Ownership Structure and Economic Growth

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Ownership Structure $\Longrightarrow$ Economic Growth?

- BlackRock, Vanguard, State Street, and Fidelity control 30% of votes of S&P 500 firms Share
- Top 10 chaebols account for half of stock market capitalization and exports in Korea

- Firms maximizing shareholder values $\Longrightarrow$
  Partially internalize externalities for commonly owned firm

- Ownership structure (common ownership, cross ownership, M&A, FDI, ...) $\Longrightarrow$
  Economic growth?
  - Business stealing effect
  - Technology spillover effect

Quantitative Schumpeterian Growth Model with Ownership Structure

- Existing Schumpeterian growth models:
  - Lack of strategic interaction (e.g., monopolistic competition)
  - Very few firms in Markov perfect equilibrium
- This paper: Many oligopolists $+$ Ownership structure
  - Exercise: Public patenting firms in the US ($\simeq$ 1000 firms)
- Quantify three inter-firm networks:
  - Ownership structure
  - Product market rivalry
  - Technology spillover
- Common ownership in the US:
  - Internalization of business stealing $\Longrightarrow g \downarrow\downarrow$
  - Internalization of technology spillover $\Longrightarrow g \uparrow$

Literature

- Competition & Innovation:
  d'Aspremont and Jacquemin (1988); Aghion et al. (2001, 2005); Acemoglu and Akcigit (2012); Aghion et al. (2013); Bloom et al. (2013); Lopez and Vives (2019); Peters (2020); Akcigit and Ates (2021, 2023); Liu et al. (2022); Cavenaile et al. (2023); Anton et al. (2023, 2024); Kini et al. (2024); Hopenhayn and Okumura (2024)
  Quantitative endogenous growth models matched to firm-(pair-)level data
  Ownership structure $\Longrightarrow$ economic growth.
- Hedonic Demand / Empirical IO:
  Lancaster (1966); Rosen (1974); Berry et al. (1995); Nevo (2001); Pellegrino (2024); Ederer and Pellegrino (2024)
  Dynamic general equilibrium / R&D
- Oligopoly / Common Ownership / Market Power:
  Rubinstein and Yaari (1983); Rotemberg (1984); Neary (2003); Atkeson and Burstein (2008); Gutierrez and Philippon (2017); He and Huang (2017); Azar et al. (2018, 2022); Autor et al. (2020); Baqaee and Farhi (2020); De Loecker et al. (2020); Azar and Vives (2021); Edmond et al. (2023)

Simple Static Partial Equilibrium Model

- Firm $i \in \{1, \dots, n\}$ chooses quantity q_i and R&D effort x_i
- Inverse demand: $\mathbf{p}(\mathbf{q}) = \mathbf{b} - \boldsymbol{\Sigma}\mathbf{q}$
- CRS production technology with marginal cost: $\mathbf{m}(\mathbf{x}) = \mathbf{a} - \boldsymbol{\Omega}\mathbf{x}$
- Cost of R&D: $c_i(x_i) = \frac{1}{2}x_i^2$

Common Ownership Weights

- $\mathbf{K} = \left[\kappa_{ij}\right]$: common ownership weights that firm i places on the profit of firm j
- More overlapping ownership b/w firm i and $j \Longrightarrow$ Higher κ_{ij}

Proportional Influece

- Diagonal elements κ_{ii} are normalized to 1 for all firm i
- $\mathbf{K} = \mathbf{I}$: dispersed ownership (each firm maximizes its own value)
- $\mathbf{K} = [\mathbf{1}]$: monopoly (maximizes total producer surplus)

Cournot & R&D Game

- Firm i's profit:

$$\begin{aligned}\pi_i(\mathbf{q},\mathbf{x}) &= q_i[p_i - m_i] - c_i(x_i) \\ &= q_i\left[b_i - \sum_{j=1}^{n}\sigma_{ij}q_j - a_i + \sum_{j=1}^{n}\omega_{ij}x_j\right] - \frac{1}{2}x_i^2\end{aligned}$$

- Given $\{q_j, x_j\}_{j\neq i}$, firm i chooses q_i and x_i to maximize $\sum_j \kappa_{ij}\pi_j(\mathbf{q},\mathbf{x})$

Impact of Common Ownership on R&D

- Comparative statics where $\{q_k, x_k\}_{k \neq i}$ are held constant:

$$\frac{\partial x_i}{\partial \kappa_{ij}} = \frac{q_j}{2 - \omega_{ii}^2}(2\omega_{ij} - \omega_{ii}\sigma_{ij}) \qquad \forall\ j \neq i$$

  - SOC: $2 > \omega_{ii}^2$

- Internalize business stealing effect: $\partial^2 x_i / \partial \kappa_{ij} \partial \sigma_{ij} < 0$
- Internalize technology spillover effect: $\partial^2 x_i / \partial \kappa_{ij} \partial \omega_{ij} > 0$

Overview of Identification Strategy

Networks	Measurement
Common ownership weight $\mathbf{K}$	Institutional investor holdings in 13f filings (Backus et al., 2021)
Product market rivalry $\mathbf{\Sigma}$	Product proximity (Hoberg and Phillips, 2016): Text analysis of business description in 10k filings
Technology spillover $\mathbf{\Omega}$	Technology proximity (Bloom et al., 2013): Patent classification

Oligopolistic Schumpeterian Growth Model

- Linear quadratic aggregator (final good):

$$Y_t = \mathbf{q}_t^T \mathbf{b}_t - \frac{1}{2}\mathbf{q}_t^T \mathbf{\Sigma} \mathbf{q}_t$$

