a01-fnn

April 7, 2024

[1]:
```
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# IE 678 Deep Learning, University of Mannheim
# Author: Rainer Gemulla
```

[2]:
```
%matplotlib ipympl

import math
import matplotlib.pyplot as plt
import torch
import torch.nn as nn
import torch.nn.functional as F

# import helper functions
import sys, os

sys.path.append(os.getcwd())
from a01helper import *  # check out the helper functions there, if you like
```

```
Warning: Cannot change to a different GUI toolkit: notebook. Using ipympl
instead.
```

1 Task 1: Implement an MLP

1.1 1a Logistic Regression

[3]:
```
# nn.Module is the superclass of all PyTorch models.
class LogisticRegression(nn.Module):
    """A logistic regression model.

    Parameters
    ----------
    D number of inputs
    C number of classes
    """

    # the definition of all parameters the model uses happens here, i.e., during
    # initialization
    def __init__(self, D, C):
```

```
        super(LogisticRegression, self).__init__()

        # Create and initialize model parameters. For (multinomial) logistic␣
↪regression,
        # we have a DxC-dimensional weight matrix W and a C-dimensional bias b.
        self.W = torch.randn(D, C) / math.sqrt(D)
        self.b = torch.randn(C) / math.sqrt(C)

        # Model parameters must be registered to PyTorch as follows. Here we␣
↪provide
        # a useful name that helps to access/analyze the model later on.
        self.register_parameter("0_weight", nn.Parameter(self.W))
        self.register_parameter("0_bias", nn.Parameter(self.b))

    # the forward function computes the model output for the provided (for this
    # assignent: single) input
    def forward(self, x):
        eta = self.W.t() @ x + self.b
        logprob = F.log_softmax(eta, dim=-1)
        return logprob
```

[4]:
```
# let's test it
logreg = LogisticRegression(3, 2)
x = torch.rand(3)  # input
logreg(x)  # output (log probabilities)
logreg(x).exp()  # output (probabilities)
```

[4]:
```
tensor([0.7939, 0.2061])
```

[5]:
```
# you can access individual parameters as follows
logreg.get_parameter("0_bias")
```

[5]:
```
Parameter containing:
tensor([-0.0527, -0.8002], requires_grad=True)
```

[6]:
```
# or all of them at once
list(logreg.named_parameters())
```

[6]:
```
[('0_weight',
  Parameter containing:
  tensor([[ 0.2860, -0.3515],
          [ 0.7918, -0.1698],
          [ 0.0949, -0.5201]], requires_grad=True)),
 ('0_bias',
  Parameter containing:
  tensor([-0.0527, -0.8002], requires_grad=True))]
```

[7]:
```
# or directly the tensors stored in the parameters
for par, value in logreg.state_dict().items():
    print(f"{par:<15}= {value}")
```

```
0_weight       = tensor([[ 0.2860, -0.3515],
        [ 0.7918, -0.1698],
        [ 0.0949, -0.5201]])
0_bias         = tensor([-0.0527, -0.8002])
```

1.2 1b MLP

[8]:
```
class MLP(nn.Module):
    """A fully-connected MLP.

    Parameters
    ----------

    sizes Contains the layer sizes. The first entry is the number of inputs,␣
 ↪the last
    entry the number of outputs. All entries in between correspond to the␣
 ↪number of
    units in the respective hidden layer. E.g., [2,5,7,1] means: 2 inputs -> 5D␣
 ↪hidden
    layer -> 7D hidden layer -> 1 output.

    phi Activation function used in every hidden layer (the output layer is␣
 ↪linear).

    """
    def __init__(self, sizes: list[int], phi=F.sigmoid):
        super().__init__()

        # let's remember the specification in this model
        self.sizes = sizes
        self.phi = phi

        # Initialize and register the parameters. Follow the naming scheme used␣
 ↪for
        # logistic regression above, i.e., the layer-i weights should be named␣
 ↪"i_weight" and
        # "i_bias".

        for i in range(1, len(sizes)):
            self.register_parameter(f'{i - 1}_weight', torch.nn.Parameter(torch.
 ↪randn(sizes[i - 1], sizes[i]) / math.sqrt(sizes[i - 1])))
            self.register_parameter(f'{i - 1}_bias', torch.nn.Parameter(torch.
 ↪randn(sizes[i]) / math.sqrt(sizes[i])))
```

```
    def num_layers(self):
        """Number of layers (excluding input layer)"""
        return len(self.sizes) - 1

    def forward(self, x):
        for i in range(0, self.num_layers() - 1):
            weight = getattr(self, f"{i}_weight")
            bias = getattr(self, f"{i}_bias")
            x = weight.t() @ x + bias
            x = self.phi(x)
        weight = getattr(self, f"{self.num_layers() - 1}_weight")
        bias = getattr(self, f"{self.num_layers() - 1}_bias")
        x = weight.t() @ x + bias
        return x
```

[9]:
```
# here you should see the correct parameter sizes
mlp = MLP([2, 3, 4, 2], torch.relu)
list(mlp.named_parameters())
```

[9]:
```
[('0_weight',
  Parameter containing:
  tensor([[ 1.3332, -0.7889, -0.1944],
          [-1.3557, -0.8821,  0.5116]], requires_grad=True)),
 ('0_bias',
  Parameter containing:
  tensor([-0.0612,  0.0403,  0.2524], requires_grad=True)),
 ('1_weight',
  Parameter containing:
  tensor([[ 0.0520,  0.9118, -0.8067,  0.4872],
          [-0.0393, -0.1332, -0.2776, -1.2511],
          [-0.8496, -1.6695, -0.0441, -0.6048]], requires_grad=True)),
 ('1_bias',
  Parameter containing:
  tensor([ 0.2837,  0.1361, -0.0046, -0.0430], requires_grad=True)),
 ('2_weight',
  Parameter containing:
  tensor([[-0.4725,  0.1120],
          [-0.6550, -0.0543],
          [ 0.1234,  0.2692],
          [ 0.1050,  0.6524]], requires_grad=True)),
 ('2_bias',
  Parameter containing:
  tensor([-0.7397, -0.8265], requires_grad=True))]
```

[10]:
```
# Test your code; we fix the parameters and check the result
with torch.no_grad():
```

```
    torch.manual_seed(0)
    for l in range(mlp.num_layers()):
        W, b = mlp.get_parameter(f"{l}_weight"), mlp.get_parameter(f"{l}_bias")
        W[:] = torch.randn(W.shape)
        b[:] = torch.randn(b.shape)

mlp(torch.tensor([-1.0, 2.0]))  # must give: [ 0.8315, -3.6792]
```

[10]:
```
tensor([ 0.8315, -3.6792], grad_fn=<AddBackward0>)
```

[11]:
```
# You can also evaluate your model on multiple inputs at once. Here "torch.func.vmap"
# produces a function that applies the provided function (mlp#forward) to each row of
# its argument (torch.tensor...).
#
# [[ 0.8315, -3.6792],
# [ 4.8448, -6.8813]]
torch.func.vmap(mlp)(torch.tensor([[-1.0, 2.0], [1.0, -2.0]]))
```

[11]:
```
tensor([[ 0.8315, -3.6792],
        [ 4.8448, -6.8813]], grad_fn=<AddBackward0>)
```

1.3 1c Batching

[12]:
```
class MLP(nn.Module):
    def __init__(self, sizes: list[int], phi=F.sigmoid):
        super().__init__()

        self.sizes = sizes
        self.phi = phi

        for i in range(1, len(sizes)):
            self.register_parameter(f'{i - 1}_weight', torch.nn.Parameter(torch.randn(sizes[i - 1], sizes[i]) / math.sqrt(sizes[i - 1])))
            self.register_parameter(f'{i - 1}_bias', torch.nn.Parameter(torch.randn(sizes[i]) / math.sqrt(sizes[i])))

    def num_layers(self):
        return len(self.sizes) - 1

    def forward(self, x):
        if x.dim == 1:
            x = x.unsqueeze(0)
        for i in range(0, self.num_layers() - 1):
            weight = getattr(self, f"{i}_weight")
            bias = getattr(self, f"{i}_bias")
```

```
            x = x @ weight + bias.unsqueeze(0)
            x = self.phi(x)
        weight = getattr(self, f"{self.num_layers() - 1}_weight")
        bias = getattr(self, f"{self.num_layers() - 1}_bias")
        x = x @ weight + bias.unsqueeze(0)
        return x
```

[13]:
```
# here you should see the correct parameter sizes
mlp = MLP([2, 3, 4, 2], torch.relu)
```

[14]:
```
# Test your code; we fix the parameters and check the result
with torch.no_grad():
    torch.manual_seed(0)
    for l in range(mlp.num_layers()):
        W, b = mlp.get_parameter(f"{l}_weight"), mlp.get_parameter(f"{l}_bias")
        W[:] = torch.randn(W.shape)
        b[:] = torch.randn(b.shape)
```

[15]:
```
# After you adapted the MLP class, you should get the same results as above.
mlp(torch.tensor([-1.0, 2.0]))  # must give: [ 0.8315, -3.6792]
```

[15]:
```
tensor([[ 0.8315, -3.6792]], grad_fn=<AddBackward0>)
```

[16]:
```
# Now without vmap. Only proceed to task 2 once this works correctly.
#
# [[ 0.8315, -3.6792],
# [ 4.8448, -6.8813]]
mlp(torch.tensor([[-1.0, 2.0], [1.0, -2.0]]))
```

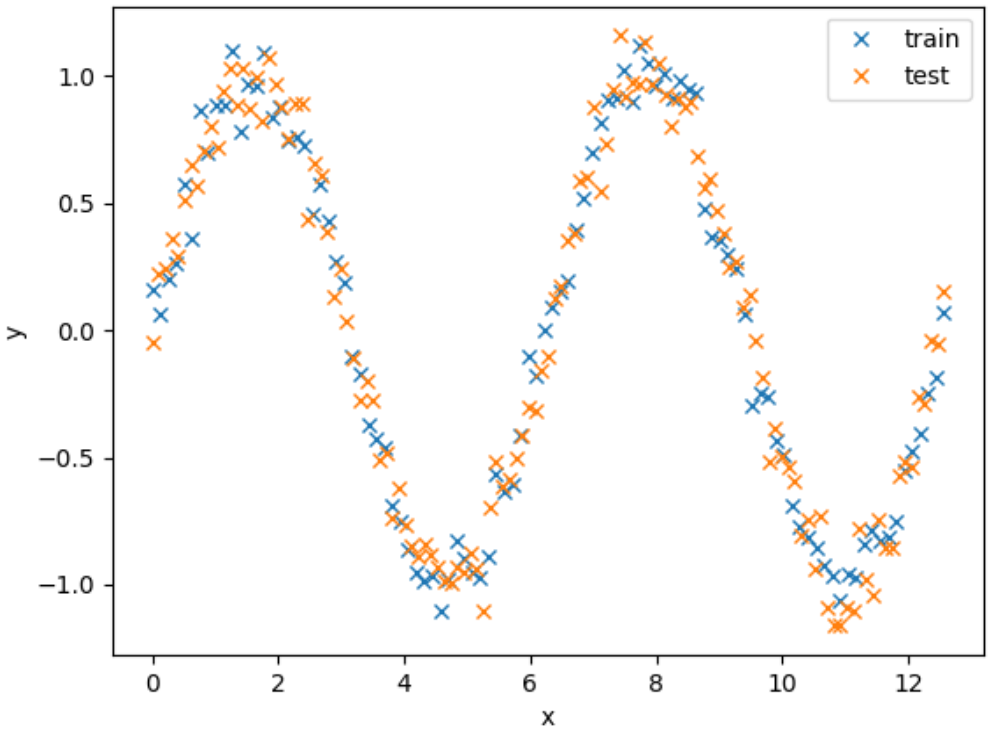
[16]:
```
tensor([[ 0.8315, -3.6792],
        [ 4.8448, -6.8813]], grad_fn=<AddBackward0>)
```

2 2 Multi-Layer Feed-Forward Neural Networks

2.1 2a Conjecture how an FNN fit will look like

[17]:
```
# here is the one-dimensional dataset that we will use
nextplot()
plot1(X1, y1, label="train")
plot1(X1test, y1test, label="test")
plt.legend()
```

