Name: Kok Jian Yu

GTID: 903550380

1. In Experiment 1, estimate the probability of winning $80 within 1000 sequential bets. Explain your reasoning thoroughly.

To win $80, we simply need to win 80 times within 1000 bets. It doesn't matter how many spins the player lost since he has unlimited money.

$$winprob = \frac{18}{38}$$

$$p(X \geq 80) = \sum_{x=80}^{1000} P(X = x) = \sum_{x=80}^{1000} \binom{1000}{x} * winprob^{x} * (1 - winprob)^{1000-x}$$

However, the above function is not calculatable using the calculator. Therefore, I am doing it with normal approximation.

$$n = 1000, p = \frac{18}{38}, q = 1 - p$$

$$n * p = 473.68 > 5$$

$$n * q = 526.32 > 5$$

$$Therefore, normal\ approximation\ is\ suitable\ to\ be\ used\ for\ this.$$

$$\mu = 483.68$$

$$\sigma = \sqrt{npq} = \frac{300}{19} = 15.79$$

$$p(X \geq 80) = p(X \geq 80 - 0.5)$$

$$z = \frac{79.5 - 483.68}{15.79} = -24.965$$

$$p(z > -24.965) \approx 100\%$$

$$Looking\ at\ the\ z\ score\ table, the\ probability\ of\ winning\ \$80\ is\ approximately\ 100\%$$

2. In Experiment 1, what is the estimated expected value of our winnings after 1000 sequential bets? Explain your reasoning thoroughly. Go here to learn about expected value: https://en.wikipedia.org/wiki/Expected_value

   In experiment 1, the result from running the simulation 1000 times is that we win $80 for every single simulation.

   Therefore, expected value = $\frac{1000}{1000} * 80 = \$80$

3. In Experiment 1, does the standard deviation reach a maximum value then stabilize or converge as the number of sequential bets increases? Explain why it does (or does not) thoroughly.

   The standard deviation only converges and stabilize on spin numbers where most of the gamblers managed to win $80 and stopped playing.

   However, on spins when most gambler have not reach $80 winnings, the standard deviation is extremely unstable, and does not show any sign of converging.

   This is due to the large difference in amount of winnings among different gamblers(simulations), which can be observed in figure 1. Due to the betting amount increasing exponentially after every loss, there are some gamblers that will be having extremely low winnings due to them betting an extremely high value. These winnings values will then heavily affect the standard deviation, thus making it extremely unstable.

4. In Experiment 2, estimate the probability of winning $80 within 1000 sequential bets. Explain your reasoning using the experiment thoroughly. (not based on plots)

   The following table describes the frequency count of winnings in experiment 2.

Winnings	Freq
80	658
64	1
-5	1
-52	1
-256	339

Therefore, probability of winning \$80 $= \frac{658}{1000} * 100 = 65.8\%$

The probability of winning is higher then losing as it requires the gambler to lose around 7 to 8 times consecutively for it to lose his bank roll. The chances of losing 7 to 8 times consecutively is low, and a single win will be able to bring the gambler's bankroll back to a net positive. Therefore, there are more wins then loss.

5. In Experiment 2, what is the estimated expected value of our winnings after 1000 sequential bets? Explain your reasoning thoroughly. (not based on plots)

   Reading from the table in Q4,

$$\begin{aligned} &expected\ winnings, E(winnings) \\ &= 80 * \frac{658}{1000} + 64 * \frac{1}{1000} + (-5) * \frac{1}{1000} + (-52) * \frac{1}{1000} + (-256) * \frac{339}{1000} \\ &= -34.137 \end{aligned}$$

6. In Experiment 2, does the standard deviation reach a maximum value then stabilize or converge as the number of sequential bets increases? Explain why it does (or does not) thoroughly.

   In experiment 2, the standard deviation increases as the number of sequential bets increases and stabilizes only when most of the gamblers(simulations) has either earned \$80 or lost \$256.

   The standard deviation here are relatively stable and doesn't really fluctuate much as experiment 1. This is because the gamblers now do not have an infinite bankroll, as such the difference between winnings are now smaller. Which leads to a smaller standard deviation.

   The standard deviation increases as the round increases as the difference between winnings will increases as the gamblers play more, as some gambler will earn more, which the others might lose more.

   It then stabilizes when the gamblers have mostly reached the gambling threshold (+80 or -256).