- Linear inverse demand:

$$\mathbf{p}_t = \mathbf{b}_t - \mathbf{\Sigma} \mathbf{q}_t$$

- CRS production technology: $q_{i,t} = a_{i,t} l_{i,t}$
- Each firm has knowledge capital $z_{i,t}$
- Each firm allocates knowledge capital to improve labor productivity and product quality:

$$\zeta a_{i,t} + b_{i,t} = z_{i,t}$$

Law of Motion of Knowledge Capital

$$d\mathbf{z}_t = \left(\underbrace{\mathbf{\Omega z}_t}_{\text{technology spillover}} + \underbrace{\mu \mathbf{x}_t}_{\text{R\&D}} \right) dt + \underbrace{\gamma \mathbf{z}_t dW_t}_{\text{shocks}}$$

- $\mathbf{\Omega} = [\omega_{ij}]$: technology spillover matrix
- $x_{i,t} = \sqrt{d_{i,t}}$
  - $d_{i,t}$: R&D cost in terms of final good
  - Innovation elasticity is 0.5
- μ, γ: positive scalars

Market Clearing and Preference

- Inelastic labor supply:

$$L = \sum_i l_{i,t}$$

- Final good market clearing:

$$C_t + \underbrace{\sum_i d_{i,t}}_{\text{R\&D cost}} = Y_t$$

- Risk neutral representative household:

$$\max \ \mathbf{E}_t \left[\int_t^{\infty} \exp\left(-\rho s\right) C_s ds \right]$$

Cournot-Nash Equilibrium

- Firm i's gross profit before subtracting dynamic R&D cost:

$$\pi_{i,t} = p_{i,t}q_{i,t} - w_t l_{i,t} = \left(b_{i,t} - \frac{1}{a_{i,t}} w_t - \sum_j \sigma_{ij} q_{j,t} \right) q_{i,t} \quad \text{where} \quad \zeta a_{i,t} + b_{i,t} = z_{i,t}$$

- Given w_t, $z_{i,t}$, and $\{q_{j,t}\}_{j \neq i}$, firm i chooses $a_{i,t}$, $b_{i,t}$, and $q_{i,t}$ to maximize $\sum_j \kappa_{ij} \pi_{j,t}$
- Quantity is a linear function of knowledge capital:

$$\mathbf{q}_t = \Bigg\{ \underbrace{2\frac{\zeta}{L}\mathbf{J}}_{\text{labor cost}} + \underbrace{\mathbf{\Sigma}}_{\text{substitutability}} + \underbrace{\mathbf{K} \circ \mathbf{\Sigma}}_{\text{ownership weight} \times \text{substitutability}} \Bigg\}^{-1} \mathbf{z}_t$$

  - $\mathbf{J}$: matrix with all elements equal to 1
- Ownership-weighted gross profits are expressed in quadratic form: $\sum_j \kappa_{ij} \pi_{j,t} = \mathbf{z}_t^T \mathbf{Q}^i \mathbf{z}_t$

Linear-Quadratic Differential Game

- Given other players' strategy $\{x_{j,t}\}_{j\neq i, t\geq 0}$, firm i chooses R&D effort $\{x_{i,t}\}_{t\geq 0}$ to maximize

$$\max_{\{x_{i,t}\}_{t\geq 0}} \quad V^i(\mathbf{z}_0) \equiv \mathbf{E}_0\left[\int_0^\infty \exp(-\rho t)\left\{\sum_j \kappa_{ij}\left(\pi_{j,t} - x_{j,t}^2\right)\right\} dt\right]$$

subject to $d\mathbf{z}_t = (\boldsymbol{\Omega}\mathbf{z}_t + \mu\mathbf{x}_t)\,dt + \gamma\mathbf{z}_t dW_t$

- Firm i's HJB equation:

$$\rho V^i(\mathbf{z}) = \max_{x_i}\left\{\mathbf{z}^T\mathbf{Q}^i\mathbf{z} - \sum_j \kappa_{ij}x_j^2 + V_{\mathbf{z}}^i(\mathbf{z})\left[\boldsymbol{\Omega}\mathbf{z} + \mu\mathbf{x}\right] + \frac{\gamma^2}{2}\mathbf{z}^T V_{\mathbf{zz}}^i(\mathbf{z})\,\mathbf{z}\right\}$$

HJB Equations $\Longrightarrow$ Riccati Equations

- Guess and verify $V^i(\mathbf{z}) = \mathbf{z}^T \mathbf{X}^i \mathbf{z}$ (for any $\mathbf{z}$)
- $\mathbf{X}^i$ is the solution of stacked algebraic Riccati equations Riccati
- All public patenting firms in the US in our dataset $\simeq$1000 firms $\Longrightarrow$
    - $1000^3 = 1$ billion undetermined coefficients

Oligopolistic Schumpeterian	Computation time	# of firms	Productivity space
Cavenaile et al. (2023)	$O(2^n)$	4	6-point grid
Our model	$O(n^4)$	≈ 1000	Continuous

BGP in Deterministic Economy

- R&D strategy: $x_{i,t} = \left(\mu \mathbf{X}_i^i\right)^T \mathbf{z}_t$ where $\mathbf{X}_i^i$ is the i th column of $\mathbf{X}^i$
- The law of motion is rewritten as $d\mathbf{z}_t = \mathbf{\Phi}\mathbf{z}_t dt + \gamma \mathbf{z}_t dW_t$ where

$$\underbrace{\mathbf{\Phi}}_{\text{technology transition matrix}} \equiv \underbrace{\mathbf{\Omega}}_{\text{spillover}} + \underbrace{\mu^2 \begin{bmatrix} \mathbf{X}_1^1 & \cdots & \mathbf{X}_n^n \end{bmatrix}^T}_{\text{R\&D}}$$

Theorem

Consider the deterministic economy ($\gamma = 0$). If the technology transition matrix $\mathbf{\Phi}$ is irreducible, then:
(i) There exists largest positive eigenvalue of $\mathbf{\Phi}$, g, and associated strictly positive eigenvector, $\mathbf{z}^*$.
(ii) There exists a globally stable BGP such that the knowledge capital growth rate of all firms is g, and the knowledge capital distribution is a scalar multiple of $\mathbf{z}^*$.

