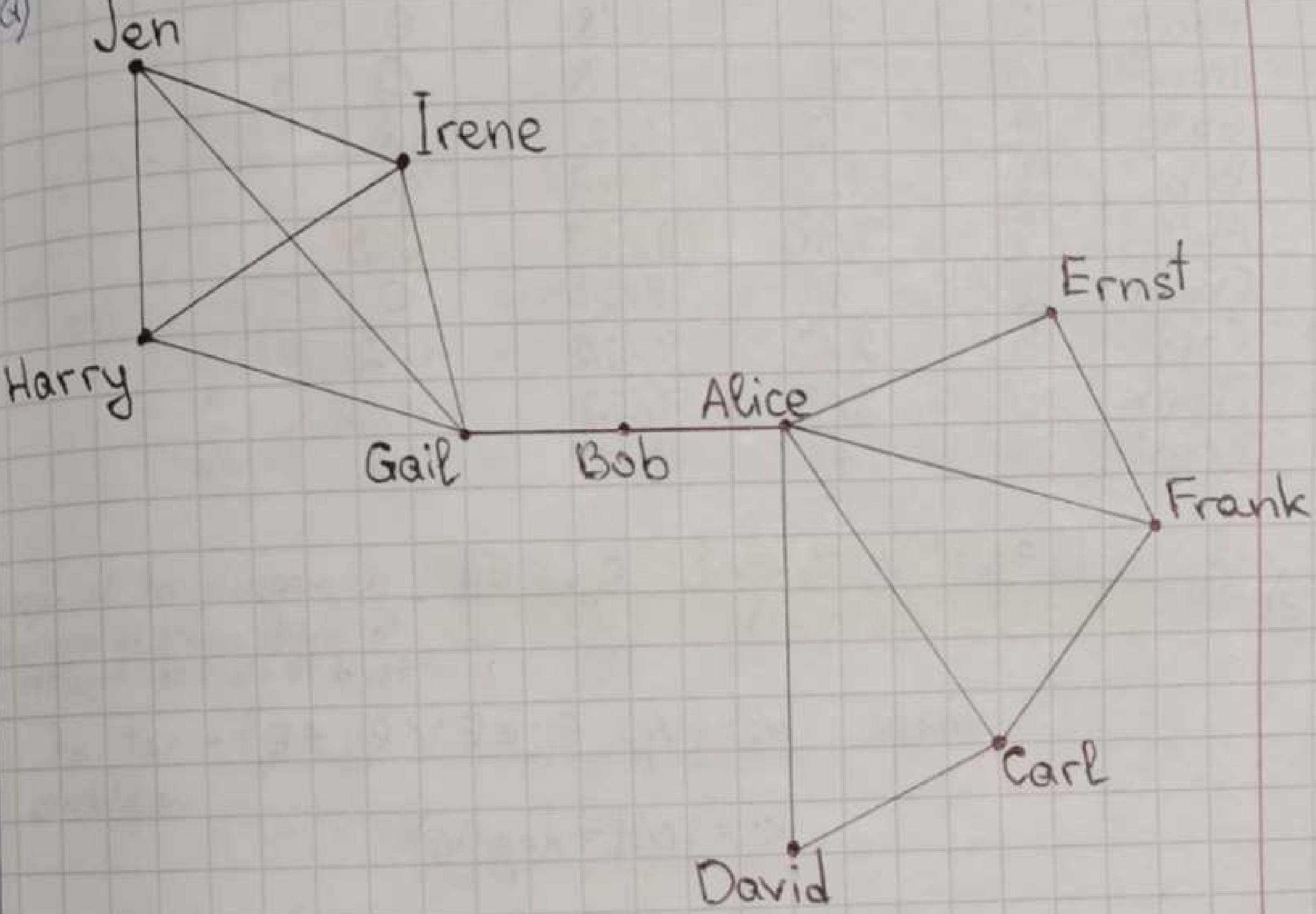
(1)

a)



b) $|V| = 10$, $|E| = 15$

c) $\text{Density} = \dfrac{|E|}{\text{maximum number of edges}} = \dfrac{|E| \cdot 2}{N(N-1)}$

