

Ownership Structure and Economic Growth

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Ownership Structure \implies Economic Growth?

- Ownership structure is concentrated
 - BlackRock, Vanguard, and State Street exercise 30% of the votes at S&P 500

Share

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 - BlackRock, Vanguard, and State Street exercise 30% of the votes at S&P 500
- Firms maximize shareholder value \implies partially internalize externalities across commonly owned firms
- Ownership structure (common ownership, cross-ownership, M&A, ...) \implies Economic growth?
 - Business-stealing effect
 - + Technology spillover effect

[Share](#)

Quantitative Schumpeterian Growth Model with Ownership Structure

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 - Monopolistic competition
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 - Many oligopolists engage in a dynamic R&D game
 - Three inter-firm networks:
 1. Ownership structure
 2. Product-market rivalry
 3. Technology spillovers

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 - Three inter-firm networks:
 1. Ownership structure
 2. Product-market rivalry
 3. Technology spillovers
- Commonly owned firms that are close in ...
 - product space \implies internalize the business-stealing effect \implies R&D \downarrow
 - technology space \implies internalize technology spillovers \implies R&D \uparrow

Identification and Findings

- Identify networks for publicly listed patenting firms in the U.S. (> 700 firms)

Network	Measurement
Ownership structure	Investor holdings from 13F filings (Backus et al., 2021)
Product-market rivalry	Product proximity (Hoberg and Phillips, 2016): Based on business descriptions in 10-K filings
Technology spillovers	Technology proximity (Jaffe, 1986; Bloom et al., 2013): Based on patent classifications

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- The rise of common ownership from 1999 to 2017 $\implies g \downarrow$ by 0.11 p.p., CE welfare \downarrow by 0.54%
- Internalization of business-stealing > Internalization of technology spillover

Related Literature

- Competition & Innovation:

d'Aspremont and Jacquemin (1988); Kamien et al. (1992); Aghion et al. (2001, 2005); Acemoglu and Akcigit (2012); Aghion et al. (2013); Bloom et al. (2013); Lopez and Vives (2019); Peters (2020); Akcigit and Ates (2021, 2023); Liu et al. (2022); Cavenaile et al. (2023), **Hopenhayn and Okumura (2024)**

[Quantitative Schumpeterian growth model with ownership structure](#)

- Hedonic Demand / Empirical IO:

Lancaster (1966); Rosen (1974); Berry et al. (1995); Nevo (2001), **Pellegrino (2024); Ederer and Pellegrino (2024)**
[Dynamic general equilibrium / R&D](#)

- Oligopoly / Common Ownership / Market Power:

Rubinstein and Yaari (1983); Rotemberg (1984); Neary (2003); Atkeson and Burstein (2008); Gutierrez and Philippon (2017); He and Huang (2017); Azar et al. (2018, 2022); Autor et al. (2020); Baqaee and Farhi (2020); De Loecker et al. (2020); Azar and Vives (2021); Edmond et al. (2023), **Anton et al. (2023, 2025); Kini et al. (2024)**

Demand and Technology

- Firm $i \in \{1, \dots, n\}$ has knowledge capital $z_{i,t}$ and produces a single differentiated product

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$$p_{i,t} = b_{i,t} - \sum_{j \neq i} \sigma_{ij} q_{j,t} - q_{i,t}$$

- $\Sigma = [\sigma_{ij}]$: product-market rivalry matrix (network) ($\sigma_{ii} = 1$)

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- Each firm allocates knowledge capital to improve labor productivity and product quality:

$$\zeta a_{i,t} + b_{i,t} = z_{i,t}$$

Common Ownership Weights (Networks)

- $K = [\kappa_{ij}]$: common ownership weights that firm i places on the value of firm j ($\kappa_{ii} = 1$)
- More overlapping ownership between firms i and $j \implies$ higher κ_{ij}
- $K = I$: dispersed ownership (each firm maximizes its own value)
- $K = \mathbf{1}_{n \times n}$: monopoly (maximizes total producer surplus)

Proportional Influence

Law of Motion of Knowledge Capital

$$\dot{z}_t = \underbrace{\Omega z_t}_{\text{Tech Spillover}} + \underbrace{\mu x_t}_{\text{R\&D}} - \underbrace{\delta z_t}_{\text{Depreciation}}$$

- $\Omega = [\omega_{ij}]$: technology spillover matrix (network)
- $x_{i,t} = \sqrt{d_{i,t}}$
 - $d_{i,t}$: R&D input in terms of final good
 - Innovation elasticity is 0.5
- μ, δ : positive scalars
- Can incorporate idiosyncratic & aggregate shocks (not today)

Market Clearing and Preference

- Inelastic labor supply:

$$L = \sum_i l_{i,t}$$

- Linear-quadratic aggregator:

$$Y_t = \mathbf{q}_t^T \mathbf{b}_t - \frac{1}{2} \mathbf{q}_t^T \mathbf{\Sigma} \mathbf{q}_t$$

- Final good market clearing:

$$C_t + \underbrace{\sum_i d_{i,t}}_{\text{R\&D input}} = Y_t$$

- Risk-neutral representative household:

$$U_t = \int_t^{\infty} \exp(-\rho s) C_s ds$$

Static Cournot Game

- Firm i 's static objective is given by:

$$\phi_{i,t} = \sum_j \kappa_{ij} \pi_{j,t}$$

where the profit (before R&D cost) of firm i is given by:

$$\pi_{i,t} = p_{i,t} q_{i,t} - w_t l_{i,t} = q_{i,t} \left(b_{i,t} - \sum_{j \neq i} \sigma_{ij} q_{j,t} - q_{i,t} - \frac{w_t}{a_{i,t}} \right)$$