[17]:
```
<matplotlib.legend.Legend at 0x146a6eaf0>
```



2.2 2b Train with 2 hidden units

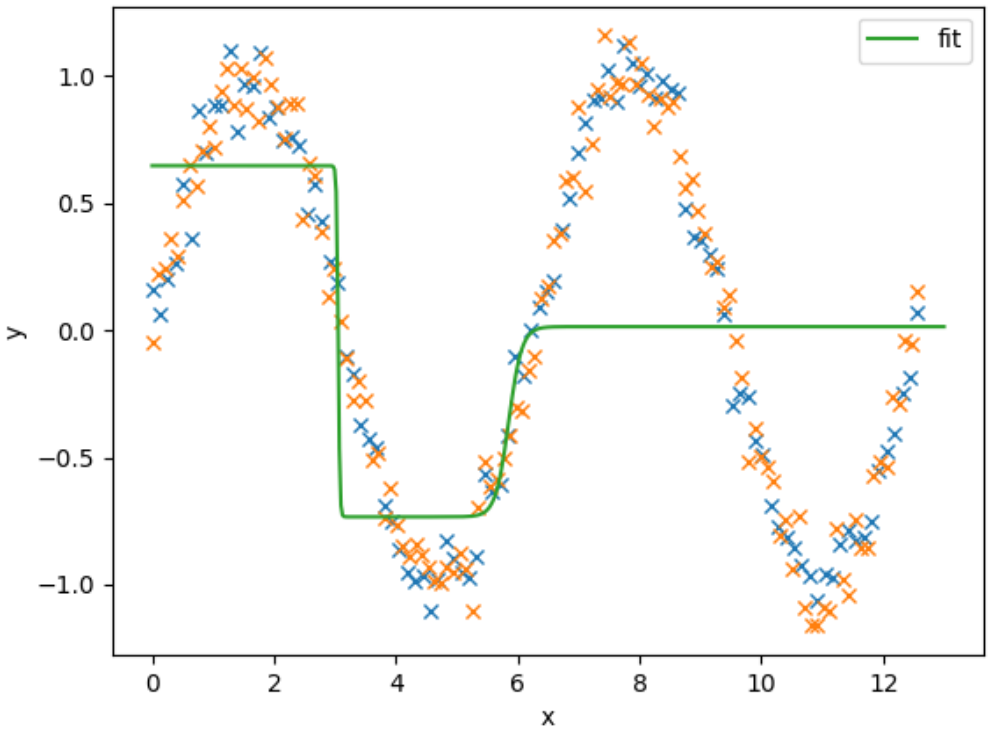
[18]:
```
# Training code. You do not need to modify this code.
train_bfgs = lambda model, **kwargs: train_scipy(X1, y1, model, **kwargs)

def train1(hidden_sizes, nreps=10, phi=F.sigmoid, train=train_bfgs, **kwargs):

    """Train an FNN.

    hidden_sizes is a (possibly empty) list containing the sizes of the hidden␣
 ↪layer(s).
    nreps refers to the number of repetitions.

    """

    best_model = None
    best_cost = math.inf
    for rep in range(nreps):
        model = MLP([1] + hidden_sizes + [1], phi)  # that's your model!
        print(f"X1 shape: {X1.shape}")
```

```
        print(f"Repetition {rep: 2d}: ", end="")
        model = train(model, **kwargs)
        mse = F.mse_loss(y1, model(X1)).item()
        if mse < best_cost:
            best_model = model
            best_cost = mse
        print(f"best_cost={best_cost:.3f}")

    return best_model
```

[19]:
```
# Let's fit the model with one hidden layer consisting of 2 units.
model = train1([2], nreps=1)
print("Training error:", F.mse_loss(y1, model(X1)).item())
print("Test error    :", F.mse_loss(y1test, model(X1test)).item())
```

```
X1 shape: torch.Size([100, 1])
Repetition  0: Optimization terminated successfully.
         Current function value: 0.293673
         Iterations: 120
         Function evaluations: 240
         Gradient evaluations: 233
best_cost=0.294
Training error: 0.29367321729660034
Test error    : 0.30551230907440186
```

[20]:
```
# plot the data and the fit
nextplot()
plot1(X1, y1, label="train")
plot1(X1test, y1test, label="test")
plot1fit(torch.linspace(0, 13, 500).unsqueeze(1), model)
```



[21]:
```
# The weight matrices and bias vectors can be read out as follows. If you want,␣
 ↪use
# these parameters to compute the output of the network (on X1) directly and␣
 ↪compare to
# vmap(model)(X1).
for par, value in model.state_dict().items():
    print(f"{par:<15}= {value}")
```

```
0_weight       = tensor([[75.7091,  9.2566]])
0_bias         = tensor([-231.3285,  -54.1847])
1_weight       = tensor([[-1.3810],
        [ 0.7480]])
1_bias         = tensor([0.6471])
```

[22]:
```
# now repeat this multiple times
for i in range (0, 5):
    model = train1([2], nreps=1)
    print("Training error:", F.mse_loss(y1, model(X1)).item())
    print("Test error    :", F.mse_loss(y1test, model(X1test)).item())
    nextplot()
    plot1(X1, y1, label="train")
```

```
    plot1(X1test, y1test, label="test")
    plot1fit(torch.linspace(0, 13, 500).unsqueeze(1), model)
```

```
X1 shape: torch.Size([100, 1])
Repetition  0:          Current function value: 0.079572
         Iterations: 387
         Function evaluations: 521
         Gradient evaluations: 511
best_cost=0.080
Training error: 0.07957355678081512
Test error    : 0.08671201020479202
```

```
/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
```

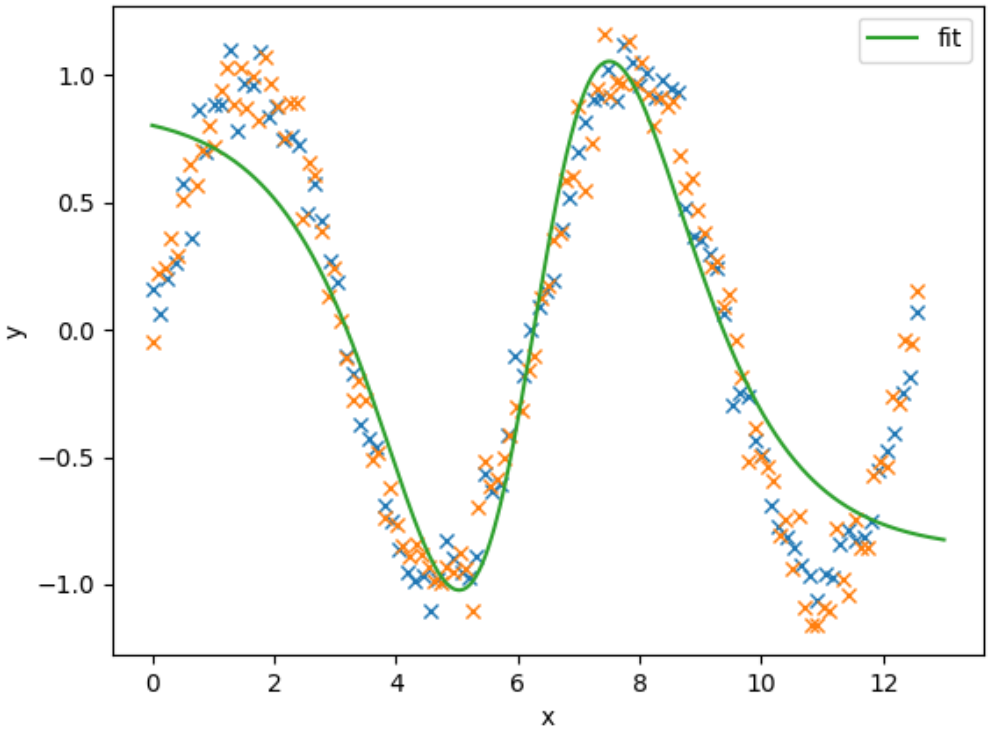
```
X1 shape: torch.Size([100, 1])
Repetition  0: Optimization terminated successfully.
         Current function value: 0.303902
         Iterations: 143
         Function evaluations: 155
         Gradient evaluations: 155
best_cost=0.304
Training error: 0.30390220880508423
Test error    : 0.3037970960140228
X1 shape: torch.Size([100, 1])
Repetition  0:
```

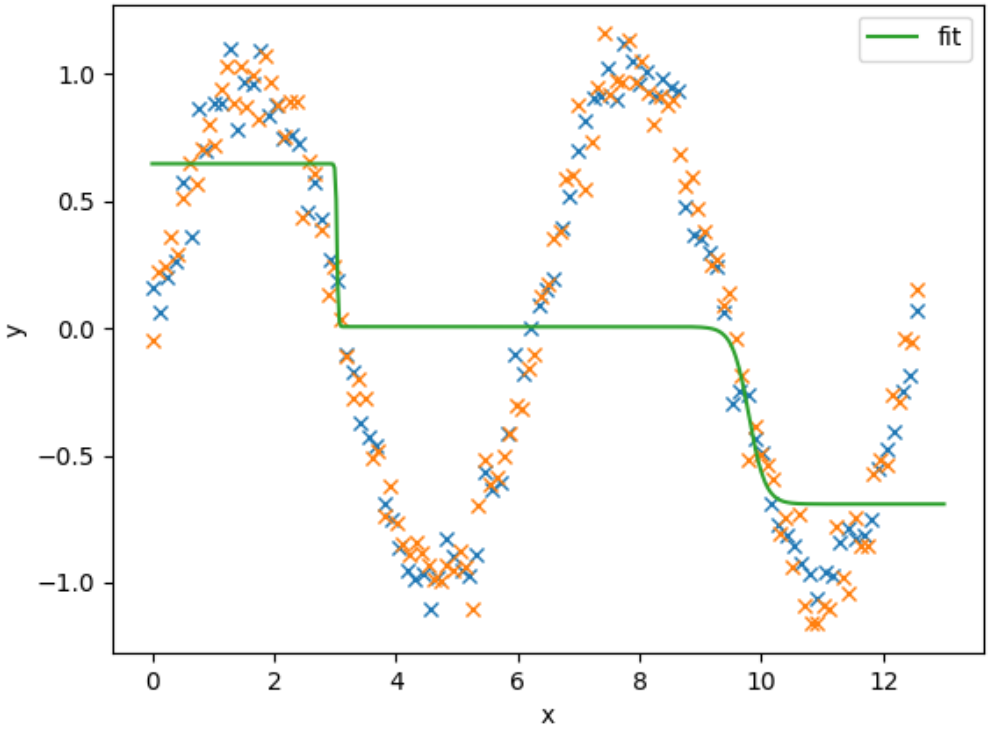
```
/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
```

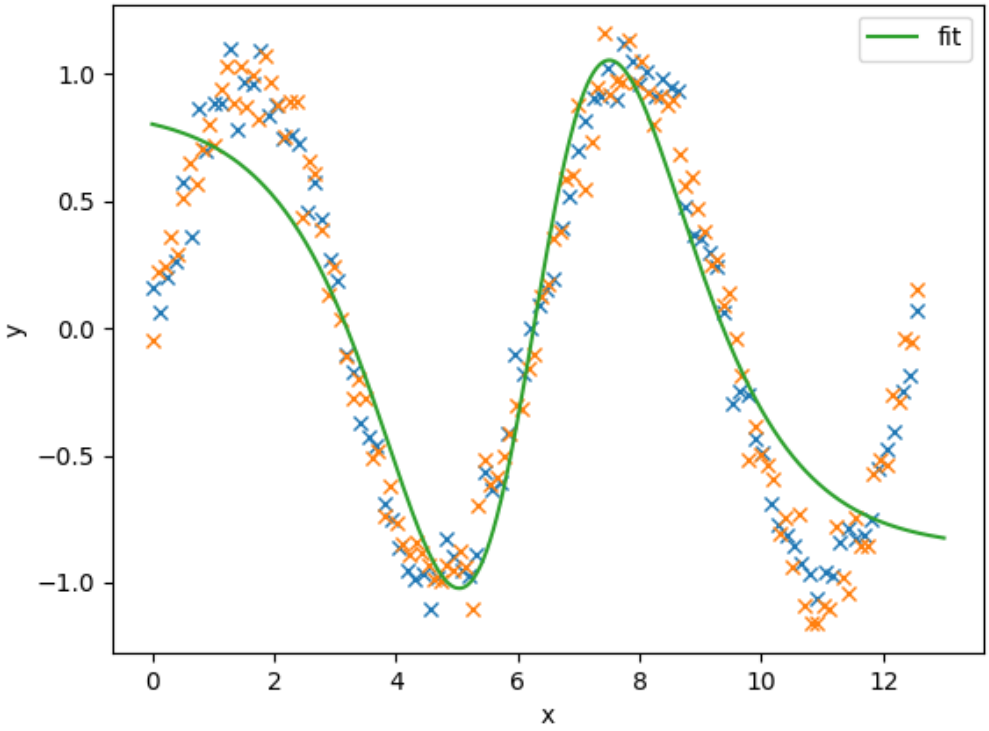
```
         Current function value: 0.079573
         Iterations: 400
         Function evaluations: 562
         Gradient evaluations: 551
best_cost=0.080
Training error: 0.07957581430673599
Test error    : 0.0867120772600174
X1 shape: torch.Size([100, 1])
Repetition  0:          Current function value: 0.286909
         Iterations: 302
         Function evaluations: 458
         Gradient evaluations: 445
```

