Pendulum Lab

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***Abstract*— Our research objective is to determine how the length and mass both independently affects the period of a free-swinging pendulum.**

I. Theory

- This lab is set up the restoring force caused by gravity on a pendulum which creates an approximated Simple Harmonic Motion that allows for a period of motion. The period of a period can be described mathematically by the equation $T = 2\pi\sqrt{\frac{l}{g}}$. While this equation is directly affected by l, which represent the radius from the top of the string to the center of mass of the mass, it lacks any reference to mass which would imply that mass is irrelevant in affecting the period.

II. Procedure

1) We set up a ring stand on top of a table to maximize the length of the pendulum to test the effect of length on the period.
2) We tied a mass holder to one end of the string and cut the end of the string so when we died the other end to the ring stand it was hanging just above the floor.
3) We raised the mass holder without any added mass 5 using a protractor in order to use the small angle approximation.
4) With a countdown from 3, the mass was released and three individuals started respective timers.
5) After 10 full swings, all three members stopped their times at the peak of the final swing.
6) All three times were recorded, and the timing process was repeated for another trial without changing the mass.
7) Following the collection of 6 data points to be averaged, measured from two trials without added mass, 200 grams were added and steps 3-6 were repeated.
8) This was followed by another addition of mass until all 5 variations of masses were tested (+0g, +200g, +400g, +500g, +1000g).
9) Next, to test the effect of length on the period a similar procedure was used.
10) We measured and recorded the length from the knot tied to the ring stand to the bottom of the mass holder.
11) Then we proceeded to gather 6 data points from 2 trials as described in steps 3-6.
12) After recording the time it takes for 10 periods to occur at each length the string was cut and retied at a different length.
13) This was repeated until all 5 lengths were tested (102.4cm, 60.43cm, 38.54cm, 72.93cm).

III. Data

Period (s)	T2 (s^2)	Mass Added (kg)
2.641	6.972	0
2.657	7.061	0.2
2.663	7.089	0.4
2.663	7.092	0.5
2.673	7.145	1
STD T^2:	0.063417016734662	
STD T:	0.011939314550863	

TABLE I
THE TABLE DISPLAYS AVERAGED DATA TAKEN SHOWING THE RELATIONSHIP BETWEEN THE PERIOD AND THE MASS OF THE PENDULUM AND ITS ERROR.

Period (s)	T2 (s^2)	Length of Pendulum (m)
1.153	1.330	0.3854
1.506	2.269	0.6043
1.723	2.969	0.7293
1.920	3.685	0.8574
2.014	4.057	1.0241

TABLE II
THE TABLE DISPLAYS AVERAGED DATA TAKEN SHOWING THE RELATIONSHIP BETWEEN THE PERIOD AND THE LENGTH OF THE PENDULUM.

IV. Analysis

A. Theoretical Prediction

From the free body diagram of the pendulum, the linear force vector parallel to the velocity of the pendulum is $F = mg\sin(\theta)$. When the angle is small, it can be approximated to $F = mg\theta = \frac{g}{l}ms$ in which $w^2 = \frac{g}{l}$. With this Simple Harmonic Motion approximation, the period can be calculated as $T = 2\pi\sqrt{\frac{l}{g}}$ or $T^2 \propto \frac{l}{g}$. Therefore, the slope of the T^2 as a function of l/g should be $\frac{4\pi^2}{g}$.

B. Experimental Result

In experiment one, we compared the mass added m with the squared period of the pendulum T^2. (See Fig. 1) Because the R^2 correlation coefficient 0.84 is less than 0.95 and the slope 0.15 is less than 0.25, the data suggest a weak correlation between the mass of the pendulum and the period of the pendulum. Because the squared period of the

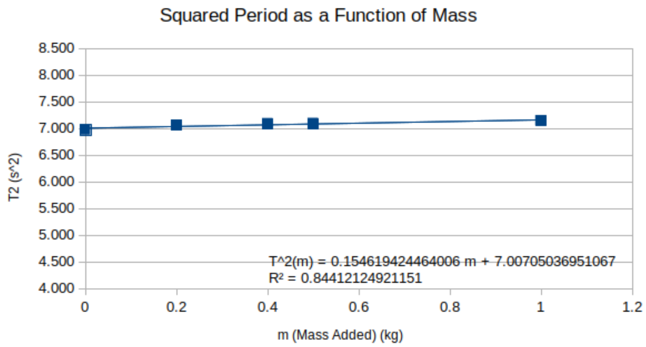

Fig. 1. The graph shows the relationship between the squared period and the mass of the pendulum. The squared period of the pendulum stays relatively constant as we increase the mass. The correlation coefficient 0.84 and slope 0.15 are very small. The graph indicates weak or no correlation between the squared period of the pendulum and its length.