- Proof: Perron–Frobenius Theorem
- "$\mathbf{\Phi}$ is irreducible" $\Longleftrightarrow$ "All firms are directly or indirectly connected technologically"

BGP in Deterministic Economy

- In BGP, $\mathbf{a}_t$, $\mathbf{b}_t$, $\mathbf{z}_t$, and $\mathbf{q}_t$ grow at the same rate

Technology Choice:	$\zeta a_{i,t} + b_{i,t} = z_{i,t}$
Linear Production Technology:	$q_{i,t} = a_{i,t} l_{i,t}$
Inelastic Labor Supply:	$L = \sum_i l_{i,t}$

- Linear and quadratic term in $\mathbf{q}_t$ of output grow at the same rate: Equilibrium Summary

$$Y_t = \mathbf{q}_t^T \mathbf{b}_t - \frac{1}{2} \mathbf{q}_t^T \mathbf{\Sigma} \mathbf{q}_t$$

Aggregation

- Apply Ito's lemma to

Decomposition

$$\begin{aligned} \log Y_t &= \log\left(\mathbf{z}_t^T \mathbf{Q} \mathbf{z}_t\right) \\ d\mathbf{z}_t &= \mathbf{\Phi} \mathbf{z}_t dt + \gamma \mathbf{z}_t dW_t \end{aligned} \implies g_t$$

- Expected utility is expressed in quadratic form:

X

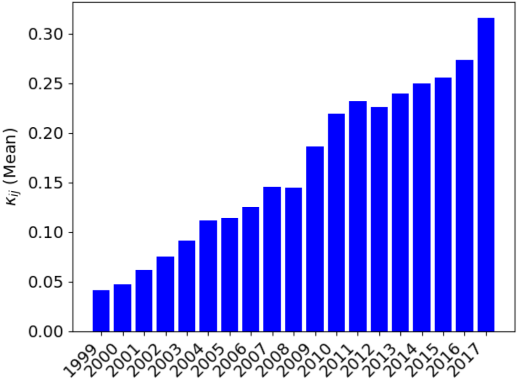
$$\mathbf{E}_t \left[\int_t^\infty \exp\left(-\rho s\right) C_s ds \middle| \mathbf{z}_t \right] = \mathbf{z}_t^T X \mathbf{z}_t$$

- Solve the equilibrium once $\implies$ Can compute expected growth and utility for any $\mathbf{z}_t$

Common Ownership Weight

- Backus et al. (2021) construct a dataset on investors' holdings based on Form 13F
- Baseline: Rotemberg (1984) proportional influence assumption

Proportional Influece



Product Substitutability

- Hoberg and Phillips (2016) estimates product similarity using business descriptions in 10-K
- Pellegrino (2024) estimates α to align with the cross-price elasticity of demand

$$\underbrace{\sigma_{ij}}_{\text{substitutabiliy}} = \alpha \times \text{product similarity b/w } i \text{ and } j \quad (i \neq j)$$

Technological Proximity

- Technological profile of firm i
  - The vector of the share of patents held by firm i in each technology class
  - Baseline: group-level classifications (≈ 4000)
- Jaffe (1986) technology proximity measure $\tilde{\omega}_{ij}$:
  - Cosine similarity of the technological profiles b/w firm i and j

Initial Distribution of Knowledge Capital

1. Gross profit (before subtracting R&D cost) $\pi_{i,t} \Longleftarrow$ "Revenue" $-$ "Costs of goods sold"
   - Deflated by GDP deflator for inter-temporal comparison
2. Quantity $\mathbf{q}_t \Longleftarrow \pi_{i,t} = \sum_j \kappa_{ij}\sigma_{ij}q_{i,t}q_{j,t}$
3. $\zeta/L \Longleftarrow$ match the cost share of the sample firms
4. Knowledge capital $\mathbf{z}_t \Longleftarrow \mathbf{q}_t = \left\{2\frac{\zeta}{L}J + \Sigma + K \circ \Sigma\right\}^{-1}\mathbf{z}_t$

Technology Spillover $\Omega = \beta \times$ Technology Proximity $\tilde{\Omega}$ First Stage

$$z_{i,t+1} - z_{i,t} = \beta \sum_{j \neq i} \tilde{\omega}_{ij,t} z_{j,t} + \text{Year FE}_t + \epsilon_{i,t}$$

	(1)	(2)	(3)
	$z_{i,t+1} - z_{i,t}$	$z_{i,t+1} - z_{i,t}$	$z_{i,t+1} - z_{i,t}$
$\sum_{j \neq i} \tilde{\omega}_{ij,t} z_{j,t}$	0.000191***	0.000152***	0.000140***
	(0.000035)	(0.000035)	(0.000039)
$\sqrt{\text{R\&D Expenditure}}$		.037**	
		(0.021)	
Year Fixed Effects	✓	✓	✓
IV			✓
IV 1st Stage F-statistics			4176
No. observations	16,324	15,173	14,181

SEs clustered by years and 4-digit naics industries are reported in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

- IV: User cost of R&D, driven by state-level tax variations (Wilson, 2009; Bloom et al., 2013)