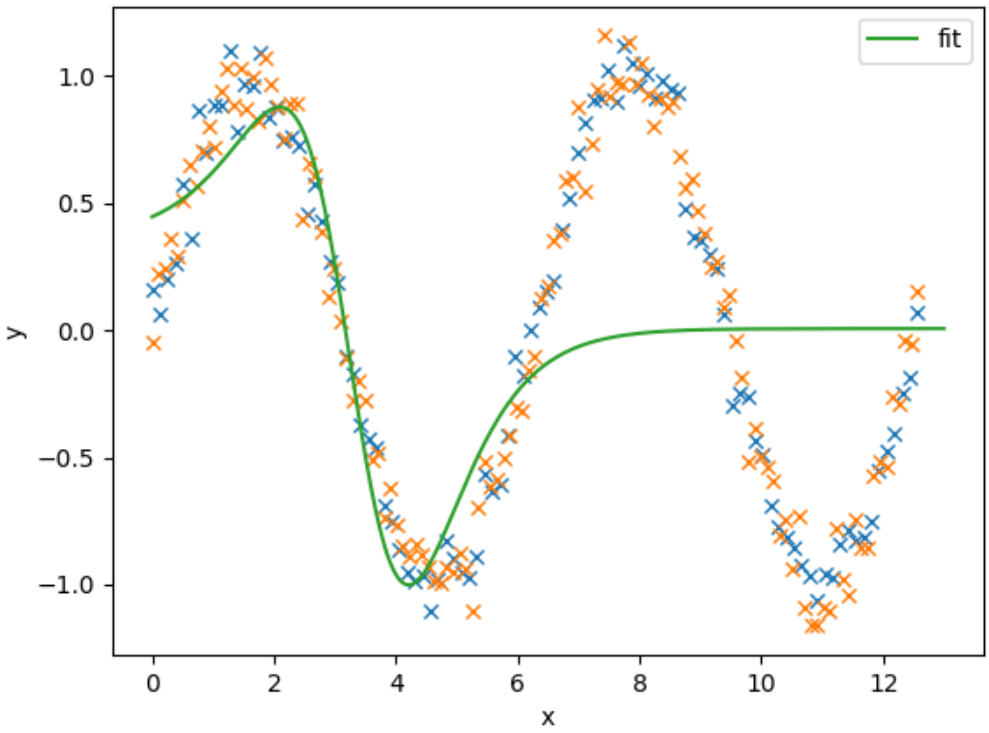
```
best_cost=0.287
Training error: 0.28690817952156067
Test error    : 0.29484879970550537
X1 shape: torch.Size([100, 1])
Repetition  0:          Current function value: 0.079573
         Iterations: 336
         Function evaluations: 470
         Gradient evaluations: 458
best_cost=0.080
Training error: 0.07956987619400024
Test error    : 0.0867103710770607
```

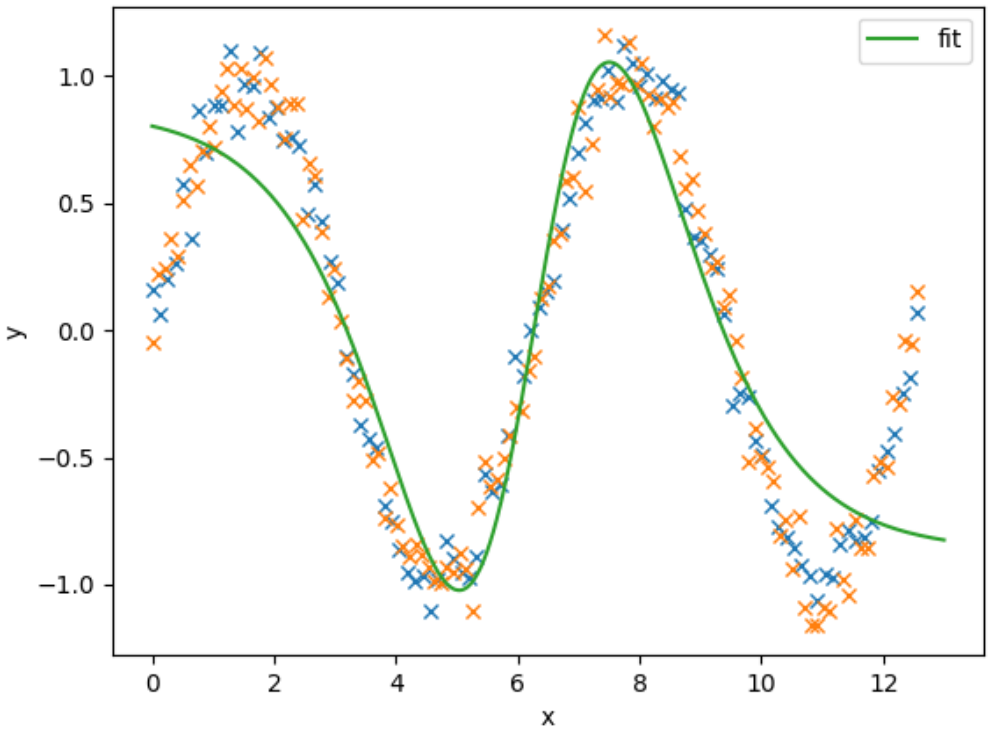
```
/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
```




fit
1.0
0.5
0.0
−0.5
−1.0
y
0
2
4
6
8
10
12
x


fit
1.0
0.5
0.0
−0.5
−1.0
y
0
2
4
6
8
10
12
x


fit
1.0
0.5
0.0
−0.5
−1.0
y
0
2
4
6
8
10
12
x



[23]:
```
# From now on, always train multiple times (nreps=10 by default) and report␣
  ↪best model.
model = train1([2], nreps=10)

print("Training error:", F.mse_loss(y1, model(X1)).item())
print("Test error    :", F.mse_loss(y1test, model(X1test)).item())
```

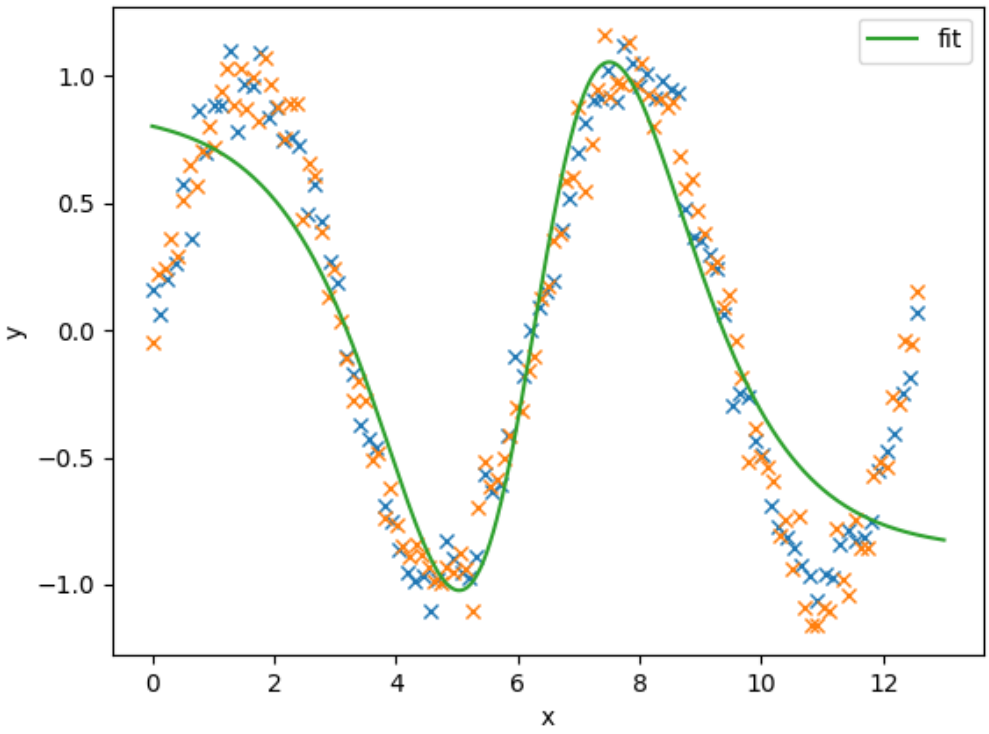
```
X1 shape: torch.Size([100, 1])
Repetition  0: Optimization terminated successfully.
         Current function value: 0.357250
         Iterations: 66
         Function evaluations: 76
         Gradient evaluations: 76
best_cost=0.357
X1 shape: torch.Size([100, 1])
Repetition  1:
```

```
/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
```

```
         Current function value: 0.438546
         Iterations: 278
         Function evaluations: 398
         Gradient evaluations: 386
best_cost=0.357
X1 shape: torch.Size([100, 1])
Repetition  2:

/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)

         Current function value: 0.286909
         Iterations: 448
         Function evaluations: 577
         Gradient evaluations: 565
best_cost=0.287
X1 shape: torch.Size([100, 1])
Repetition  3:          Current function value: 0.079573
         Iterations: 310
         Function evaluations: 459
         Gradient evaluations: 449
best_cost=0.080
X1 shape: torch.Size([100, 1])
Repetition  4:

/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)

         Current function value: 0.286909
         Iterations: 307
         Function evaluations: 413
         Gradient evaluations: 401
best_cost=0.080
X1 shape: torch.Size([100, 1])
Repetition  5:

/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
```

```
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)

         Current function value: 0.286909
         Iterations: 534
         Function evaluations: 690
         Gradient evaluations: 681
best_cost=0.080
X1 shape: torch.Size([100, 1])
Repetition  6:          Current function value: 0.079573
         Iterations: 330
         Function evaluations: 470
         Gradient evaluations: 458
best_cost=0.080
X1 shape: torch.Size([100, 1])
Repetition  7:

/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)

         Current function value: 0.079573
         Iterations: 381
         Function evaluations: 537
         Gradient evaluations: 524
best_cost=0.080
X1 shape: torch.Size([100, 1])
Repetition  8: Optimization terminated successfully.
         Current function value: 0.357250
         Iterations: 78
         Function evaluations: 80
         Gradient evaluations: 80
best_cost=0.080
X1 shape: torch.Size([100, 1])
Repetition  9:          Current function value: 0.286909
         Iterations: 324
         Function evaluations: 546
         Gradient evaluations: 533
best_cost=0.080
Training error: 0.07957330346107483
Test error    : 0.0867152065038681

/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
```

[24]:
```
# plot the data and the fit
nextplot()
plot1(X1, y1, label="train")
plot1(X1test, y1test, label="test")
plot1fit(torch.linspace(0, 13, 500).unsqueeze(1), model)
```



2.3 2c Width

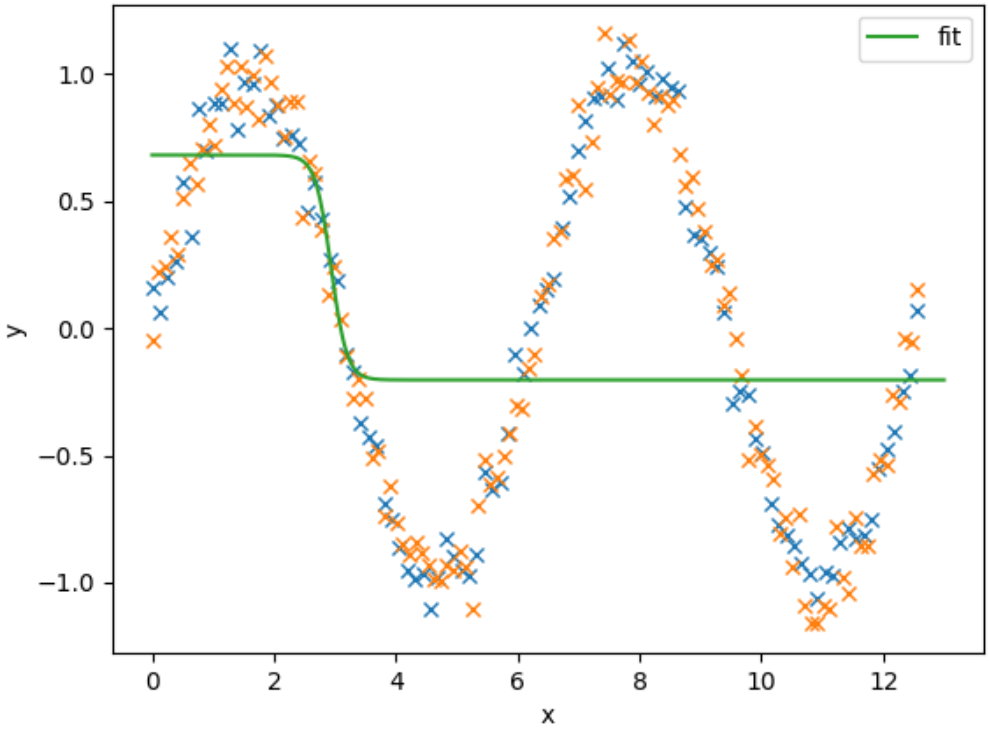
[25]:
```
# Experiment with different hidden layer sizes. To avoid recomputing
# models, you may want to save your models using torch.save(model, filename) and
# load them again using torch.load(filename).

for i in [1, 2, 3, 10, 50, 100]:
    model = train1([i], nreps = 1)
    print("Training error:", F.mse_loss(y1, model(X1)).item())
    print("Test error    :", F.mse_loss(y1test, model(X1test)).item())
    torch.save(model, f"model_{i}.pth")
```

