Stanford | ENGINEERING
Aeronautics and Astronautics

AA179 (Spring 2025)
Problem Set 2
Due on April 29, 2025, at 6 pm Pacific Time

1) Moments of inertia of planets (15 points)

1.a) Derive analytically the normalized polar (i.e., about the spin axis) moment of inertia C/MR^2 of a spherical planet that has uniform density. *M* is the mass of the planet, *R* is its radius, and *C* is the unnormalized polar moment of inertia.
Hint: *use the polar moment of inertia of an infinitely thin disk, and integrate over the height of a sphere, i.e.:*

$$C_{sphere} = \int_{-R}^{+R} dC_{disk}(z)$$

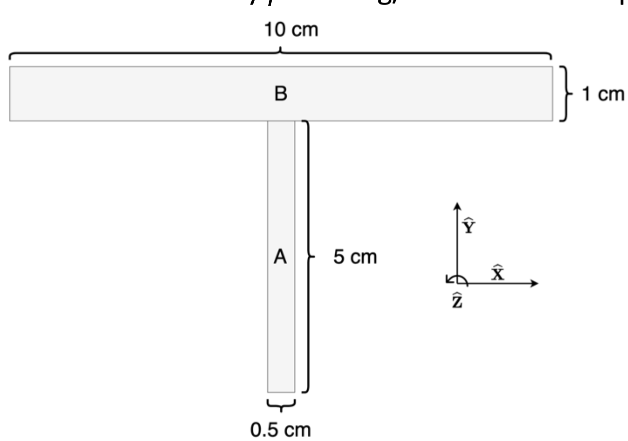
1.b) Derive the normalized polar moment of inertia C/MR^2 of a spherical planet that consists of two layers. The radius of the inner layer is one-half of the planet's radius. The density of the inner layer is three times the density of the outer layer.

Bonus problem (5 extra points)

1.c) Derive the normalized polar moment of inertia (C/Ma^2) of an ellipsoid of revolution, which is more similar to the real Earth. The two equatorial axes (*a* and *b*) are equal, and the polar axis (*c*) is shorter (c < *a* and *c* < *b*). For this problem, assume the Earth is homogeneous in density.

2) Demonstration of the Dzhanibekov effect or the Tennis racket theorem (17 points)

2.a) Explain what happens in this video and why:
https://www.youtube.com/watch?v=1x5UiwEEvpQ
2.b) Confirm your intuition by computing the inertia tensor about the system center of mass for two homogeneous cylinders each of density $\rho = 2.7$ g/cm³ that make up the letter "T".



Assume cylinder *A* has a diameter of 0.5 cm and length of 5 cm; cylinder *B* has a diameter of 1 cm and length of 10 cm. The inertia tensor for a cylinder of mass M, radius R, and length L with respect to its center of mass is:

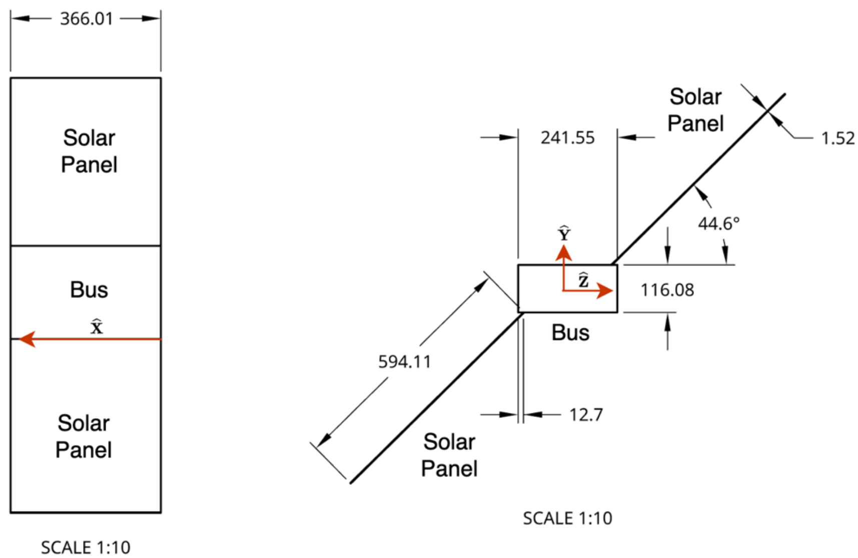
$$\mathbf{I}_{\text{cylinder}} = \begin{bmatrix} \frac{1}{2}MR^2 & 0 & 0 \\ 0 & \frac{1}{4}MR^2 + \frac{1}{12}ML^2 & 0 \\ 0 & 0 & \frac{1}{4}MR^2 + \frac{1}{12}ML^2 \end{bmatrix}$$

where I_x is the moment of inertia for the axis of the cylinder, and I_y and I_z are the moments of inertia of the cross axes. Provide your answer in units of kg m^2.