$\text{Max. numb. of edges} = \binom{N}{2} = \dfrac{N \cdot (N-1)}{2}$

$\text{Density} = \dfrac{\cancel{15}^{5} \cdot \cancel{2}^{1}}{\cancel{10}_{5} \cdot \cancel{9}_{3}} = \dfrac{1}{3}$

d) deg (v) = number of edges connected to a vertex

„Alice" is the most central node according to the "degree (has the highest degree)

	Degree	Clustering	Closeness	Betweenness
Jen	3	1	3/8	0
Harry	3	1	3/8	0
Irene	3	1	3/8	0
Gail	4	1/2	1/2	18
Bob	2	0	9/16	20
Alice	5	3/10	9/16	22
David	2	1	9/23	0
Carl	3	2/3	9/22	1/2
Frank	3	2/3	9/22	1/2
Ernst	2	1	9/23	0

e) $C_i = \frac{\delta \cdot |\{e_{jk} : v_j, v_k \in N_i, \ e_{jk} \in E\}|}{k_i(k_i - 1)}$ - a measure of the degree to which nodes in a graph tend to cluster together

where $N_i = \{v_j : e_{ij} \in E \vee e_{ji} \in E\}$ - set of neighbors

$k_i = |N_i| = \deg(v_i)$

$$\delta = \begin{cases} 2, & \text{if } G\text{- undirected} \\ 1, & \text{if } G\text{- directed} \end{cases}$$

$\{e_{jk} : v_j, v_k \in N_i, \ e_{jk} \in E\}$ - the number of edges between neighbors

$C_j = \frac{\cancel{2}\cdot\cancel{3}}{\cancel{3}\cdot\cancel{2}} = 1$

$C_h = \frac{\cancel{2}\cdot\cancel{3}}{\cancel{3}\cdot\cancel{2}} = 1$

$C_i = \frac{\cancel{2}\cdot\cancel{3}}{\cancel{3}\cdot\cancel{2}} = 1$

$C_g = \frac{\cancel{2}\cdot\cancel{3}}{{}_2\cancel{4}\cdot\cancel{3}} = \frac{1}{2}$

$C_b = \frac{2\cdot 0}{2\cdot 1} = 0$

$C_a = \frac{\cancel{2}\cdot 3}{5\cdot\cancel{4}_2} = \frac{3}{10}$

$C_d = \frac{\cancel{2}\cdot 1}{\cancel{2}\cdot 1} = 1$

$C_c = \frac{\cancel{2}\cdot 2}{3\cdot\cancel{2}} = \frac{2}{3}$

$C_f = \frac{2\cdot\cancel{2}}{3\cdot\cancel{2}} = \frac{2}{3}$

$C_e = \frac{\cancel{2}\cdot 1}{\cancel{2}\cdot 1} = 1$

$\langle C \rangle = \frac{1}{N}\sum_i C_i$ - may be interpreted as the probability that two neighbors of a randomly selected node link to each other

$$\langle C \rangle = \frac{1+1+1+\frac{1}{2}+0+\frac{3}{10}+1+\frac{2}{3}+\frac{2}{3}+1}{10} = \frac{\cancel{214}^{107}}{\cancel{30}_{15}\cdot 10} = \frac{107}{150} \approx 0{,}713$$

f) $C(v) = \frac{N-1}{\sum_y d(y,v)}$ - the closeness centrality

where $d(y,v)$ - the length of the shortest path between y, v

$$C(j) = \frac{9}{1\cdot 3+2+3+4\cdot 4} = \frac{\cancel{9}^{3}}{\cancel{24}_{8}} = \frac{3}{8}$$

$$C(h) = \frac{9}{1\cdot 3+2+3+4\cdot 4} = \frac{\cancel{9}^{3}}{\cancel{24}_{8}} = \frac{3}{8}$$

$$C(i) = \frac{9}{1\cdot 3+2+3+4\cdot 4} = \frac{\cancel{9}^{3}}{\cancel{24}_{8}} = \frac{3}{8}$$

