * x[0] + -0.69 * x[1] + -0.74 * x[2] + -1.40
    h_layer_0_0 = max(0.0, h_layer_0_0)
    h_layer_0_1 = 0.20 * x[0] + 0.29 * x[1] + -0.021 * x[2] + 1.25
    h_layer_0_1 = max(0.0, h_layer_0_1)
    h_layer_0_2 = 0.33 * x[0] + -0.57 * x[1] + 0.47 * x[2] + 1.94
    h_layer_0_2 = max(0.0, h_layer_0_2)
    h_layer_0_3 = 1.39 * x[0] + 0.94 * x[1] + 0.50 * x[2] + -1.13
    h_layer_0_3 = max(0.0, h_layer_0_3)
    h_layer_1_0 = 1.16 * h_layer_0_0 + -1.59 * h_layer_0_1 + 0.95 * h_layer_0_2 +
       -1.22 * h_layer_0_3 + -0.54
    h_layer_1_0 = max(0.0, h_layer_1_0)
    h_layer_1_1 = -0.55 * h_layer_0_0 + 1.13 * h_layer_0_1 + -0.58 * h_layer_0_2
     + -0.72 * h_layer_0_3 + 1.56
    h_layer_1_1 = max(0.0, h_layer_1_1)
    h_layer_1_2 = 1.10 * h_layer_0_0 + -1.01 * h_layer_0_1 + 0.96 * h_layer_0_2 +
       -2.84 * h_layer_0_3 + -0.02
    h_layer_1_2 = max(0.0, h_layer_1_2)
    h_layer_1_3 = 0.27 * h_layer_0_0 + 0.44 * h_layer_0_1 + 0.39 * h_layer_0_2 +
     0.15 * h_layer_0_3 + -1.24
    h_layer_1_3 = max(0.0, h_layer_1_3)
    h_layer_2_0 = -2.80 * h_layer_1_0 + -0.60 * h_layer_1_1 + 3.07 * h_layer_1_2
     + -1.63 * h_layer_1_3 + -0.36
    y_0 = h_layer_2_0

    return [y_0]
```

Is time/size of unfolded policies a good proxy?

Setup

We imitate ~ 40000 expert policies from `stable-baselines3` using various policy classes/nb parameters on various environments.

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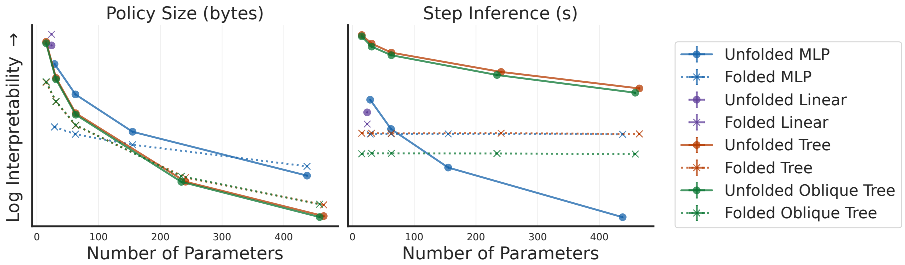

Aggregated policies interpretability on classic control environments

Perspectives

- Beliefs such as "trees are more interpretable than neural networks" should be used with caution.
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Result: for similar problems, RL struggles more when there is partial observability

Success rates over thousands of RL runs with varying hyperparameters when learning different policies in the same IBMDP[1].

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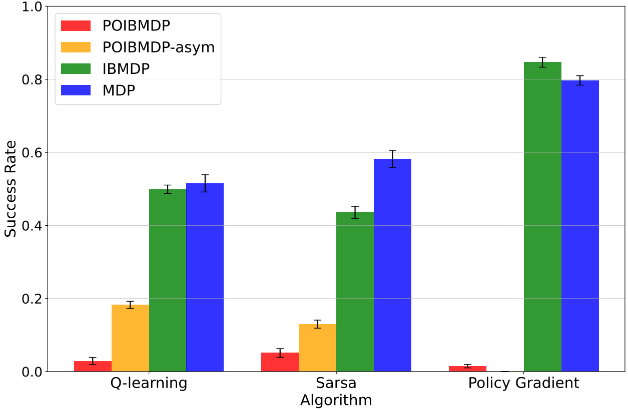

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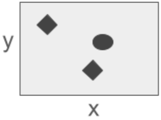
Decision tree induction as solving MDPs



Example of decision tree induction as an MDP.

