Ownership Structure and Economic Growth

Koki Okumura

UCLA

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- Two inter-firm externalities of innovation in Shumpeterian growth model:
  - – Business-stealing effect
  - + Technology spillover effect
- What is the aggregate effects of common ownership on R&D, growth, and welfare?

Quantitative Schumpeterian Growth Model with Ownership Structure

- Existing endogenous growth models are not suited for the analysis of common ownership across many firms/industries
  - Monopolistic competition w/o strategic interaction (Romer, 1990; Klette and Kortum, 2004)
  - Markov perfect equilibrium with 2–4 firms (Aghion et al., 2001; Cavenaile et al., 2023)

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- My framework is based on a new class of endogenous growth model developed by Hopenhayn and Okumura (2024)
  - Hundreds or thousands of oligopolists engage in a dynamic R&D game
  - LQ differential game avoids the curse of dimensionality
  - Two networks that govern the two externalities of innovation
    - Product market rivalry networks (Pellegrino, 2024)
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- This paper incorporates ownership structure networks into endogenous growth model
  - Overlap of networks determines the internalization of the two externalities

Identification and Findings

- Identify networks for publicly listed patenting firms in the U.S. (>700 firms)

Network	Measurement
Ownership structure	common ownership weights (Backus et al., 2021) Institutional investor shareholdings from 13F filings
Product-market rivalry	Product proximity (Hoberg and Phillips, 2016): Text analysis of business descriptions in 10-K filings
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- Commonly owned firms that are close in ...
  - product space $\Longrightarrow$ internalize business-stealing effect $\Longrightarrow$ R&D $\downarrow$
  - technology space $\Longrightarrow$ internalize technology spillovers $\Longrightarrow$ R&D $\uparrow$

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  - technology space $\Longrightarrow$ internalize technology spillovers $\Longrightarrow$ R&D $\uparrow$
- The rise of common ownership from 1999 to 2017 $\Longrightarrow$ $g \downarrow$ by 0.11 p.p., welfare $\downarrow$ by 0.54%
  - Internalization of business-stealing > Internalization of technology spillover

Related Literature

- Competition & Innovation:
  d'Aspremont and Jacquemin (1988); Kamien et al. (1992); Aghion et al. (2001, 2005); Acemoglu and Akcigit (2012); Aghion et al. (2013); Bloom et al. (2013); Lopez and Vives (2019); Peters (2020); Akcigit and Ates (2021, 2023); Liu et al. (2022); Cavenaile et al. (2023), **Hopenhayn and Okumura (2024)**
  Endogenous growth model with ownership structure networks

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- Hedonic Demand / Empirical IO:
  Lancaster (1966); Rosen (1974); Berry et al. (1995); Nevo (2001), **Pellegrino (2024); Ederer and Pellegrino (2024)**
  Dynamic general equilibrium / R&D

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- Oligopoly / Common Ownership / Market Power:
  Rubinstein and Yaari (1983); Rotemberg (1984); Neary (2003); Atkeson and Burstein (2008); Gutierrez and Philippon (2017); He and Huang (2017); Azar et al. (2018, 2022); Autor et al. (2020); Baqaee and Farhi (2020); De Loecker et al. (2020); Azar and Vives (2021); Edmond et al. (2023), **Anton et al. (2023, 2025); Kini et al. (2024)**
  Aggregate implications of common ownership for R&D allocation and growth

Preference and Production Technology (1/2)

- Risk-neutral representative household:

$$U_t = \int_t^{\infty} \exp\left(-\rho(s-t)\right) C_s ds$$

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- Linear-quadratic aggregator (Pellegrino, 2024):

$$Y_t = \boldsymbol{q}_t^T \boldsymbol{b}_t - \frac{1}{2}\boldsymbol{q}_t^T \boldsymbol{\Sigma} \boldsymbol{q}_t$$

  - $\boldsymbol{\Sigma} = \left[\sigma_{ij}\right]$: product-market rivalry matrix (networks) $(\sigma_{ii} = 1)$