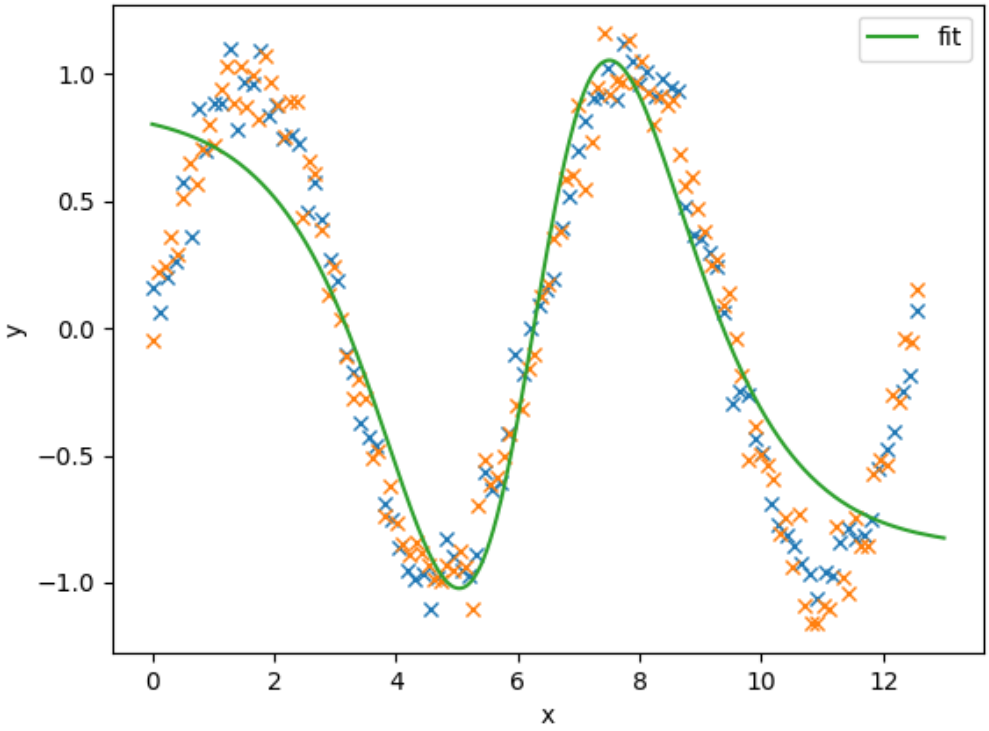
```
X1 shape: torch.Size([100, 1])
Repetition  0: Optimization terminated successfully.
         Current function value: 0.372919
```

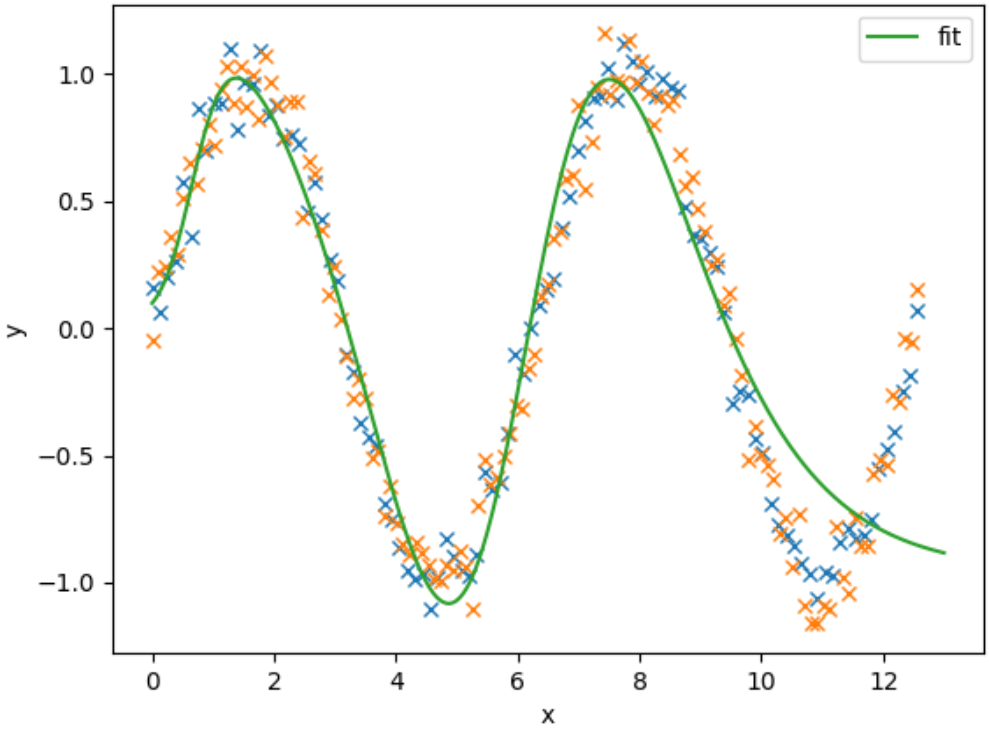
```
         Iterations: 42
         Function evaluations: 47
         Gradient evaluations: 47
best_cost=0.373
Training error: 0.3729189336299896
Test error    : 0.3743167221546173
X1 shape: torch.Size([100, 1])
Repetition  0:
/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
         Current function value: 0.079573
         Iterations: 390
         Function evaluations: 593
         Gradient evaluations: 583
best_cost=0.080
Training error: 0.07957376539707184
Test error    : 0.08670931309461594
X1 shape: torch.Size([100, 1])
Repetition  0:          Current function value: 0.049892
         Iterations: 289
         Function evaluations: 469
         Gradient evaluations: 456
best_cost=0.050
Training error: 0.04989229515194893
Test error    : 0.0598050132393837
X1 shape: torch.Size([100, 1])
Repetition  0:
/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
         Current function value: 0.005721
         Iterations: 1026
         Function evaluations: 1250
         Gradient evaluations: 1234
best_cost=0.006
Training error: 0.0057213036343455315
Test error    : 0.0150537034496665
X1 shape: torch.Size([100, 1])
Repetition  0:
```

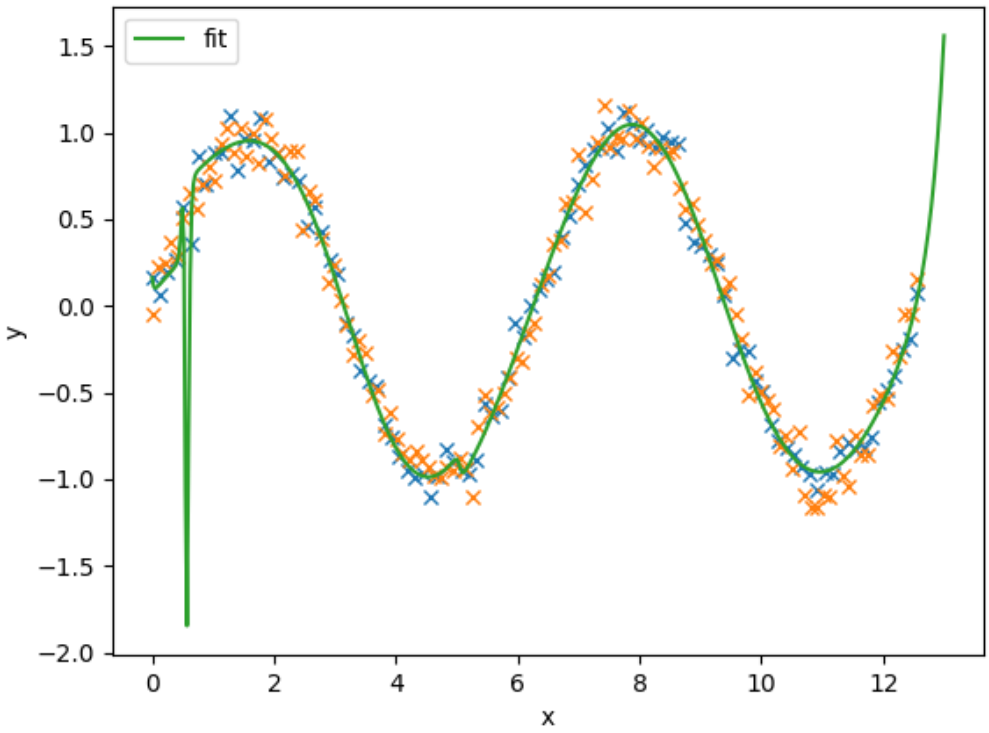
```
/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)

         Current function value: 0.003007
         Iterations: 5655
         Function evaluations: 6282
         Gradient evaluations: 6268
best_cost=0.003
Training error: 0.0030068017076700926
Test error    : 2.0873610973358154
X1 shape: torch.Size([100, 1])
Repetition  0:          Current function value: 0.001826
         Iterations: 8126
         Function evaluations: 8804
         Gradient evaluations: 8792
best_cost=0.002
Training error: 0.001825749408453703
Test error    : 4.303783893585205

/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
```

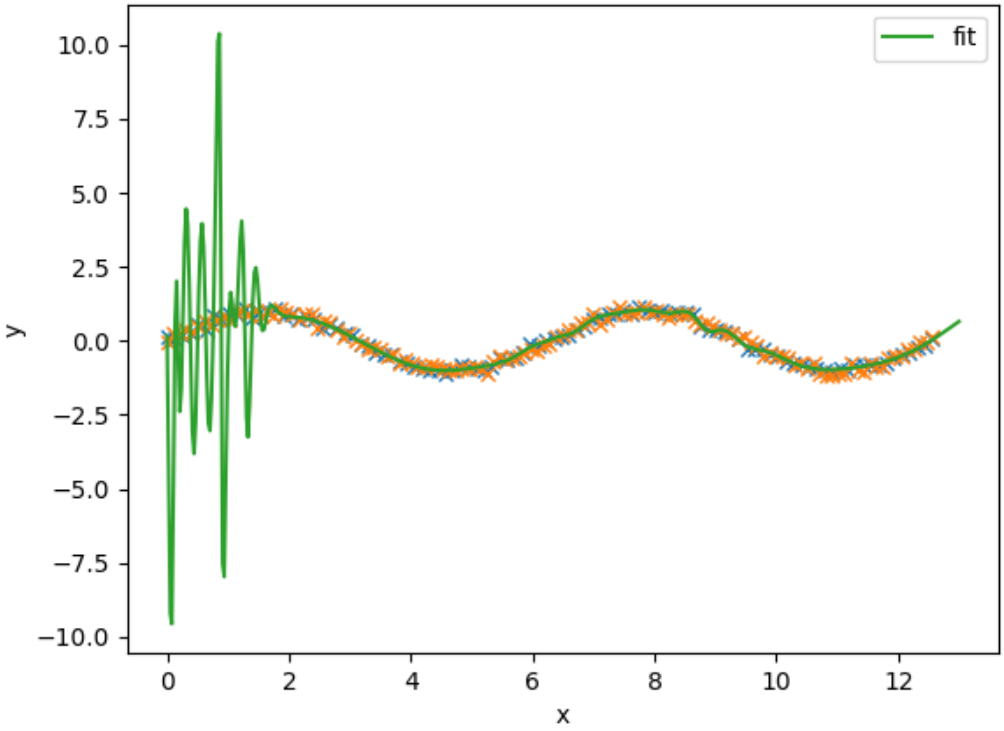
[26]:
```python
# torch.load(filename)

for i in [1, 2, 3, 10, 50, 100]:
    model = torch.load(f"model_{i}.pth")
    nextplot()
    plot1(X1, y1, label="train")
    plot1(X1test, y1test, label="test")
    plot1fit(torch.linspace(0, 13, 500).unsqueeze(1), model)
```

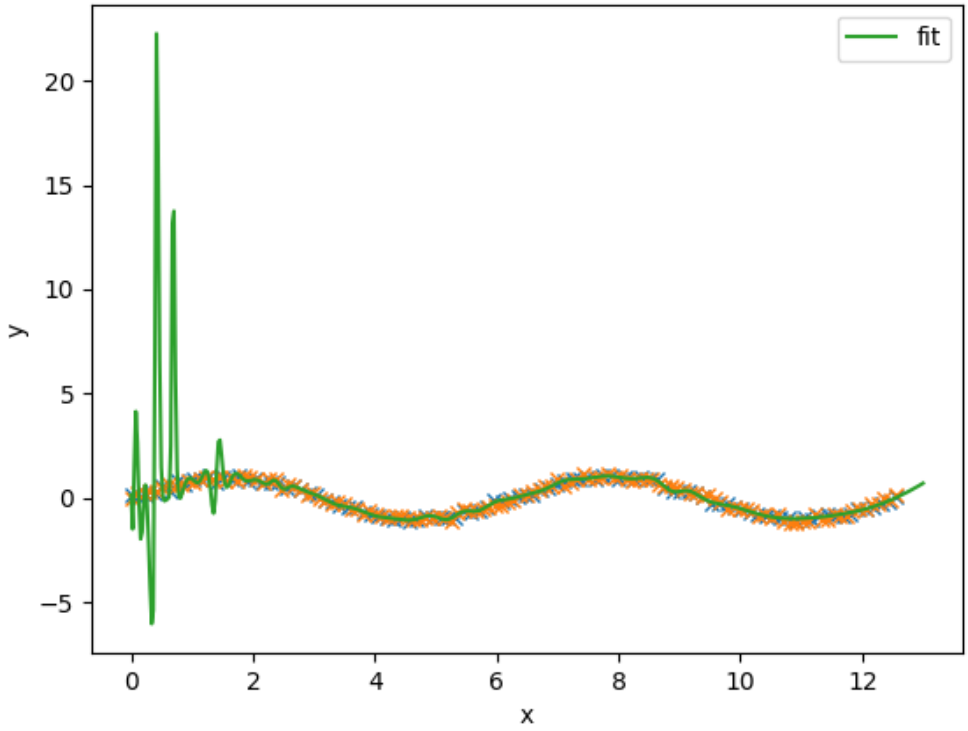

fit
1.0
0.5
0.0
−0.5
−1.0
y
0
2
4
6
8
10
12
x


fit
1.0
0.5
0.0
−0.5
−1.0
y
0
2
4
6
8
10
12
x


fit
1.0
0.5
0.0
−0.5
−1.0
y
0
2
4
6
8
10
12
x


fit
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
y
0
2
4
6
8
10
12
x


fit
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
−10.0
y
0
2
4
6
8
10
12
x