- Given w_t , $z_{i,t}$, and $\{a_{j,t}, b_{j,t}, q_{j,t}\}_{j \neq i}$ and $\zeta a_{i,t} + b_{i,t} = z_{i,t}$, firm i chooses $a_{i,t}$, $b_{i,t}$, and $q_{i,t}$ to maximize $\phi_{i,t}$

Linear-Quadratic Differential Game

- Given other firms' R&D $\{x_{j,t}\}_{j \neq i, t \geq 0}$, firm i chooses R&D $\{x_{i,t}\}_{t \geq 0}$ to maximize

$$\max_{\{x_{i,t}\}_{t \geq 0}} V^i(z_0) \equiv \int_0^\infty \exp(-\rho t) \left\{ \sum_j \kappa_{ij} (\pi_{j,t} - d_{j,t}) \right\} dt$$


subject to $\dot{z}_t = \mathbf{\Omega} z_t + \mu x_t - \delta z_t$

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- Gross profit: $\sum_j \kappa_{ij} \pi_{j,t} = z_t^T \mathbf{Q}^i z_t$ 
- R&D cost: $d_{i,t} = x_{i,t}^2$

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- Gross profit: $\sum_j \kappa_{ij} \pi_{j,t} = z_t^T \mathbf{Q}^i z_t$ ○
- R&D cost: $d_{i,t} = x_{i,t}^2$
- Firm i 's HJB equation:

$$\rho V^i(z) = \max_{x_i} \left\{ z^T \mathbf{Q}^i z - \sum_j \kappa_{ij} x_j^2 + V_z^i(z) [\mathbf{\Omega} z + \mu x - \delta z] \right\}$$

HJB Equations \implies Riccati Equations

- Guess and verify $V^i(z) = z^T X^i z$ (for any z)
- X^i is the solution of stacked algebraic Riccati equations

Riccati Equations

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Riccati Equations
- Public & patenting firms in the U.S. in our dataset > 700 firms \implies
 $\underbrace{700 \times 700}_{\text{size of } X^i} \times \underbrace{700}_n = 343 \text{ million undetermined coefficients } (< 1 \text{ min on my laptop})$

Oligopolistic Schumpeterian	Computation time	# of firms	Productivity space
Cavenaile et al. (2023)	$O(2^n)$	4	6 grid
Our model	$O(n^4)$	>700	Continuous

BGP

- Linear R&D strategy: $x_t = \mu \tilde{X} z_t$ where $\tilde{X} = [X_1^1 \cdots X_n^n]^T$ and X_i^i is the i th column of X^i
- The law of motion is rewritten as $\dot{z}_t = \Phi z_t$ where

$$\Phi \equiv \underbrace{\Omega}_{\text{Tech Spillover}} + \underbrace{\mu^2 \tilde{X}}_{\text{R\&D}} - \underbrace{\delta I}_{\text{Depreciation}}$$

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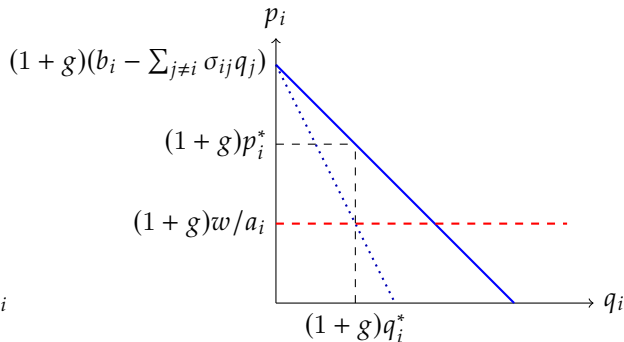
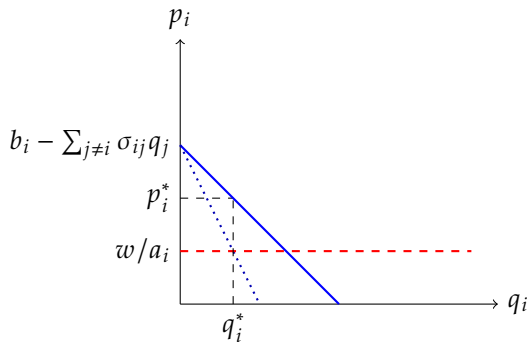
Theorem

If Φ is irreducible, then:

- (i) There exists a largest positive eigenvalue of Φ , g , and an associated positive eigenvector, z^* .
- (ii) There exists a globally stable BGP such that the knowledge capital growth rate of all firms is g , and the knowledge capital distribution is a scalar multiple of z^* .