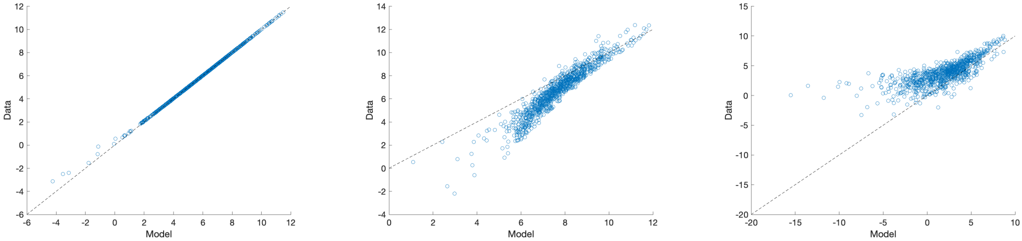
Identification: Summary

- Publicly available data $+$ Compustat

Notation	Description	Value	Source
$\Psi^T\Psi$	Product proximity		Form 10-K, Hoberg and Phillips (2016)
$\widetilde{\Omega}$	Technological proximity		USPTO, Patent classification
K	Common ownership weights		Form 13F, Backus et al. (2021)
α	Product proximity $\rightarrow$ Substitutability	.12	Pellegrino (2024)
β	Technological proximity $\rightarrow$ Spillover	.00014	Estimate the law of motion
γ	St.d. of idiosyncratic shocks	.027	Estimate the law of motion
ζ/L	Labor augmentation efficiency	.0063	Compustat, Cost of goods sold
ρ	Discount rate	.10	
μ	R&D efficiency	.05	1.6% economic growth rate

Fit b/w Model and Data

- Model (x-axis) vs Data (y-axis) : firm-level profits (targeted), sales and R&D expenditures



(a) Log of Profit (Targeted) (b) Log of Sales (c) Log of R&D Expenditure

Total Output

Total output (Social Optimum: 100)	Ownership (Baseline: 2017)				
	Dispersed	Mean=1999	Uniform	Baseline	Monopoly
Baseline	91.30	91.02	90.78	89.08	89.17
Only Business Steal $\boldsymbol{\Omega} = [0]$	91.30	91.02	90.78	89.08	89.17
Only Tech Spill $\boldsymbol{\Sigma} = \mathbf{I}$, $\zeta/L = 0$	75.00	75.00	75.00	75.00	75.00

- Inelastic labor supply $\implies$ changes in total output arise from misallocation
- Common ownership worsens product misallocation

Total R&D Expenditure

Total R&D Expenditure (Social Optimum: 100)	Ownership (Baseline: 2017)				
	Dispersed	Mean=1999	Uniform	Baseline	Monopoly
Baseline	26.16	25.90	25.56	22.36	19.36
Only Business Steal $\mathbf{\Omega} = [0]$	28.15	27.79	27.07	23.42	19.80
Only Tech Spill $\mathbf{\Sigma} = \mathbf{I}$, $\zeta/L = 0$	18.27	18.34	18.75	18.86	19.84

- Internalization of business stealing $>$ Internalization of technology spillover
- Network heterogeneity is important

Expected Growth Rate

Expected economic growth rate (%)	Ownership (Baseline: 2017)				
	Dispersed	Mean=1999	Uniform	Baseline	Monopoly
Baseline	1.796	1.793	1.791	1.753	1.713
Only Business Steal $\mathbf{\Omega} = [0]$	1.097	1.094	1.093	1.062	1.020
Only Tech Spill $\mathbf{\Sigma} = \mathbf{I}$, $\zeta/L = 0$	2.051	2.054	2.068	2.072	2.107

- 0.043 pp (2.4%) lower growth under common ownership compared to dispersed ownership

Expected Social Welfare

Expected Social Welfare (Social Optimum: 100)	Ownership (Baseline: 2017)				
	Dispersed	Mean=1999	Uniform	Baseline	Monopoly
Baseline	87.72	87.42	87.16	85.25	85.18
Only Business Steal $\mathbf{\Omega} = [0]$	88.83	88.53	88.30	86.44	86.41
Only Tech Spill $\mathbf{\Sigma} = \mathbf{I}$, $\zeta/L = 0$	68.81	68.82	68.88	68.89	69.02

- 2.8% CE welfare loss under common ownership compared to dispersed ownership

Firm Value Share

Firm value share (%)	Ownership (Baseline: 2017)				
	Dispersed	Mean=1999	Uniform	Baseline	Monopoly
Baseline	28.74	33.43	29.63	34.34	40.92
Only Business Steal $\boldsymbol{\Omega} = [0]$	27.91	28.80	32.60	33.51	40.14
Only Tech Spill $\boldsymbol{\Sigma} = \mathbf{I},\ \zeta/L = 0$	64.82	64.81	64.76	64.74	64.63

- Firm value share is 5.6% lower under common ownership compared to dispersed ownership

When Common Ownership Affects only R&D Decisions

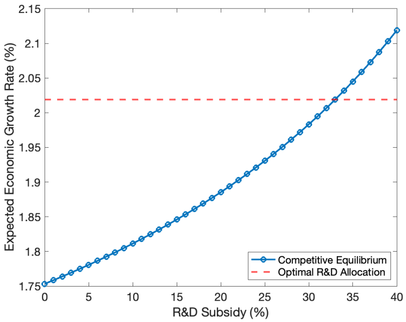
- Investors may only influence R&D decisions (d'Aspremont and Jacquemin, 1988)

	Ownership Structure		
	Dispersed	Common R&D	Baseline
Output (Social Optimum: 100)	91.30	91.30	89.08
R&D Expenditure (Social Optimum: 100)	26.17	19.76	22.36
Growth Rate (%)	1.796	1.726	1.753
Social Welfare (Social Optimum: 100)	87.72	87.49	85.25
Firm Value Share (%)	28.74	29.04	34.34

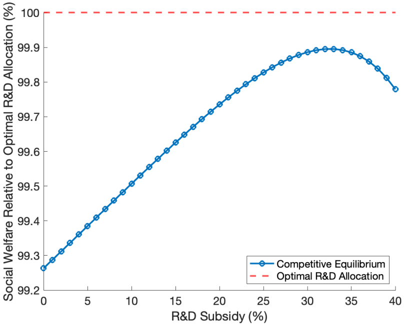
- Lowest R&D expenditure and growth rate
- Intermediate social welfare

Uniform R&D Subsidy

Social Optimum



(a) Expected Growth



(b) Expected Social Welfare

- Optimal rate is $s = 33\%$, which increases g by 0.25 pp (14%)
- CE Welfare loss relative to optimal R&D allocation is reduced to 0.1% (Initially 0.7%)