7. Include figures 1 through 5.

Figure 1

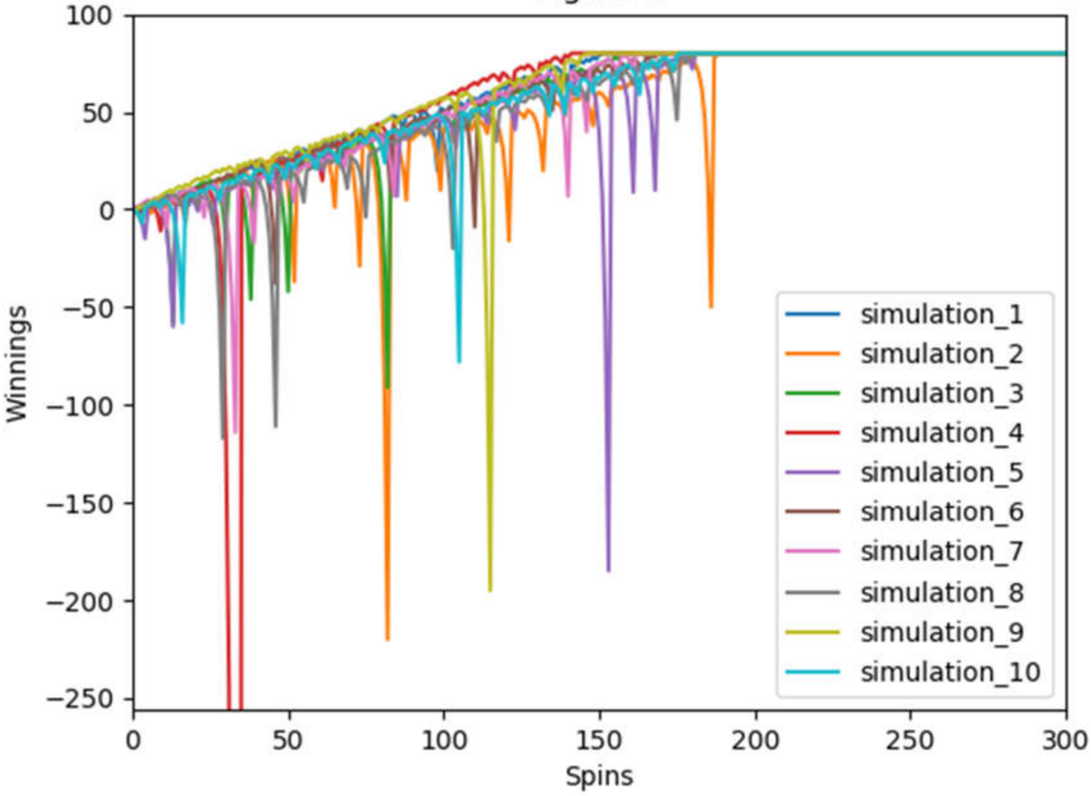


Figure 2