- Proof: Perron–Frobenius Theorem
- “ Φ is irreducible” \iff “All firms are directly or indirectly connected technologically”

CES on BGP despite Non-CES Demand



- a_i , b_i , $q_i (= a_i l_i)$, p_i , and w/a_i grow at the same rate g
- (i) (consumer surplus / producer surplus) and (ii) (cost / revenue) stay the same
- Demand elasticity is constant on BGP despite linear demand

Lifetime Utility

- Lifetime utility is expressed in quadratic form:

$$\int_t^{\infty} \exp(-\rho s) C_s ds = \mathbf{z}_t^T \mathbf{X} \mathbf{z}_t$$

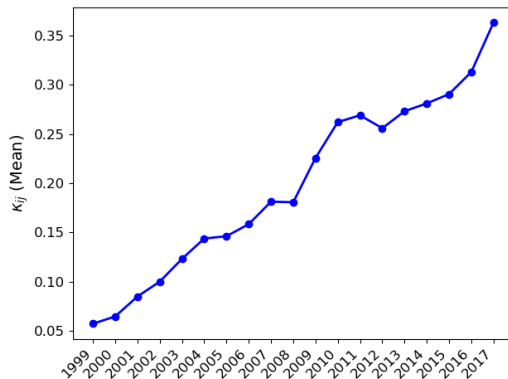
x

- Solve the equilibrium once \implies we can compute lifetime utility for any initial \mathbf{z}_t
- This property holds even if we introduce idiosyncratic / aggregate shocks
- In the exercise, we focus on the transition dynamics starting from the observed initial \mathbf{z}_t

Common Ownership Weights K

- Backus et al. (2021) construct a dataset on investors' holdings based on Form 13F
- Baseline: Rotemberg (1984) proportional influence

Proportional Influence



Product-Market Rivalry Σ

- Hoberg and Phillips (2016) estimates product proximity using business descriptions in 10-K
- Pellegrino (2024) estimates α to align with the cross-price elasticity of demand

$$\underbrace{\sigma_{ij}}_{\text{substitutability}} = \alpha \times \text{product proximity between } i \text{ and } j \quad (i \neq j)$$

micro estimates

Technological Proximity $\tilde{\Omega}$

- Technological profile of firm i
 - The vector of the share of patents held by firm i in each technology class
 - Baseline: group-level patent classifications (≈ 4000), five years window
- Jaffe (1986) technological proximity measure $\tilde{\omega}_{ij}$:
 - Cosine similarity of the technological profiles between firms i and j
 - Impose $\sum_{j \neq i} \tilde{\omega}_{ij} = 1$ for each i

Distribution of Knowledge Capital z_t

Variables	Identification
$\pi_{i,t}$	Gross profit (before R&D cost) = Revenue – Cost of goods sold
q_t	$\pi_{i,t} = \sum_j \kappa_{ij} \sigma_{ij} q_{i,t} q_{j,t}$
ζ/L	Matches sample firms' cost share (average markup)
z_t	$z_t = \left\{ 2 \frac{\zeta}{L} \mathbf{1}_{n \times n} + \Sigma + K \circ \Sigma \right\} q_t$

Technology Spillover $\Omega = \beta \times \text{Technological Proximity } \tilde{\Omega}$ First Stage

$$\log z_{i,t+1} - \log z_{i,t} = \beta \sum_{j \neq i} \tilde{\omega}_{ij,t} \frac{z_{j,t}}{z_{i,t}} + \text{Controls}_{i,t} + \epsilon_{i,t}$$

	(1)	(2)	(3)
	$\log z_{i,t+1} - \log z_{i,t}$	$\log z_{i,t+1} - \log z_{i,t}$	$\log z_{i,t+1} - \log z_{i,t}$
$\sum_{j \neq i} \tilde{\omega}_{ij,t} \frac{z_{j,t}}{z_{i,t}}$	0.026** (0.010)	0.024** (0.010)	0.073* (0.038)
$\frac{x_{i,t}}{z_{i,t}}$		0.514*** (0.063)	
Firm & Year FEs	✓	✓	✓
Controls	✓	✓	✓
IV			✓
Observations	14,576	14,576	14,576

SEs clustered by years and 4-digit NAICS industries are reported in parentheses. Control variables include $\log z_{i,t}$, firm fixed effects, and year fixed effects. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

- IV: User cost of R&D, driven by federal and state-specific rules variations (Bloom et al., 2013)

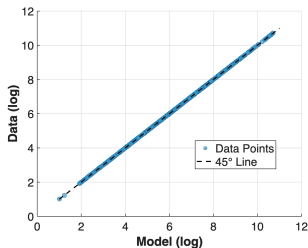
Identification: Summary

- Publicly available data + Compustat

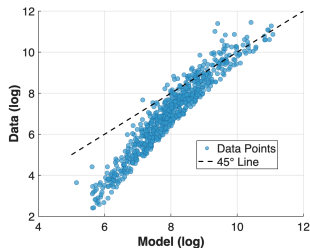
Notation	Description	Value	Source
Σ	Product proximity		Form 10-K, Hoberg and Phillips (2016)
$\tilde{\Omega}$	Technological proximity		USPTO, Patent classification
K	Common ownership weights		Form 13F, Backus et al. (2021)
α	Product proximity \rightarrow Substitutability	0.120	Pellegrino (2024)
β	Technological proximity \rightarrow Spillovers	0.024	Estimated from the law of motion
ζ/L	Labor-augmenting efficiency	0.004	Compustat, Cost of goods sold
ρ	Discount rate	0.100	
μ	R&D efficiency	0.066	2.6% R&D share (moment match)
δ	Depreciation rate	0.017	1.2% economic growth rate (moment match)

Model vs. Data: Firm-level Profits, Sales, and R&D

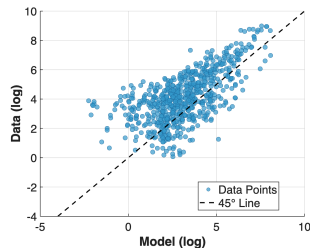
- Comparison of firm-level model-generated values (x -axis) with observed data (y -axis)



(a) Log of Profit (Targeted)