2.4 2d Distributed representations

[27]:
```
# train a model to analyze
model = train1([2])
```

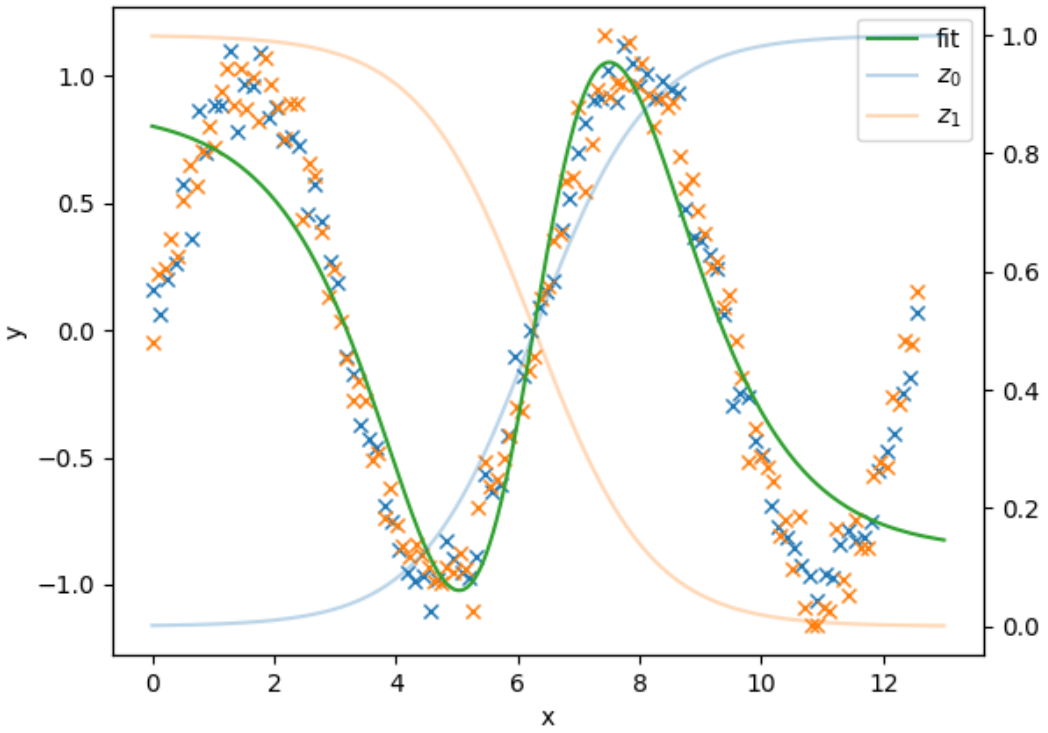
```
X1 shape: torch.Size([100, 1])
Repetition  0:          Current function value: 0.079573
         Iterations: 399
         Function evaluations: 542
         Gradient evaluations: 533
best_cost=0.080
X1 shape: torch.Size([100, 1])
Repetition  1: Optimization terminated successfully.
         Current function value: 0.301865
         Iterations: 131
         Function evaluations: 161
         Gradient evaluations: 161
best_cost=0.080
X1 shape: torch.Size([100, 1])
Repetition  2:
```

```
/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)

         Current function value: 0.079573
         Iterations: 384
         Function evaluations: 538
         Gradient evaluations: 526
best_cost=0.080
X1 shape: torch.Size([100, 1])
Repetition  3:          Current function value: 0.079573
         Iterations: 379
         Function evaluations: 534
         Gradient evaluations: 523
best_cost=0.080
X1 shape: torch.Size([100, 1])
Repetition  4: Optimization terminated successfully.
         Current function value: 0.372457
         Iterations: 131
         Function evaluations: 137
         Gradient evaluations: 137
best_cost=0.080
X1 shape: torch.Size([100, 1])
Repetition  5:

/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)

         Current function value: 0.079573
         Iterations: 402
         Function evaluations: 536
         Gradient evaluations: 525
best_cost=0.080
X1 shape: torch.Size([100, 1])
Repetition  6: Optimization terminated successfully.
         Current function value: 0.277769
         Iterations: 86
         Function evaluations: 97
```

```
         Gradient evaluations: 97
best_cost=0.080
X1 shape: torch.Size([100, 1])
Repetition  7: Optimization terminated successfully.
         Current function value: 0.357250
         Iterations: 85
         Function evaluations: 90
         Gradient evaluations: 90
best_cost=0.080
X1 shape: torch.Size([100, 1])
Repetition  8:          Current function value: 0.079572
         Iterations: 392
         Function evaluations: 619
         Gradient evaluations: 605
best_cost=0.080
X1 shape: torch.Size([100, 1])
Repetition  9:          Current function value: 0.079573
         Iterations: 403
         Function evaluations: 646
         Gradient evaluations: 633
best_cost=0.080
```

```
/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
```

[28]:
```
# plot the fit as well as the outputs of each neuron in the hidden
# layer (scale for the latter is shown on right y-axis)
nextplot()
plot1(X1, y1, label="train")
plot1(X1test, y1test, label="test")
plot1fit(torch.linspace(0, 13, 500).unsqueeze(1), model, hidden=True,␣
 ↪scale=False)
```



[29]:
```
# train a model to analyze
model = train1([3])
```

```
X1 shape: torch.Size([100, 1])
Repetition  0:          Current function value: 0.007324
         Iterations: 422
         Function evaluations: 535
         Gradient evaluations: 523
best_cost=0.007
X1 shape: torch.Size([100, 1])
Repetition  1:

/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)

         Current function value: 0.079572
```

```
         Iterations: 397
         Function evaluations: 577
         Gradient evaluations: 565
best_cost=0.007
X1 shape: torch.Size([100, 1])
Repetition  2:          Current function value: 0.007324
         Iterations: 543
         Function evaluations: 689
         Gradient evaluations: 677
best_cost=0.007
X1 shape: torch.Size([100, 1])
Repetition  3:

/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)

         Current function value: 0.049886
         Iterations: 317
         Function evaluations: 514
         Gradient evaluations: 503
best_cost=0.007
X1 shape: torch.Size([100, 1])
Repetition  4: Optimization terminated successfully.
         Current function value: 0.042560
         Iterations: 155
         Function evaluations: 172
         Gradient evaluations: 172
best_cost=0.007
X1 shape: torch.Size([100, 1])
Repetition  5:          Current function value: 0.079573
         Iterations: 364
         Function evaluations: 561
         Gradient evaluations: 548
best_cost=0.007
X1 shape: torch.Size([100, 1])
Repetition  6:

/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
```

```
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
```

```
         Current function value: 0.079573
         Iterations: 343
         Function evaluations: 561
         Gradient evaluations: 548
best_cost=0.007
X1 shape: torch.Size([100, 1])
Repetition  7:          Current function value: 0.049901
         Iterations: 221
         Function evaluations: 320
         Gradient evaluations: 308
best_cost=0.007
X1 shape: torch.Size([100, 1])
Repetition  8:
```

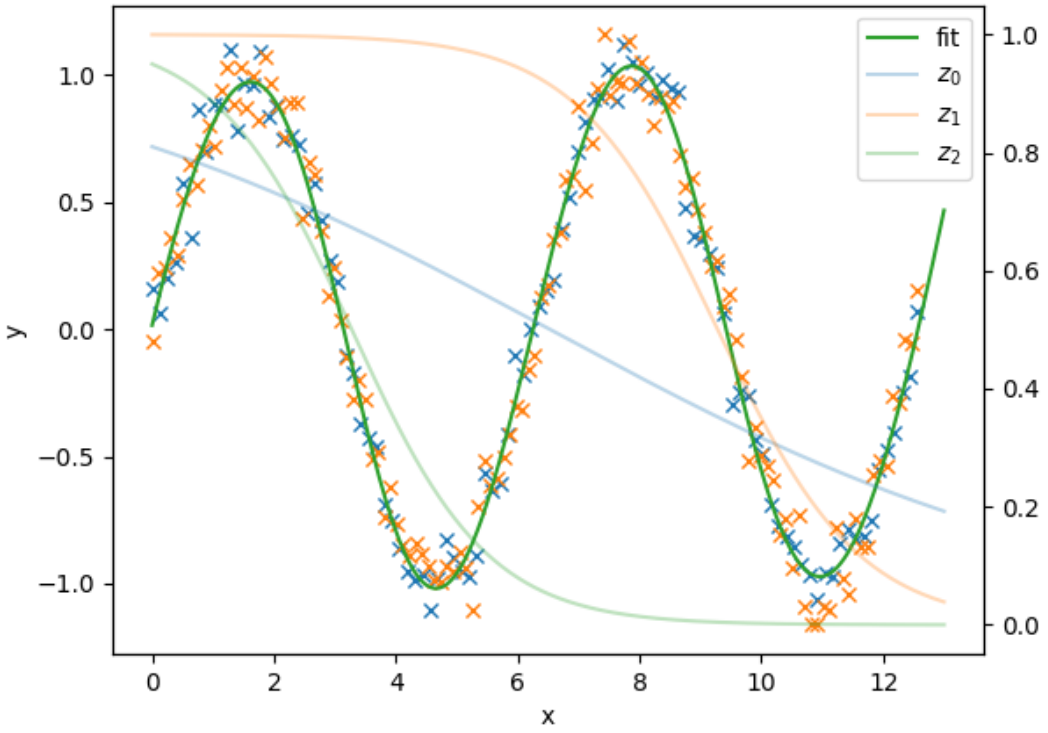
```
/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
```

```
         Current function value: 0.049877
         Iterations: 563
         Function evaluations: 811
         Gradient evaluations: 799
best_cost=0.007
X1 shape: torch.Size([100, 1])
Repetition  9:          Current function value: 0.053823
         Iterations: 540
         Function evaluations: 743
         Gradient evaluations: 728
best_cost=0.007
```

```
/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
```

[30]:
```
# plot the fit as well as the outputs of each neuron in the hidden
# layer (scale for the latter is shown on right y-axis)
nextplot()
plot1(X1, y1, label="train")
plot1(X1test, y1test, label="test")
```

```
plot1fit(torch.linspace(0, 13, 500).unsqueeze(1), model, hidden=True,␣
 ↪scale=False)
```



[31]:
```
# train a model to analyze
model = train1([10])
```

```
X1 shape: torch.Size([100, 1])
Repetition  0:

/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)

         Current function value: 0.006069
         Iterations: 2361
         Function evaluations: 2702
         Gradient evaluations: 2683
best_cost=0.006
X1 shape: torch.Size([100, 1])
Repetition  1:
```

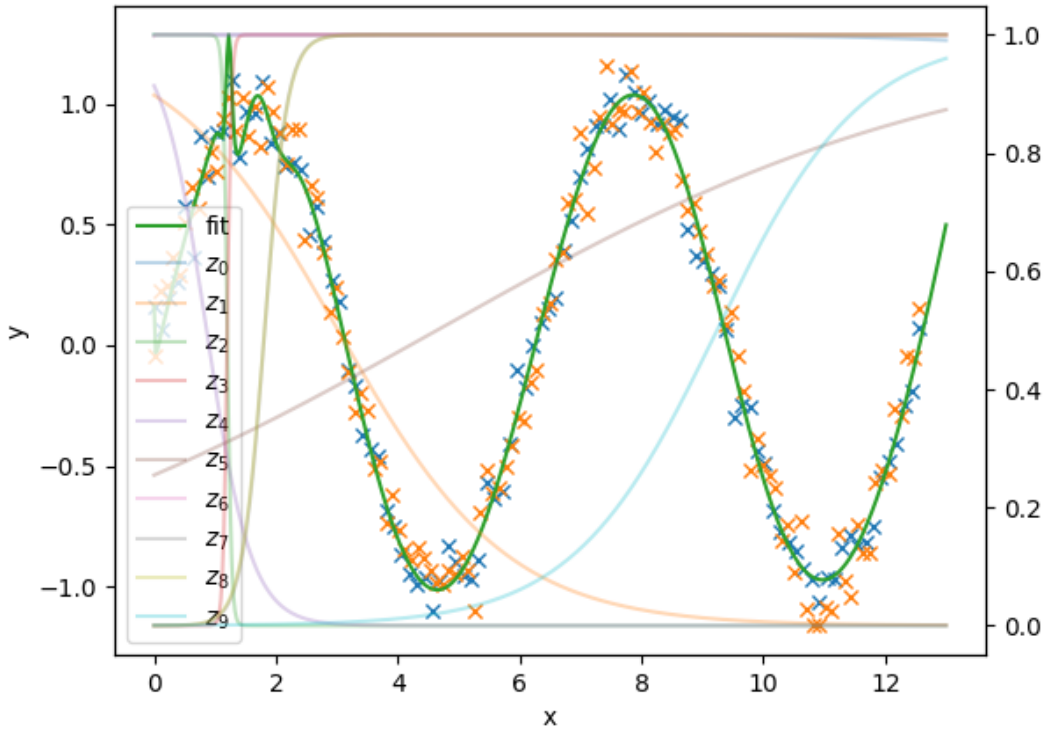
```
/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)

         Current function value: 0.006260
         Iterations: 2624
         Function evaluations: 3005
         Gradient evaluations: 2995
best_cost=0.006
X1 shape: torch.Size([100, 1])
Repetition  2:          Current function value: 0.006688
         Iterations: 587
         Function evaluations: 696
         Gradient evaluations: 686
best_cost=0.006
X1 shape: torch.Size([100, 1])
Repetition  3:

/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)

         Current function value: 0.006667
         Iterations: 1257
         Function evaluations: 1487
         Gradient evaluations: 1475
best_cost=0.006
X1 shape: torch.Size([100, 1])
Repetition  4:

/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)

         Current function value: 0.006232
         Iterations: 1648
         Function evaluations: 1908
         Gradient evaluations: 1897
best_cost=0.006
X1 shape: torch.Size([100, 1])
Repetition  5:

/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
```

```
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)

         Current function value: 0.006114
         Iterations: 1489
         Function evaluations: 1719
         Gradient evaluations: 1707
best_cost=0.006
X1 shape: torch.Size([100, 1])
Repetition  6:

/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)

         Current function value: 0.006576
         Iterations: 2629
         Function evaluations: 2955
         Gradient evaluations: 2945
best_cost=0.006
X1 shape: torch.Size([100, 1])
Repetition  7:

/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)

         Current function value: 0.006076
         Iterations: 1852
         Function evaluations: 2128
         Gradient evaluations: 2116
best_cost=0.006
X1 shape: torch.Size([100, 1])
Repetition  8:

/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)

         Current function value: 0.006614
         Iterations: 2289
         Function evaluations: 2825
         Gradient evaluations: 2807
best_cost=0.006
X1 shape: torch.Size([100, 1])
Repetition  9:          Current function value: 0.006805
         Iterations: 1034
         Function evaluations: 1288
```

```
         Gradient evaluations: 1275
best_cost=0.006
```