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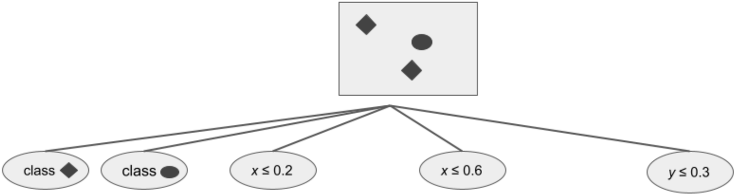

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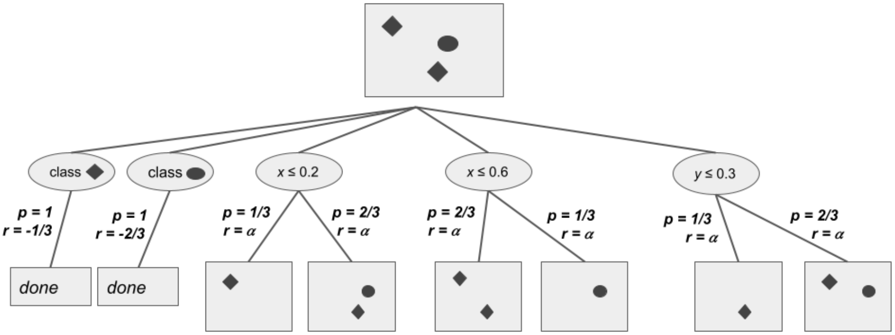

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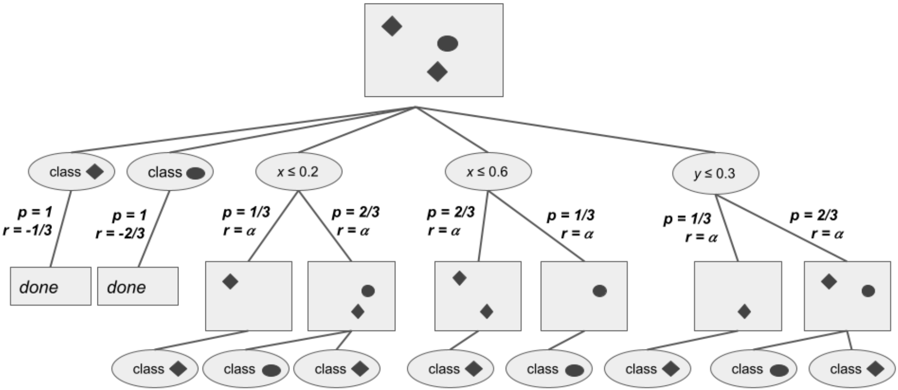

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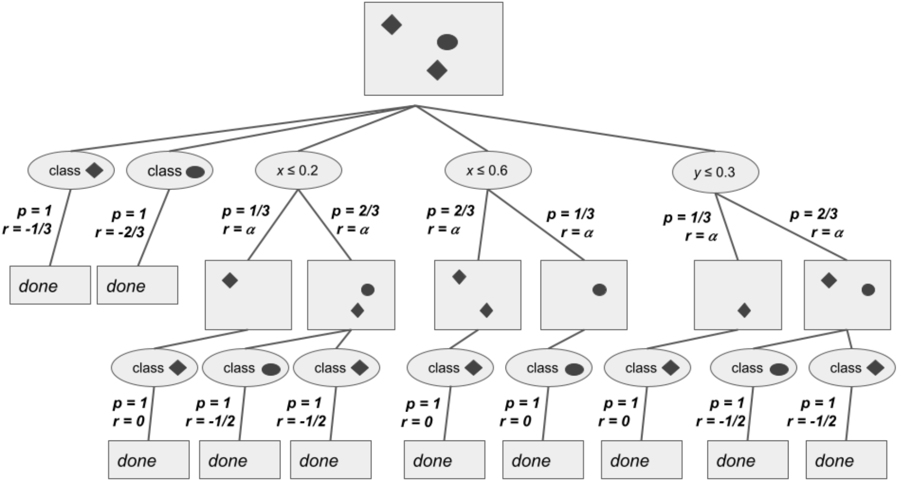

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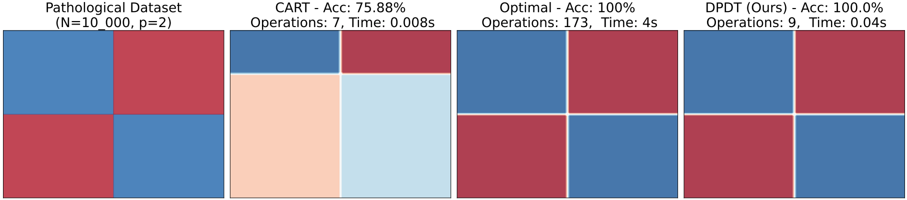
Fast like greedy trees, accurate like optimal trees



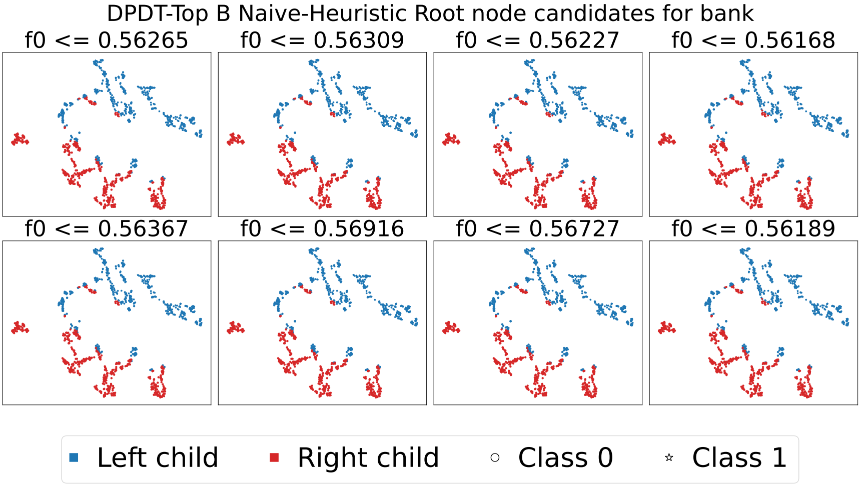
Comparison of greedy, optimal, and DPDT depth-2 trees on the checkersboard dataset.

Fast like greedy trees, accurate like optimal trees

Comparison of accuracies and operations for depth-3 trees.

	Accuracy						Operations					
Dataset	Opt	Greedy	DPDT				Opt	Greedy	DPDT			
			CART$^-$	CART$^+$	TopB$^-$	TopB$^+$			CART$^-$	CART$^+$	TopB$^-$	TopB$^+$