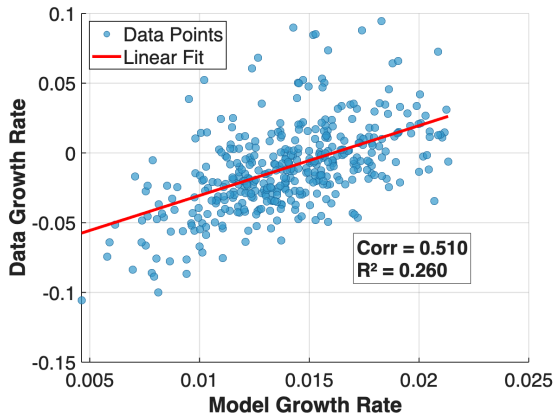
(b) Log of Sales



(c) Log of R&D Expenditure

Model vs. Data: Firm-level Growth Rates

- Data: Average growth rate of $z_{i,t}$ between 2010 and 2017
- Model: Networks and initial knowledge capital in 2010



Counterfactual Ownership Structures

Ownership Structure	Description
Baseline	Observed common ownership structure in 2017
Dispersed	$K^D = I$
Mean=1999	$\kappa_{ij,2017}^{M1999} = \text{const} \times \kappa_{ij,2017}$ and $E \left[\kappa_{ij,2017}^{M1999} \right] = E \left[\kappa_{ij,1999} \right]$ for $j \neq i$
Uniform	$\kappa_{ij,2017}^U = E \left[\kappa_{ij,2017} \right]$ for $j \neq i$
Monopoly	$K^M = \mathbf{1}_{n \times n}$

- For the moment, assume:
 - Ownership structure only affects R&D decisions
 - Product-market competition: firms maximize static profits (dispersed ownership)

Total R&D Expenditure (Optimal R&D: 100)

Total R&D in 2017 (Optimal R&D: 100)	Ownership (Baseline: 2017)				
	Dispersed	Mean=1999	Uniform	Baseline	Monopoly
Baseline	40.48	38.68	31.56	28.26	21.39
Only Business Steal $\Omega = [0]$					
Only Tech Spill $\Sigma = I, \zeta/L = 0$					

- Internalization of business-stealing > internalization of technology spillovers

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Baseline	40.48	38.68	31.56	28.26	21.39
Only Business Steal $\Omega = [0]$	52.04	49.73	41.51	36.40	30.69
Only Tech Spill $\Sigma = I, \zeta/L = 0$					

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Only Tech Spill $\Sigma = I, \zeta/L = 0$	13.61	14.25	18.33	19.33	27.77

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Total R&D, Growth Rate, Social Welfare, Firm Value Share

	Ownership (Baseline: 2017)				
	Dispersed	Mean=1999	Uniform	Baseline	Monopoly
Total R&D (Optimal R&D: 100)	40.48	38.68	31.56	28.26	21.39
Economic Growth Rate (%)					
Welfare (Optimal R&D: 100)					
Firm Value Share (%)					

- The rise of common ownership from 1999 to 2017 $\implies g \downarrow$ by 0.11 p.p., CE welfare \downarrow by 0.54%

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Economic Growth Rate (%)	1.32	1.31	1.24	1.20	1.11
Welfare (Optimal R&D: 100)					
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Economic Growth Rate (%)	1.32	1.31	1.24	1.20	1.11
Welfare (Optimal R&D: 100)	94.91	94.86	94.52	94.35	93.47
Firm Value Share (%)					

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Economic Growth Rate (%)	1.32	1.31	1.24	1.20	1.11
Welfare (Optimal R&D: 100)	94.91	94.86	94.52	94.35	93.47
Firm Value Share (%)	26.63	26.72	27.20	27.24	27.82

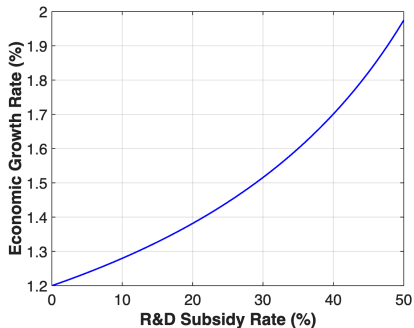
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When Common Ownership Affects Both R&D and Production

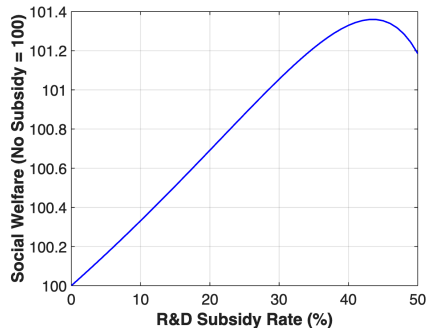
Production ownership structure R&D ownership structure	Dispersed Dispersed	Dispersed Common	Common Common
Output (Dispersed: 100)	100.00	100.00	97.26
Total R&D (Dispersed: 100)	100.00	69.81	86.36
Economic Growth Rate (%)	1.323	1.200	1.288
Welfare (Dispersed: 100)	100.00	99.41	97.28
Firm Value Share (%)	26.63	27.24	34.10

- Less product market competition \Rightarrow Private return on R&D \uparrow

Optimal Uniform R&D Subsidy



(a) Economic Growth



(b) Social Welfare

- Optimal rate is $s = 43\%$, which increases g by 0.57 pp and CE welfare by 1.36%
- c.f. Welfare gains from optimal R&D allocation: 6.0%

Which Firms' R&D Should be Subsidized?