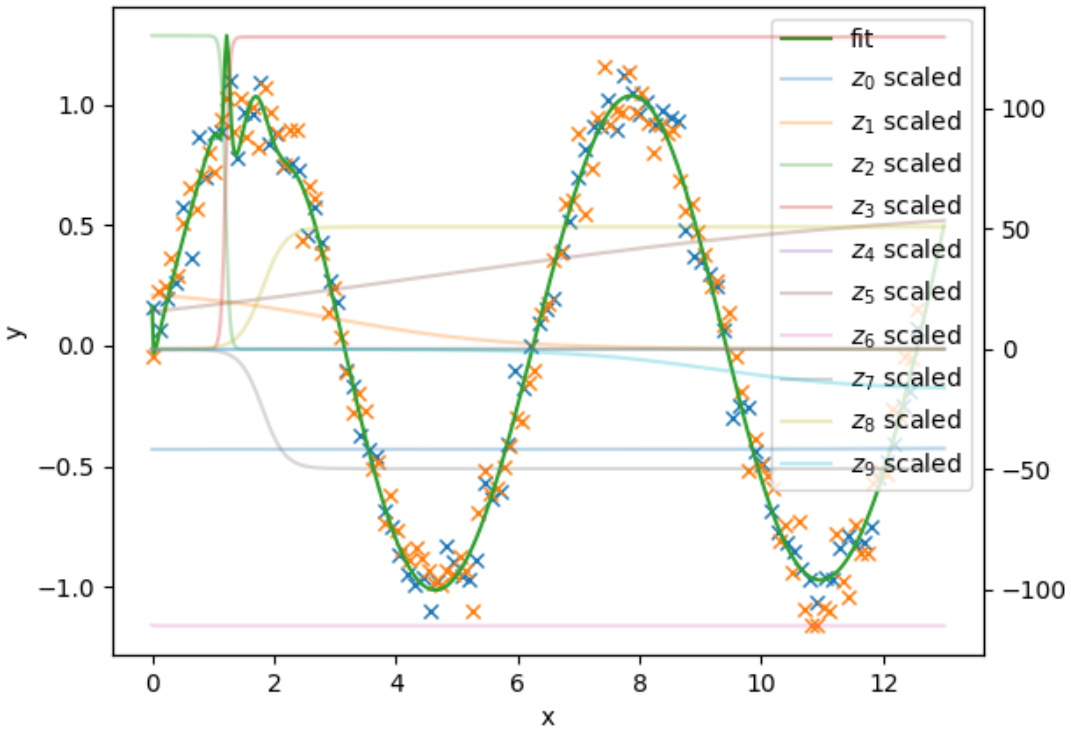
```
/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
```

[32]:
```
# plot the fit as well as the outputs of each neuron in the hidden
# layer (scale for the latter is shown on right y-axis)
nextplot()
plot1(X1, y1, label="train")
plot1(X1test, y1test, label="test")
plot1fit(torch.linspace(0, 13, 500).unsqueeze(1), model, hidden=True,␣
 ↪scale=False)
```



[33]:
```
# plot the fit as well as the outputs of each neuron in the hidden layer, scaled
# by its weight for the output neuron (scale for the latter is shown on right
# y-axis)

nextplot()
```

```
plot1(X1, y1, label="train")
plot1(X1test, y1test, label="test")
plot1fit(torch.linspace(0, 13, 500).unsqueeze(1), model, hidden=True,␣
  ↪scale=True)
```



2.5 2e Experiment with different optimizers (optional)

[34]:
```
# PyTorch provides many gradient-based optimizers; see
# https://pytorch.org/docs/stable/optim.html. You can use a PyTorch optimizer
# as follows.
train_adam = lambda model, **kwargs: fnn_train(
    X1, y1, model, optimizer=torch.optim.Adam(model.parameters(), lr=0.01),␣
  ↪**kwargs
)
model = train1([50], nreps=1, train=train_adam, max_epochs=5000, tol=1e-8,␣
  ↪verbose=True)
```

```
X1 shape: torch.Size([100, 1])
Repetition  0: Epoch     0: cost=   0.561
Epoch     1: cost=   0.547
Epoch     2: cost=   0.508
```

```
Epoch     3: cost=   0.491
Epoch     4: cost=   0.482
Epoch     5: cost=   0.464
Epoch     6: cost=   0.450
Epoch     7: cost=   0.447
Epoch     8: cost=   0.445
Epoch     9: cost=   0.439
Epoch    10: cost=   0.437
Epoch    11: cost=   0.440
Epoch    12: cost=   0.443
Epoch    13: cost=   0.443
Epoch    14: cost=   0.443
Epoch    15: cost=   0.445
Epoch    16: cost=   0.446
Epoch    17: cost=   0.445
Epoch    18: cost=   0.443
Epoch    19: cost=   0.441
Epoch    20: cost=   0.441
Epoch    21: cost=   0.438
Epoch    22: cost=   0.436
Epoch    23: cost=   0.434
Epoch    24: cost=   0.433
Epoch    25: cost=   0.431
Epoch    26: cost=   0.430
Epoch    27: cost=   0.429
Epoch    28: cost=   0.428
Epoch    29: cost=   0.428
Epoch    30: cost=   0.428
Epoch    31: cost=   0.427
Epoch    32: cost=   0.427
Epoch    33: cost=   0.427
Epoch    34: cost=   0.426
Epoch    35: cost=   0.426
Epoch    36: cost=   0.425
Epoch    37: cost=   0.425
Epoch    38: cost=   0.424
Epoch    39: cost=   0.423
Epoch    40: cost=   0.422
Epoch    41: cost=   0.421
Epoch    42: cost=   0.421
Epoch    43: cost=   0.420
Epoch    44: cost=   0.419
Epoch    45: cost=   0.419
Epoch    46: cost=   0.418
Epoch    47: cost=   0.418
Epoch    48: cost=   0.417
Epoch    49: cost=   0.417
Epoch    50: cost=   0.416
```

```
Epoch    51: cost=   0.416
Epoch    52: cost=   0.415
Epoch    53: cost=   0.415
Epoch    54: cost=   0.414
Epoch    55: cost=   0.414
Epoch    56: cost=   0.413
Epoch    57: cost=   0.412
Epoch    58: cost=   0.412
Epoch    59: cost=   0.411
Epoch    60: cost=   0.411
Epoch    61: cost=   0.410
Epoch    62: cost=   0.410
Epoch    63: cost=   0.409
Epoch    64: cost=   0.409
Epoch    65: cost=   0.408
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Epoch    67: cost=   0.407
Epoch    68: cost=   0.407
Epoch    69: cost=   0.406
Epoch    70: cost=   0.406
Epoch    71: cost=   0.405
Epoch    72: cost=   0.405
Epoch    73: cost=   0.404
Epoch    74: cost=   0.403
Epoch    75: cost=   0.403
Epoch    76: cost=   0.402
Epoch    77: cost=   0.402
Epoch    78: cost=   0.401
Epoch    79: cost=   0.401
Epoch    80: cost=   0.400
Epoch    81: cost=   0.400
Epoch    82: cost=   0.399
Epoch    83: cost=   0.399
Epoch    84: cost=   0.398
Epoch    85: cost=   0.398
Epoch    86: cost=   0.397
Epoch    87: cost=   0.397
Epoch    88: cost=   0.396
Epoch    89: cost=   0.396
Epoch    90: cost=   0.395
Epoch    91: cost=   0.395
Epoch    92: cost=   0.394
Epoch    93: cost=   0.394
Epoch    94: cost=   0.393
Epoch    95: cost=   0.393
Epoch    96: cost=   0.393
Epoch    97: cost=   0.392
Epoch    98: cost=   0.392
```

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Epoch    99: cost=   0.391
Epoch   100: cost=   0.391
Epoch   101: cost=   0.390
Epoch   102: cost=   0.390
Epoch   103: cost=   0.389
Epoch   104: cost=   0.389
Epoch   105: cost=   0.388
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Epoch   114: cost=   0.384
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Epoch   116: cost=   0.383
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Epoch   118: cost=   0.382
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Epoch   123: cost=   0.380
Epoch   124: cost=   0.380
Epoch   125: cost=   0.379
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Epoch   130: cost=   0.377
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Epoch   132: cost=   0.377
Epoch   133: cost=   0.376
Epoch   134: cost=   0.376
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Epoch   142: cost=   0.372
Epoch   143: cost=   0.372
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Epoch   145: cost=   0.371
Epoch   146: cost=   0.371
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Epoch   147: cost=   0.370
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Epoch   193: cost=   0.346
Epoch   194: cost=   0.345
```

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Epoch   195: cost=   0.345
Epoch   196: cost=   0.344
Epoch   197: cost=   0.343
Epoch   198: cost=   0.343
Epoch   199: cost=   0.342
Epoch   200: cost=   0.341
Epoch   201: cost=   0.341
Epoch   202: cost=   0.340
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Epoch   206: cost=   0.337
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Epoch   213: cost=   0.331
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Epoch   226: cost=   0.319
Epoch   227: cost=   0.318
Epoch   228: cost=   0.317
Epoch   229: cost=   0.315
Epoch   230: cost=   0.314
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Epoch   233: cost=   0.311
Epoch   234: cost=   0.310
Epoch   235: cost=   0.309
Epoch   236: cost=   0.308
Epoch   237: cost=   0.306
Epoch   238: cost=   0.305
Epoch   239: cost=   0.304
Epoch   240: cost=   0.303
Epoch   241: cost=   0.301
Epoch   242: cost=   0.300
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Epoch   243: cost=   0.299
Epoch   244: cost=   0.297
Epoch   245: cost=   0.296
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Epoch   248: cost=   0.292
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Epoch   252: cost=   0.286
Epoch   253: cost=   0.285
Epoch   254: cost=   0.283
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Epoch   256: cost=   0.280
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Epoch   271: cost=   0.260
Epoch   272: cost=   0.255
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Epoch   274: cost=   0.254
Epoch   275: cost=   0.250
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Epoch   280: cost=   0.242
Epoch   281: cost=   0.240
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Epoch   283: cost=   0.237
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Epoch   285: cost=   0.234
Epoch   286: cost=   0.232
Epoch   287: cost=   0.231
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Epoch   291: cost=   0.224
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Epoch   339: cost=   0.156
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Epoch   387: cost=   0.119
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Epoch   435: cost=   0.101
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Epoch   531: cost=   0.083
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Epoch  2018: cost=   0.022
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Epoch  2354: cost=   0.014
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Epoch  3075: cost=   0.008
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Epoch  3315: cost=   0.007
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Epoch  3319: cost=   0.008
Epoch  3320: cost=   0.009
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Epoch  3398: cost=   0.007
Epoch  3399: cost=   0.007
Epoch  3400: cost=   0.007
best_cost=0.007
```