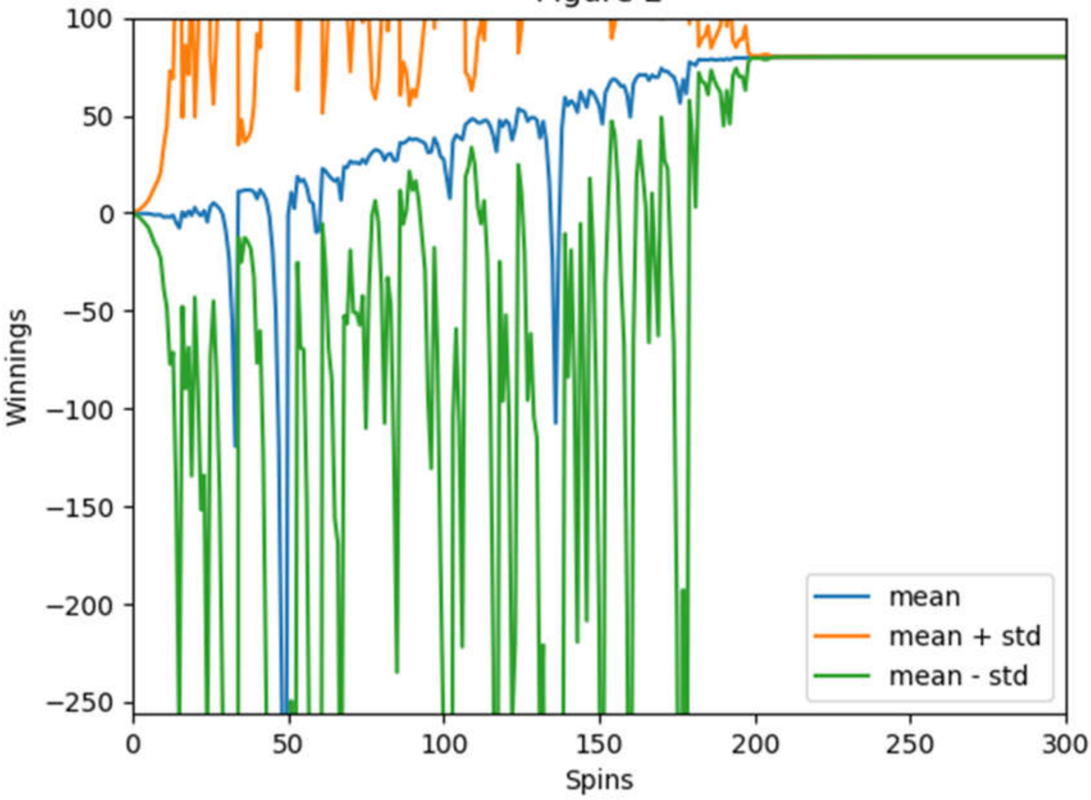


Figure 3

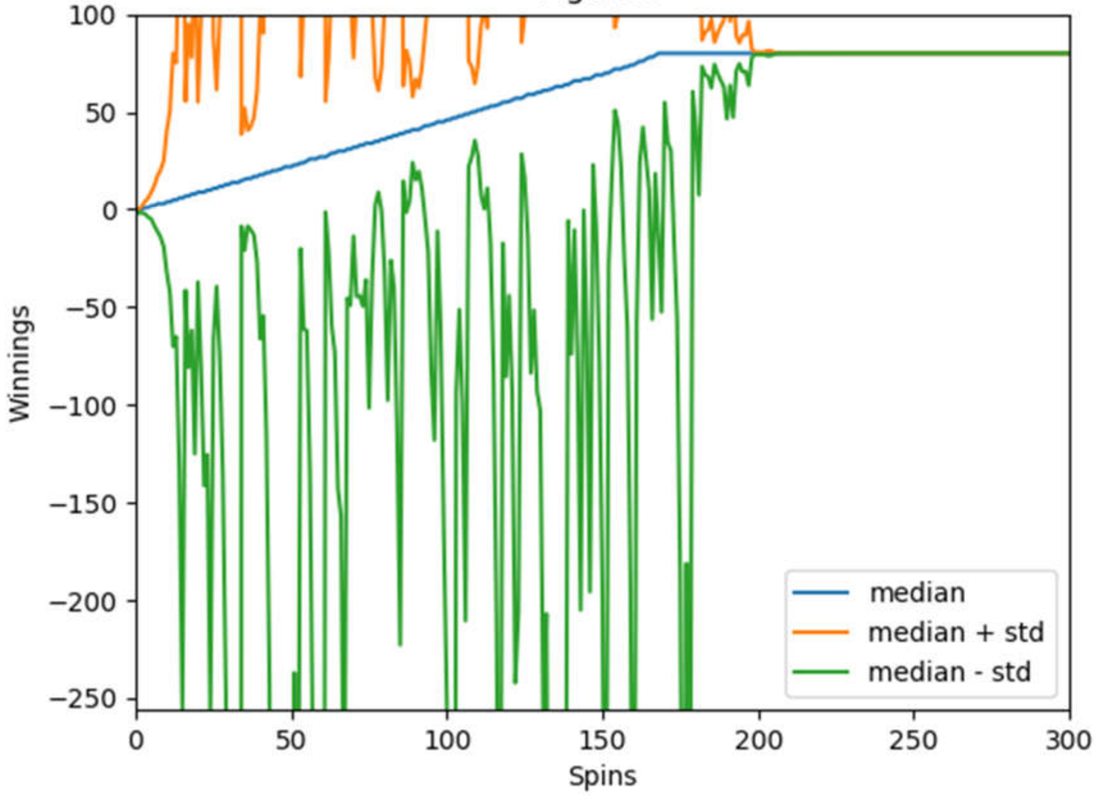
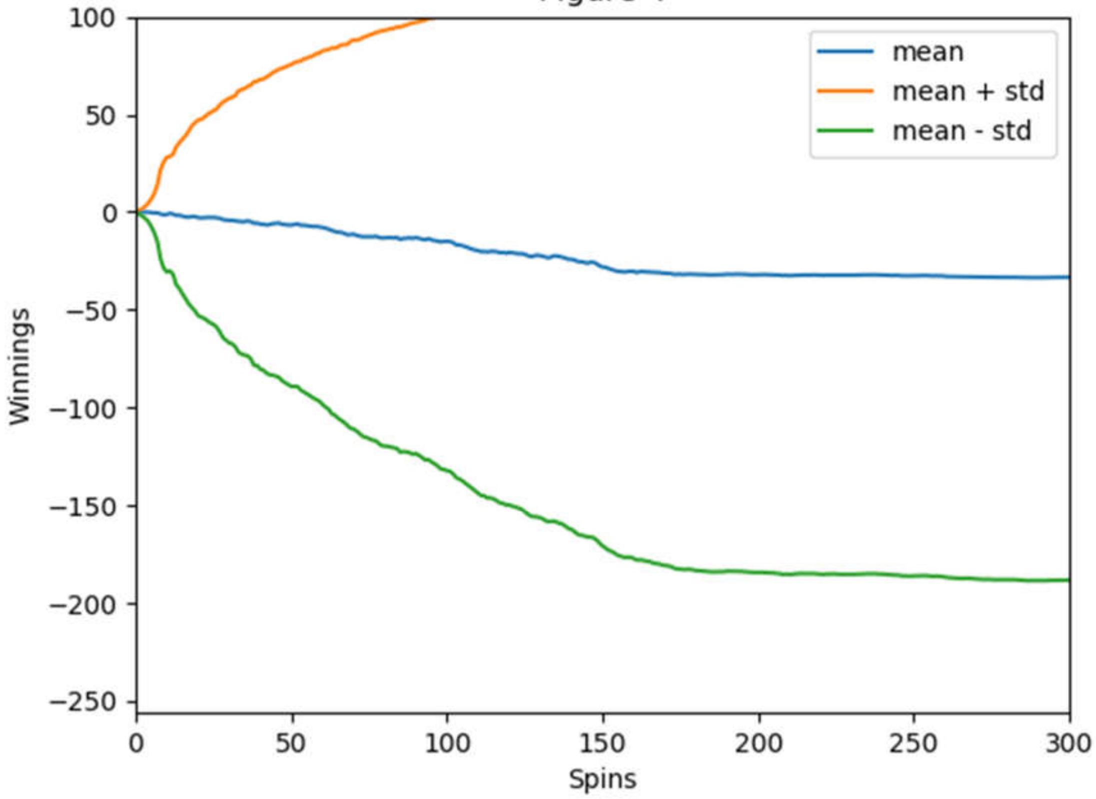