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- Each firm i has knowledge capital $z_{i,t}$
- Firms allocates knowledge capital to improve labor productivity and product quality:

$$\zeta a_{i,t} + b_{i,t} = z_{i,t}$$

Law of Motion of Knowledge Capital

$$\dot{z}_t = \underbrace{\mathbf{\Omega} z_t}_{\text{Tech Spillover}} + \underbrace{\mu x_t}_{\text{R\&D}} - \underbrace{\delta z_t}_{\text{Depreciation}}$$

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- $\Omega = [\omega_{ij}]$: technology spillover matrix (networks)
- $x_{i,t} = \sqrt{d_{i,t}}$
  - $d_{i,t}$: R&D input in terms of final good
  - Innovation elasticity $d \log(\text{firm value}) / d \log(\text{R\&D cost}) = 0.5$

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- μ, δ: positive scalars
- Can incorporate idiosyncratic & aggregate shocks (not today)

Market Clearing

- Final good market clearing:

$$C_t + \underbrace{\sum_i d_{i,t}}_{\text{R\&D input}} = Y_t$$

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- Inelastic production labor supply:

$$L = \sum_i l_{i,t}$$

Common Ownership Weights (Networks)

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- More overlapping ownership between firms i and $j \implies$ higher κ_{ij}
- $\boldsymbol{K} = \boldsymbol{I}$: dispersed ownership (each firm maximizes its own value)
- $\boldsymbol{K} = \mathbf{1}_{n\times n}$: monopoly (maximizes total producer surplus)

Proportional Influence (Rotemberg, 1984)

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where

$$\kappa_{ij} \equiv \frac{\boldsymbol{s}_i^T \boldsymbol{s}_j}{\boldsymbol{s}_i^T \boldsymbol{s}_i} = \cos\left(\boldsymbol{s}_i, \boldsymbol{s}_j\right) \sqrt{\frac{\text{Investor HHI}_j}{\text{Investor HHI}_i}} \quad \text{where} \quad \boldsymbol{s}_i \equiv [s_{i1}, \ldots, s_{io}, \ldots, s_{in_o}]^T$$

Markov Perfect Equilibrium

- Given other firms' strategies, firm i chooses $\{a_{i,t}, b_{i,t}, q_{i,t}, x_{i,t}\}_{t\geq 0}$ to maximize

$$\max_{\{a_{i,t},b_{i,t},q_{i,t},x_{i,t}\}_{t\geq 0}} V^i(z_0) \equiv \int_0^\infty \exp(-\rho t) \sum_j \kappa_{ij} \left(\underbrace{\pi_{j,t}}_{\text{Gross Profit}} - \underbrace{d_{j,t}}_{\text{R\&D Cost}} \right) dt$$

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- Markov perfect equilibrium can be solved by the following steps:
  1. Static Game: For each t, choose $\{a_{i,t}, b_{i,t}, q_{i,t}\}$ to maximize $\sum_j \kappa_{ij}\pi_{j,t}$.
  2. Dynamic Game: Given the static strategy profile, choose $\{x_{i,t}\}$ to maximize $V^i(z_0)$

Static Cournot Game

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where the profit (before R&D cost) of firm i is given by:

$$\pi_{i,t} = p_{i,t} q_{i,t} - w_t l_{i,t} = q_{i,t} \left(b_{i,t} - \sum_{j \neq i} \sigma_{ij} q_{j,t} - q_{i,t} - \frac{w_t}{a_{i,t}} \right)$$

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Assumption

Given w_t, $z_{i,t}$, and $\{a_{j,t}, b_{j,t}, q_{j,t}\}_{j\neq i}$ and $\zeta a_{i,t} + b_{i,t} = z_{i,t}$, firm i chooses $a_{i,t}$, $b_{i,t}$, and $q_{i,t}$ to maximize $\sum_j \kappa_{ij}\pi_{j,t}$

