Innovation Networks and R&D Allocation

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Theory Part

- Innovation network (knowledge spillovers) across sectors
- Endogenous growth
- Log linear
- Optimal R&D allocation
- Impact of R&D allocation on welfare

Empirical Part & Application

- Measure innovation network across sectors using patent citation
- Find knowledge spillovers through the innovation network
  - Upstream patents $\uparrow \rightarrow$ Downstream patents $\uparrow$
- Optimal R&D allocation v.s. Actual R&D allocation

Preference and Production Technology

$$V_t = \int_t^\infty e^{-\rho(s-t)} \ln y_s \mathrm{d}s$$

$$y_t = \prod_{i=1}^{K} y_{it}^{\beta_i}$$

$$\ln y_{it} = \int_0^1 \ln\left(q_{it}(\nu) l_{it}(\nu)\right) d\nu$$

- $i = 1, ..., K$: sectors
- $\sum_{i=1}^K \beta_i = 1$
- ν: variety
- $q_{it}(\nu)$: quality of variety
- $l_{it}(\nu)$: production worker

Innovation Process

- Sectorial average quality

$$\ln q_{it} \equiv \int_0^1 \ln q_{it}(\nu)\mathrm{d}\nu$$

- A flow of new innovation

$$n_{it} = s_{it}\eta_i\chi_{it}, \quad \chi_{it} \equiv \prod_{j=1}^{K} q_{jt}^{\omega_{ij}}$$

  - s_{it}: R&D resources (scientists)
  - η_i: exogenous
  - $\sum_{j=1}^{K} \omega_{ij} \equiv 1$

- The law of motion for each sector's knowledge stock

$$\dot{q}_{it}/q_{it} = \lambda \ln\left(n_{it}/q_{it}\right)$$

  - λ: innovation step size

Resources

- Market clearing for production workers and scientists

$$\sum_{i=1}^{K} \ell_{it} = \bar{\ell}, \quad \ell_{it} \equiv \int_0^1 \ell_{it}(\nu) d\nu$$

$$\sum_{i=1}^{K} s_i = \bar{s}$$

Planner's Problem

- **Lemma 1** (Planner's allocation of production workers): $\ell_{it}(\nu) = \ell_{it} = \beta_i \bar{\ell}$
- Innovation network $\mathbf{\Omega} \equiv [\omega_{ij}]$
- Share of scientists $\gamma_t \equiv [\gamma_{it}]_{i=1}^K = [s_{it}/\bar{s}]_{i=1}^K$
- Planner's problem in vector forms

$$\max_{\{\gamma_t\} \text{ s.t. } \gamma_t' \mathbf{1} = 1 \forall t} \int_0^\infty e^{-\rho t} \beta' \ln \boldsymbol{q}_t \, \mathrm{d}t$$

$$\text{s.t. } \mathrm{d} \ln \boldsymbol{q}_t / \mathrm{d}t = \lambda \cdot (\ln \boldsymbol{\eta} + \ln \bar{s} + \ln \gamma_t + (\mathbf{\Omega} - \boldsymbol{I}) \ln \boldsymbol{q}_t)$$

Optimal R&D Allocation

- **Proposition 1** (Optimal R&D allocation): Starting from any initial knowledge stock $\boldsymbol{q}_0$, the optimal R&D allocation is time-invariant and follows, along the entire time path,

$$\gamma' = \frac{\rho}{\rho+\lambda}\beta' \left(\boldsymbol{I} - \frac{\boldsymbol{\Omega}}{1+\rho/\lambda} \right)^{-1}$$

- Note

$$\gamma' \propto \beta' \left(\boldsymbol{I} + \frac{\boldsymbol{\Omega}}{1+\rho/\lambda} + \left(\frac{\boldsymbol{\Omega}}{1+\rho/\lambda} \right)^2 + \cdots \right)$$

  - β': direct impact
  - $\boldsymbol{\Omega}$: innovation network
  - ρ/λ: the society's effective discount rate

R&D Allocation and Economic Growth

- Innovation centrality $\boldsymbol{a}$: $\boldsymbol{a}' = \boldsymbol{a}'\Omega$ and $\sum_{i=1}^{K} a_i = 1$
- **Lemma 2** (Growth rate): Consider a BGP (every sector growth at the same rate). Suppose the R&D allocation is $\boldsymbol{b}$ (i.e. $s_i/\overline{s} = b_i$) and time-invariant. Then, the growth rate is

$$g(\boldsymbol{b}) = \text{ const } + \lambda \cdot \boldsymbol{a}' \ln \boldsymbol{b}$$

