Introduction
000

Simple Model
00000

Growth Model
000

Equilibrium
0000000

Identification
0000000

Exercise
00000000

Conclusion
0

Ownership Structure and Economic Growth

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Ownership Structure $\Longrightarrow$ Economic Growth?

- Ownership structure is concentrated   `Share`
    - BlackRock, Vanguard, State Street, and Fidelity control 30% of votes of S&P 500 firms
    - Top 4 chaebols account for 55% of stock market capitalization in Korea

- Firms maximize shareholder values $\Longrightarrow$
  Partially internalize externalities for commonly owned firm

- Ownership structure (common ownership, cross ownership, M&A, FDI, ...) $\Longrightarrow$
  Economic growth?
    - Business stealing effect
    - Technology spillover effect

Quantitative Schumpeterian Growth Model with Ownership Structure

- Existing Schumpeterian growth models:
    - Monopolistic competition (no strategic interaction)
    - Very few firms in Markov perfect equilibrium

- This paper: Many oligopolists
    - Overlapping ownership and technology spillover across industries

- Quantify three inter-firm networks for publicly listed patenting firms in the US ($\simeq$ 1000 firms)
    - Ownership structure
    - Product market rivalry
    - Technology spillover

- Common ownership in the US:
    - Internalization of business stealing $\implies g \downarrow\downarrow$
    - Internalization of technology spillover $\implies g \uparrow$

Literature

- Competition & Innovation:
  d'Aspremont and Jacquemin (1988); Kamien et al. (1992); Aghion et al. (2001, 2005); Acemoglu and Akcigit (2012); Aghion et al. (2013); Bloom et al. (2013); Lopez and Vives (2019); Peters (2020); Akcigit and Ates (2021, 2023); Liu et al. (2022); Cavenaile et al. (2023); Anton et al. (2023, 2024); Kini et al. (2024); Hopenhayn and Okumura (2024)
  Quantitative Schumpeterian growth model with ownership structure

- Hedonic Demand / Empirical IO:
  Lancaster (1966); Rosen (1974); Berry et al. (1995); Nevo (2001); Pellegrino (2024); Ederer and Pellegrino (2024)
  Dynamic general equilibrium / R&D

- Oligopoly / Common Ownership / Market Power:
  Rubinstein and Yaari (1983); Rotemberg (1984); Neary (2003); Atkeson and Burstein (2008); Gutierrez and Philippon (2017); He and Huang (2017); Azar et al. (2018, 2022); Autor et al. (2020); Baqaee and Farhi (2020); De Loecker et al. (2020); Azar and Vives (2021); Edmond et al. (2023)

Simple Model

- Static partial equilibrium model of oligopolistic competition in production and R&D
  - d'Aspremont and Jacquemin (1988); Kamien et al. (1992); Leahy and Neary (1997); Lopez and Vives (2019); Anton et al. (2024)

- Firm $i \in \{1, \ldots, n\}$ chooses quantity q_i and R&D effort x_i

- Linear inverse demand: $p(q) = b - \Sigma q$ $\quad (\Sigma = [\sigma_{ij}]_{n \times n}, \sigma_{ii} = 1)$

- CRS production technology with marginal cost: $m(x) = \overline{m} - \Omega x$ $\quad (\Omega = [\omega_{ij}]_{n \times n})$

- Quadratic R&D cost: $c(x_i) = \frac{1}{2} x_i^2$

Common Ownership Weights

- $K = \left[\kappa_{ij}\right]_{n \times n}$: common ownership weights that firm i places on the value of firm j $\quad (\kappa_{ii} = 1)$

- More overlapping ownership b/w firm i and $j \implies$ Higher κ_{ij}

  Proportional Influence

- $K = I$: dispersed ownership (each firm maximizes its own value)

- $K = \mathbf{1}_{n \times n}$: monopoly (maximizes total producer surplus)

Cournot & R&D Game

- Firm i's profit:

$$\pi_i(\boldsymbol{q}, \boldsymbol{x}) = [p_i(\boldsymbol{q}) - m_i(\boldsymbol{x})]q_i - c(x_i)$$

$$= \left[b_i - \sum_{j=1}^{n} \sigma_{ij} q_j - \overline{m}_i + \sum_{j=1}^{n} \omega_{ij} x_j \right] q_i - \frac{1}{2} x_i^2$$

- Given $\left\{ q_j, x_j \right\}_{j \neq i}$, firm i chooses q_i and x_i to maximize $\sum_j \kappa_{ij} \pi_j(\boldsymbol{q}, \boldsymbol{x})$

Impact of Common Ownership on R&D

- Comparative statics where $\{q_k, x_k\}_{k \neq i}$ are held constant:

$$\frac{\partial x_i}{\partial \kappa_{ij}} = \frac{q_j}{2 - \omega_{ii}^2}(2\omega_{ij} - \omega_{ii}\sigma_{ij}) \qquad \forall \; j \neq i$$

  - SOC: $2 > \omega_{ii}^2$

- Internalize business stealing effect: $\partial^2 x_i / \partial \kappa_{ij} \partial \sigma_{ij} < 0$

- Internalize technology spillover effect: $\partial^2 x_i / \partial \kappa_{ij} \partial \omega_{ij} > 0$

Preview of Identification Strategy

Networks	Measurement
Common ownership K	Institutional investor holdings (Backus et al., 2021)
Product market rivalry Σ	Product proximity (Hoberg and Phillips, 2016):
	Text analysis of business description
Technology spillover Ω	Technological proximity (Jaffe, 1986; Bloom et al., 2013):
	Patent classification

Schumpeterian Growth Model

- Linear quadratic aggregator (final good):

$$Y_t = \boldsymbol{q}_t^T \boldsymbol{b}_t - \frac{1}{2} \boldsymbol{q}_t^T \boldsymbol{\Sigma} \boldsymbol{q}_t$$