	Private R&D x	Social / Private value of R&D
Initial knowledge capital z	0.122*** (0.000848)	-0.000212 (0.000158)
Ownership structure centrality	-7.16*** (0.239)	0.580*** (0.0446)
Product market centrality	-16.4*** (0.286)	-1.06*** (0.0532)
Technology spillover centrality	5.45*** (0.423)	1.48*** (0.0788)
Intercept	-22.4*** (0.255)	0.900*** (0.0475)
Observations	740	740
R^2	0.976	0.555

Conclusion

- Quantitative Schumpeterian growth model with ownership structure
 - Utilizes micro data and computational capabilities
- Common ownership in the U.S.:
 1. Internalization of business-stealing effect $\Rightarrow g \downarrow \downarrow$
 2. Internalization of technology spillover effect $\Rightarrow g \uparrow$
- Potential applications:
 - Conglomerate: e.g., Chaebols (Korea), Zaibatsu / Keiretsu (Japan)
 - FDI and international technology diffusion
 - Technology licensing

[Back](#)

Meta	
Vanguard	7.55%
Blackrock	6.50%
Fidelity	5.38%
Accel IX LP	3.88%
State Street	3.40%

[Back](#)

Shareholding percentage	Microsoft	Google	Amazon
OpenAI (ChatGPT)	49%	—	—
Anthropic (Claude)	—	14%	23%

Technology & Product Proximity: Example

Tesla vs. Ford	
Technology Proximity	0.11
Product Proximity	0.15

Apple vs. Intel	
Technology Proximity	0.57
Product Proximity	0.00

- $o \in \{1, 2, \dots, n_o\}$: owners
- s_{io} : the proportion of shares in firm i owned by owner o where $\sum_o s_{io} = 1$
- \widehat{V}_i : value of firm i
- $\widetilde{V}_o \equiv \sum_i s_{io} \widehat{V}_i$: value of owner o
- Firm i 's objective:

$$\sum_o s_{io} \widetilde{V}_o \propto \sum_j \kappa_{ij} \widehat{V}_j$$

$$\kappa_{ij} \equiv \frac{\mathbf{s}_i^T \mathbf{s}_j}{\mathbf{s}_i^T \mathbf{s}_i} = \cos(\mathbf{s}_i, \mathbf{s}_j) \sqrt{\frac{IH H I_j}{IH H I_i}} \quad \text{where} \quad \mathbf{s}_i \equiv [s_{i1}, \dots, s_{i0}, \dots, s_{in_0}]^T$$

Empirical Literature: Common Ownership \implies R&D

- Anton et al. (2025):
 - Dependent variables: R&D, citation-weighted patents, market value of patents
 - + Interaction term between common ownership and technology proximity
 - Interaction term between common ownership and product proximity
- Kini et al. (2024): DiD that exploits mergers between financial institutions
 - Dependent variables: Investments, new product development
 - + Post (merger) \times treatment (common owner) \times technology proximity

R&D Externalities

1. Business-stealing effect
 - Innovators steal the business (profits) of other firms
2. Technology spillover effect
 - Innovation improves the productivity of other firms
3. Appropriability effect (market power)
 - Innovators cannot appropriate the entire consumer surplus

Generalized Hedonic-Linear Demand (Pellegrino, 2024)

- $i \in \{1, 2, \dots, n\}$: firms / products
- 1 unit of product i provides
 - 1 unit of idiosyncratic characteristic $k \in \{1, 2, \dots, n\}$
 - $\psi_{k,i}$ unit of shared characteristic $k \in \{n+1, n+2, \dots, n+n_k\}$ where $\sum_k \psi_{k,i}^2 = 1$
- Aggregate each characteristic:

$$y_{k,t} = \begin{cases} q_{k,t} & k = 1, 2, \dots, n \\ \sum_i \psi_{k,i} q_{i,t} & k = n+1, n+2, \dots, n+n_k \end{cases}$$

- Linear-quadratic aggregator over characteristics:

$$Y_t = (1 - \alpha) \sum_{k=1}^n \left(\underbrace{\hat{b}_{k,t} y_{k,t} - \frac{1}{2} y_{k,t}^2}_{\text{idiosyncratic characteristic}} \right) + \alpha \sum_{k=n+1}^{n+n_k} \left(\underbrace{\hat{b}_{k,t} y_{k,t} - \frac{1}{2} y_{k,t}^2}_{\text{shared characteristic}} \right)$$

Generalized Hedonic-Linear Demand (Pellegrino, 2024)

- Quality:

$$b_i = (1 - \alpha) \hat{b}_i + \alpha \sum_{k=n+1}^{n+n_k} \psi_k \hat{b}_k$$

- Inverse demand:

$$\frac{p}{P} = b - \Sigma q$$

- Inverse cross-price elasticity of demand:

$$\frac{\partial \log p_i}{\partial \log q_j} = -\frac{q_j}{p_i} \cdot \sigma_{ij}$$

- Cross-price elasticity of demand:

$$\frac{\partial \log q_i}{\partial \log p_j} = -\frac{p_j}{q_i} (\Sigma^{-1})_{ij}$$

Characteristics of Static Equilibrium

- Equilibrium quantities:

$$q_i^* = \frac{1}{2}z_i - \sqrt{\zeta w_t} - \frac{1}{2} \sum_{j \neq i} (\sigma_{ij} + \kappa_{ij} \sigma_{ij}) q_j^*$$

- Assume $\{q_j^*\}_{j \neq i}$ are held constant
- By the Envelope Theorem, the marginal value of knowledge capital (R&D incentive) is given by:

$$\frac{\partial \phi_i^*}{\partial z_i} = q_i^*$$

- Greater overlap between ownership and product-market rivalry networks \implies R&D \downarrow

$$\frac{\partial^2 (\partial \phi_i^* / \partial z_i)}{\partial \kappa_{ij} \partial \sigma_{ij}} = \frac{\partial^2 q_i^*}{\partial \kappa_{ij} \partial \sigma_{ij}} = -\frac{1}{2} q_j^* < 0$$