- The R&D allocation that maximize the growth rate is $\boldsymbol{a}$
  - $\boldsymbol{a} = \arg\max_{\boldsymbol{b}} \boldsymbol{a}' \ln \boldsymbol{b} \quad \text{s.t.} \;\; \boldsymbol{b} \geq 0, \; \mathbf{1}'\boldsymbol{b} = 1$
- **Proposition 2** : $\lim_{\rho/\lambda \to \infty} \gamma = \beta$, $\lim_{\rho/\lambda \to 0} \gamma = \boldsymbol{a}$

The Impact of R&D Allocation on Welfare

- **Proposition 4** (Welfare gain): The consumption-equivalent welfare impact of adopting the optimal R&D is

$$\mathcal{L}(\boldsymbol{b}) = \exp\left(\frac{\lambda}{\rho}\gamma'(\ln\gamma - \ln\boldsymbol{b})\right)$$

  - $\boldsymbol{b}$: initial R&D allocation
  - γ: optimal R&D allocation
  - $\gamma'(\ln\gamma - \ln\boldsymbol{b})$: relative entropy

Knowledge Spillovers from Abroad: Optimal R&D

- Domestic innovation benefits from foreign knowledge spillovers

$$n_{it} = s_{it}\eta_i\chi_{it}, \quad \text{where } \chi_{it} = \prod_{j=1}^{K}\left[(q_{jt})^{x_{ij}}\left(q_{jt}^{f}\right)^{1-x_{ij}}\right]^{\omega_{ij}}$$

- **Proposition 6** (Optimal R&D allocation with knowledge spillovers from abroad):

$$\gamma' = \xi^{-1}\frac{\rho}{\rho+\lambda}\beta'\left(\boldsymbol{I} - \frac{\boldsymbol{\Omega}\circ \boldsymbol{X}}{1+\rho/\lambda}\right)^{-1}$$

  - A constant $\xi \equiv \frac{\rho}{\rho+\lambda}\beta'\left(\boldsymbol{I} - \frac{\boldsymbol{\Omega}\circ\boldsymbol{X}}{1+\rho/\lambda}\right)^{-1}\mathbf{1}$ ensures $\sum_i \gamma_i = 1$
  - $\boldsymbol{\Omega}\circ\boldsymbol{X} = [\omega_{ij}x_{ij}]$
- As if the planner is impatient relative to the closed economy.

Knowledge Spillovers from Abroad: Welfare Gain

- **Proposition 7** (Welfare gain with knowledge spillovers from abroad):

$$\mathcal{L}(\boldsymbol{b};\xi) = \exp\left(\xi \frac{\lambda}{\rho}\gamma'(\ln\gamma - \ln \boldsymbol{b})\right)$$

- ξ is a measure of R&D self-sufficiency
  - $\xi \leq 1$ in open economy
  - $\xi = 1$ only if closed economy

Other Theoretical Extensions

- General functional forms + First order approximation
  $y_t = \mathcal{Y}(\{y_{it}\}), \chi_{it} = \mathcal{X}_i(\{q_{jt}\})$
  $\rightarrow \beta_{it} \equiv \frac{\partial \ln \mathcal{Y}(\{y_{it}\})}{\partial \ln y_{it}}, \omega_{ijt} \equiv \frac{\partial \ln \chi_i(\{q_{it}\})}{\partial \ln q_{jt}}$

- Production network
  $\rightarrow \gamma' \propto \hat{\beta}' \left(\boldsymbol{I} - \frac{\Omega}{1+\rho/\lambda} \right)^{-1}, \hat{\beta}_i \equiv \frac{\partial \ln y_t}{\partial \ln q_{it}}$

- Semi endogenous growth
  $\dot{q}_{it}/q_{it} = \lambda \ln \left(n_{it}/q_{it}^{1+\kappa} \right)$
  $\rightarrow \hat{\rho} = \rho + \kappa\lambda$

- Decentralized equilibrium
  $\rightarrow \gamma = \beta$

Data

- Patent
  - US: USPTO
  - Global: Google Patents
- Production and final use
  - US: Bureau of Labor Statistics (BLS)
  - Global: World Input-Output Database (WIOD)
- Sectoral R&D Allocation
  - US: Compustat
  - Global: Worldscope and Datastream