- Linear inverse demand:

$$\boldsymbol{p}_t = \boldsymbol{b}_t - \boldsymbol{\Sigma} \boldsymbol{q}_t$$

- CRS production technology (intermediate good):

$$q_{i,t} = a_{i,t} l_{i,t}$$

- Each firm has knowledge capital (state variable): $z_{i,t}$

- Each firm allocates knowledge capital to improve labor productivity and product quality:

$$\zeta a_{i,t} + b_{i,t} = z_{i,t}$$

Law of Motion of Knowledge Capital

$$dz_t = \left(\underbrace{\Omega z_t}_{\text{tech spillover}} + \underbrace{\mu x_t}_{\text{R\&D}} \right) dt + \underbrace{\gamma z_t \circ dW_t}_{\text{shocks}}$$

- $\Omega = [\omega_{ij}]$: technology spillover matrix

- $x_{i,t} = \sqrt{d_{i,t}}$
  - $d_{i,t}$: R&D input in terms of final good
  - Innovation elasticity is 0.5

- μ, γ: positive scalars

Market Clearing and Preference

- Inelastic labor supply:

$$L = \sum_i l_{i,t}$$

- Final good market clearing:

$$C_t + \underbrace{\sum_i d_{i,t}}_{\text{R\&D input}} = Y_t$$

- Risk neutral representative household:

$$\max \ E_t \left[\int_t^\infty \exp\left(-\rho s\right) C_s ds \right]$$

Cournot-Nash Equilibrium

- Firm i's gross profit before subtracting dynamic R&D cost: Diagram

$$\pi_{i,t} = p_{i,t}q_{i,t} - w_t l_{i,t} = \left(b_{i,t} - \sum_j \sigma_{ij}q_{j,t} - \frac{w_t}{a_{i,t}} \right) q_{i,t} \quad \text{where} \quad \zeta a_{i,t} + b_{i,t} = z_{i,t}$$

- Given w_t, $z_{i,t}$, and $\left\{ q_{j,t} \right\}_{j \neq i}$, firm i chooses $a_{i,t}$, $b_{i,t}$, and $q_{i,t}$ to maximize $\sum_j \kappa_{ij}\pi_{j,t}$

- Quantity is a linear function of knowledge capital:

$$q(z_t) = \left\{ \underbrace{2\frac{\zeta}{L}\mathbf{1}_{n \times n}}_{\text{labor cost}} + \underbrace{\boldsymbol{\Sigma}}_{\text{substitutability}} + \underbrace{\boldsymbol{K} \circ \boldsymbol{\Sigma}}_{\text{ownership} \times \text{substitutability}} \right\}^{-1} z_t$$

- Ownership-weighted gross profits are expressed in quadratic form: $\sum_j \kappa_{ij}\pi_{j,t} = z_t^T Q^i z_t$ ○

Linear-Quadratic Differential Game

- Given other firms' R&D $\left\{x_{j,t}\right\}_{j\neq i, t\geq 0}$, firm i chooses R&D $\left\{x_{i,t}\right\}_{t\geq 0}$ to maximize

$$\max_{\left\{x_{i,t}\right\}_{t\geq 0}} \quad V^i\left(z_0\right) \equiv E_0\left[\int_0^\infty \exp\left(-\rho t\right)\left\{\sum_j \kappa_{ij}\left(\pi_{j,t} - x_{j,t}^2\right)\right\}dt\right]$$

  subject to $dz_t = \left(\Omega z_t + \mu x_t\right)dt + \gamma z_t \circ dW_t$

- Firm i's HJB equation:

$$\rho V^i\left(z\right) = \max_{x_i}\left\{z^T Q^i z - \sum_j \kappa_{ij} x_j^2 + V_z^i\left(z\right)\left[\Omega z + \mu x\right] + \frac{\gamma^2}{2}z^T V_{zz}^i\left(z\right)z\right\}$$

HJB Equations $\Longrightarrow$ Riccati Equations

- Guess and verify $V^i(z) = z^T X^i z$ (for any z)

- X^i is the solution of stacked algebraic Riccati equations Riccati

- All public patenting firms in the US in our dataset $\simeq 1000$ firms $\Longrightarrow$
  $1000^3 = 1$ billion undetermined coefficients (20 seconds on my laptop)

Oligopolistic Schumpeterian	Computation time	# of firms	Productivity space
Cavenaile et al. (2023)	$O(2^n)$	4	6 grid
Our model	$O(n^4)$	≈ 1000	Continuous

Balanced Growth Path

- R&D strategy: $x_{i,t} = \left(\mu X_i^i\right)^T z_t$ where X_i^i is the i th column of X^i
- The law of motion is rewritten as $dz_t = \Phi z_t dt + \gamma z_t \circ dW_t$ where

$$\Phi \equiv \underbrace{\Omega}_{\text{Tech Spillover}} + \underbrace{\mu^2 \left[X_1^1 \cdots X_n^n\right]^T}_{\text{R\&D}}$$

Theorem

Consider the deterministic economy ($\gamma = 0$). If Φ is irreducible, then:
(i) There exists the largest positive eigenvalue of Φ, g, and associated positive eigenvector, z^*.
(ii) There exists a globally stable BGP such that the knowledge capital growth rate of all firms is g, and the knowledge capital distribution is a scalar multiple of z^*.