room	**0.992**	0.968	0.991	**0.992**	0.990	**0.992**	10^6	15	286	16100	111	16100
bean	**0.871**	0.777	0.812	0.853	0.804	0.841	$5 \cdot 10^6$	15	295	25900	112	16800
eeg	**0.708**	0.666	0.689	0.706	0.684	0.699	$2 \cdot 10^6$	13	289	26000	95	11000
avila	**0.585**	0.532	0.574	**0.585**	0.563	0.572	$3 \cdot 10^7$	9	268	24700	60	38900
magic	**0.831**	0.801	0.822	0.828	0.807	0.816	$6 \cdot 10^6$	15	298	28000	70	4190
htru	**0.981**	0.979	0.979	0.980	0.979	0.980	$6 \cdot 10^7$	15	295	25300	55	2180
occup.	**0.994**	0.989	0.991	**0.994**	0.990	0.992	$7 \cdot 10^5$	13	280	16300	33	510
skin	**0.969**	0.966	0.966	0.966	0.966	0.966	$7 \cdot 10^4$	15	301	23300	20	126
fault	**0.682**	0.553	0.672	0.674	0.672	0.673	$9 \cdot 10^8$	13	295	24200	111	16800
segment	**0.887**	0.574	0.812	0.879	0.786	0.825	$2 \cdot 10^6$	7	220	16300	68	11400
page	**0.971**	0.964	0.970	0.970	0.964	0.965	10^7	15	298	22400	701	4050
bidding	**0.993**	0.981	0.985	**0.993**	0.985	**0.993**	$3 \cdot 10^5$	13	256	9360	58	2700