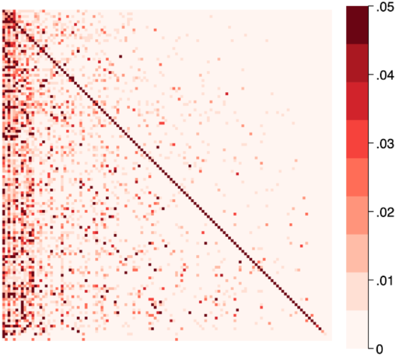
Innovation Network Ω

- Innovation network from sector j to sector i is measured by

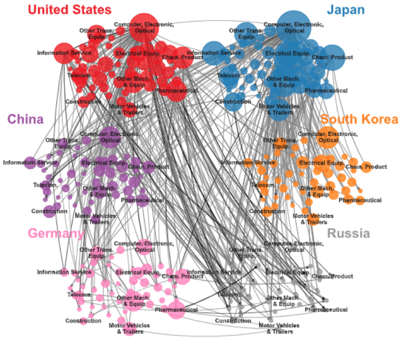
$$\omega_{ijt} \equiv \frac{\text{Cites }_{ijt}}{\sum_{k=1}^{K} \text{ Cites }_{ikt}}$$

Innovation Network Ω

(a) IPC-to-IPC (131×131) network Ω



(b) The global innovation network



- 3-digit International Patent Classification (IPC) class
- Innovation network follows a hierarchical structure

Innovation Network is Highly Correlated over Time

Time Period	All years	2020	2010	2000	1990	1980
All years		0.98	0.98	0.97	0.90	0.89
2020	0.95		0.97	0.93	0.86	0.85
2010	0.96	0.97		0.96	0.88	0.87
2000	0.93	0.92	0.96		0.92	0.90
1990	0.90	0.80	0.84	0.90		0.91
1980	0.81	0.77	0.81	0.87	0.89	

- Bottom half: the Pearson correlations
- Top half: the Pearson correlations of the rank values

Innovation Network is Highly Correlated across Countries

Countries	All	US	Japan	China	South Korea	Germany	Russia	France	UK	Canada	Netherlands
All		0.98	0.87	0.87	0.84	0.89	0.63	0.86	0.92	0.88	0.81
US	0.95		0.84	0.86	0.82	0.88	0.64	0.85	0.92	0.88	0.80
Japan	0.86	0.83		0.88	0.89	0.85	0.63	0.87	0.86	0.84	0.83
China	0.85	0.86	0.87		0.88	0.85	0.66	0.85	0.87	0.86	0.82
South Korea	0.78	0.77	0.83	0.84		0.84	0.64	0.84	0.85	0.82	0.84
Germany	0.85	0.87	0.81	0.80	0.72		0.64	0.83	0.87	0.83	0.81
Russia	0.62	0.63	0.62	0.62	0.55	0.61		0.65	0.64	0.64	0.66
France	0.91	0.86	0.79	0.77	0.72	0.82	0.57		0.86	0.85	0.83
UK	0.87	0.89	0.85	0.85	0.80	0.86	0.64	0.80		0.88	0.82
Canada	0.86	0.88	0.79	0.81	0.71	0.81	0.59	0.80	0.81		0.81
Netherlands	0.84	0.85	0.79	0.82	0.75	0.79	0.58	0.78	0.79	0.81	

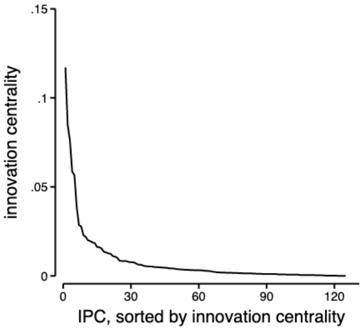
- Bottom half: the Pearson correlations
- Top half: the Person correlations of the rank values

Innovation Network Weakly Correlates with I-O Networks

US	Japan	China	South Korea	Germany	Russia	France	UK	Canada	Netherlands
0.32	0.28	0.35	0.31	0.23	0.19	0.36	0.41	0.29	0.22

