Math 531: Problem Set 6

Due: Wednesday, 03 April 2024

Directions: Work all the problems below. Give complete answers and provide your reasoning when you think it is appropriate or useful to do so. In any case, follow any problem-specific instructions that are given. Note that these problems are all proofs, in which both the mathematics and the valid, correct composition and presentation are being graded. So, be careful and thorough in both of these aspects. Please do not submit first drafts of arguments, since these will often contain superfluous material. Edit with an eye for conciseness; write only what is necessary to establish the result. Use complete sentences and the appropriate balance of verbal and non-verbal language. There are numerous examples of such in the textbook – do not ignore these! Partial credit will be given where appropriate.

1. PROVE: the product of an irrational number and a nonzero rational number is irrational.
2. PROVE: $\sqrt{2} + \sqrt{3}$ is an irrational number.
3. PROVE: there do not exist three distinct real numbers a, b and c such that all of the numbers $a + b + c$, ab, ac, bc and abc are equal.
4. Let a, b, c, d be real numbers. PROVE: at most four of the numbers ab, ac, ad, bc, bd and cd are negative.
5. PROVE: there are no integers $a \geq 2$ and $n \geq 1$ such that $a^2 + 1 = 2^n$.
6. PROVE: there do not exist real numbers a and b in the open interval $(0, 1)$ such that $4a(1 - b) > 1$ and $4b(1 - a) > 1$.