Static Profits

- Gross profit: $\pi_{i,t} = \sum_j \kappa_{ij} \sigma_{ij} q_{i,t} q_{j,t}$
- Firms choose labor productivity and product quality: $\zeta a_{i,t} = \sqrt{\zeta w_t}$, $b_{i,t} = z_{i,t} - \sqrt{\zeta w_t}$
- Labor market clearing: $L = \sum_i \frac{q_{i,t}}{a_{i,t}} \implies \sqrt{\zeta w_t} = \frac{\zeta}{L} \sum_i q_{i,t}$
- $q_t = N z_t$ where $N \equiv \left\{ 2 \frac{\zeta}{L} J + \Sigma + K \circ \Sigma \right\}^{-1}$
- N_i : the i th row of N
- Ownership weighted profit:

$$\sum_j \kappa_{ij} \frac{\pi_{j,t}}{P_t} = \sum_j \kappa_{ij} \sum_h \kappa_{jh} \sigma_{jh} q_{j,t} q_{h,t} = z_t^T Q^i z_t$$

where

$$Q^i = \frac{1}{2} \sum_j \kappa_{ij} \sum_h \kappa_{jh} \sigma_{jh} \left(N_j^T N_h + N_h^T N_j \right)$$

Riccati Equations

- $V^i(\mathbf{z}) = \mathbf{z}^T \mathbf{X}^i \mathbf{z}$ where \mathbf{X}^i is the solution of the stacked Riccati equation

$$0 = \mathbf{Q}^i - \mu^2 \sum_j \kappa_{ij} \mathbf{X}_j^j \left(\mathbf{X}_j^j \right)^T + \left(\mathbf{\Phi} - \frac{1}{2} \left(\rho - \gamma^2 \right) \mathbf{I} \right)^T \mathbf{X}^i + \mathbf{X}^i \left(\mathbf{\Phi} - \frac{1}{2} \left(\rho - \gamma^2 \right) \mathbf{I} \right)$$

- $\mathbf{X}_i^i \equiv$ the i th column of \mathbf{X}^i
- $\mathbf{\Phi} \equiv \mathbf{\Omega} + \mu^2 \begin{bmatrix} \mathbf{X}_1^1 & \cdots & \mathbf{X}_n^n \end{bmatrix}^T$
- Algorithm: Given $\begin{bmatrix} \mathbf{X}_\tau^1 & \cdots & \mathbf{X}_\tau^n \end{bmatrix}$, update $\begin{bmatrix} \mathbf{X}_{\tau-\Delta}^1 & \cdots & \mathbf{X}_{\tau-\Delta}^n \end{bmatrix}$ by

$$-\frac{\mathbf{X}_\tau^i - \mathbf{X}_{\tau-\Delta}^i}{\Delta} = \mathbf{Q}^i - \mu^2 \sum_j \kappa_{ij} \mathbf{X}_{j,\tau}^j \left(\mathbf{X}_{j,\tau}^j \right)^T + \left(\mathbf{\Phi}_\tau - \frac{1}{2} \left(\rho - \gamma^2 \right) \mathbf{I} \right)^T \mathbf{X}_\tau^i + \mathbf{X}_\tau^i \left(\mathbf{\Phi}_\tau - \frac{1}{2} \left(\rho - \gamma^2 \right) \mathbf{I} \right)$$

Summary of Equilibrium

Description	Expression
Production strategy	$q_t = N z_t$
R&D strategy	$x_t = \mu \tilde{X} z_t$
Law of motion	$dz_t = (\Omega z_t + \mu x_t) dt + \gamma z_t dW_t$
Profit of final producers	$\Pi_t^F / P_t = q_t^T \left(\frac{1}{2} \Sigma \right) q_t$
Total operating profit of firms	$\Pi_t / P_t = q_t^T \left(\frac{1}{2} \Sigma \circ (K + K^T) \right) q_t$
Labor income	$w_t L / P_t = q_t^T \left(\frac{\zeta}{L} J \right) q_t$
Output	$Y_t = q_t^T \left(\frac{\zeta}{L} J + \frac{1}{2} \Sigma + \frac{1}{2} \Sigma \circ (K + K^T) \right) q_t$
Consumption	$C_t = Y_t - x_t^T x_t$

Example: Symmetric Equilibrium

Assumption

- No common ownership, symmetric product substitutability, symmetric technology spillover:
 $\kappa_{ij} = 0, \sigma_{ij} = \sigma, \omega_{ij} = \omega \quad \forall i \neq j$
- R&D strategy: $x_{i,t}^* = \mu \left(\tilde{x}_1 z_{i,t} + \tilde{x}_2 \sum_{j \neq i} z_j \right)$
 - \tilde{x}_1 : market size effect (> 0)
 - \tilde{x}_2 : strategic substitutability (< 0) / complementarity (> 0)
- Growth rate: $g = \underbrace{(n-1)\omega}_{\text{Tech Spillover}} + \underbrace{\mu^2 (\tilde{x}_1 + (n-1)\tilde{x}_2)}_{\text{R\&D}}$
- Stability (irreducibility) requires $\omega + \mu^2 \tilde{x}_2 > 0$
 - Tech spillover (ω) must be strong relative to strategic substitutability ($\tilde{x}_2 < 0$)

