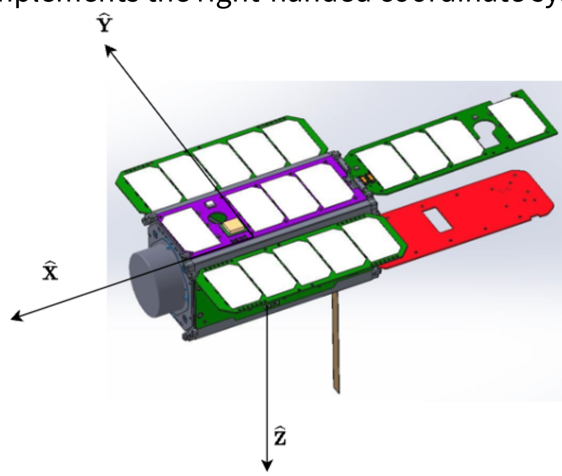
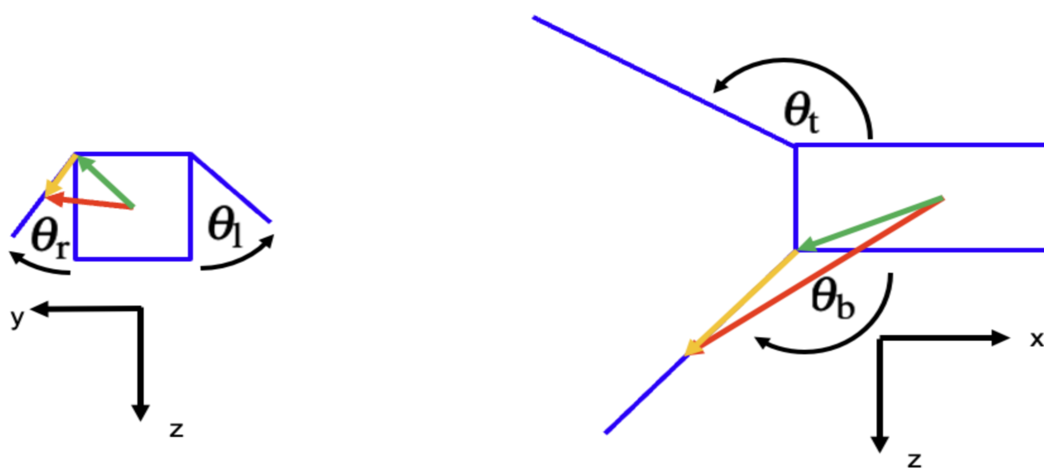
Stanford | ENGINEERING
Aeronautics and Astronautics

AA179 (Spring 2025)
Problem Set 3
Due on May 13, 2025, at 6 pm Pacific Time

In this homework problem, we will explore design choices for the CubeSat QB50 that was discussed in the lectures. For our purposes, the CubeSat consists of 5 components: main chassis and 4 solar panels. We will model the chassis as a box. The longest dimension of the box measures 22 cm and the other two dimensions are 8 cm. The mass of the chassis is 1.6 kg. Each solar panel is modeled as a plate, with the same dimensions (22 cm by 8 cm). The mass of each panel is 0.1 kg. The drawing below shows the geometry of the CubeSat QB50 along with the spacecraft-fixed coordinate system. X-axis is in the orbital velocity direction. Z-axis is in the nadir direction. Y-axis complements the right-handed coordinate system.



The solar panels in the figure above are shown in a fully deployed state. However, we have a freedom in how they are deployed. The deployment of solar panels is defined by the deployment angles θ_i that are shown in the figure below.



Thus, fully deployed corresponds to is $\theta_r = \theta_l = 90°$, and $\theta_b = \theta_t = 180°$. In addition, the negative Z side of the chassis also has a solar panel.

1) Rotational Stability (100 points)

Our first task is to analyze the rotational stability of the spacecraft. In order to do so, you will need to follow the steps described in Lecture_08_QB50.pptx using the code provided in Jupyter notebook lecture8_python_qb50_for_hw3_2025.ipynb.

1.a) Derive the inertia tensor in the spacecraft-fixed frame about the spacecraft's center of mass (COM) for the fully deployed solar panels. Is this inertia tensor diagonal? If not, what does it tell you about alignment of the principal inertia axes and the spacecraft coordinate axes?
1.b) Derive the principal moments of inertia of the spacecraft for the fully deployed solar panels.
1.c) Derive the principal axes of the spacecraft for the fully deployed solar panels. Are the principal axes the same as the spacecraft coordinate axes (from the figures above)?
1.d) Our goal is to achieve a stable rotation around the Y spacecraft axis. What can you conclude about the rotation stability for configuration with fully deployed panels?

Hint: It is not a secret that we've had a problem here. We saw it in the lecture. How do we solve it? We need to move mass closer to the Z-axis to reduce I_z. We can play with solar panels to achieve that.

1.e) Compute how I_y and I_z change as a function deployment angles θ_i and plot the results. Since there are four angles, there are four degrees of freedom in this problem. Let's simplify this problem by keeping the deployment of the solar panels symmetric. The top panels and side panels will be deployed together leading to $\theta_r = \theta_l$ and $\theta_b = \theta_t$. Thus, you can plot the principal moments of inertia as a function of two variables (not four) θ_r and θ_b. Here is a piece of code that you would need to use to make a plot of a function of two variables:

```
# create a grid of values for the deployments angles
theta_top  = np.linspace(0.1, 180, 100) * deg2rad
theta_side = np.linspace(0.1, 90, 100)  * deg2rad
theta_top, theta_side = np.meshgrid(theta_top, theta_side)

# initialize a grid of values
dI = np.zeros(theta_top.shape)

# call function
for i in range(dI.shape[0]):
    for j in range(dI.shape[1]):
        dI[i,j] = Iyz_diff(theta_top[i,j],theta_side[i,j]) # You will make Iyz_diff.
```

Hint: Your goal here is to write a function that outputs $I_y - I_z$. This difference needs to be positive for stable rotation around the spacecraft Y-axis. Writing this function requires putting nearly all the steps in the Jupiter notebook into a single functional block that outputs $I_y - I_z$.