Dynamic R&D Game $\Longrightarrow$ Linear-Quadratic Differential Game

- Given other firms' R&D $\{x_{j,t}\}_{j\neq i, t\geq 0}$, firm i chooses R&D $\{x_{i,t}\}_{t\geq 0}$ to maximize

$$\max_{\{x_{i,t}\}_{t\geq 0}} \quad V^i(z_0) \equiv \int_0^\infty \exp(-\rho t) \sum_j \kappa_{ij} \left(\pi_{j,t} - d_{j,t}\right) dt$$

  - Gross profit: $\sum_j \kappa_{ij}\pi_{j,t} = \boldsymbol{z}_t^T \boldsymbol{Q}^i \boldsymbol{z}_t$ Q
  - R&D cost: $\sum_j \kappa_{ij} d_{j,t} = \sum_j \kappa_{ij} x_{j,t}^2$
  - Law of motion: $\dot{\boldsymbol{z}}_t = \boldsymbol{\Omega} \boldsymbol{z}_t + \mu \boldsymbol{x}_t - \delta \boldsymbol{z}_t$

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- Firm i's HJB equation:

$$\rho V^i(\boldsymbol{z}) = \max_{x_i} \left\{ \boldsymbol{z}^T \boldsymbol{Q}^i \boldsymbol{z} - \sum_j \kappa_{ij} x_j^2 + V_{\boldsymbol{z}}^i(\boldsymbol{z}) \left[\boldsymbol{\Omega}\boldsymbol{z} + \mu \boldsymbol{x} - \delta \boldsymbol{z}\right] \right\}$$

HJB Equations $\Longrightarrow$ Riccati Equations

- Guess and verify $V^i(z) = z^T X^i z$ (for any z)
- X^i is the solution of stacked algebraic Riccati equations

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  Riccati Equations

- Public & patenting firms in the U.S. in our dataset > 700 firms $\Longrightarrow$ $\underbrace{700 \times 700}_{\text{size of } X^i} \times \underbrace{700}_{n} = 343$ million undetermined coefficients (< 1 min on my laptop)

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Oligopolistic Schumpeterian	Computation time	# of firms	Productivity space
Cavenaile et al. (2023)	$O(2^n)$	4	6 grid
Our model	$O(n^4)$	>700	Continuous

Transition

BGP

- Linear R&D strategy: $x_t = \mu \widetilde{\boldsymbol{X}} z_t$ where $\widetilde{\boldsymbol{X}} = \begin{bmatrix} \boldsymbol{X}_1^1 & \cdots & \boldsymbol{X}_n^n \end{bmatrix}^T$ and $\boldsymbol{X}_i^i$ is the ith column of $\boldsymbol{X}^i$
- The law of motion is rewritten as $\dot{z}_t = \boldsymbol{\Phi} z_t$ where

$$\boldsymbol{\Phi} \equiv \underbrace{\boldsymbol{\Omega}}_{\text{Tech Spillover}} + \underbrace{\mu^2 \widetilde{\boldsymbol{X}}}_{\text{R\&D}} - \underbrace{\delta \boldsymbol{I}}_{\text{Depreciation}}$$

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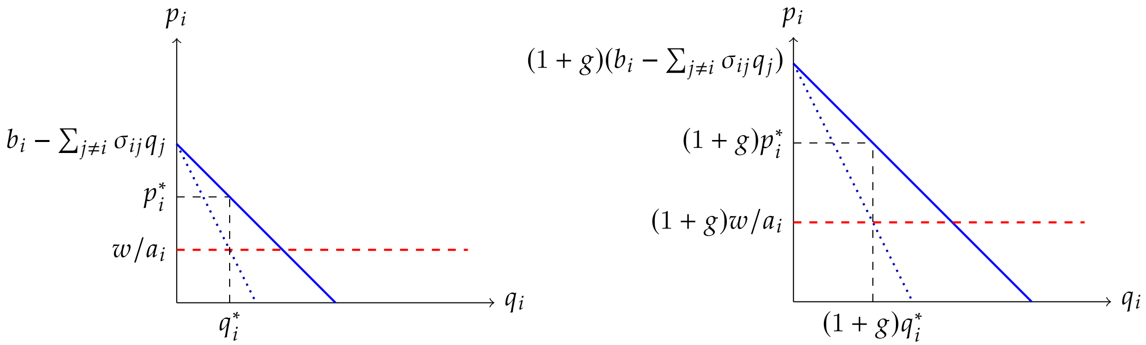
Theorem