Output and Expected Utility

- Output: $Y_t = \mathbf{q}_t^T \mathbf{Q} \mathbf{q}_t$ where

$$\mathbf{Q} = \frac{\zeta}{L} \mathbf{J} + \frac{1}{2} \mathbf{\Sigma} + \frac{1}{2} \mathbf{\Sigma} \circ (\mathbf{K} + \mathbf{K}^T)$$

- Expected utility:

$$V(\mathbf{z}_t) \equiv E_t \left[\int_t^\infty \exp(-\rho s) C_s ds \middle| \mathbf{z}_t \right] = \mathbf{z}_t^T \mathbf{X} \mathbf{z}_t$$

where \mathbf{X} is the solution of the Lyapunov equation (obtained from households' HJB equation):

$$0 = \mathbf{Q} - \mu^2 \tilde{\mathbf{X}}^T \tilde{\mathbf{X}} + \mathbf{X} \left(\mathbf{\Phi} - \frac{1}{2} (\rho - \gamma^2) \mathbf{I} \right) + \left(\mathbf{\Phi} - \frac{1}{2} (\rho - \gamma^2) \mathbf{I} \right)^T \mathbf{X}$$

Social Optimum

- Static optimal allocation: $q_t^* = N^* z_t$ where $N^* \equiv \left\{ 2 \frac{\zeta}{L} J + \Sigma \right\}^{-1}$
- Optimal output: $Y_t^* = z_t^T Q^* z_t$ where $Q^* = \frac{1}{2} N^*$
- Optimal expected utility:

$$V^*(z_t) \equiv E_t \left[\int_t^\infty \exp(-\rho s) C_s ds \middle| z_t \right] = z_t^T X^* z_t,$$

where X^* is the solution of the Riccati equation (obtained from planner's HJB equation):

$$0 = Q^* - \mu^2 (X^*)^2 + X^* \left(\Phi^* - \frac{1}{2} (\rho - \gamma^2) I \right) + \left(\Phi^* - \frac{1}{2} (\rho - \gamma^2) I \right) X^*$$

- Optimal R&D: $x_t^* = \mu X^* z_t$
- Optimal technology transition matrix: $\Phi^* = \Omega + \mu^2 X^*$

Property of BGP

- On the BGP, a_t , b_t , z_t , and q_t grow at the same rate

Knowledge Capital: $\zeta a_{i,t} + b_{i,t} = z_{i,t}$

Linear Production Technology: $q_{i,t} = a_{i,t} l_{i,t}$

Inelastic Labor Supply: $L = \sum_i l_{i,t}$

- The linear and quadratic terms in q_t of output grow at the same rate:

$$Y_t = q_t^T b_t - \frac{1}{2} q_t^T \Sigma q_t$$

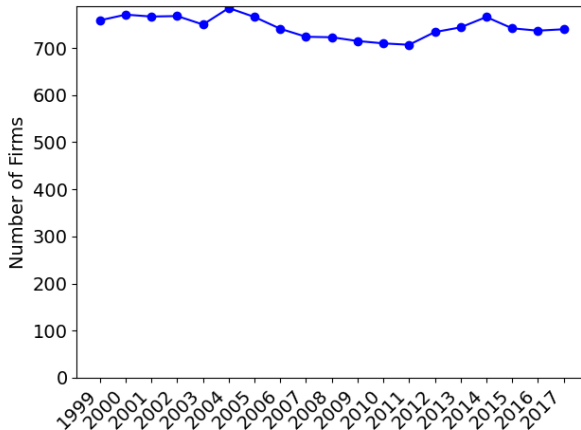
Growth Decomposition

- Aggregate output: $Y_t = \mathbf{z}_t^T \mathbf{Q} \mathbf{z}_t$
- $d\mathbf{z}_t/dt = \mathbf{\Phi} \mathbf{z}_t$ where $\mathbf{\Phi} = \mathbf{\Omega} + \mu^2 \tilde{\mathbf{X}} - \delta \mathbf{I}$

$$\frac{d \log Y_t}{dt} = \underbrace{\frac{\mathbf{z}_t^T (\mathbf{Q} \mathbf{\Omega} + \mathbf{\Omega} \mathbf{Q}) \mathbf{z}_t}{Y_t}}_{\text{Tech Spillover}} + \underbrace{\frac{\mu^2 \mathbf{z}_t^T (\mathbf{Q} \tilde{\mathbf{X}} + \tilde{\mathbf{X}}^T \mathbf{Q}) \mathbf{z}_t}{Y_t}}_{\text{R\&D}} - \underbrace{2\delta}_{\text{Depreciation}}$$

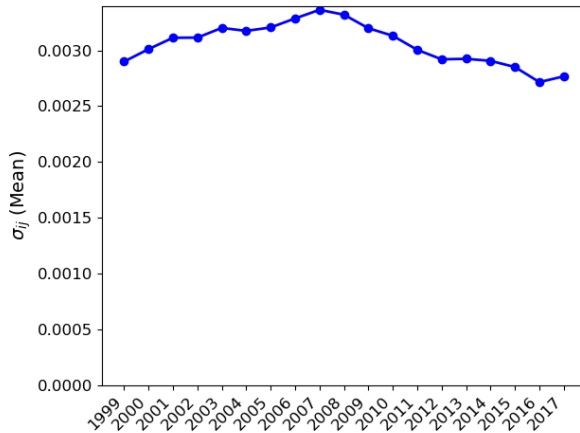
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Number of Sample Firms