[35]:
```
# Experiment with different number of layers and activation functions. Here is
# an example with three hidden layers (of sizes 4, 5, and 6) and ReLU␣
 ↪activations.
#
# You can also plot the outputs of the hidden neurons in the first layer (using
# the same code above).
model = train1([4, 5, 6], nreps=50, phi=F.relu)
```

```python
nextplot()
plot1(X1, y1, label="train")
plot1(X1test, y1test, label="test")
plot1fit(torch.linspace(0, 13, 500).unsqueeze(1), model)
print("Training error:", F.mse_loss(y1, model(X1)).item())
print("Test error    :", F.mse_loss(y1test, model(X1test)).item())
```

```
X1 shape: torch.Size([100, 1])
Repetition  0:
```

```
/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
```

```
         Current function value: 0.027642
         Iterations: 106
         Function evaluations: 226
         Gradient evaluations: 221
best_cost=0.028
X1 shape: torch.Size([100, 1])
Repetition  1:          Current function value: 0.372736
         Iterations: 20
         Function evaluations: 93
         Gradient evaluations: 89
best_cost=0.028
X1 shape: torch.Size([100, 1])
Repetition  2:          Current function value: 0.082316
         Iterations: 77
         Function evaluations: 175
         Gradient evaluations: 169
best_cost=0.028
X1 shape: torch.Size([100, 1])
Repetition  3:
```

```
/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
```

```
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)

         Current function value: 0.372829
         Iterations: 54
         Function evaluations: 224
         Gradient evaluations: 219
best_cost=0.028
X1 shape: torch.Size([100, 1])
Repetition  4:          Current function value: 0.356289
         Iterations: 62
         Function evaluations: 159
         Gradient evaluations: 151
best_cost=0.028
X1 shape: torch.Size([100, 1])
Repetition  5:

/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)

         Current function value: 0.436769
         Iterations: 19
         Function evaluations: 107
         Gradient evaluations: 98
best_cost=0.028
X1 shape: torch.Size([100, 1])
Repetition  6:          Current function value: 0.082545
         Iterations: 87
         Function evaluations: 201
         Gradient evaluations: 196
best_cost=0.028
X1 shape: torch.Size([100, 1])
Repetition  7:

/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)

         Current function value: 0.355800
         Iterations: 47
         Function evaluations: 226
         Gradient evaluations: 220
best_cost=0.028
```

```
X1 shape: torch.Size([100, 1])
Repetition  8:
/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)

         Current function value: 0.098439
         Iterations: 131
         Function evaluations: 239
         Gradient evaluations: 227
best_cost=0.028
X1 shape: torch.Size([100, 1])
Repetition  9:          Current function value: 0.355687
         Iterations: 139
         Function evaluations: 261
         Gradient evaluations: 257
best_cost=0.028
X1 shape: torch.Size([100, 1])
Repetition  10: Optimization terminated successfully.
         Current function value: 0.101445
         Iterations: 53
         Function evaluations: 66
         Gradient evaluations: 66
best_cost=0.028
X1 shape: torch.Size([100, 1])
Repetition  11:
/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)

         Current function value: 0.356116
         Iterations: 52
         Function evaluations: 161
         Gradient evaluations: 155
best_cost=0.028
X1 shape: torch.Size([100, 1])
Repetition  12:          Current function value: 0.435001
         Iterations: 46
```

```
         Function evaluations: 123
         Gradient evaluations: 119
best_cost=0.028
X1 shape: torch.Size([100, 1])
Repetition  13:

/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)

         Current function value: 0.435068
         Iterations: 41
         Function evaluations: 131
         Gradient evaluations: 125
best_cost=0.028
X1 shape: torch.Size([100, 1])
Repetition  14:

/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)

         Current function value: 0.007341
         Iterations: 212
         Function evaluations: 447
         Gradient evaluations: 433
best_cost=0.007
X1 shape: torch.Size([100, 1])
Repetition  15:          Current function value: 0.355830
         Iterations: 46
         Function evaluations: 140
         Gradient evaluations: 135
best_cost=0.007
X1 shape: torch.Size([100, 1])
Repetition  16:

/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
```

```
/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)

         Current function value: 0.353423
         Iterations: 154
         Function evaluations: 435
         Gradient evaluations: 415
best_cost=0.007
X1 shape: torch.Size([100, 1])
Repetition  17:          Current function value: 0.405736
         Iterations: 15
         Function evaluations: 95
         Gradient evaluations: 90
best_cost=0.007
X1 shape: torch.Size([100, 1])
Repetition  18:          Current function value: 0.085500
         Iterations: 113
         Function evaluations: 241
         Gradient evaluations: 234
best_cost=0.007
X1 shape: torch.Size([100, 1])
Repetition  19:

/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)

         Current function value: 0.082287
         Iterations: 96
         Function evaluations: 187
         Gradient evaluations: 183
best_cost=0.007
X1 shape: torch.Size([100, 1])
Repetition  20:          Current function value: 0.434775
         Iterations: 18
         Function evaluations: 113
         Gradient evaluations: 102
best_cost=0.007
X1 shape: torch.Size([100, 1])
Repetition  21:

/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
```

```
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)

         Current function value: 0.356477
         Iterations: 34
         Function evaluations: 122
         Gradient evaluations: 118
best_cost=0.007
X1 shape: torch.Size([100, 1])
Repetition  22:          Current function value: 0.355919
         Iterations: 54
         Function evaluations: 154
         Gradient evaluations: 149
best_cost=0.007
X1 shape: torch.Size([100, 1])
Repetition  23:

/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)

         Current function value: 0.008464
         Iterations: 251
         Function evaluations: 460
         Gradient evaluations: 451
best_cost=0.007
X1 shape: torch.Size([100, 1])
Repetition  24:

/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)

         Current function value: 0.008703
         Iterations: 185
         Function evaluations: 276
         Gradient evaluations: 270
best_cost=0.007
X1 shape: torch.Size([100, 1])
Repetition  25:

/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
```

```
/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)

         Current function value: 0.355581
         Iterations: 138
         Function evaluations: 261
         Gradient evaluations: 255
best_cost=0.007
X1 shape: torch.Size([100, 1])
Repetition  26:          Current function value: 0.085421
         Iterations: 55
         Function evaluations: 161
         Gradient evaluations: 155
best_cost=0.007
X1 shape: torch.Size([100, 1])
Repetition  27:          Current function value: 0.357612
         Iterations: 46
         Function evaluations: 123
         Gradient evaluations: 116
best_cost=0.007
X1 shape: torch.Size([100, 1])
Repetition  28:

/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)

         Current function value: 0.434378
         Iterations: 34
         Function evaluations: 122
         Gradient evaluations: 116
best_cost=0.007
X1 shape: torch.Size([100, 1])
Repetition  29:

/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)

         Current function value: 0.083644
         Iterations: 205
         Function evaluations: 328
```

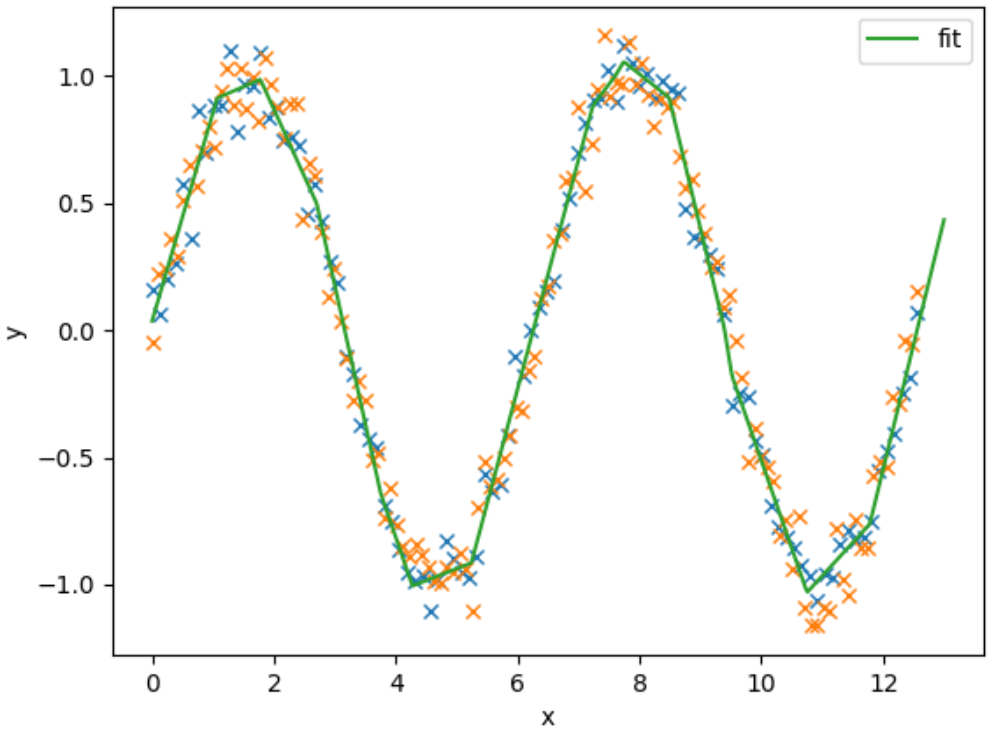
```
         Gradient evaluations: 323
best_cost=0.007
X1 shape: torch.Size([100, 1])
Repetition  30:

/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)

         Current function value: 0.434433
         Iterations: 28
         Function evaluations: 111
         Gradient evaluations: 107
best_cost=0.007
X1 shape: torch.Size([100, 1])
Repetition  31:          Current function value: 0.356297
         Iterations: 72
         Function evaluations: 166
         Gradient evaluations: 161
best_cost=0.007
X1 shape: torch.Size([100, 1])
Repetition  32:

/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)

         Current function value: 0.005862
         Iterations: 382
         Function evaluations: 528
         Gradient evaluations: 523
best_cost=0.006
X1 shape: torch.Size([100, 1])
Repetition  33:          Current function value: 0.308671
         Iterations: 68
         Function evaluations: 180
         Gradient evaluations: 175
best_cost=0.006
X1 shape: torch.Size([100, 1])
Repetition  34:
```

```
/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)

         Current function value: 0.436922
         Iterations: 60
         Function evaluations: 163
         Gradient evaluations: 155
best_cost=0.006
X1 shape: torch.Size([100, 1])
Repetition  35:          Current function value: 0.435347
         Iterations: 37
         Function evaluations: 109
         Gradient evaluations: 101
best_cost=0.006
X1 shape: torch.Size([100, 1])
Repetition  36:

/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)

         Current function value: 0.371726
         Iterations: 35
         Function evaluations: 118
         Gradient evaluations: 113
best_cost=0.006
X1 shape: torch.Size([100, 1])
Repetition  37:          Current function value: 0.008174
         Iterations: 108
         Function evaluations: 221
         Gradient evaluations: 217
best_cost=0.006
X1 shape: torch.Size([100, 1])
Repetition  38:

/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
```

```
/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)

         Current function value: 0.355790
         Iterations: 129
         Function evaluations: 232
         Gradient evaluations: 227
best_cost=0.006
X1 shape: torch.Size([100, 1])
Repetition  39: Optimization terminated successfully.
         Current function value: 0.435275
         Iterations: 35
         Function evaluations: 52
         Gradient evaluations: 52
best_cost=0.006
X1 shape: torch.Size([100, 1])
Repetition  40:          Current function value: 0.435287
         Iterations: 18
         Function evaluations: 99
         Gradient evaluations: 94
best_cost=0.006
X1 shape: torch.Size([100, 1])
Repetition  41:

/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)

         Current function value: 0.372387
         Iterations: 57
         Function evaluations: 177
         Gradient evaluations: 173
best_cost=0.006
X1 shape: torch.Size([100, 1])
Repetition  42:          Current function value: 0.356134
         Iterations: 110
         Function evaluations: 214
         Gradient evaluations: 209
best_cost=0.006
X1 shape: torch.Size([100, 1])
Repetition  43: Optimization terminated successfully.
         Current function value: 0.098100
```

```
         Iterations: 147
         Function evaluations: 169
         Gradient evaluations: 169
best_cost=0.006
X1 shape: torch.Size([100, 1])
Repetition  44:          Current function value: 0.356624
         Iterations: 28
         Function evaluations: 115
         Gradient evaluations: 103
best_cost=0.006
X1 shape: torch.Size([100, 1])
Repetition  45: Optimization terminated successfully.
         Current function value: 0.506238
         Iterations: 14
         Function evaluations: 16
         Gradient evaluations: 16
best_cost=0.006
X1 shape: torch.Size([100, 1])
Repetition  46:

/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)

         Current function value: 0.092225
         Iterations: 89
         Function evaluations: 209
         Gradient evaluations: 204
best_cost=0.006
X1 shape: torch.Size([100, 1])
Repetition  47:

/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)

         Current function value: 0.355652
         Iterations: 168
         Function evaluations: 310
         Gradient evaluations: 306
best_cost=0.006
X1 shape: torch.Size([100, 1])
Repetition  48:
```

```
/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)

         Current function value: 0.008766
         Iterations: 145
         Function evaluations: 251
         Gradient evaluations: 243
best_cost=0.006
X1 shape: torch.Size([100, 1])
Repetition  49:          Current function value: 0.007130
         Iterations: 196
         Function evaluations: 295
         Gradient evaluations: 288
best_cost=0.006
Training error: 0.005861788988113403
Test error    : 0.010837704874575138

/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
```