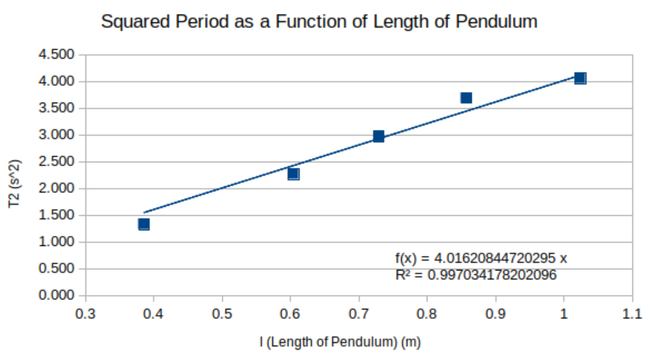

Fig. 2. The graph shows the relationship between the squared period and the length of the pendulum with fixed intercept at (0, 0). The squared period of the pendulum increases as we increase the length. The correlation coefficient 0.99 and slope 4.01 are big. The graph indicates a direct linear relationship between the squared period of the pendulum and its length.

pendulum is mass independent, the experimental data match with the theoretical prediction. Using this assumption, we calculated our sample standard deviation of $\sigma_{T^2} = 0.063$ and $\sigma_T = 0.012$. This insignificant error further suggest that there are no relationship between the mass of the pendulum to its period.

In experiment two, we compared the length of the pendulum with the squared period of the pendulum T^2. (See Fig. 2) Because R^2 correlation coefficient 0.99 is greater than 0.95 and the slope 4.01 is greater than 0.25, the data suggest a strong correlation between the length of the pendulum and the period of the pendulum. Because the length is directly proportional to the squared period of the pendulum, the experimental data match with the theoretical prediction. Using our theory $slope = \frac{4\pi^2}{g}$, we calculated our experimental gravitational constant g to be $9.829773062 \approx 9.83$. The percent error of the gravitational constant is $\frac{9.829773062 - g_{theoretical}}{g_{theoretical}} = 0.003038068 \approx 0.0030$. This insignificant error further suggest that our Simple Harmonic Motion is accurate.

V. Source of Error

There are some sources of error in our experiment:

- As we add mass, the center of mass of the pendulum system goes up in y-direction, our measurement of the length will no longer be accurate, making the actual length l smaller than expected. The experimental values of the period are expected to decrease due to this effect. Our experimental method minimized this error by using longer length so that $\frac{\delta l}{l}$ can be negligible.
- Some of the data may not be consistent with the equation $T = 2\pi\sqrt{\frac{l}{g}}$ because of the small angle approximation for an ideal pendulum may create errors. The actual pendulum period may be amplitude-dependent. This error has no effect on our conclusion as we tried to use a constant small angle of $\theta_0 = 5°$, making the period proportional to $2\pi\sqrt{\frac{l}{g}}$.[1] The approximation does not affect our measurement of the period.
- Human reaction random error may significantly add noise to our data since our experimental method allowed human to time the period of the pendulum. We tried to minimize the effect of this error by recording the time of 10 periodic cycles so that $\frac{\delta T}{T}$ can be negligible.
- Air resistance may be a damping factor in the pendulum system, reducing the period of the pendulum (especially at $\theta = 0$ when the linear velocity is large). However, as the mass of the pendulum increase, the drag force will have a limited effect on the pendulums speed. Our experimental method minimized this error by setting 1kg mass as the constant mass when changing the length of the pendulum.

VI. Conclusion

In this experiment, we accomplished our research objective by discovering the effect of the mass and the length of the pendulum on the period of the pendulum and verifying the theoretical Simple Harmonic Approximation of the pendulum motion.

We swing the pendulum with the different mass of the pendulum and length of the string and observe the relationship between the mass of the pendulum, the length of the string and squared period of the pendulum. The linearity of the length-squared-period graph with a big positive slope clearly suggests a direct relationship between the length of the pendulum and its period whereas the low correlation coefficient of the mass-squared-period graph suggests weak or no relationship between the mass of the pendulum and its period. Specifically, the gravitational constant we calculated using our experimental value is $g_{exp} = 9.83$ with a percent error of 0.0030 (or 0.3%) which suggests our slope of T^2 as a function of l matches with our theoretical prediction. The low standard deviation of T using pendulums with different mass suggests that the period is independent of the mass of the pendulum. Our experiment was considered successful because our experimental data is consistent with the Simple Harmonic Motion theory.

REFERENCES

[1] 3.5 Pendulum period - MIT, 3.5 Pendulum period - MIT. [Online]. Available: http://web.mit.edu/18.098/book/extract2009-01-16.pdf. [Accessed: 03-May-2019]. This website explains different approximation of the pendulum system.