Conclusion

- Quantitative Schumpeterian growth model with ownership structure
  - Match to data at firm (pair) level

- Common ownership in the US:
  1. Internalization of business stealing effect $\Longrightarrow g \downarrow\downarrow$
  2. Internalization of technology spillover effect $\Longrightarrow g \uparrow$

- Application:
  - Chaebols in Korea
  - Zaibatsu (pre-WWII) and cross-shareholding (late 20th century) in Japan
  - FDI / multinational companies and international technology diffusion

Share of Top 5 Shareholders in Largest Market Cap Firms

Back

Microsoft	
Vanguard	9.20%
Blackrock	7.75%
Steven Ballmer	4.48%
State Street	3.97%
Fidelity	2.66%

Nvidia	
Vanguard	8.93%
BlackRock	7.74%
Fidelity	4.12%
State Street	3.97%
Jensen Huang	3.80%

Apple	
Vanguard	9.29%
Blackrock	7.48%
State Street	3.96%
Fidelity	2.27%
Geode Capital	2.26%

Google	
Vanguard	7.36%
Blackrock	6.47%
State Street	3.39%
Fidelity	3.01%
Sergey Brin	2.99%

Amazon	
Jeffrey Bezos	8.58%
Vanguard	7.77%
Blackrock	6.50%
State Street	3.44%
Fidelity	3.10%

Meta	
Vanguard	7.55%
Blackrock	6.50%
Fidelity	5.38%
Accel IX LP	3.88%
State Street	3.40%

Equity Investments by Big tech in AI Startups

Back

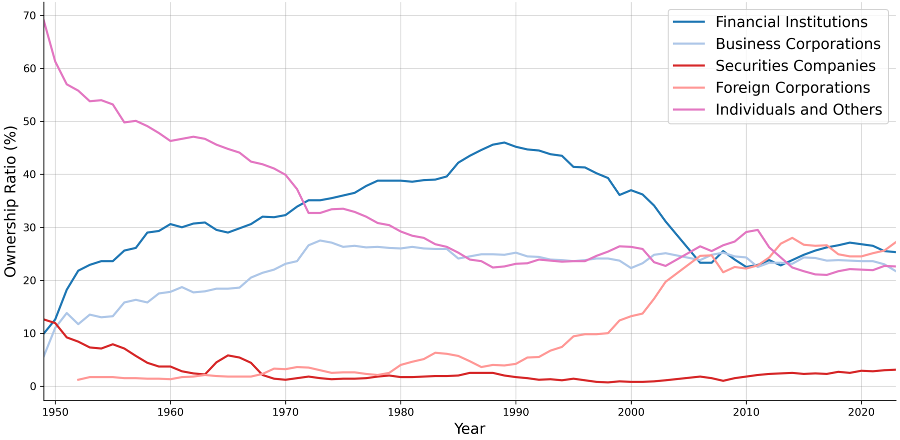
Shareholding percentage	Microsoft	Google	Amazon
OpenAI (ChatGPT)	49%	—	—
Anthropic (Claude)	—	14%	23%

Technology & Product Proximity: Example

Tesla vs. Ford	
Technology Proximity	0.11
Product Proximity	0.15

Apple vs. Intel	
Technology Proximity	0.57
Product Proximity	0.00

Ownership Ratio by Holder Types in Japan



Back to Identification

Rotemberg (1984) Proportional Influence

- $o \in \{1, 2, ..., n_o\}$: owners
- s_{io}: the proportion of shares in firm i owned by owner o where $\sum_o s_{io} = 1$
- $\hat{V}_{i,t}$: value of firm i
- $\tilde{V}_{o,t} \equiv \sum_i s_{io} \hat{V}_{i,t}$: value of owner o
- Firms' objective:

$$\sum_o s_{io} \tilde{V}_{o,t} \propto \sum_j \kappa_{ij} \hat{V}_{j,t}$$

where

$$\kappa_{ij} \equiv \mathbf{s}_i^T \mathbf{s}_j / \mathbf{s}_i^T \mathbf{s}_i \quad \text{where} \quad \mathbf{s}_i \equiv \left[s_{i1}, ..., s_{io}, ..., s_{in_o} \right]^T$$

Back

Total Surplus

- Total surplus for product i:

$$ts_i(\mathbf{q},\mathbf{x}) = \pi_i(\mathbf{q},\mathbf{x}) + cs_i(\mathbf{q}) = q_i \left[b_i - \frac{1}{2}\sum_{j=1}^{n} \sigma_{ij} q_j - a_i + \sum_{j=1}^{n} \omega_{ij} x_j \right] - \frac{1}{2} x_i^2$$

R&D Externalities

1. Business stealing effect
   - Innovators steel the business (profits) of other firms
2. Technology spillover effect
   - Innovation improves the productivity of other firms
3. Appropriability effect (market power)
   - Innovators cannot appropriate the entire consumer surplus

R&D Allocation and Externalities

- Firms maximize common owner weighted profits:

$$\mathbf{x}^* = (\mathbf{K} \circ \boldsymbol{\Omega})[\boldsymbol{\Sigma} + \mathbf{K} \circ \boldsymbol{\Sigma} - \boldsymbol{\Omega}(\mathbf{K} \circ \boldsymbol{\Omega})]^{-1}(\mathbf{b} - \mathbf{a})$$

- Firms maximize common owner weighted total surplus ($\star$):

$$\mathbf{x}^*_{TS} = (\mathbf{K} \circ \boldsymbol{\Omega}) \left[\frac{1}{2}(\boldsymbol{\Sigma} + \mathbf{K} \circ \boldsymbol{\Sigma}) - \boldsymbol{\Omega}(\mathbf{K} \circ \boldsymbol{\Omega}) \right]^{-1} (\mathbf{b} - \mathbf{a})$$