$$C(g) = \frac{9}{1\cdot 4+2+3\cdot 4} = \frac{\cancel{9}^{1}}{\cancel{18}_{2}} = \frac{1}{2}$$

$$C(b) = \frac{9}{1\cdot 2+2\cdot 7} = \frac{9}{16}$$

$$C(a) = \frac{9}{1\cdot 5+2+3\cdot 3} = \frac{9}{16}$$

$$C(d) = \frac{9}{1\cdot 2+2\cdot 3+3+4\cdot 3} = \frac{9}{23}$$

$$C(c) = \frac{9}{1\cdot 3+2\cdot 2+3+4\cdot 3} = \frac{9}{22}$$

$$C(f) = \frac{9}{1\cdot 3+2\cdot 2+3+4\cdot 3} = \frac{9}{22}$$

$$c(e) = \frac{9}{1 \cdot 2 + 2 \cdot 3 + 3 + 4 \cdot 3} = \frac{9}{23}$$

„Bob" and „Alice" are the most central node according to the closeness centrality (have the highest coefficient)

g) $B(v) = \sum_{v \neq s \neq t} \frac{\sigma_{st}(v)}{\sigma_{st}}$ - the betweenness centrality

where σ_{st} - number of the shortest paths between s and t

$\sigma_{st}(v)$ - number of the shortest paths between s and t that include v

$$B(j) = \frac{0}{1} + \frac{0}{1} + \frac{0}{1} + \dots = 0$$

$$B(h) = \frac{0}{1} + \frac{0}{1} + \dots = 0$$

$$B(i) = \frac{0}{1} + \frac{0}{1} + \dots = 0$$

same as for Harry

$$B(g) = \underbrace{\overset{h-b}{\frac{1}{1}} + \overset{h-a}{\frac{1}{1}} + \overset{h-d}{\frac{1}{1}} + \overset{h-c}{\frac{1}{1}} + \overset{h-f}{\frac{1}{1}} + \overset{h-e}{\frac{1}{1}}}_{h} + \underbrace{\frac{1}{1} \cdot 6}_{j} + \underbrace{\frac{1}{1} \cdot 6}_{i} + 0 = 18$$

$$B(b) = \underbrace{\overset{h-a}{\frac{1}{1}} + \overset{h-d}{\frac{1}{1}} + \overset{h-c}{\frac{1}{1}} + \overset{h-f}{\frac{1}{1}} + \overset{h-e}{\frac{1}{1}}}_{h} + \underbrace{\frac{1}{1} \cdot 5}_{j} + \underbrace{\frac{1}{1} \cdot 5}_{i} + \underbrace{\frac{1}{1} \cdot 5}_{g} + 0 = 20$$

$$B(a) = \underbrace{\overset{d-c}{\frac{0}{1}} + \overset{d-f}{\frac{1}{2}} + \overset{d-e}{\frac{1}{1}} + \overset{d-b}{\frac{1}{1}} + \overset{d-g}{\frac{1}{1}} + \overset{d-h}{\frac{1}{1}} + \overset{d-j}{\frac{1}{1}} + \overset{d-i}{\frac{1}{1}}}_{d} + \underbrace{\overset{c-e}{\frac{1}{2}} + \frac{1}{1} \cdot 5}_{c} +$$

$$+ \underbrace{\frac{1}{1} \cdot 5}_{f} + \underbrace{\frac{1}{1} \cdot 5}_{e} + 0 = 22$$

$$B(d) = \frac{0}{1} + \cdots = 0$$

$$B(c) = \underbrace{\overset{d-a}{\frac{0}{1}} + \overset{d-f}{\frac{1}{2}} + \overset{d-e}{\frac{0}{1}}}_{d} + 0 = \frac{1}{2}$$

$$B(f) = \underbrace{\frac{0}{1}}_{d} + \underbrace{\overset{c-a}{\frac{0}{1}} + \overset{c-e}{\frac{1}{2}}}_{c} + 0 = \frac{1}{2}$$

$$B(e) = \frac{0}{1} + \cdots = 0$$

"Alice" is the most central node according to the betweenness centrality (has the highest coefficient)