If $\mathbf{\Phi}$ is irreducible, then:
(i) There exists a largest positive eigenvalue of $\mathbf{\Phi}$, g, and an associated positive eigenvector, z^*.
(ii) There exists a globally stable BGP such that the knowledge capital growth rate of all firms is g, and the knowledge capital distribution is a scalar multiple of z^*.

- Proof: Perron–Frobenius Theorem
- "$\mathbf{\Phi}$ is irreducible" $\Longleftrightarrow$ "All firms are directly or indirectly connected technologically"

Symmetric Example

CES on BGP despite Non-CES Demand



- a_i, b_i, $q_i (= a_i l_i)$, p_i, and w/a_i grow at the same rate g
- (i) (consumer surplus / producer surplus) and (ii) (cost / revenue) stay the same
- Demand elasticity is constant on BGP despite linear demand

Lifetime Utility

- Lifetime utility is expressed in quadratic form:

$$\int_t^{\infty} \exp\left(-\rho s\right) C_s ds = z_t^T X z_t$$

X

- Solve the equilibrium once $\Longrightarrow$ we can compute lifetime utility for any initial z_t
- This property holds even if we introduce idiosyncratic / aggregate shocks
- In the exercise, we focus on the transition dynamics starting from the observed initial z_t

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- Equilibrium quantity $q_i = \frac{1}{2} z_i - \frac{1}{2} \sum_{k \neq i} \sigma_{ik} q_k - \sqrt{\zeta w_t}$ and profit $\pi_i = q_i^2$

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$$\text{Tech Spillover Externalities:} \quad \frac{\partial \dot{z}_j}{\partial z_i} = \omega_{ij}, \quad \frac{\partial \pi_j}{\partial z_j} = 2q_j \frac{\partial q_j}{\partial z_j} = q_j$$

$$z_i \uparrow \quad \underbrace{\Longrightarrow}_{\text{strong if } \omega_{ij} \text{ is large}} \quad z_j \uparrow \quad \Longrightarrow \quad \pi_j \uparrow$$

Intuition of Externality

- Assume firms choose static variables $\{a_{i,t}, b_{i,t}, q_{i,t}\}$ to maximize static profits
- Equilibrium quantity $q_i = \frac{1}{2}z_i - \frac{1}{2}\sum_{k\neq i}\sigma_{ik}q_k - \sqrt{\zeta w_t}$ and profit $\pi_i = q_i^2$

Tech Spillover Externalities: $$\frac{\partial \dot{z}_j}{\partial z_i} = \omega_{ij}, \quad \frac{\partial \pi_j}{\partial z_j} = 2q_j \frac{\partial q_j}{\partial z_j} = q_j$$

$$z_i \uparrow \underbrace{\Longrightarrow}_{\text{strong if } \omega_{ij} \text{ is large}} z_j \uparrow \Longrightarrow \pi_j \uparrow$$

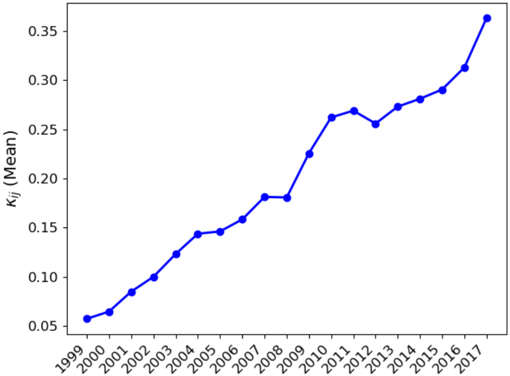
Business Stealing Externalities: $$\frac{\partial \pi_j}{\partial z_i} = 2q_j \frac{\partial q_j}{\partial z_i} = 2q_j\left(-\frac{1}{2}\sigma_{ji}\frac{\partial q_i}{\partial z_i}\right) = -\frac{1}{2}\sigma_{ji}q_j$$