- $\mathbf{K} = [1]$ in ($\star$) $\Longrightarrow$ Social Optimum
- Externalities: (i) Appropriability, (ii) Business stealing, (iii) Technology spillover

Generalized Hedonic-Linear Demand (Pellegrino, 2024)

- $i \in \{1, 2, ..., n\}$: firms / products
- 1 unit of product i provides
  - 1 unit of idiosyncratic characteristic $k \in \{1, 2, ..., n\}$
  - $\psi_{k,i}$ unit of shared characteristic $k \in \{n+1, n+2, ..., n+n_k\}$ where $\sum_k \psi_{k,i}^2 = 1$
- Aggregate each characteristic:

$$y_{k,t} = \begin{cases} q_{k,t} & k = 1, 2, ..., n \\ \sum_i \psi_{k,i} q_{i,t} & k = n+1, n+2, ..., n+n_k \end{cases}$$

- Linear quadratic aggregator over characteristics:

$$Y_t = (1-\alpha) \sum_{k=1}^{n} \left(\underbrace{\hat{b}_{k,t} y_{k,t} - \frac{1}{2} y_{k,t}^2}_{\text{idiosyncratic characteristic}} \right) + \alpha \sum_{k=n+1}^{n+n_k} \left(\underbrace{\hat{b}_{k,t} y_{k,t} - \frac{1}{2} y_{k,t}^2}_{\text{shared characteristic}} \right)$$

Generalized Hedonic-Linear Demand (Pellegrino, 2024)

- Quality:

$$b_i = (1 - \alpha)\,\hat{b}_i + \alpha \sum_{k=n+1}^{n+n_k} \psi_k \hat{b}_k$$

- Inverse demand:

$$\frac{\mathbf{p}}{P} = \mathbf{b} - \mathbf{\Sigma q}$$

- Inverse cross price elasticity of demand:

$$\frac{\partial \log p_i}{\partial \log q_j} = -\frac{q_j}{p_i} \cdot \sigma_{ij}$$

- Cross price elasticity of demand:

$$\frac{\partial \log q_i}{\partial \log p_j} = -\frac{p_j}{q_i} (\mathbf{\Sigma}^{-1})_{ij}$$

Static Profits

- Real gross profit: $\frac{\pi_{i,t}}{P_t} = \sum_j \kappa_{ij}\sigma_{ij}q_{i,t}q_{j,t}$
- Firms choose labor productivity and product quality: $\zeta a_{i,t} = \sqrt{\zeta \frac{w_t}{P_t}}$, $b_{i,t} = z_{i,t} - \sqrt{\zeta \frac{w_t}{P_t}}$
- Labor market clearing: $L = \sum_i \frac{q_{i,t}}{a_{i,t}} \Longrightarrow \sqrt{\zeta \frac{w_t}{P_t}} = \frac{\zeta}{L} \sum_i q_{i,t}$
- $\mathbf{q}_t = \mathbf{N}\mathbf{z}_t$ where $N \equiv \left\{2\frac{\zeta}{L}\mathbf{J} + \mathbf{\Sigma} + \mathbf{K} \circ \mathbf{\Sigma}\right\}^{-1}$
- $\mathbf{N}_i$: the i th row of $\mathbf{N}$
- Ownership weighted profit:

$$\sum_j \kappa_{ij} \frac{\pi_{j,t}}{P_t} = \sum_j \kappa_{ij} \sum_h \kappa_{jh}\sigma_{jh}q_{j,t}q_{h,t} = \mathbf{z}_t^T \mathbf{Q}^i \mathbf{z}_t$$

where

$$\mathbf{Q}^i = \frac{1}{2} \sum_j \kappa_{ij} \sum_h \kappa_{jh}\sigma_{jh} \left(N_j^T N_h + N_h^T N_j\right)$$

Back

Riccati Equations

- $V^i(\mathbf{z}) = \mathbf{z}^T \mathbf{X}^i \mathbf{z}$ where $\mathbf{X}^i$ is the solution of the stacked Riccati equation

$$0 = \mathbf{Q}^i - \mu^2 \sum_j \kappa_{ij} \mathbf{X}_j^j \left(\mathbf{X}_j^j\right)^T + \left(\mathbf{\Phi} - \frac{1}{2}\left(\rho - \gamma^2\right)\mathbf{I}\right)^T \mathbf{X}^i + \mathbf{X}^i \left(\mathbf{\Phi} - \frac{1}{2}\left(\rho - \gamma^2\right)\mathbf{I}\right)$$

  - $\mathbf{X}_i^i \equiv$ the i th column of $\mathbf{X}^i$
  - $\mathbf{\Phi} \equiv \mathbf{\Omega} + \mu^2 \begin{bmatrix} \mathbf{X}_1^1 & \cdots & \mathbf{X}_n^n \end{bmatrix}^T$

- Algorithm: Given $\begin{bmatrix} \mathbf{X}_\tau^1 & \cdots & \mathbf{X}_\tau^n \end{bmatrix}$, update $\begin{bmatrix} \mathbf{X}_{\tau-\Delta}^1 & \cdots & \mathbf{X}_{\tau-\Delta}^n \end{bmatrix}$ by

$$-\frac{\mathbf{X}_\tau^i - \mathbf{X}_{\tau-\Delta}^i}{\Delta} = \mathbf{Q}^i - \mu^2 \sum_j \kappa_{ij} \mathbf{X}_{j,\tau}^j \left(\mathbf{X}_{j,\tau}^j\right)^T + \left(\mathbf{\Phi}_\tau - \frac{1}{2}\left(\rho - \gamma^2\right)\mathbf{I}\right)^T \mathbf{X}_\tau^i + \mathbf{X}_\tau^i \left(\mathbf{\Phi}_\tau - \frac{1}{2}\left(\rho - \gamma^2\right) I\right)$$