- Proof: Perron–Frobenius Theorem
- "Φ is irreducible" $\Longleftrightarrow$ "All firms are directly or indirectly connected technologically"

Intuition of Why the Model Has the BGP

- On the BGP, a_t, b_t, z_t, and q_t grow at the same rate

Technological Choice:	$\zeta a_{i,t} + b_{i,t} = z_{i,t}$
Linear Production Technology:	$q_{i,t} = a_{i,t} l_{i,t}$
Inelastic Labor Supply:	$L = \sum_i l_{i,t}$

- Linear and quadratic term in q_t of output grow at the same rate: (Equilibrium Summary)

$$Y_t = q_t^T b_t - \frac{1}{2} q_t^T \Sigma q_t$$

Partial Equilibrium Diagram and BGP
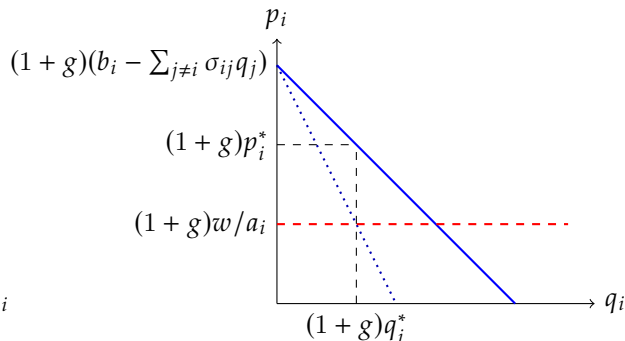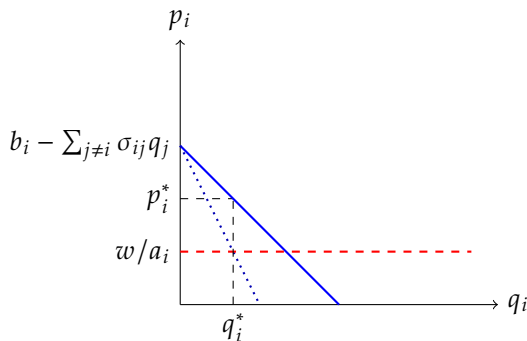
- a_i, b_i, $q_i(= a_i l_i)$, p_i, and w/a_i grow at the same rate of g
- (i) (consumer surplus / producer surplus) and (ii) (cost / revenue) stay the same

Back

Expected Growth Rate and Utility

- Apply Itô's lemma:

  Decomposition

$$
\begin{aligned}
\log Y_t &= \log\left(z_t^T Q z_t\right) \\
dz_t &= \Phi z_t dt + \gamma z_t \circ dW_t
\end{aligned}
\implies \quad E_t\left[\,g_t \middle|\, z_t\right]
$$

- Expected utility is expressed in quadratic form:

  X

$$
E_t\left[\int_t^\infty \exp\left(-\rho s\right) C_s ds \middle|\, z_t\right] = z_t^T X z_t
$$

- Solve the equilibrium once $\implies$ Can compute expected growth and utility for any z_t

Common Ownership Weight K

- Backus et al. (2021) construct a dataset on investors' holdings based on Form 13F
- Baseline: Rotemberg (1984) proportional influence

`Proportional Influence`
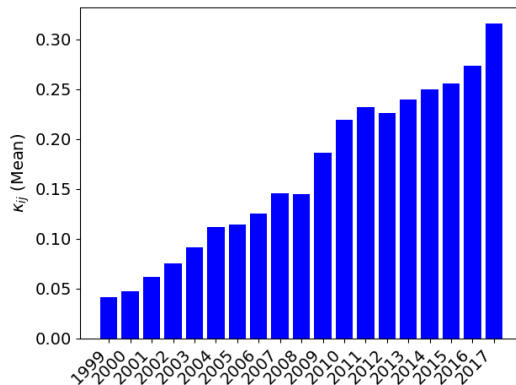
Product Market Rivalry Σ

- Hoberg and Phillips (2016) estimates product proximity using business descriptions in 10-K

- Pellegrino (2024) estimates α to align with the cross-price elasticity of demand micro estimates

$$\underbrace{\sigma_{ij}}_{\text{substitutability}} = \alpha \times \text{product proximity b/w } i \text{ and } j \quad (i \neq j)$$

Technological Proximity $\widetilde{\Omega}$

- Technological profile of firm i
  - The vector of the share of patents held by firm i in each technology class
  - Baseline: group-level patent classifications (≈ 4000)

- Jaffe (1986) technological proximity measure $\tilde{\omega}_{ij}$:
  - Cosine similarity of the technological profiles b/w firm i and j

Distribution of Knowledge Capital z_t

Variables	Identification
$\pi_{i,t}$	Gross profit (before R&D cost) = Revenue − Cost of goods sold
$\boldsymbol{q}_t$	$\pi_{i,t} = \sum_j \kappa_{ij}\, \sigma_{ij}\, q_{i,t}\, q_{j,t}$
ζ/L	Matches sample firms' cost share (average markup)
$\boldsymbol{z}_t$	$\boldsymbol{z}_t = \left\{ 2\dfrac{\zeta}{L}\boldsymbol{J} + \boldsymbol{\Sigma} + \boldsymbol{K} \circ \boldsymbol{\Sigma} \right\} \boldsymbol{q}_t$

Technology Spillover $\Omega = \beta \times$Technological Proximity $\widetilde{\Omega}$ ꜰɪʀꜱᴛ ꜱᴛᴀɢᴇ

$$z_{i,t+1} - z_{i,t} = \beta \sum_{j \neq i} \tilde{\omega}_{ij,t} z_{j,t} + \text{Year FE}_t + \epsilon_{i,t}$$

	(1)	(2)	(3)
	$z_{i,t+1} - z_{i,t}$	$z_{i,t+1} - z_{i,t}$	$z_{i,t+1} - z_{i,t}$
$\sum_{j \neq i} \tilde{\omega}_{ij,t} z_{j,t}$	0.000191***	0.000152***	0.000140***
	(0.000035)	(0.000035)	(0.000039)
$\sqrt{\text{R\&D Expenditure}}$		0.037**	
		(0.021)	
Year Fixed Effects	✓	✓	✓
IV			✓
IV 1st Stage F-statistics			4176
No. observations	16,324	15,173	14,181

SEs clustered by years and 4-digit NAICS industries are reported in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

- IV: Firm-specific tax price of R&D from federal and state-specific rules (Bloom et al., 2013)