1.f) Using the plot that you created, deployment of which solar panels would have a larger effect on the moments of inertia? Describe why.

1.g) If the side panels are fully deployed (i.e., $\theta_r = \theta_l = 90°$) and assuming $\theta_t = \theta_b$, what range of θ_t makes the spacecraft rotationally stable around the spacecraft's Y axis? You can find the answer from the figure by plotting a contour line for $I_y - I_z = 0$. Below is a hint for plotting contour lines.

```
plt.contour(theta_top * rad2deg, theta_side * rad2deg, dI,
levels=np.array([0.0]),colors='w')
```

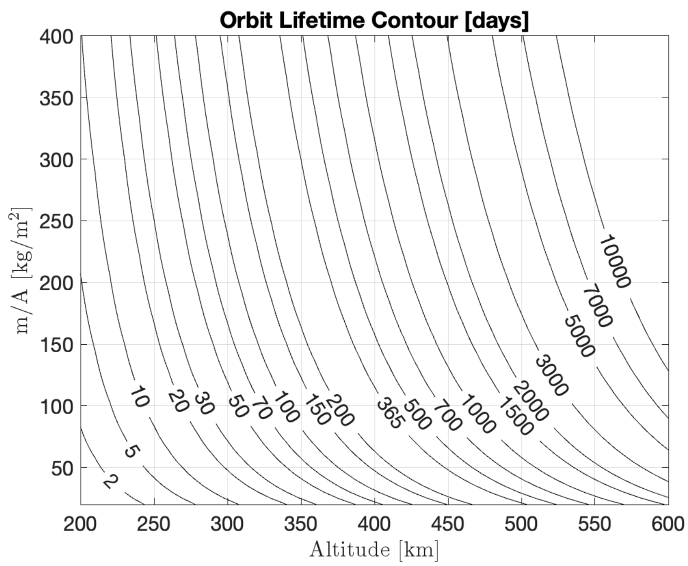
2) Solar Power (25 points)

We use the solar panels for power. In this problem set, we make a simplification: we assume that on average the solar light comes from the negative Z-direction. Also, notice that one panel (the one controlled by θ_b) is in shadow of the other panel. In fact, it is not really a solar panel. It acts as a boom for the magnetometer. Thus, it does not generate any power. We assumed that the panel efficiency is 30%. The amount of Solar flux is 1370 W/m^2. The available power is computed as Top-projected Area * Solar Flux * Efficiency. The amount of power needed to run all the spacecraft systems is 21.7 W.

2.a) With symmetric deployment of panels ($\theta_r = \theta_l$ and $\theta_b = \theta_t$), describe qualitatively what effect changing the solar panel deployment angle would have on the amount of available power?
2.b) What is the total available power when the solar panels are fully deployed? Is it sufficient for the spacecraft?
2.c) For the fully deployed left and right panels, what is the θ_t (again assuming $\theta_b = \theta_t$) that provides rotational stability and sufficient power? Refer to the code snipped in 1.e) for creating a plot of two variables. This time, you would need to plot the available power as a function of the deployment angles.

3) Atmospheric Drag (25 points)

The figure below provides the spacecraft lifetime as a function of the initial orbit altitude and the mass-to-frontal-area ratio. The orbit lifetime is defined here as the time it takes for the spacecraft altitude to decrease to 100 km. The y-axis of the figure is the spacecraft mass-to-frontal-area-ratio (spacecraft area projected onto the Y-Z plane, see spacecraft drawing). This ratio is controlled by the solar panel deployment angles. You can assume that the side panels do not contribute to the atmospheric drag. The spacecraft mass is assumed to remain constant. The spacecraft is assumed to be deployed from an initial altitude of 410 km.



3.a) What is the spacecraft lifetime for the fully deployed panels?
3.b) Describe the spacecraft configuration that corresponds to the maximum atmospheric drag and, therefore, minimum spacecraft lifetime. Assume the spacecraft maintains its nominal attitude (rotation around the spacecraft Y-axis with Z-axis of the spacecraft always pointing nadir). What is this minimum spacecraft lifetime?
3.c) If we want to achieve a 1-year mission duration, can we simultaneously satisfy the attitude requirement (stable rotation around the spacecraft Y-axis) and sufficient power requirement (at least 21.7 W)? Describe a configuration where all these requirements are satisfied. Again, assume symmetric deployment of panels ($\theta_r = \theta_l$ and $\theta_b = \theta_t$).

Hint: all design requirements in this homework are reduced to plotting various quantities (moment of inertia differences, power, mass-to-area ratio) as a function of two solar panel deployment angles (again, assuming symmetric deploying) and finding the part of that parameter space where all the constraints are met.

Bonus question (5 points)
3.e) What effect would asymmetric deployment of solar panels have on the attitude dynamics of the spacecraft. *Hint: The spacecraft target orbit is LEO, where atmospheric drag force is not negligible.*