raisin	**0.894**	0.869	0.879	0.886	0.875	0.883	$4 \cdot 10^6$	15	295	20900	48	1440
rice	**0.938**	0.933	0.934	0.937	0.933	0.936	$2 \cdot 10^7$	15	298	25500	49	1470
wilt	**0.996**	0.993	0.994	0.995	0.994	0.994	$3 \cdot 10^5$	13	274	11300	33	465
bank	**0.983**	0.933	0.971	0.980	0.951	0.974	$6 \cdot 10^4$	13	271	7990	26	256

CART generates more diverse splits than Top B



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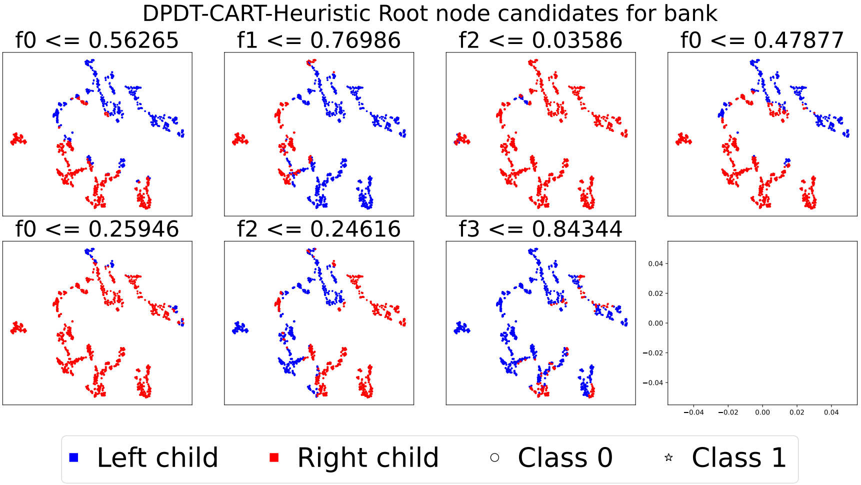

Why generating candidate splits with CART?

Theorem (DPDT trees are not worse than greedy trees)

The greedy tree is always a solution of the MDPs we solve. Because we solve the MDPs exactly with DP, if the greedy tree is the best solution, DPDT will find it.

Theorem (DPDT trees can be strictly better than greedy trees)

There exist a depth budget D and a dataset for which DPDT trees are strictly better than greedy trees.[a]

[a] cf. checkersboard dataset.