Summary of Equilibrium

Description	Expression
Production strategy	$\mathbf{q}_t = N\mathbf{z}_t$
R&D strategy	$\mathbf{x}_t = \mu\tilde{X}\mathbf{z}_t$
Law of motion	$d\mathbf{z}_t = (\Omega\mathbf{z}_t + \mu\mathbf{x}_t)\,dt + \gamma\mathbf{z}_t dW_t$
Profit of final producers	$\Pi_t^F/P_t = \mathbf{q}_t^T\left(\frac{1}{2}\Sigma\right)\mathbf{q}_t$
Total operating profit of firms	$\Pi_t/P_t = \mathbf{q}_t^T\left(\frac{1}{2}\Sigma \circ (K + K^T)\right)\mathbf{q}_t$
Labor income	$w_t L/P_t = \mathbf{q}_t^T\left(\frac{\zeta}{L}J\right)\mathbf{q}_t$
Output	$Y_t = \mathbf{q}_t^T\left(\frac{\zeta}{L}J + \frac{1}{2}\Sigma + \frac{1}{2}\Sigma \circ (K + K^T)\right)\mathbf{q}_t$
Consumption	$C_t = Y_t - \mathbf{x}_t^T\mathbf{x}_t$

Back

Output and Expected Utility

- Output: $Y_t = \mathbf{q}_t^T Q \mathbf{q}_t$ where

$$Q = \frac{\zeta}{L} J + \frac{1}{2}\Sigma + \frac{1}{2}\Sigma \circ \left(K + K^T\right)$$

- Expected utility:

$$V(\mathbf{z}_t) \equiv \mathbf{E}_t \left[\int_t^\infty \text{exp}\left(-\rho s\right) C_s ds \middle| \mathbf{z}_t \right] = \mathbf{z}_t^T X \mathbf{z}_t$$

where X is the solution of the Lyapunov equation (obtained from households' HJB equation):

$$0 = Q - \mu^2 \tilde{X}^T \tilde{X} + X\left(\Phi - \frac{1}{2}\left(\rho - \gamma^2\right) I\right) + \left(\Phi - \frac{1}{2}\left(\rho - \gamma^2\right) I\right)^T X$$

Back

Social Optimum

- Static optimal allocation: $\mathbf{q}_t^* = N^* \mathbf{z}_t$ where $N^* \equiv \left\{2\frac{\zeta}{L}J + \Sigma\right\}^{-1}$
- Optimal output: $Y_t^* = \mathbf{z}_t^T Q^* \mathbf{z}_t$ where $Q^* = \frac{1}{2}N^*$
- Optimal expected utility:

$$V^*\left(\mathbf{z}_t\right) \equiv \mathbf{E}_t\left[\int_t^\infty \text{exp}\left(-\rho s\right) C_s ds \middle| \mathbf{z}_t\right] = \mathbf{z}_t^T X^* \mathbf{z}_t,$$

where X^* is the solution of the Riccati equation (obtained from planner's HJB equation):

$$0 = Q^* - \mu^2\left(X^*\right)^2 + X^*\left(\Phi^* - \frac{1}{2}\left(\rho - \gamma^2\right)I\right) + \left(\Phi^* - \frac{1}{2}\left(\rho - \gamma^2\right)I\right)X^*$$

- Optimal R&D: $\mathbf{x}_t^* = \mu X^* \mathbf{z}_t$
- Optimal technology transition matrix: $\Phi^* = \Omega + \mu^2 X^*$

Back

Stochastic Process of Output

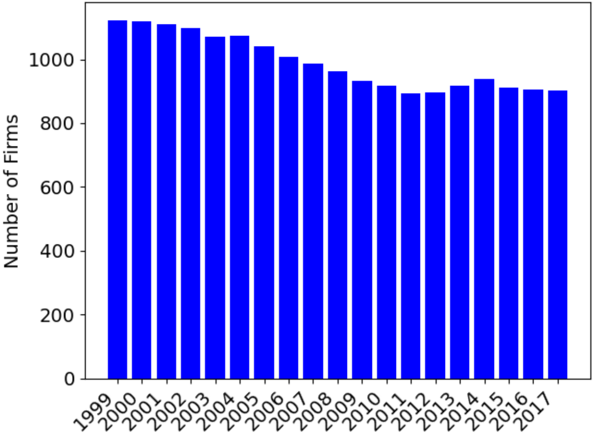
- Applying It's lemma,

$$d\log Y_t = \left[\frac{\mathbf{z}_t^T\left(Q\Phi + \Phi^T Q\right)\mathbf{z}_t}{Y_t} + \gamma^2\left\{\frac{\sum_i z_{i,t}^2 Q_{ii}}{Y_t} - \frac{2\mathbf{z}_t^T Q \,\text{diag}\left(\mathbf{z}_t^2\right) Q\mathbf{z}_t}{Y_t^2}\right\}\right] dt + \frac{2\gamma\mathbf{z}_t^T Q \,\text{diag}\left(\mathbf{z}_t\right)}{Y_t} dW_t$$

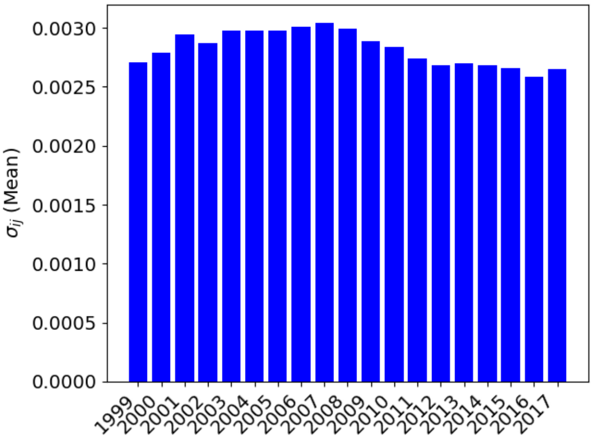
where $Y_t = \mathbf{z}_t^T Q \mathbf{z}_t$ and $\Phi = \Omega + \mu^2 \widetilde{X}$