Figure 4



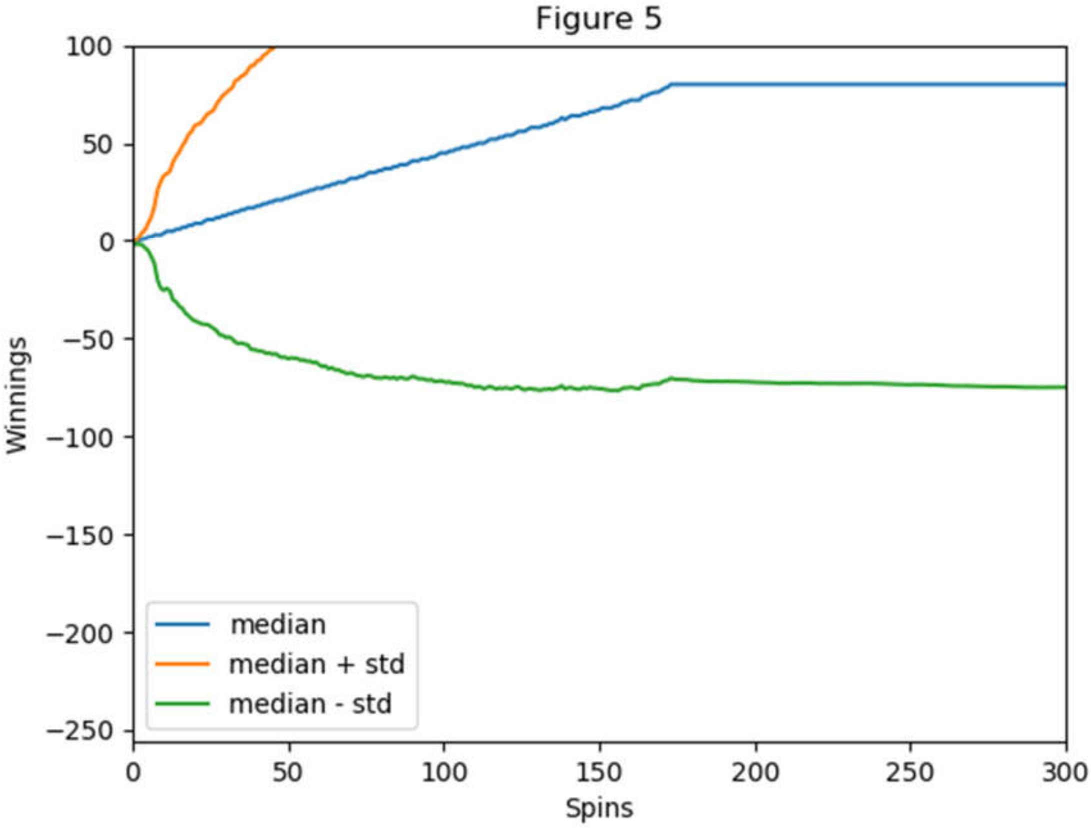


Figure 5
100
50
0
−50
−100
−150
−200
−250
Winnings
0
50
100
150
200
250
300
Spins
median
median + std
median - std