Characteritzation of Io's orbit around Jupiter

March 23, 2020

In the figure we have a representation of Io's orbit around Jupiter. One of the main goals of this lab session is to renderize a view of the Jupiter-Io system as seen from a distant observer.

Let's define three working reference systems as follows:

{B}≡Base system, at rest, with origin at Jupiter's center, with the 3 axis displayed in the figure; the orientation is the same as that of the observer, which has chosen to take the y axis parallel to Jupiter rotation axis.

{J}≡System associated to Jupiter, rotating with it around the y axis; and

{I}≡System associated to Io, its axis labelled as x', y', z' at the figure. The movement of Io along its circular orbit takes place in a plane different from the $\{x,\, z\}$ plane of the observer. The orbit plane cuts the $\{x, z\}$ plane of the observer along a line, displayed as a solid line in the figure. The highest point of Io's orbit above the observer's plane occurs at a time that we will take as the time origin $t = 0$ and forms an angle δ with the $\{x,\, z\}$ observer's plane, as shown in the figure. At all times $t > 0$, Io's rotation movement is locked: always the same face is pointing towards Jupiter (with the opposite face always pointing away of it). This implies that if we take the x' axis of {I} as shown in the figure, it will continue pointing directly away of Jupiter at all times.

Taking into account the above considerations, complete the following steps:

1) Find out a sequence of transformations wich applied to {B} generate the systems {J} and {I}, respectively.

2) Write on paper the (matrix) equations that allow for finding the coordinates of a point P in the {B} system at any time t , assuming such coordinates are known in the systems {J} or {I}, in terms of *Delta*, *Gamma* and the data (provided in the file *construccio.pov*).

w_{Jup} ≡Jupiter's rotation angular velocity

w_{Orb} ≡Io's orbital angular velocity

w_{Io} ≡ Io's angular velocity ≡ Io's orbital angular velocity

R_{Jup} ≡Jupiter's radius

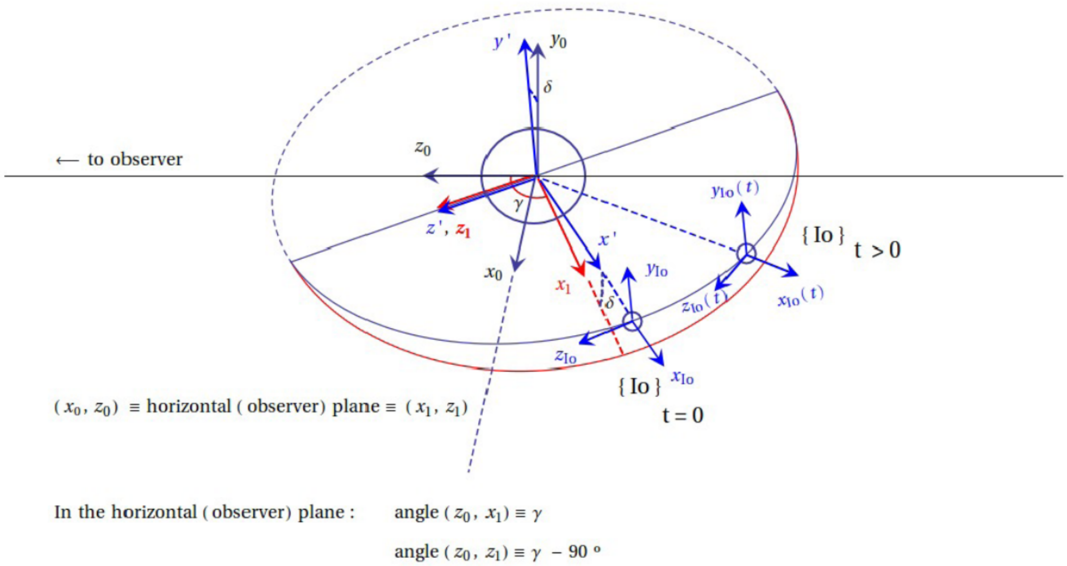
R_{Io} ≡ Io's radius

R_{orb} ≡ Io's orbit radius

You will need such equations for the calculation of *point #4.*

3) In the file *construccio.pov*, apply the appropriate transformations identified in section 1 for Jupiter and Io objects and to its respective systems {J} and {I}, so that all of them are located properly in the observer system {B}. Then

generate an animated sequence displaying one revolution of Io around Jupiter, showing also the reference frames attached to the two objects (note that in the file *construccio.pov* both *the orbit radius and satellite radius have been modified -i.e. they are not set to scale- so that the visualization is easier, as an artificially 'big' Io is renderized*).



4) Display of Io's transit:

Introduce the same transformations in the file *transit.pov* (this time the orbit and satellite radius *correct values have been restored*, the size of the satellite is now tiny compared to Jupiter's size) and make an mpg animation with a time span $0.68 < t < 0.69$ (A few still images at different times can be checked with the *clock* variable being set via a command line option, as in, for example, povray +K0.68 transit.pov, should we wish to set the clock value to 0.68 . Remember that time is used in units of one orbit period).

Although the size of Io is very small, the position's camera has been set very close to it during the transit (provided the transformations you entered are correct). *Using the matrix equations found in point #2, compute the position of Io at* $t = 0.685$ in the observer {B} reference frame, and using the camera location defined in the *transit.pov* file, compute and report the ratios of the distances camera-Jupiter/camera-Io.

5) Moving observer:

Place the Sun (light-source) in the position <-2000,100,25000>. As for the observer location (camera position in the code), modify (see the note below) the z coordinate so that at $t_0 = 0.68$ we have $z = 22$ whilst at $t_1 = 0.69$ it's $z = 6.6$. Generate again the mpg animation with a time span $0.68 < t < 0.69$. Suggestion: make z depend on time via $z(t) = A + B\,(t_1 - t)^n$ with both $n = 2$ and $n = 4$ and chose A and B such that the above conditions for z at times t_0

and t_1 are fulfilled.

Make a brief hand written report of the several points, including the calculations of points #2 and #4, and submit it together with the requested mpg files and the modified *transit.pov* file.