3) Moments of inertia of spacecraft (18 points)

The measured inertia tensor of a spacecraft about its center of mass and expressed in spacecraft-fixed coordinates is equal to:

$$\mathbf{I} = \begin{bmatrix} 0.1717 & 0 & 0 \\ 0 & 0.2557 & -0.04922 \\ 0 & -0.04922 & 0.1932 \end{bmatrix} \text{ kg m}^2$$



3.a) What are the principal moments of inertia?

3.b) What are the unit vectors of the principal axes?

3.c) Draw principal axes on top of the schematic diagram above. Doing this by hand is fine but it must be drawn accurately with respect angles shown on the diagram. Ideally, you would use a protractor for task.

4) Juno spinning spacecraft (20 points)

The Juno spacecraft is spinning at a rate of 2 revolutions per minute about a rotation axis that has a moment of inertia $I_z = 4000$ kg m^2. Juno consists of the main body modeled as a cylinder with a radius of 2 meters and a height of 1 meter, as well as 3 thin solar panels that extend 10 meters beyond the main body. A thruster is located at 2-meter distance from the rotation axis and can provide a thrust of 15 N. The thrust direction is perpendicular to both the rotation axis and the direction to the center of mass of the spacecraft.

4.a) How long does the thruster need to work to spin up the spacecraft to 10 revolutions per minute? Express your answer in seconds.
4.b) If one were to place the thruster at the end of a solar panel, how much time would it take to spin up from 2 to 10 revolutions per minute? Express your answer in seconds.

Bonus problem (5 extra points):
4.c) Discuss the downsides of putting thrusters at the end of solar panels from the standpoint of spacecraft design.

5) Attitude control (30 points)
The spacecraft orbits the Earth on a circular orbit with a radius of *R* and has the following inertia tensor about its center of mass:

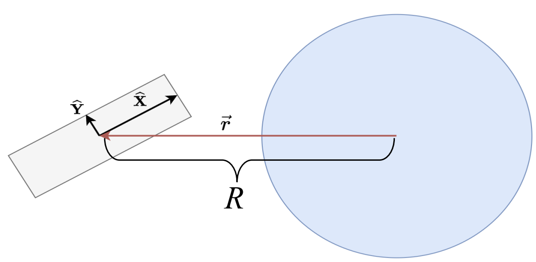
$$\mathbf{I} = \begin{bmatrix} 100 & 0 & 0 \\ 0 & 200 & 0 \\ 0 & 0 & 300 \end{bmatrix} \text{ kg m}^2$$

The gravity gradient torque can be estimated from the following expression:

$$\vec{\tau} = \frac{3GM}{R^5} \vec{r} \times \boldsymbol{I} \cdot \vec{r} \,,$$

where $\vec{r}$ is the position of the spacecraft's center of mass with respect to the planet center, $\mathbf{I}$ is the inertia tensor of the spacecraft about its center of mass, and $\vec{\tau}$ is the torque. $G = 6.67430 \cdot 10^{-11}$ N m²/kg² is the gravitational constant, and $M = 5.9722 \cdot 10^{24}$ kg is the mass of the Earth.

5.a) The spacecraft is oriented such that $\vec{r}$ has no $\hat{Z}$ component when expressed in principal coordinates of the spacecraft. A diagram of the scenario is given below:



Show that the gravity gradient torque reduces to the following scalar expression under this condition:

$$\tau_z = \frac{3GM}{R^5}(I_y - I_x)yx,$$

where x and y are the measures of $\vec{r}$ in principal coordinates $\hat{X}$ and $\hat{Y}$.
5.b) What spacecraft attitude would maximize the z-component of the gravity gradient torque expression (τ_z) from the previous problem? Draw this configuration.
5.c) The attitude control system uses magnetorquers and can produce a torque of 0.0002 N m or 0.2 mN m. Down to what altitude could the attitude control system counteract the gravity gradient torque in the most pessimistic case?

Hint: *use the worst-case attitude configuration you found in the previous part.*