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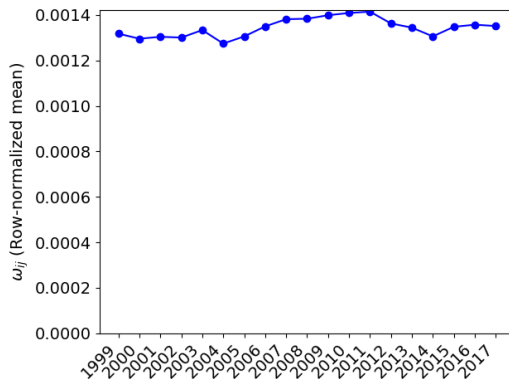
Trend of Product Substitutability



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Technological Proximity

- Merge USPTO data with Compustat firms using DISCERN 2 dataset (Arora et al., 2024)
- Jaffe measure, group-level patent classification, stacked over 5 years



Correlation Across Networks

	K	Σ	Ω
K	1.0000	-0.0035	0.0115
Σ	-0.0035	1.0000	0.2542
Ω	0.0115	0.2542	1.0000

- K : Ownership network
- Σ : Product substitutability network
- Ω : Technological proximity network

Microeconomic Estimates vs. GHL (Pellegrino, 2024) (1/2)

Market	Firm i	Firm j	Micro Estimate	GHL
Auto	Ford	Ford	-4.320	-5.197
Auto	Ford	General Motors	0.034	0.056
Auto	Ford	Toyota	0.007	0.017
Auto	General Motors	Ford	0.065	0.052
Auto	General Motors	General Motors	-6.433	-4.685
Auto	General Motors	Toyota	0.008	0.005
Auto	Toyota	Ford	0.018	0.025
Auto	Toyota	General Motors	0.008	0.008
Auto	Toyota	Toyota	-3.085	-4.851

Microeconomic Estimates vs. GHL (Pellegrino, 2024) (2/2)

Market	Firm i	Firm j	Micro Estimate	GHL
Cereals	Kellogg's	Kellogg's	-3.231	-1.770
Cereals	Kellogg's	Quaker Oats	0.033	0.023
Cereals	Quaker Oats	Kellogg's	0.046	0.031
Cereals	Quaker Oats	Quaker Oats	-3.031	-1.941
Computers	Apple	Apple	-11.979	-8.945
Computers	Apple	Dell	0.018	0.025
Computers	Dell	Apple	0.027	0.047
Computers	Dell	Dell	-5.570	-5.110

First Stage

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	R&D (1)
State tax credit component of R&D user cost	-1.16*** (0.29)
Federal tax credit component of R&D user cost	-34.29*** (3.64)
Firm fixed effects	✓
Year fixed effects	✓
No. of observations	16197

SEs clustered by years and 4-digit NAICS industries are reported in parentheses.

- IV: User cost of R&D, driven by federal and state-specific rules variations (Wilson, 2009; Bloom et al., 2013)

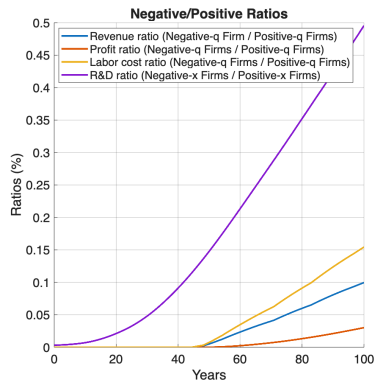
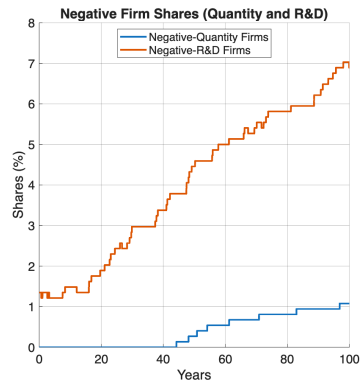
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Negative R&D and Output

- Issue with the model: negative output and R&D
 - Inada condition is not satisfied
 - Non-negativity constraint makes model intractable

Negative R&D and Quantity

- Firms with negative values are negligible along the transition path
- The weight on consumption 100 years and beyond is 0.00454% ($\rho = 0.1$)



Alternative Corporate Governance Models: Ederer and Pellegrino (2024)

1. Super-proportional influence: $\tilde{\kappa}_{ij} = \frac{\sum_o s_{io} \gamma_{io} s_{jo}}{\sum_o s_{io} \gamma_{io} s_{io}}$ where $\gamma_{io} = \sqrt{s_{io}}$
2. Blockholder influence: $\tilde{\kappa}_{ij} = \frac{\sum_o s_{io} b_{io} s_{jo}}{\sum_o s_{io} s_{jo}} \quad (i \neq j)$, where $b_{io} = 1$ if $s_{io} > 5\%$
3. Governance frictions and entrenchment
 - Azar and Ribeiro (2021) estimate an objective function where the manager of firm i discounts other firms' profit by τ_i

Alternative Corporate Governance Models

Ownership Structure in 2017						
	Dispersed Ownership	Baseline: Proportional Influence	Super Proportional Influence	Blockholder Influence	Governance Frictions (Uniform)	Governance Frictions (Firm-Specific)
Total R&D Expenditure	100.00	69.81	68.97	77.45	90.32	90.41
Expected Growth Rate (%)	1.323	1.200	1.194	1.234	1.287	1.289
Expected Social Welfare	100.00	99.41	99.37	99.59	99.86	99.86
Firm Value Share (%)	26.63	27.24	27.24	27.09	26.82	26.84

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