Identification: Summary

- Publicly available data + Compustat

Notation	Description	Value	Source
$\boldsymbol{\Sigma}$	Product proximity		Form 10-K, Hoberg and Phillips (2016)
$\widetilde{\boldsymbol{\Omega}}$	Technological proximity		USPTO, Patent classification
K	Common ownership weights		Form 13-F, Backus et al. (2021)
α	Product proximity $\rightarrow$ Substitutability	0.12	Pellegrino (2024)
β	Technological proximity $\rightarrow$ Spillover	0.00014	Estimate the law of motion
γ	Std. of idiosyncratic shocks	0.027	Estimate the law of motion
ζ/L	Labor augmentation efficiency	0.0063	Compustat, Cost of goods sold
ρ	Discount rate	0.10	$>$ risk free rates, $<$ private R&D returns
μ	R&D efficiency	0.05	1.7% economic growth rate

Fit b/w Model and Data

- Comparison of firm-level model-generated values (x-axis) with observed data (y-axis)
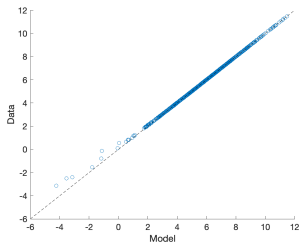
(a) Log of Profit (Targeted)   (b) Log of Sales   (c) Log of R&D Expenditure
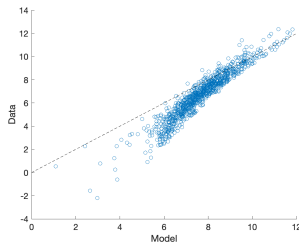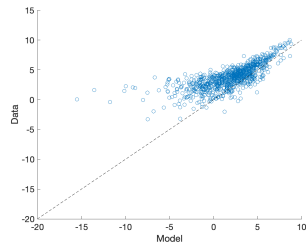
Counterfactual Ownership Structures

Ownership Structure	Description
Baseline	Observed ownership structure in 2017
Dispersed	$K^D = I$
Mean=1999	$\kappa_{ij,2017}^{M1999} = \text{const} \times \kappa_{ij,2017}$ and $E\left[\kappa_{ij,2017}^{M1999}\right] = E\left[\kappa_{ij,1999}\right]$ for $j \neq i$
Uniform	$\kappa_{ij,2017}^{U} = E\left[\kappa_{ij,2017}\right]$ for $j \neq i$
Monopoly	$K^M = 1$

Total Output

Total Output in 2017 (Social Optimum: 100)	Ownership (Baseline: 2017)				
	Dispersed	Mean=1999	Uniform	Baseline	Monopoly
Baseline	91.30	91.02	90.78	89.08	89.17
Only Business Steal $\mathbf{\Omega} = [0]$	91.30	91.02	90.78	89.08	89.17
Only Tech Spill $\mathbf{\Sigma} = \mathbf{I}$, $\zeta/L = 0$	75.00	75.00	75.00	75.00	75.00

- Inelastic labor supply $\implies$ Changes arise from product misallocation
- Common ownership exacerbates product misallocation

Introduction
000

Simple Model
00000

Growth Model
000

Equilibrium
0000000

Identification
0000000

Exercise
00●00000

Conclusion
0

Total R&D Expenditure

Total R&D in 2017 (Social Optimum: 100)	Ownership (Baseline: 2017)				
	Dispersed	Mean=1999	Uniform	Baseline	Monopoly
Baseline	27.08	26.83	26.44	23.21	18.48
Only Business Steal $\mathbf{\Omega} = [0]$	29.08	28.72	27.95	24.23	18.85
Only Tech Spill $\mathbf{\Sigma} = \mathbf{I}$, $\zeta/L = 0$	18.27	18.34	18.75	18.86	19.84

- Internalization of business stealing > Internalization of technology spillover
- Network heterogeneity is important

Expected Growth Rate

Expected Economic Growth Rate in 2017 (%)	Ownership (Baseline: 2017)				
	Dispersed	Mean=1999	Uniform	Baseline	Monopoly
Baseline	1.796	1.793	1.791	1.753	1.713
Only Business Steal $\mathbf{\Omega} = [0]$	1.097	1.094	1.093	1.062	1.020
Only Tech Spill $\mathbf{\Sigma} = \mathbf{I}$, $\zeta/L = 0$	2.051	2.054	2.068	2.072	2.107

- In baseline, the expected growth rate is
  - 0.043 pp (2.4%) lower compared to dispersed ownership, and
  - 0.040 pp (2.2%) lower compared to common ownership level in 1999.

Expected Social Welfare

Expected Social Welfare (Social Optimum: 100)	Ownership (Baseline: 2017)				
	Dispersed	Mean=1999	Uniform	Baseline	Monopoly
Baseline	87.72	87.42	87.16	85.25	85.18
Only Business Steal $\mathbf{\Omega} = [0]$	88.83	88.53	88.30	86.44	86.41
Only Tech Spill $\mathbf{\Sigma} = \mathbf{I}$, $\zeta/L = 0$	68.81	68.82	68.88	68.89	69.02

- In the baseline, the consumption-equivalent welfare loss is
  - 2.8% compared to dispersed ownership, and
  - 2.5% compared to the common ownership level in 1999.

Firm Value Share

Firm Value Share in 2017 (%)	Ownership (Baseline: 2017)				
	Dispersed	Mean=1999	Uniform	Baseline	Monopoly
Baseline	28.74	29.63	33.43	33.34	40.92
Only Business Steal $\mathbf{\Omega} = [0]$	27.91	28.80	32.60	33.51	40.14
Only Tech Spill $\mathbf{\Sigma} = \mathbf{I}, \zeta/L = 0$	64.82	64.81	64.76	64.74	64.63

- In baseline, firm value share is
  - 5.6% lower compared to dispersed ownership, and
  - 4.7% lower compared to common ownership level in 1999.