Description	Expression
Tech Spillover	$\mathbf{z}_t^T\left(Q\Omega + \Omega Q\right)\mathbf{z}_t / Y_t$
R&D	$\mu^2 \mathbf{z}_t^T\left(Q\widetilde{X} + \widetilde{X}^T Q\right)\mathbf{z}_t / Y_t$
Ito	$\gamma^2\left\{\sum_i z_{i,t}^2 Q_{ii} / Y_t - 2\mathbf{z}_t^T Q \text{diag}\left(\mathbf{z}_t^2\right) Q\mathbf{z}_t / Y_t^2\right\}$
Total	$\mathbf{E}\left[d\log Y_t\right]$

Number of Sample Firms

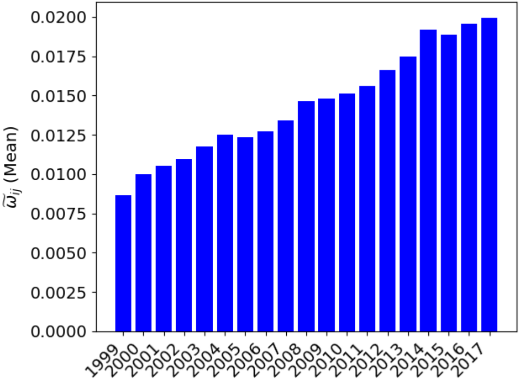


Trend of Product Substitutability

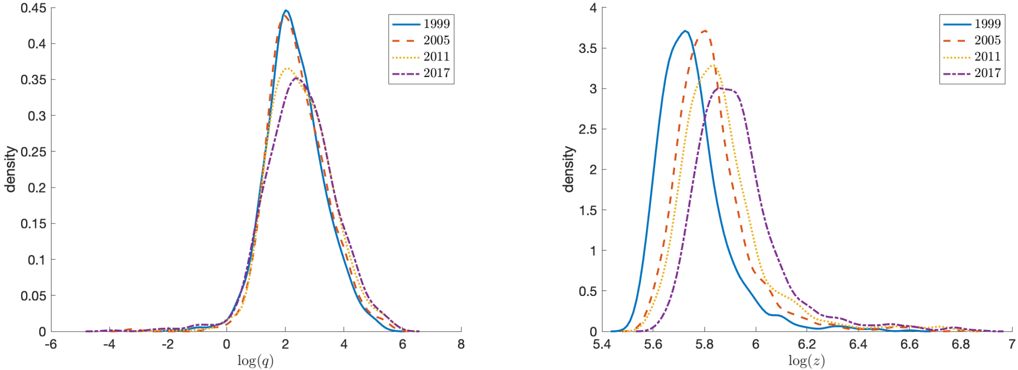


Technological Proximity

- Merge USPTO data with Compustat firms using DISCERN 2 dataset (Arora et al., 2024)
- Jaffe measure, Group-level patent classification, Stack for 5 years



Distributions of Estimated Knowledge Capital and Quantity



(a) Log of Quantity

(b) Log of Knowledge Capital

First Stage Back

	(1)
Dependent Variable:	$z_{i,t}$
User cost of R&D	-39.495^{***}
	(4.7044)
Year Fixed Effects	✓
No. observations	12,947

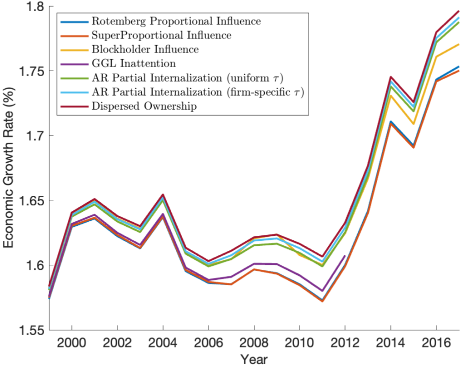
SEs clustered by years and 4-digit naics industries are reported in parentheses.

- IV: User cost of R&D, driven by state-level tax variations (Wilson, 2009; Bloom et al., 2013)

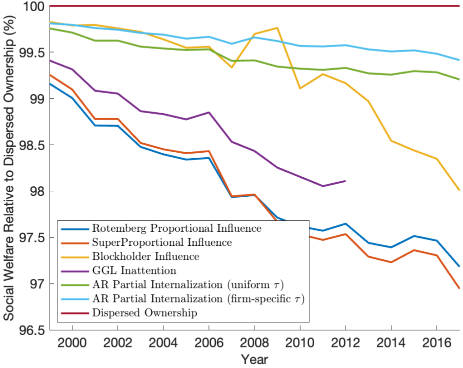
Alternative Corporate Governance Models: Ederer and Pellegrino (2024)

1. Super-proportional influence: $\tilde{\kappa}_{ij} = \frac{\sum_{z=1}^{Z} s_{iz}\gamma_{iz}s_{jz}}{\sum_{z=1}^{Z} s_{iz}\gamma_{iz}s_{iz}}$ where $\gamma_{iz} = \sqrt{s_{iz}}$
2. Blockholder influence: $\tilde{\kappa}_{ij} = \frac{\sum_{z=1}^{Z} s_{iz}b_{iz}s_{jz}}{\sum_{z=1}^{Z} s_{iz}s_{jz}} \quad (i \neq j)$, where $b_{iz} = 1$ if $s_{iz} > 5\%$
3. Rational investor inattention
   - Gilje et al. (2020) (GGL) estimate the probability that an investor votes against Institutional Shareholders Service recommendations
   - Utilize the estimate to capture the investor's level of attention
4. Governance frictions and entrenchment
   - Azar and Ribeiro (2021) (AR) estimate an objective function where the manager of firm i discounts other firms' profit by τ_i

Alternative Corporate Governance Models



(a) Expected Growth



(b) Expected Social Welfare

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