Innovation Centrality Across Sectors *a*

(a) Innovation Centrality Across IPCs

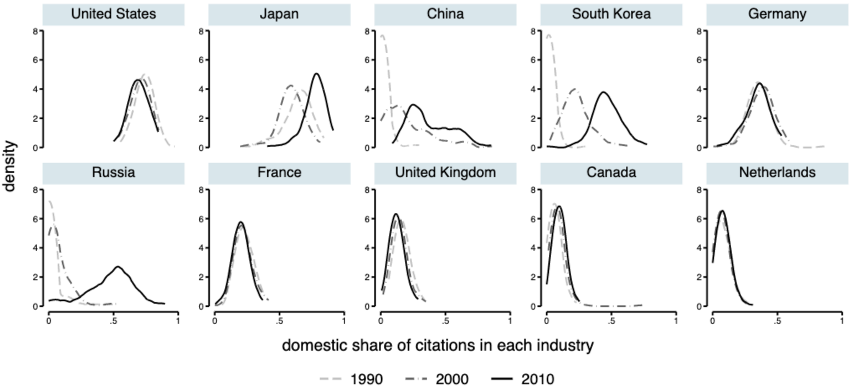


(b) Top 10 IPCs By Innovation Centrality a_i

1	A61	medical or veterinary science; hygiene
2	G06	computing; calculating or counting
3	H01	basic electric elements
4	H04	electric communication technique
5	G01	measuring; testing
6	B60	vehicles in general
7	G02	optics
8	B01	physical or chemical processes or apparatus in general
9	C08	organic macromolecular compounds; their preparation or chemical working-up; compositions based thereon
10	F16	engineering elements or units; general measures for producing and maintaining effective functioning of machines or installations; thermal insulation in general

- Innovation centrality is highly heterogeneous across sectors
- $a_{1\text{st}}/a_{5\text{th}} = 2$, $a_{1\text{st}}/a_{20\text{th}} = 10$, $a_{1\text{st}}/a_{50\text{th}} = 30$

Domestic Citation Shares $\boldsymbol{X}$



- citations made to domestic patents / total citations made by patents
- The US and Japan rely relatively more on domestic knowledge

US Evidence of Innovation Spillovers Through Network

$Y=$	ln(Patents)				ln(Cites)			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$Knowledge_{it}^{Up}$	0.555***	0.605***	0.509***	0.583**	0.790***	0.840***	0.756***	0.917***
	(0.174)	(0.194)	(0.169)	(0.269)	(0.197)	(0.207)	(0.192)	(0.289)
$ln(R\&D\ Stock)_{i,t-1}$	0.426***	0.433***	0.410***	0.408***	0.340***	0.347***	0.328***	0.206
	(0.100)	(0.101)	(0.096)	(0.111)	(0.114)	(0.114)	(0.111)	(0.133)
$Knowledge_{it}^{Down}$		-0.112				-0.110		
		(0.152)				(0.095)		
$Knowledge_{it}^{Up,IO}$			0.258				0.198	
			(0.165)				(0.203)	
Specification	OLS	OLS	OLS	IV 2nd Stage	OLS	OLS	OLS	IV 2nd Stage
IV 1st Stage F-statistics				427				427
R^2	0.916	0.917	0.917	0.169	0.900	0.900	0.900	0.092
No. of Sectors	95	95	95	95	95	95	95	95
No. of Obs	1900	1900	1900	1140	1900	1900	1900	1140
Fixed Effects	Sector, Year				Sector, Year			

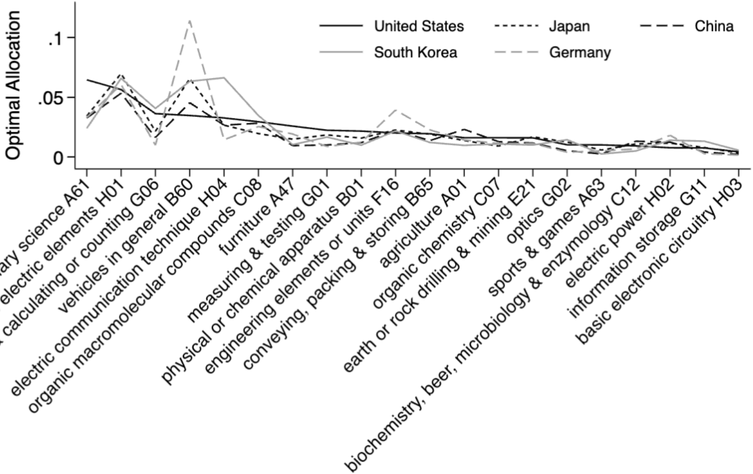
- Knowledge $_{it}^{\text{Up}} \equiv \sum_{j\neq i} \sum_{\tau=1}^{10} \omega_{ij,t-\tau} \ln n_{j,t-\tau}$
- Knowledge $_{it}^{\text{Down}} \equiv \sum_{k\neq i} \sum_{\tau=1}^{10} \omega_{ki,t-\tau} \ln n_{k,t-\tau}$
- $\omega_{ij,t}$: innovation network
- $n_{i,t}$: patent counts
- IV: R&D cost shifters (state-level tax, Wilson (2009))