$$z_i \uparrow \Longrightarrow q_i \uparrow \underbrace{\Longrightarrow}_{\text{strong if } \sigma_{ij} \text{ is large}} q_j \downarrow \Longrightarrow \pi_j \downarrow$$

Common Ownership Weights K

- Backus et al. (2021) construct a dataset on investors' holdings based on Form 13F
- Baseline: proportional influence (Rotemberg, 1984)

Proportional Influence



Product-Market Rivalry Σ

- Hoberg and Phillips (2016) estimates product proximity using business descriptions in 10-K
- Pellegrino (2024) estimates α to align with the cross-price elasticity of demand

$$\underbrace{\sigma_{ij}}_{\text{substitutability}} = \alpha \times \text{product proximity between } i \text{ and } j \quad (i \neq j)$$

micro estimates

Technological Proximity $\widetilde{\Omega}$

- Technological profile of firm i
  - The vector of the share of patents held by firm i in each technology class
  - Baseline: group-level patent classifications (≈ 4000), five years window

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- Technological profile of firm i
  - The vector of the share of patents held by firm i in each technology class
  - Baseline: group-level patent classifications (≈ 4000), five years window
- Jaffe (1986) technological proximity measure $\tilde{\omega}_{ij}$:
  - Cosine similarity of the technological profiles between firms i and j
  - Impose $\sum_{j \neq i} \tilde{\omega}_{ij} = 1$ for each i

Distribution of Knowledge Capital z_t

Variables	Identification
$\pi_{i,t}$	Gross profit (before R&D cost) = Revenue − Cost of goods sold

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ζ/L	Matches sample firms' cost share (average markup)
$\boldsymbol{z}_t$	$\boldsymbol{z}_t = \left\{ 2\frac{\zeta}{L}\mathbf{1}_{n\times n} + \boldsymbol{\Sigma} + \boldsymbol{K} \circ \boldsymbol{\Sigma} \right\} \boldsymbol{q}_t$

Technology Spillover $\Omega = \beta \times$Technological Proximity $\tilde{\Omega}$ First Stage

$$\log z_{i,t+1} - \log z_{i,t} = \beta \sum_{j \neq i} \tilde{\omega}_{ij,t} \frac{z_{j,t}}{z_{i,t}} + \text{Controls}_{i,t} + \epsilon_{i,t}$$

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	(1)	(2)	(3)
	$\log z_{i,t+1} - \log z_{i,t}$	$\log z_{i,t+1} - \log z_{i,t}$	$\log z_{i,t+1} - \log z_{i,t}$
$\sum_{j \neq i} \tilde{\omega}_{ij,t} \frac{z_{j,t}}{z_{i,t}}$	0.026**	0.024**	0.073*
	(0.010)	(0.010)	(0.038)
$\frac{x_{i,t}}{z_{i,t}}$		0.514***	
		(0.063)	
Firm & Year FEs	✓	✓	✓
Controls	✓	✓	✓
IV			✓
Observations	14,576	14,576	14,576

SEs clustered by years and 4-digit NAICS industries are reported in parentheses. Control variables include $\log z_{i,t}$, firm fixed effects, and year fixed effects. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

- IV: User cost of R&D, driven by federal and state-specific rules variations (Bloom et al., 2013)