When Common Ownership Affects only R&D Decisions

- Common ownership only influences R&D decisions (cf. d'Aspremont and Jacquemin (1988))

	Ownership Structure		
	Dispersed	Common R&D	Baseline
Output (Social Optimum: 100)	91.30	91.30	89.08
R&D Expenditure (Social Optimum: 100)	26.17	19.76	22.36
Expected Growth Rate (%)	1.796	1.726	1.753
Expected Social Welfare (Social Optimum: 100)	87.72	87.49	85.25
Firm Value Share (%)	28.74	29.04	34.34

- Lowest R&D expenditure and expected growth rate
- Intermediate social welfare and firm value share

Introduction
000

Simple Model
00000

Growth Model
000

Equilibrium
0000000

Identification
0000000

Exercise
0000000●

Conclusion
0

Optimal Uniform R&D Subsidy Social Optimum
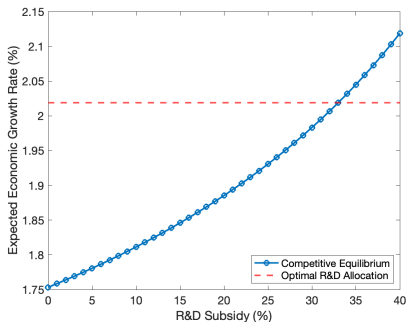
(a) Expected Growth
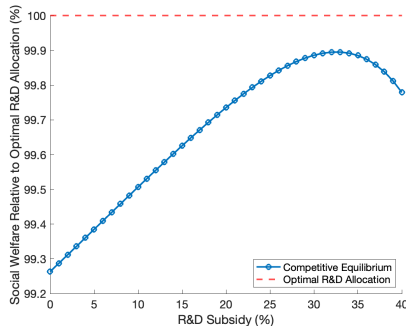
(b) Expected Social Welfare

- Optimal rate is s =33%, which increases g by 0.25 pp (14%)

Introduction
000

Simple Model
00000

Growth Model
000

Equilibrium
0000000

Identification
0000000

Exercise
00000000

Conclusion
●

Conclusion

- Quantitative Schumpeterian growth model with ownership structure
  - Utilize micro data and computational capability

- Common ownership in the US:
  1. Internalization of business stealing effect $\implies g \downarrow\downarrow$
  2. Internalization of technology spillover effect $\implies g \uparrow$

- Potential applications:
  - Chaebols in Korea
  - Zaibatsu (pre-WWII) and cross-shareholding (late 20th century) in Japan
  - FDI / multinational companies and international technology diffusion
  - Technology licensing

Share of Top 5 Shareholders in Largest Market Cap Firms Back

Microsoft	
Vanguard	9.20%
Blackrock	7.75%
Steven Ballmer	4.48%
State Street	3.97%
Fidelity	2.66%

Nvidia	
Vanguard	8.93%
BlackRock	7.74%
Fidelity	4.12%
State Street	3.97%
Jensen Huang	3.80%

Apple	
Vanguard	9.29%
Blackrock	7.48%
State Street	3.96%
Fidelity	2.27%
Geode Capital	2.26%

Google	
Vanguard	7.36%
Blackrock	6.47%
State Street	3.39%
Fidelity	3.01%
Sergey Brin	2.99%

Amazon	
Jeffrey Bezos	8.58%
Vanguard	7.77%
Blackrock	6.50%
State Street	3.44%
Fidelity	3.10%

Meta	
Vanguard	7.55%
Blackrock	6.50%
Fidelity	5.38%
Accel IX LP	3.88%
State Street	3.40%

Equity Investments by Big Tech in AI Startups `Back`

Shareholding percentage	Microsoft	Google	Amazon
OpenAI (ChatGPT)	49%	—	—
Anthropic (Claude)	—	14%	23%

Technology & Product Proximity: Example

Tesla vs. Ford	
Technology Proximity	0.11
Product Proximity	0.15

Apple vs. Intel	
Technology Proximity	0.57
Product Proximity	0.00

Rotemberg (1984) Proportional Influence

- $o \in \{1, 2, ..., n_o\}$: owners
- s_{io}: the proportion of shares in firm i owned by owner o where $\sum_o s_{io} = 1$
- $\widehat{V}_i$: value of firm i
- $\widetilde{V}_o \equiv \sum_i s_{io} \widehat{V}_i$: value of owner o
- Firms' objective:

$$\sum_o s_{io} \widetilde{V}_o \propto \sum_j \kappa_{ij} \widehat{V}_j$$

where

$$\kappa_{ij} \equiv \frac{\boldsymbol{s}_i^T \boldsymbol{s}_j}{\boldsymbol{s}_i^T \boldsymbol{s}_i} = \cos\left(\boldsymbol{s}_i, \boldsymbol{s}_j\right) \sqrt{\frac{IHHI_j}{IHHI_i}} \quad \text{where} \quad \boldsymbol{s}_i \equiv [s_{i1}, ..., s_{io}, ..., s_{in_o}]^T$$

Total Surplus

- Total surplus for product i:

$$ts_i(\boldsymbol{q}, \boldsymbol{x}) = \pi_i(\boldsymbol{q}, \boldsymbol{x}) + cs_i(\boldsymbol{q}) = q_i \left[b_i - \frac{1}{2} \sum_{j=1}^{n} \sigma_{ij} q_j - \overline{m}_i + \sum_{j=1}^{n} \omega_{ij} x_j \right] - \frac{1}{2} x_i^2$$

R&D Externalities

1. Business stealing effect
   - Innovators steel the business (profits) of other firms

2. Technology spillover effect
   - Innovation improves the productivity of other firms

3. Appropriability effect (market power)
   - Innovators cannot appropriate the entire consumer surplus

R&D Allocation and Externalities

- Firms maximize common owner weighted profits:

$$x^* = (K \circ \Omega)[\Sigma + K \circ \Sigma - \Omega(K \circ \Omega)]^{-1}(b - \overline{m})$$

