Stanford | ENGINEERING
Aeronautics and Astronautics

Problem Set 5
Total of 150 points
Due on June 10, 2024, at 10:00 pm, Pacific Time

1) Unidentified Flying Object (30 Points + 5 Bonus)
An amateur astronomer (shown in the picture) has discovered an unknown object in deep space and has determined its position and velocity with respect to the Ecliptic Sun-centric J2000 reference frame. Here are the positions and velocities with respect to the Ecliptic Sun-centric J2000 reference frame:

Epoch	2024-May-29 00:00:00.0000 TDB
X (km)	-5.654521158368091E+07
Y (km)	-4.141584370434758E+09
Z (km)	-2.516187548793956E+09
X (km/sec)	1.894813995825346E+00
Y (km/sec)	-2.900052383928433E+01
Z (km/sec)	-1.587424775496980E+01

1.a) Compute the semi-major axis of the object's orbit. Show your calculations.

1.b) Compute the orbit's eccentricity and show your work. Verify that it is consistent with your semi-major axis.

1.c) Classify the orbit (elliptical, parabolic, or hyperbolic). What does this imply about the object's origin?

Bonus (5 Points):
1.d) What is that object?

2) Where is My Car? (35 Points + 5 Bonus)
An Earth-based observation network detects an unknown object traveling through the solar system. Positions and velocities for this unknown object with respect to the Sun-centric J2000 reference frame are given in pset5_input.txt. To extract positions velocities, you can use the following python code:

```
data = np.loadtxt('/PATH/pset5_input.txt', usecols = (1,2,3,4,5,6), delimiter = ',')
x = data[:,0] * 1e3 # we multiply by 1e3 to get meters/sec
y = data[:,1] * 1e3
z = data[:,2] * 1e3
vx = data[:,3] * 1e3
vy = data[:,4] * 1e3
vz = data[:,5] * 1e3
```

You will need to replace "PATH" with your own path to the directory that contains the file. This will give you positions (x, y, and z) and velocities (vx, vy, and vz) in meters and meters per second respectively.

You would also need a time variable for plotting variables as a function of time. You can use the following code for time:

```
import numpy as np
t = np.arange(0, 60, 1)
```

This will give you time in days since 2018-Feb-07 03:00:00 UTC.

2.a) Using coord2elem function provided in canvas (see src/kepler.py), compute all the orbital elements for all 60 times. Plot the time evolution for the semimajor axis, eccentricity and inclination as a function of time. Semimajor axis should be plotted in Astronomical Units or AUs. Inclination should be plotted in degrees. Note that this inclination is referenced to the Earth orbital plane (a.k.a. the ecliptic plane). Is this an unperturbed Keplerian orbit?

Hint: 1 AU = 149,597,870,700 meters, which is the Earth's orbit semimajor axis.

2.b) Compute the pericenter and apocenter distances at the last time step. Compare these distances to the sizes of planetary orbits. What can you say about the orbit of the object with respect to the planets of the Solar System?

Bonus (5 points):
2.c) Identify the object.

3) Mini Project: Earth-Mars Transfer and Mars Surface Imaging
You are part of a mission design team working on a Mars orbiter tasked with high-resolution surface imaging. Your job is to plan the spacecraft's transfer from Earth to Mars, place the orbiter into an appropriate orbit and develop an imaging strategy balancing resolution and surface coverage.

Phase 1: Earth-Mars Transfer (30 Points)
Your mission begins with a Hohmann transfer from Earth to Mars. To simplify analysis, we assume both Earth and Mars move in circular, coplanar orbits. The orbital position of Mars and Earth are given by:

$$\begin{aligned} x(t) &= a\cos L(t) \\ y(t) &= a\sin L(t) \end{aligned} \tag{1}$$

where a is the planet's orbital radius, and $L(t)$ is its ecliptic longitude given by:

$$L(t) = n(t - t_0) + L_0 \tag{2}$$

where n is the mean motion of the planet, and L_0 is the ecliptic longitude at t_0. You are provided with the following parameters of the Mars' and Earth's orbits:

a_{Earth}	1.00 AU
a_{Mars}	1.52 AU
t_0	2025-05-27 00:00:00 UTC
$L_{0,Earth}$	245.62°
$L_{0,Mars}$	173.71°

3.1.a) Compute the total Δv required for a Hohmann transfer from Earth to Mars. Assume your transfer goes from the center of Earth to the center of Mars. Note that for this problem we neglect the Mars orbit insertion Δv.

3.1.b) Compute the time of flight for the above Hohmann transfer. Present your answer in Earth days.