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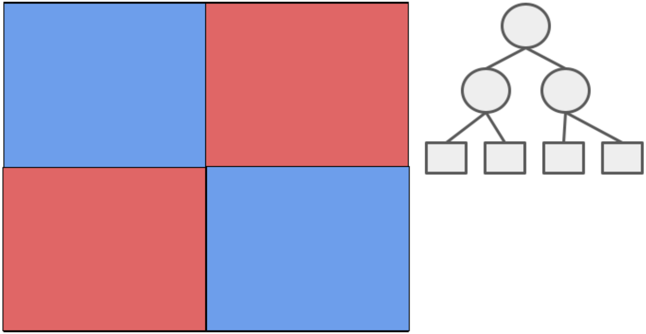

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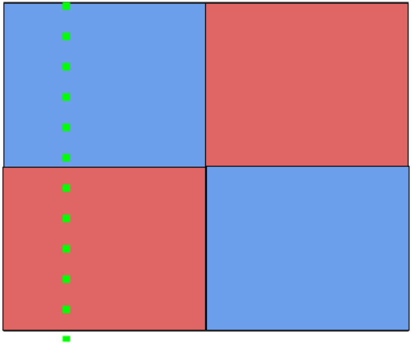

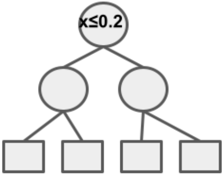

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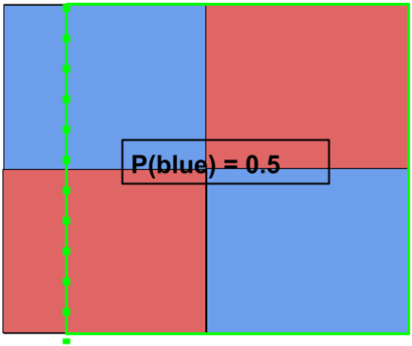

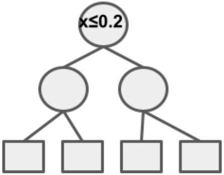

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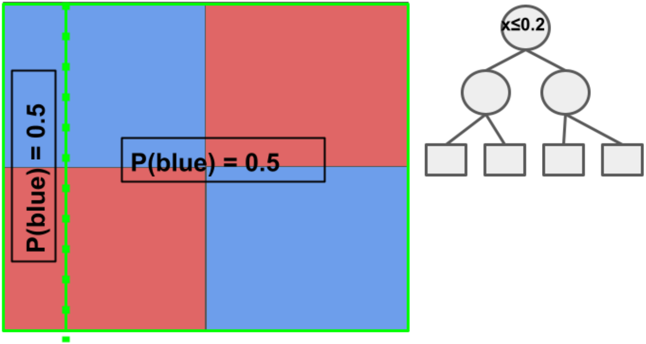

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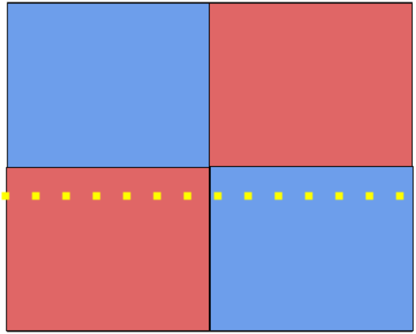

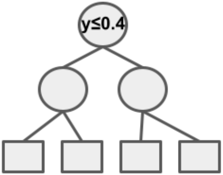

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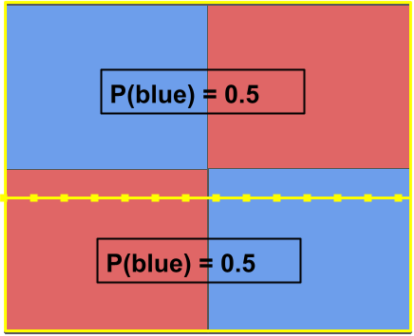

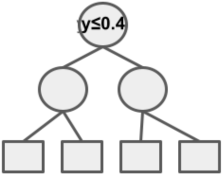

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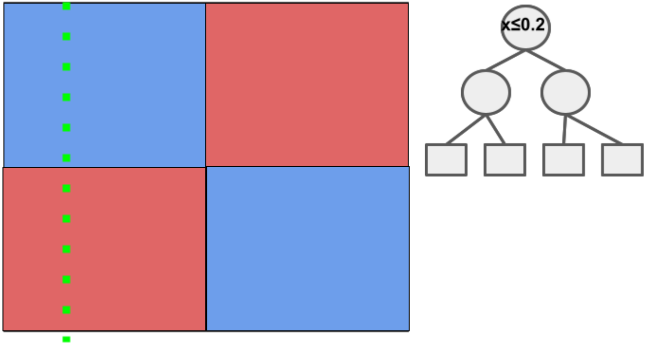

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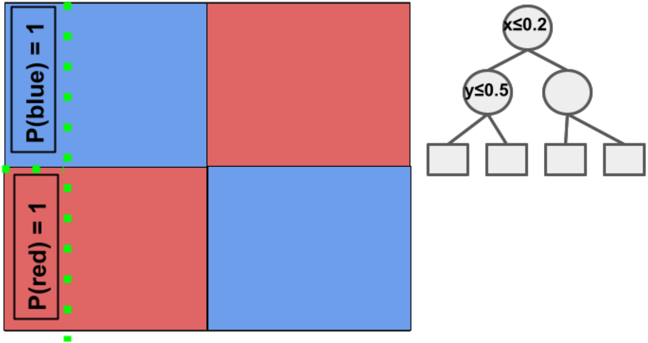

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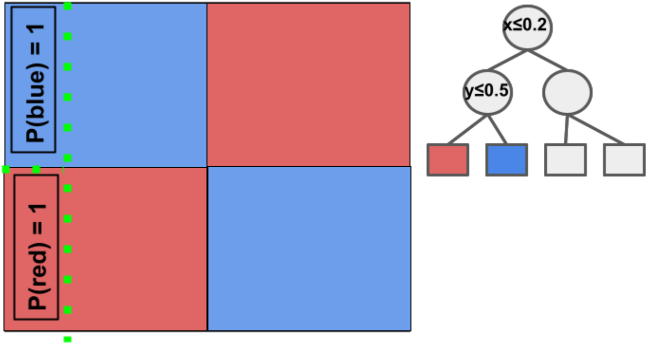

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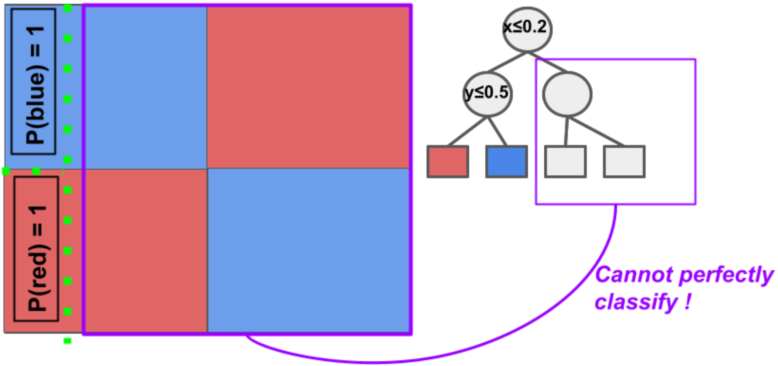