- Firms maximize common owner weighted total surplus ($\star$):

$$x^*_{TS} = (K \circ \Omega) \left[\frac{1}{2}(\Sigma + K \circ \Sigma) - \Omega(K \circ \Omega) \right]^{-1} (b - \overline{m})$$

- $K = \mathbf{1}_{n \times n}$ in ($\star$) $\implies$ Social Optimum

- Externalities: (i) Appropriability, (ii) Business stealing, (iii) Technology spillover

Generalized Hedonic-Linear Demand (Pellegrino, 2024)

- $i \in \{1, 2, ..., n\}$: firms / products
- 1 unit of product i provides
  - 1 unit of idiosyncratic characteristic $k \in \{1, 2, ..., n\}$
  - $\psi_{k,i}$ unit of shared characteristic $k \in \{n+1, n+2, ..., n+n_k\}$ where $\sum_k \psi_{k,i}^2 = 1$
- Aggregate each characteristic:

$$y_{k,t} = \begin{cases} q_{k,t} & k = 1, 2, ..., n \\ \sum_i \psi_{k,i} q_{i,t} & k = n+1, n+2, ..., n+n_k \end{cases}$$

- Linear quadratic aggregator over characteristics:

$$Y_t = (1 - \alpha) \sum_{k=1}^{n} \left(\underbrace{\hat{b}_{k,t} y_{k,t} - \frac{1}{2} y_{k,t}^2}_{\text{idiosyncratic characteristic}} \right) + \alpha \sum_{k=n+1}^{n+n_k} \left(\underbrace{\hat{b}_{k,t} y_{k,t} - \frac{1}{2} y_{k,t}^2}_{\text{shared characteristic}} \right)$$

Generalized Hedonic-Linear Demand (Pellegrino, 2024)

- Quality:

$$b_i = (1 - \alpha)\hat{b}_i + \alpha \sum_{k=n+1}^{n+n_k} \psi_k \hat{b}_k$$

- Inverse demand:

$$\frac{p}{P} = b - \Sigma q$$

- Inverse cross price elasticity of demand:

$$\frac{\partial \log p_i}{\partial \log q_j} = -\frac{q_j}{p_i} \cdot \sigma_{ij}$$

- Cross price elasticity of demand:

$$\frac{\partial \log q_i}{\partial \log p_j} = -\frac{p_j}{q_i}(\Sigma^{-1})_{ij}$$

Static Profits

- Gross profit: $\pi_{i,t} = \sum_j \kappa_{ij} \sigma_{ij} q_{i,t} q_{j,t}$
- Firms choose labor productivity and product quality: $\zeta a_{i,t} = \sqrt{\zeta w_t}$, $b_{i,t} = z_{i,t} - \sqrt{\zeta w_t}$
- Labor market clearing: $L = \sum_i \frac{q_{i,t}}{a_{i,t}} \Longrightarrow \sqrt{\zeta w_t} = \frac{\zeta}{L} \sum_i q_{i,t}$
- $q_t = N z_t$ where $N \equiv \left\{ 2\frac{\zeta}{L} J + \Sigma + K \circ \Sigma \right\}^{-1}$
- N_i: the i th row of N
- Ownership weighted profit:

$$\sum_j \kappa_{ij} \frac{\pi_{j,t}}{P_t} = \sum_j \kappa_{ij} \sum_h \kappa_{jh} \sigma_{jh} q_{j,t} q_{h,t} = z_t^T Q^i z_t$$

where

$$Q^i = \frac{1}{2} \sum_j \kappa_{ij} \sum_h \kappa_{jh} \sigma_{jh} \left(N_j^T N_h + N_h^T N_j \right)$$

Back

Riccati Equations

- $V^i(z) = z^T X^i z$ where X^i is the solution of the stacked Riccati equation

$$0 = Q^i - \mu^2 \sum_j \kappa_{ij} X_j^j \left(X_j^j \right)^T + \left(\mathbf{\Phi} - \frac{1}{2} \left(\rho - \gamma^2 \right) I \right)^T X^i + X^i \left(\mathbf{\Phi} - \frac{1}{2} \left(\rho - \gamma^2 \right) I \right)$$

  - $X_i^i \equiv$ the i th column of X^i
  - $\mathbf{\Phi} \equiv \mathbf{\Omega} + \mu^2 \begin{bmatrix} X_1^1 & \cdots & X_n^n \end{bmatrix}^T$

- Algorithm: Given $\begin{bmatrix} X_\tau^1 & \cdots & X_\tau^n \end{bmatrix}$, update $\begin{bmatrix} X_{\tau-\Delta}^1 & \cdots & X_{\tau-\Delta}^n \end{bmatrix}$ by

$$-\frac{X_\tau^i - X_{\tau-\Delta}^i}{\Delta} = Q^i - \mu^2 \sum_j \kappa_{ij} X_{j,\tau}^j \left(X_{j,\tau}^j \right)^T + \left(\mathbf{\Phi}_\tau - \frac{1}{2} \left(\rho - \gamma^2 \right) I \right)^T X_\tau^i + X_\tau^i \left(\mathbf{\Phi}_\tau - \frac{1}{2} \left(\rho - \gamma^2 \right) I \right)$$