Global Evidence of Innovation Spillovers Through Network

$Y=$	ln(Patents)				ln(Cites)			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$Knowledge^{Up}_{mit}$	0.162***	0.188***	0.159***	0.226**	0.352***	0.393***	0.350***	0.453***
	(0.055)	(0.056)	(0.055)	(0.113)	(0.077)	(0.080)	(0.078)	(0.143)
$ln(R\&D\ Stock)_{mi,t-1}$	0.043***	0.043***	0.043***	0.079***	0.084***	0.084***	0.083***	0.083***
	(0.013)	(0.013)	(0.013)	(0.020)	(0.018)	(0.018)	(0.018)	(0.030)
$Knowledge^{Down}_{mit}$		-0.059				-0.094		
		(0.039)				(0.062)		
$Knowledge^{Up,IO}_{mit}$			0.070				-0.054	
			(0.065)				(0.068)	
Specification	OLS	OLS	OLS	IV 2nd Stage	OLS	OLS	OLS	IV 2nd Stage
IV 1st Stage F-statistics				148				148
R^2	0.968	0.968	0.968	0.035	0.943	0.943	0.943	0.028
No. of Country x Sectors	570	570	556	282	570	570	556	282
No. of Obs	11014	11014	10774	4587	11014	11014	10774	4587
Fixed Effects	Country x Sector, Country x Year, Sector x Year				Country x Sector, Country x Year, Sector x Year			

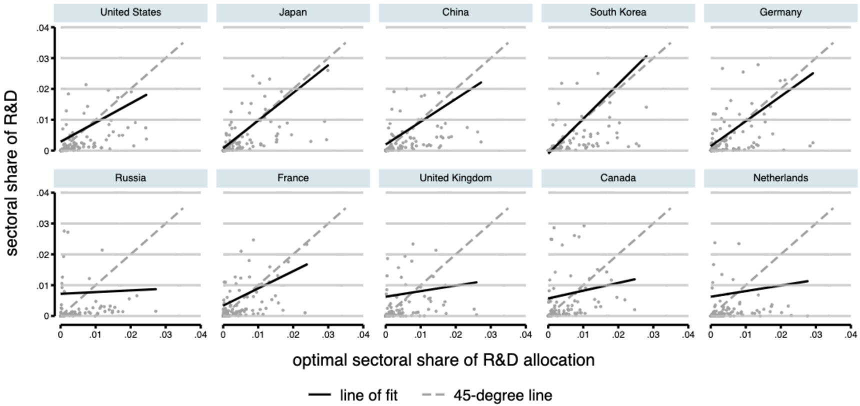
- $\text{Knowledge}^{\text{Up}}_{mit} \equiv \sum_{cj \neq mi} \sum_{\tau=1}^{10} \frac{\text{Cites}^{N}_{mi \to cj,t-\tau}}{\sum_{c'=1}^{N} \sum_{k=1}^{K} \text{Cites}_{mi \to c'k,t-\tau}} \ln n_{cj,t-\tau}$
- IV: R&D cost shifter (Thomson 2017)

Optimal R&D Allocations



- $\gamma' = \xi^{-1} \frac{\rho}{\rho+\lambda} \beta' \left(\boldsymbol{I} - \frac{\boldsymbol{\Omega} \circ \boldsymbol{X}}{1+\rho/\lambda} \right)^{-1}$

Actual R&D Allocation v.s. Optimal R&D Allocation



Consumption Gains by Moving to Optimal R&D Allocation

	US	Japan	China	South Korea	Germany	Russia	France	UK	Canada	Netherlands
2000	9.98	4.24	5.78	5.25	4.79	13.70	5.17	7.55	7.22	6.70
2005	8.85	5.04	5.26	3.92	4.11	11.18	5.38	8.17	7.29	5.45
2010	8.04	5.64	5.60	4.24	4.09	16.76	5.38	8.15	6.21	10.22
	Sweden	**Switzerland**	**Italy**	**Finland**	**India**	**Australia**	**Belgium**	**Austria**	**Denmark**	**European Union**
2000	6.65	5.18	5.04	5.39	10.91	5.72	5.72	6.52	5.93	5.91
2005	5.53	4.10	4.57	5.63	8.33	4.19	5.62	8.50	5.30	5.04
2010	6.20	3.67	4.40	7.95	6.21	7.30	6.73	9.87	5.39	5.76

- The welfare gain (%) of moving from the actual allocation to optimal allocation: $\exp\left(\frac{\lambda}{\rho}\gamma'(\ln\gamma - \ln \boldsymbol{b})\right)$
- The calculation focuses on improving allocation in top 50 IPC classes by total patents