3.1.c) Compute the required angle between Earth and Mars ($L_{Earth} - L_{Mars}$) at the time of launch from the Earth for a successful Hohmann transfer to Mars. When should be the launch to have such an angle between the Earth and Mars? Please provide a launch date and time.

3.1.d) Compute the time between the two successive Hohmann transfer opportunities between the Earth and Mars. Explain your calculations and provide your answer in Earth days.

Phase 2: Choosing Orbits for Consistent Imaging (35 Points)

Once the spacecraft reaches Mars, it performs an orbit insertion burn and is captured into an elliptical parking orbit with the following parameters:

Pericenter altitude q	400 km
Apocenter altitude Q	5000 km
Inclination with respect to Martian equator	20°

Also, you are given the following parameters of Mars:

Mars equatorial radius	3376 km
J_2 gravity coefficient of Mars	$1.96 \cdot 10^{-3}$
Mass of Mars	$6.39 \cdot 10^{23}$ kg
Mars rotation period	24 h 37 m 22.7 s

Consistent lighting is crucial for Mars surface observations. To achieve this, you plan to use a Sun-synchronous orbit (SSO), which requires careful tuning of orbital inclination to exploit Mars's oblateness. Your goal is to transfer to a **circular Sun-synchronous orbit** with an orbital altitude of **400 km.**

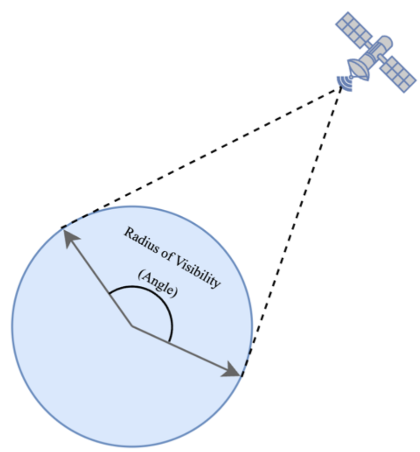
3.2.a) At the desired orbital altitude of 400 km, calculate the inclination needed to make the orbit Sun-synchronous. Is this orbit prograde or retrograde? Explain your answer.

Hint: You would need to exploit secular perturbation due to the Earth's J_2 in the longitude of the ascending node: $\dot{\Omega} = -\frac{3}{2} n J_2 \left(\frac{R_e}{p}\right)^2 \cos i$, where n is the mean motion, R_e is the equatorial radius of the Earth, p is the semilatus rectum and J_2 is the degree-2 zonal coefficient due to the oblateness of Mars. Remember that for a Sun-synchronous orbit, the rate of change of Ω is equal to the mean motion of the planet around the Sun.

3.2.b) Detail an efficient maneuver sequence to transition from the initial elliptical parking orbit to the desired circular, Sun-synchronous orbit. Where should you perform the inclination change and circularization burn, and in what order should you perform them? Justify your choices and compute the total required amount of Δv.

Phase 3: Optimizing Mars observations (20 points)

Your spacecraft is equipped with a high-resolution camera and will be used to monitor Martian surface. You must now balance altitude, resolution, and field of view. First, you must understand what portion of Mars each one can observe from orbit. The radius of visibility is the angle observable from a satellite for a given altitude, and is defined in the figure below:



3.3.a) Compute and plot the radius of visibility as a function of altitude from 400 km to the altitude of the Mars Stationary Orbit.

3.3.b) Your satellite must balance wide coverage with high-resolution imaging. Discuss the fundamental tradeoff between the radius of visibility and image resolution. How does this tradeoff influence your orbit design?

3.3.c) Your mission requires 1-meter resolution of the Martian surface from orbit. What is the Instantaneous Field of View (IFOV) required to achieve 10-meter resolution? Provide your answer in microradians.

Hint: IFOV is not a super intuitive term. IFOV refers to the angular size of one detector element (a.k.a., pixel). In this question, we are concerned with finding the pixel angular size that would correspond to a 10-meter resolution.

3.3.d) What Field of View (FOV) is needed to cover a 5 km-wide surface footprint in the nadir direction? How would this footprint change if you point your camera toward the limb of the planet (i.e., at the edge of the zone of visibility). Provide your answers in degrees.

4) Effect of the Earth oblateness and pear-shapeness (bonus problem 20 points)

4.a) Mathematically explain why J_2 perturbations have a secular component, but J_3 only causes periodic perturbations.

5) Effect of 3rd body perturbation (bonus problem 15 points)

5.a) Provide intuition using geometric reasoning for the effect of 3rd body perturbation on the time evolution of orbital elements. Think of the Earth's Moon affecting the satellite moving around the Earth. Confirm your intuition by numerical integration and plotting orbital elements as a function of time.