Back

Summary of Equilibrium

Description	Expression
Production strategy	$\boldsymbol{q}_t = N\boldsymbol{z}_t$
R&D strategy	$\boldsymbol{x}_t = \mu \tilde{X} \boldsymbol{z}_t$
Law of motion	$d\boldsymbol{z}_t = \left(\Omega \boldsymbol{z}_t + \mu \boldsymbol{x}_t\right) dt + \gamma \boldsymbol{z}_t dW_t$
Profit of final producers	$\Pi_t^F / P_t = \boldsymbol{q}_t^T \left(\frac{1}{2}\Sigma\right) \boldsymbol{q}_t$
Total operating profit of firms	$\Pi_t / P_t = \boldsymbol{q}_t^T \left(\frac{1}{2}\Sigma \circ \left(K + K^T\right)\right) \boldsymbol{q}_t$
Labor income	$w_t L / P_t = \boldsymbol{q}_t^T \left(\frac{\zeta}{L}J\right) \boldsymbol{q}_t$
Output	$Y_t = \boldsymbol{q}_t^T \left(\frac{\zeta}{L}J + \frac{1}{2}\Sigma + \frac{1}{2}\Sigma \circ \left(K + K^T\right)\right) \boldsymbol{q}_t$
Consumption	$C_t = Y_t - \boldsymbol{x}_t^T \boldsymbol{x}_t$

Back

Output and Expected Utility

- Output: $Y_t = \boldsymbol{q}_t^T Q \boldsymbol{q}_t$ where

$$Q = \frac{\zeta}{L} J + \frac{1}{2} \Sigma + \frac{1}{2} \Sigma \circ \left(K + K^T \right)$$

- Expected utility:

$$V\left(\boldsymbol{z}_t\right) \equiv \boldsymbol{E}_t \left[\int_t^\infty \exp\left(-\rho s\right) C_s ds \middle| \boldsymbol{z}_t \right] = \boldsymbol{z}_t^T X \boldsymbol{z}_t$$

where X is the solution of the Lyapunov equation (obtained from households' HJB equation):

$$0 = Q - \mu^2 \tilde{X}^T \tilde{X} + X \left(\Phi - \frac{1}{2} \left(\rho - \gamma^2 \right) I \right) + \left(\Phi - \frac{1}{2} \left(\rho - \gamma^2 \right) I \right)^T X$$

Back

Social Optimum

- Static optimal allocation: $\boldsymbol{q}_t^* = N^* \boldsymbol{z}_t$ where $N^* \equiv \left\{ 2\frac{\zeta}{L} J + \Sigma \right\}^{-1}$
- Optimal output: $Y_t^* = \boldsymbol{z}_t^T Q^* \boldsymbol{z}_t$ where $Q^* = \frac{1}{2} N^*$
- Optimal expected utility:

$$V^* (\boldsymbol{z}_t) \equiv \boldsymbol{E}_t \left[\int_t^\infty \exp\left(-\rho s\right) C_s ds \,\middle|\, \boldsymbol{z}_t \right] = \boldsymbol{z}_t^T X^* \boldsymbol{z}_t,$$

where X^* is the solution of the Riccati equation (obtained from planner's HJB equation):

$$0 = Q^* - \mu^2 \left(X^*\right)^2 + X^* \left(\Phi^* - \frac{1}{2}\left(\rho - \gamma^2\right) I\right) + \left(\Phi^* - \frac{1}{2}\left(\rho - \gamma^2\right) I\right) X^*$$

- Optimal R&D: $\boldsymbol{x}_t^* = \mu X^* \boldsymbol{z}_t$
- Optimal technology transition matrix: $\Phi^* = \Omega + \mu^2 X^*$

Stochastic Process of Output

- Applying Itô's lemma,

$$d \log Y_t = \left[\frac{z_t^T \left(Q\Phi + \Phi^T Q \right) z_t}{Y_t} + \gamma^2 \left\{ \frac{\sum_i z_{i,t}^2 Q_{ii}}{Y_t} - \frac{2 z_t^T Q \operatorname{diag} \left(z_t^2 \right) Q z_t}{Y_t^2} \right\} \right] dt + \frac{2\gamma z_t^T Q \operatorname{diag} \left(z_t \right)}{Y_t} dW_t$$

where $Y_t = z_t^T Q z_t$ and $\Phi = \Omega + \mu^2 \widetilde{X}$

Tech Spillover	$\dfrac{z_t^T \left(Q\Omega + \Omega Q \right) z_t}{Y_t}$
R&D Contribution	$\dfrac{\mu^2 z_t^T \left(Q\widetilde{X} + \widetilde{X}^T Q \right) z_t}{Y_t}$
Itô Correction	$\gamma^2 \left\{ \dfrac{\sum_i z_{i,t}^2 Q_{ii}}{Y_t} - \dfrac{2 z_t^T Q \operatorname{diag} \left(z_t^2 \right) Q z_t}{Y_t^2} \right\}$
Total	$E \left[d \log Y_t \right]$

# Number of Sample Firms
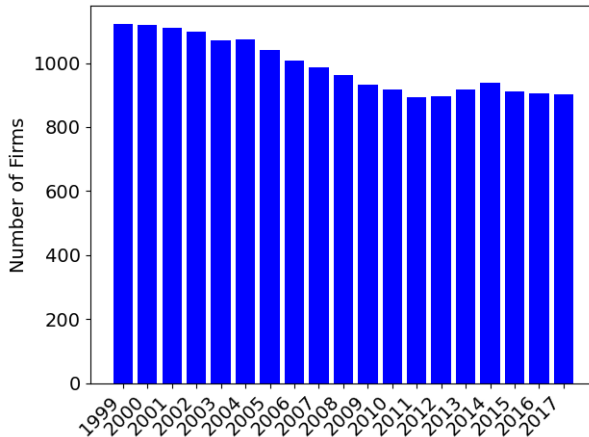
# Trend of Product Substitutability
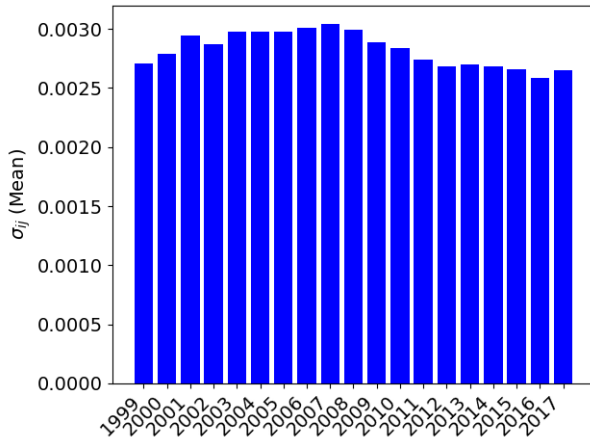
Technological Proximity