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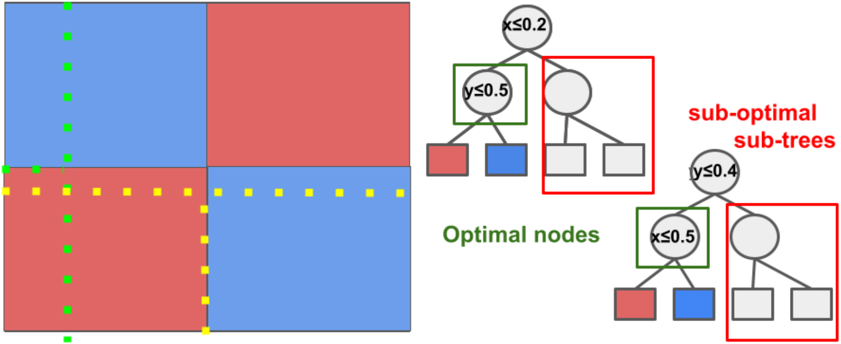

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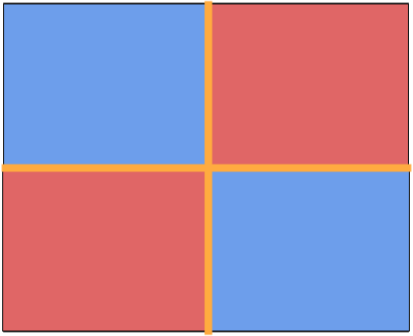
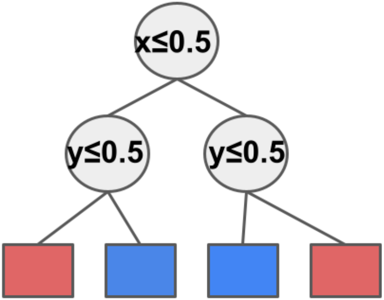
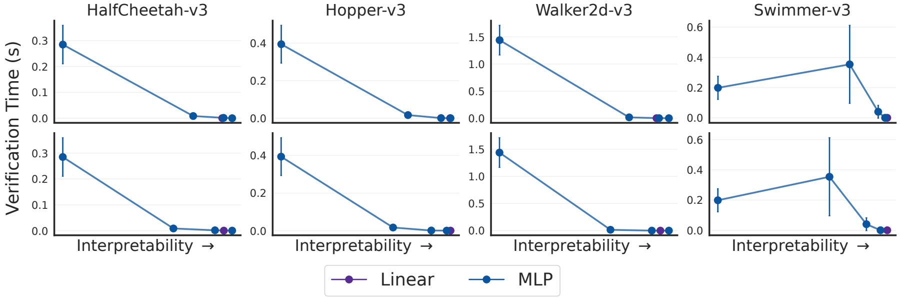


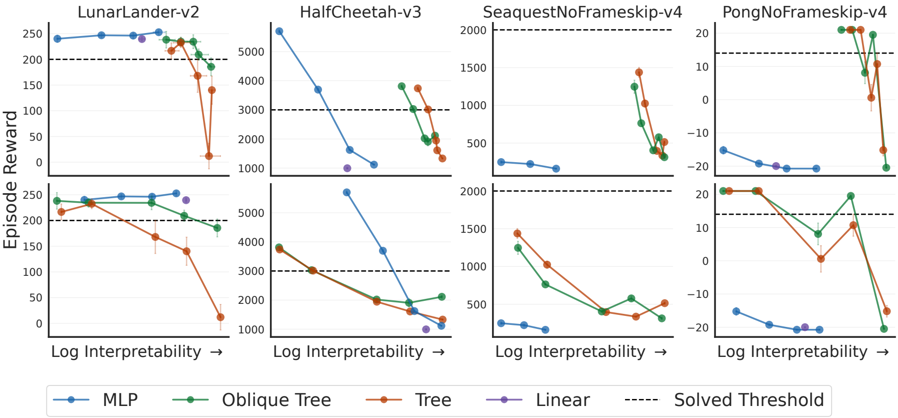


Result: verification time does scale with step inference time



Verification time as a function of policy interpretability. Top row, interpretability is measured with step inference times. Bottom row, the interpretability is measured with policy size.

Result: there is no dominating policy class for all environments



Interpretability-Performance trade-offs. Top row, interpretability is measured with step inference times. Bottom row, the interpretability is measured with policy size.

We propose policy unfolding

```
# Decision tree for Mountain Car
def play(x):
    if x[1] <= -0.2597:
        if x[1] <= -0.6378:
            return 0
        else:
            if x[0] <= -1.0021:
                return 2
            else:
                return 0
    else:
        if x[1] <= -0.0508:
            if x[0] <= 0.2979:
                if x[0] <= 0.0453:
                    return 2
                else:
                    if x[1] <=
  -0.2156:
                        return 0
                    else:
                        return 2
            else:
                return 0
        else:
            return 2
```

```
# Small ReLU MLP for Pendulum
def play(x):
    h_layer_0_0 = 1.238*x[0]+0.971*x
      [1]
                  +0.430*x[2]+0.933
    h_layer_0_0 = max(0, h_layer_0_0
     )
    h_layer_0_1 = -1.221*x[0]+1.001
                  *x[1]-0.423*x[2]
                  +0.475
    h_layer_0_1 = max(0, h_layer_0_1
     )
    h_layer_1_0 = -0.109*h_layer_0_0
                  -0.377*h_layer_0_1
                  +1.694
    h_layer_1_0 = max(0, h_layer_1_0
     )
    h_layer_1_1 = -3.024*h_layer_0_0
                  -1.421*h_layer_0_1
                  +1.530
    h_layer_1_1 = max(0, h_layer_1_1
     )

    h_layer_2_0 = -1.790*h_layer_1_0
                  +2.840*h_layer_1_1
                  +0.658
    y_0 = h_layer_2_0
    return [y_0]
```