Identification: Summary

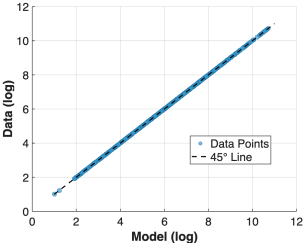
- Publicly available data + Compustat

Notation	Description	Value	Source
$\boldsymbol{\Sigma}$	Product proximity		Form 10-K, Hoberg and Phillips (2016)
$\widetilde{\boldsymbol{\Omega}}$	Technological proximity		USPTO, Patent classification
$\boldsymbol{K}$	Common ownership weights		Form 13F, Backus et al. (2021)
α	Product proximity $\longrightarrow$ Substitutability	0.120	Pellegrino (2024)
β	Technological proximity $\longrightarrow$ Spillovers	0.024	Estimated from the law of motion
ζ/L	Labor-augmenting efficiency	0.004	Compustat, Cost of goods sold
ρ	Discount rate	0.100	
μ	R&D efficiency	0.066	2.6% R&D share (moment match)
δ	Depreciation rate	0.017	1.2% economic growth rate (moment match)

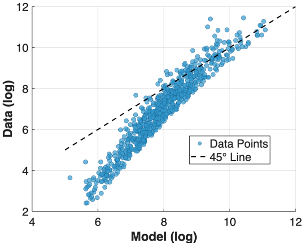
Network Correlation

Model vs. Data: Firm-level Profits, Sales, and R&D

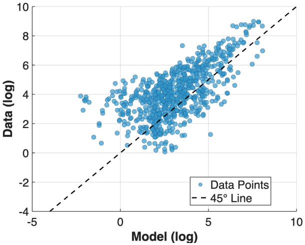
- Comparison of firm-level model-generated values (x-axis) with observed data (y-axis)



(a) Log of Profit (Targeted)



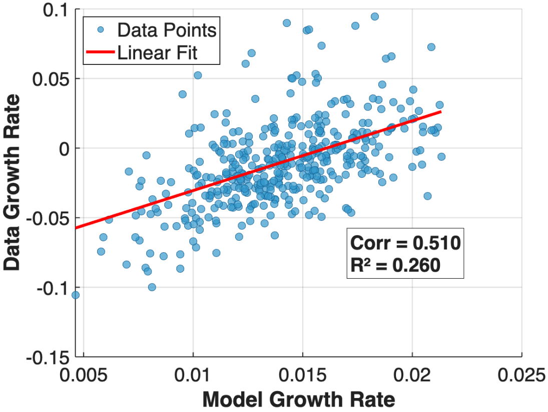
(b) Log of Sales



(c) Log of R&D Expenditure

Model vs. Data: Firm-level Growth Rates

- Data: Average growth rate of $z_{i,t}$ between 2010 and 2017
- Model: Networks and initial knowledge capital in 2010



Comparison with Anton et al. (2025)

- Anton et al. (2025) estimates the effect of interaction b/w common ownership and technology/product proximity on R&D

$$\log\left(1+\text{R\&D}_{it}/A_{it}\right) = \gamma_1 \log\left(\sum_{j\neq i}\kappa_{ijt}\text{tech proximity}_{ijt}G_{jt}\right) + \gamma_2 \log\left(\sum_{j\neq i}\kappa_{ijt}\text{product proximity}_{ijt}G_{jt}\right)$$
$$+\,\text{Controls}_{it} + \text{Firm FEs}_i + \text{Year FEs}_t + \varepsilon_{ijt}$$

	$\text{R\&D}_{it}$	A_{it}	G_{it}	Sample period
Anton et al. (2025)	Observed R&D	Assets	R&D stocks	1985–2015
Our model	Model-generated R&D	Knowledge capital	Knowledge capital	1999–2017

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	Anton et al. (2025)	Our model
$\log\left(\sum_{j\neq i}\kappa_{ijt}\text{tech proximity}_{ijt}G_{jt}\right)$	0.00513^{**} (0.00226)	0.00194^{***} (0.000272)
$\log\left(\sum_{j\neq i}\kappa_{ijt}\text{product proximity}_{ijt}G_{jt}\right)$	-0.00457^{**} (0.00222)	-0.00547^{***} (0.000693)