- Merge USPTO data with Compustat firms using DISCERN 2 dataset (Arora et al., 2024)
- Jaffe measure, Group-level patent classification, Stack for 5 years
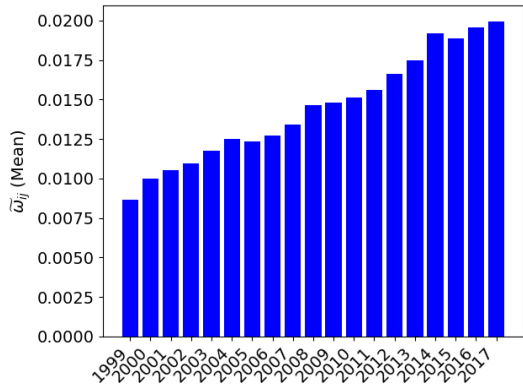
Distributions of Estimated Knowledge Capital and Quantity
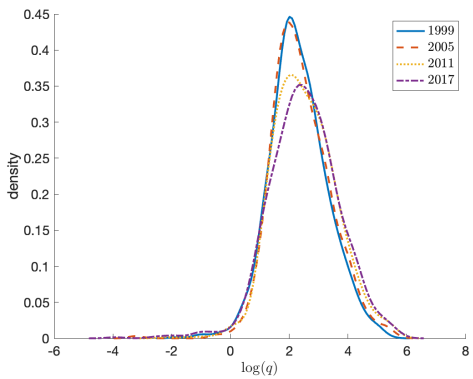
(a) Log of Quantity
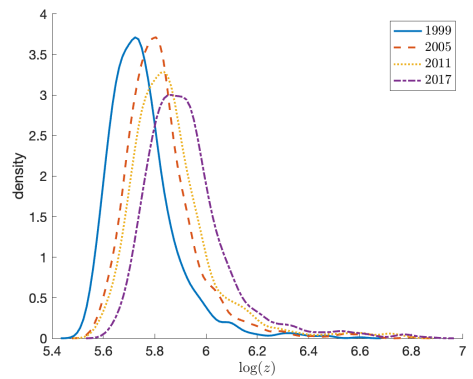
(b) Log of Knowledge Capital

Microeconometric Estimates vs. GHL (Pellegrino, 2024) (1/2)

Market	Firm i	Firm j	Micro Estimate	GHL
Auto	Ford	Ford	-4.320	-5.197
Auto	Ford	General Motors	0.034	0.056
Auto	Ford	Toyota	0.007	0.017
Auto	General Motors	Ford	0.065	0.052
Auto	General Motors	General Motors	-6.433	-4.685
Auto	General Motors	Toyota	0.008	0.005
Auto	Toyota	Ford	0.018	0.025
Auto	Toyota	General Motors	0.008	0.008
Auto	Toyota	Toyota	-3.085	-4.851

Back

Microeconometric Estimates vs. GHL (Pellegrino, 2024) (2/2)

Market	Firm i	Firm j	Micro Estimate	GHL
Cereals	Kellogg's	Kellogg's	-3.231	-1.770
Cereals	Kellogg's	Quaker Oats	0.033	0.023
Cereals	Quaker Oats	Kellogg's	0.046	0.031
Cereals	Quaker Oats	Quaker Oats	-3.031	-1.941
Computers	Apple	Apple	-11.979	-8.945
Computers	Apple	Dell	0.018	0.025
Computers	Dell	Apple	0.027	0.047
Computers	Dell	Dell	-5.570	-5.110

First Stage (Back)

	(1)
Dependent Variable:	$z_{i,t}$
User cost of R&D	-39.495^{***}
	(4.7044)
Year Fixed Effects	✓
No. observations	12,947

SEs clustered by years and 4-digit NAICS industries are reported in parentheses.

- IV: User cost of R&D, driven by state-level tax variations (Wilson, 2009; Bloom et al., 2013)

Alternative Corporate Governance Models: Ederer and Pellegrino (2024)

1. Super-proportional influence: $\tilde{\kappa}_{ij} = \frac{\sum_{z=1}^{Z} s_{iz} \gamma_{iz} s_{jz}}{\sum_{z=1}^{Z} s_{iz} \gamma_{iz} s_{iz}}$ where $\gamma_{iz} = \sqrt{s_{iz}}$

2. Blockholder influence: $\tilde{\kappa}_{ij} = \frac{\sum_{z=1}^{Z} s_{iz} b_{iz} s_{jz}}{\sum_{z=1}^{Z} s_{iz} s_{jz}}$ $(i \neq j)$, where $b_{iz} = 1$ if $s_{iz} > 5\%$

3. Governance frictions and entrenchment
   - Azar and Ribeiro (2021) (AR) estimate an objective function where the manager of firm i discounts other firms' profit by τ_i

Alternative Corporate Governance Models

		Ownership Structure in 2017				
	Dispersed Ownership	Baseline: Proportional Influence	Super Proportional Influence	Blockholder Influence	Governance Frictions (Uniform)	Governance Frictions (Firm-Specific)
Total Output	100.00	97.57	97.36	98.25	99.31	99.49
Total R&D Expenditure	100.00	85.73	84.48	91.05	97.57	98.70
Expected Growth Rate (%)	1.796	1.753	1.750	1.771	1.788	1.791
Expected Social Welfare	100.00	97.18	96.94	98.01	99.21	99.41
Firm Value Share (%)	28.74	34.32	34.32	32.79	30.65	30.78

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