3 3 Backpropagation

[36]:
```
# Let's fit the model with one hidden layer consisting of 50 units.
model = train1([50], nreps=1)
print("Training error:", F.mse_loss(y1, model(X1)).item())
print("Test error    :", F.mse_loss(y1test, model(X1test)).item())

# Extract parameters
pars = dict(model.named_parameters())
W1 = pars["0_weight"].data  # 1x50
b1 = pars["0_bias"].data  # 50
W2 = pars["1_weight"].data  # 50x1
b2 = pars["1_bias"].data  # 1
```

```
X1 shape: torch.Size([100, 1])
Repetition  0:          Current function value: 0.004436
         Iterations: 3943
         Function evaluations: 4298
         Gradient evaluations: 4289
best_cost=0.004
Training error: 0.004435778129845858
Test error    : 26.959814071655273
```

```
/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/scipy/optimize/_minimize.py:708: OptimizeWarning: Desired error not
necessarily achieved due to precision loss.
  res = _minimize_bfgs(fun, x0, args, jac, callback, **options)
```

3.1 3a Forward pass

[37]:
```
# Compute results of forward pass on an example x (i.e., z1, z2, z3, z4, yhat,␣
 ↪l) using Pytorch
x = X1test[1, :]
y = y1test[1, :]
print(f"x={x}, y={y}, yhat={model(x).detach()}, l={torch.nn.
 ↪MSELoss()(y,model(x))}")
```

```
x=tensor([0.1030]), y=tensor([0.2253]), yhat=tensor([[-1.3225]]),
l=2.3957338333129883
```

```
/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/torch/nn/modules/loss.py:535: UserWarning: Using a target size
(torch.Size([1, 1])) that is different to the input size (torch.Size([1])). This
will likely lead to incorrect results due to broadcasting. Please ensure they
have the same size.
  return F.mse_loss(input, target, reduction=self.reduction)
```

[38]:
```
# Now do this by hand (including all intermediate values). You should get the␣
 ↪same
# results as above.
z1 = W1.t() * x
z2 = z1 + b1.unsqueeze(1)
z3 = 1/ (1 + torch.exp(-1 * z2))
z4 = torch.zeros(1, 1)
for i in range(W2.shape[0]):
    z4 += W2[i] * z3[i]
y_hat = z4 + b2
l = (y - y_hat)**2
print(f"x={x}, y={y}, yhat={y_hat}, l={l}")
```

```
x=tensor([0.1030]), y=tensor([0.2253]), yhat=tensor([[-1.3226]]),
l=tensor([[2.3958]])
```

3.2 3b Backward pass

[39]:
```
# Compute results of backward pass on example output (i.e., delta_x, delta_W1,␣
 ↪delta_z1,
# delta_b1, delta_z2, delta_z3, delta_W2, delta_z4, delta_b2, delta_yhat,␣
 ↪delta_l, delta_y)

delta_l = 1
delta_y = (2 * (y - y_hat)).squeeze(1)
delta_yhat = -2 * (y - y_hat)
delta_b2 = (delta_yhat * 1).squeeze(1)
delta_z4 = delta_yhat * 1
delta_W2 = delta_z4 * z3
delta_z3 = delta_z4 * W2
delta_z2 = delta_z3 * z3 * (1 - z3)
delta_b1 = delta_z2.squeeze(1)
delta_z1 = delta_z2

delta_x = torch.zeros(1, 1)
for i in range(W1.shape[0]):
    delta_x += delta_z1[i] * W1[0][i]
delta_x = delta_x.squeeze(1)

delta_W1 = (delta_z1 * x).t()
```

[40]:
```
# Use PyTorch's backprop
x.requires_grad = True
y.requires_grad = True
if x.grad is not None:
    x.grad.zero_()
if y.grad is not None:
```

```
    y.grad.zero_()
model.zero_grad()
t_yhat = model(x)
t_yhat.retain_grad()
t_l = torch.nn.MSELoss()(t_yhat, y)
t_l.backward()
t_delta_l = 1
t_delta_y = y.grad
t_delta_yhat = t_yhat.grad
t_delta_b2 = model.get_parameter("1_bias").grad
t_delta_W2 = model.get_parameter("1_weight").grad
t_delta_b1 = model.get_parameter("0_bias").grad
t_delta_W1 = model.get_parameter("0_weight").grad
t_delta_x = x.grad
```

```
/Users/ngkokteng/PycharmProjects/Deep Learning/.venv/lib/python3.9/site-
packages/torch/nn/modules/loss.py:535: UserWarning: Using a target size
(torch.Size([1])) that is different to the input size (torch.Size([1, 1])). This
will likely lead to incorrect results due to broadcasting. Please ensure they
have the same size.
  return F.mse_loss(input, target, reduction=self.reduction)
```

[41]:
```
# Check if equal (show squared error)
for v in ["y", "yhat", "b2", "W2", "b1", "W1", "x"]:
    print(f'{v}, squared error={torch.
 ↪sum((eval("t_delta_"+v)-eval("delta_"+v))**2)}')
```

```
y, squared error=2.3283064365386963e-10
yhat, squared error=2.3283064365386963e-10
b2, squared error=2.3283064365386963e-10
W2, squared error=5.1774509124413726e-09
b1, squared error=1.7092266091367492e-07
W1, squared error=1.8068877505328373e-09
x, squared error=1213.602294921875
```

[42]:
```
# Check if equal (show actual values)
for v in ["l", "y", "yhat", "b2", "W2", "b1", "W1", "x"]:
    print(f'{v}, pytorch={eval("t_delta_"+v)}, you={eval("delta_"+v)}')
```

```
l, pytorch=1, you=1
y, pytorch=tensor([3.0956]), you=tensor([3.0956])
yhat, pytorch=tensor([[-3.0956]]), you=tensor([[-3.0956]])
b2, pytorch=tensor([-3.0956]), you=tensor([-3.0956])
W2, pytorch=tensor([[-3.0885e+00],
        [-2.7294e+00],
        [-3.0956e+00],
        [-3.0860e+00],
        [-1.9720e-05],
        [-2.1162e-01],
```

```
[-1.5810e-05],
[-1.9257e-26],
[-3.0956e+00],
[-2.8082e+00],
[-6.7645e-03],
[-2.8477e+00],
[-6.4975e-18],
[-1.4684e+00],
[-2.8181e-05],
[-1.2377e-06],
[-3.0956e+00],
[-1.3328e-31],
[-6.4777e-07],
[-3.0956e+00],
[-1.2281e+00],
[-2.8658e+00],
[-1.9031e-02],
[-3.0956e+00],
[-4.3156e-05],
[-2.6180e-01],
[-5.3807e-03],
[-2.3027e+00],
[-1.3252e-06],
[-3.1496e-01],
[-3.0956e+00],
[-1.3970e+00],
[-2.8697e+00],
[-3.0956e+00],
[-4.4417e-01],
[-3.0951e+00],
[-3.0956e+00],
[-3.0314e+00],
[-3.0948e+00],
[-2.0257e-02],
[-1.8734e-22],
[-4.9265e-08],
[-3.0286e+00],
[-2.2877e-08],
[-3.0338e+00],
[-3.0954e+00],
[-1.8084e-06],
[-4.2464e-06],
[-2.8255e-01],
[-3.0149e+00]]), you=tensor([[-3.0885e+00],
[-2.7295e+00],
[-3.0956e+00],
[-3.0860e+00],
[-1.9720e-05],
```

```
        [-2.1162e-01],
        [-1.5810e-05],
        [-1.9257e-26],
        [-3.0956e+00],
        [-2.8083e+00],
        [-6.7645e-03],
        [-2.8478e+00],
        [-6.4975e-18],
        [-1.4684e+00],
        [-2.8181e-05],
        [-1.2377e-06],
        [-3.0956e+00],
        [-1.3328e-31],
        [-6.4777e-07],
        [-3.0956e+00],
        [-1.2281e+00],
        [-2.8658e+00],
        [-1.9031e-02],
        [-3.0956e+00],
        [-4.3156e-05],
        [-2.6180e-01],
        [-5.3807e-03],
        [-2.3027e+00],
        [-1.3252e-06],
        [-3.1496e-01],
        [-3.0956e+00],
        [-1.3970e+00],
        [-2.8697e+00],
        [-3.0956e+00],
        [-4.4418e-01],
        [-3.0951e+00],
        [-3.0956e+00],
        [-3.0315e+00],
        [-3.0948e+00],
        [-2.0257e-02],
        [-1.8734e-22],
        [-4.9266e-08],
        [-3.0286e+00],
        [-2.2878e-08],
        [-3.0338e+00],
        [-3.0954e+00],
        [-1.8084e-06],
        [-4.2464e-06],
        [-2.8255e-01],
        [-3.0149e+00]])
b1, pytorch=tensor([ 3.5041e-02,  1.5967e+01, -3.4076e-05, -2.4785e-01,
1.3148e-03,
        -2.7232e-01,  4.1549e-04,  8.6091e-25,  0.0000e+00,  9.2368e+00,
```

```
         5.3845e-01,  1.6630e+01,  4.4990e-16,  3.2996e+01, -1.2648e-03,
        -6.2552e-05,  0.0000e+00, -7.9921e-30,  4.1968e-05, -0.0000e+00,
        -1.1424e+01, -6.4931e+00,  5.5412e-01,  0.0000e+00, -3.2869e-03,
        -6.1175e+00,  3.5463e-01, -6.5427e+01,  3.2628e-05, -1.9352e+01,
         0.0000e+00,  1.2294e+01, -7.0916e+00, -0.0000e+00,  1.1568e+01,
         1.3491e-02,  0.0000e+00, -1.5278e+00, -3.1560e-02,  3.4543e-01,
         1.7002e-20, -6.1738e-07, -2.2016e+00, -2.2857e-06, -2.8186e+00,
         1.7621e-02,  1.4582e-04, -2.6997e-04, -1.0979e+01,  3.6433e+00]),
you=tensor([ 3.5041e-02,  1.5968e+01, -3.4076e-05, -2.4786e-01,  1.3148e-03,
        -2.7232e-01,  4.1549e-04,  8.6091e-25,  0.0000e+00,  9.2368e+00,
         5.3845e-01,  1.6630e+01,  4.4990e-16,  3.2996e+01, -1.2648e-03,
        -6.2553e-05,  0.0000e+00, -7.9922e-30,  4.1968e-05, -0.0000e+00,
        -1.1424e+01, -6.4931e+00,  5.5412e-01,  0.0000e+00, -3.2869e-03,
        -6.1176e+00,  3.5463e-01, -6.5427e+01,  3.2628e-05, -1.9352e+01,
         0.0000e+00,  1.2294e+01, -7.0916e+00, -0.0000e+00,  1.1568e+01,
         1.3491e-02,  0.0000e+00, -1.5279e+00, -3.1560e-02,  3.4543e-01,
         1.7002e-20, -6.1738e-07, -2.2017e+00, -2.2857e-06, -2.8186e+00,
         1.7621e-02,  1.4582e-04, -2.6997e-04, -1.0979e+01,  3.6433e+00])
W1, pytorch=tensor([[ 3.6093e-03,  1.6447e+00, -3.5099e-06, -2.5530e-02,
1.3543e-04,
         -2.8050e-02,  4.2797e-05,  8.8676e-26,  0.0000e+00,  9.5141e-01,
          5.5462e-02,  1.7129e+00,  4.6341e-17,  3.3987e+00, -1.3027e-04,
         -6.4431e-06,  0.0000e+00, -8.2321e-31,  4.3228e-06,  0.0000e+00,
         -1.1767e+00, -6.6881e-01,  5.7076e-02,  0.0000e+00, -3.3856e-04,
         -6.3012e-01,  3.6528e-02, -6.7392e+00,  3.3608e-06, -1.9933e+00,
          0.0000e+00,  1.2663e+00, -7.3046e-01,  0.0000e+00,  1.1916e+00,
          1.3896e-03,  0.0000e+00, -1.5737e-01, -3.2508e-03,  3.5580e-02,
          1.7513e-21, -6.3592e-08, -2.2678e-01, -2.3543e-07, -2.9033e-01,
          1.8150e-03,  1.5020e-05, -2.7808e-05, -1.1308e+00,  3.7527e-01]]),
you=tensor([[ 3.6093e-03,  1.6447e+00, -3.5099e-06, -2.5530e-02,  1.3543e-04,
         -2.8050e-02,  4.2797e-05,  8.8677e-26,  0.0000e+00,  9.5142e-01,
          5.5462e-02,  1.7129e+00,  4.6341e-17,  3.3987e+00, -1.3027e-04,
         -6.4431e-06,  0.0000e+00, -8.2322e-31,  4.3229e-06, -0.0000e+00,
         -1.1767e+00, -6.6881e-01,  5.7077e-02,  0.0000e+00, -3.3856e-04,
         -6.3013e-01,  3.6528e-02, -6.7392e+00,  3.3608e-06, -1.9933e+00,
          0.0000e+00,  1.2663e+00, -7.3046e-01, -0.0000e+00,  1.1916e+00,
          1.3896e-03,  0.0000e+00, -1.5737e-01, -3.2508e-03,  3.5580e-02,
          1.7513e-21, -6.3592e-08, -2.2678e-01, -2.3543e-07, -2.9033e-01,
          1.8150e-03,  1.5020e-05, -2.7808e-05, -1.1309e+00,  3.7527e-01]])
x, pytorch=tensor([-34.3910]), you=tensor([0.4458])
```