Counterfactual Ownership Structures

Ownership Structure	Description
Baseline	Observed common ownership structure in 2017

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Uniform	$\kappa_{ij,2017}^{U} = \boldsymbol{E}\left[\kappa_{ij,2017}\right]$ for $j \neq i$
Monopoly	$\boldsymbol{K}^{M} = \mathbf{1}_{n \times n}$

Total R&D Expenditure

- For the moment, assume that ownership structure only affects R&D decisions
  - Product-market competition: firms maximize gross profits (dispersed ownership)

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Total R&D in 2017 (Optimal R&D: 100)	Ownership Structure				
	Dispersed	Mean=1999	Uniform	Baseline	Monopoly
Baseline	40.48	38.68	31.56	28.26	21.39
Only Business Steal $\boldsymbol{\Omega} = [0]$					
Only Tech Spill $\boldsymbol{\Sigma} = \boldsymbol{I},\ \zeta/L = 0$					

- Internalization of business-stealing > internalization of technology spillovers

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Total R&D, Growth Rate, Social Welfare, Firm Value Share

	Ownership Structure				
	Dispersed	Mean=1999	Uniform	Baseline	Monopoly
Total R&D (Optimal R&D: 100)	40.48	38.68	31.56	28.26	21.39
Economic Growth Rate (%)					
CE Welfare (Optimal R&D: 100)					
Firm Value Share (%)					

- The rise of common ownership from 1999 to 2017 $\Longrightarrow$
- The results are qualitatively the same under different corporate governance assumptions

Corporate Governance Assumptions

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Firm Value Share (%)					

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Firm Value Share (%)	26.63	26.72	27.20	27.24	27.82

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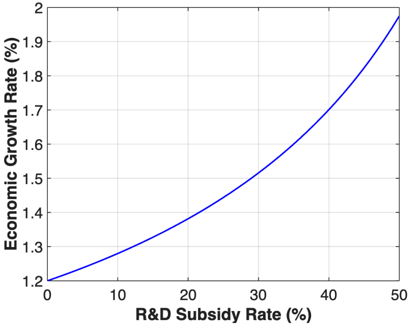
Corporate Governance Assumptions

When Common Ownership Affects Both R&D and Production

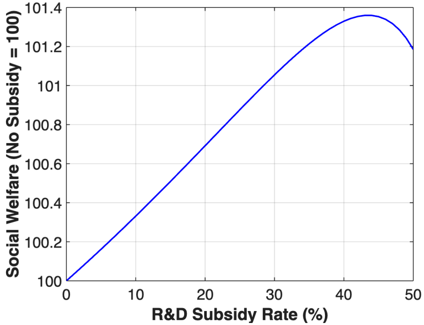
Production ownership structure R&D ownership structure	Dispersed Dispersed	Dispersed Common	Common Common
Total Output (Dispersed: 100)	100.00	100.00	97.26
Total R&D (Dispersed: 100)	100.00	69.81	86.36
Economic Growth Rate (%)	1.323	1.200	1.288
CE Welfare (Dispersed: 100)	100.00	99.41	97.28
Firm Value Share (%)	26.63	27.24	34.10

- Less product market competition $\Longrightarrow$ Private return on R&D $\uparrow$

Optimal Uniform R&D Subsidy



(a) Economic Growth



(b) Social Welfare

- Optimal rate is $s = 43\%$, which increases g by 0.57 pp and CE welfare by 1.36%
- c.f. Welfare gains from optimal R&D allocation: 6.0%

Which Firms' R&D Should be Subsidized?

	Private R&D x	Social / Private value of R&D
Initial knowledge capital z	0.122***	-0.000212
	(0.000848)	(0.000158)
Product market centrality	-16.4***	-1.06***
	(0.286)	(0.0532)
Technology spillover centrality	5.45***	1.48***
	(0.423)	(0.0788)
Ownership structure centrality	-7.16***	0.580***
	(0.239)	(0.0446)
Intercept	-22.4***	0.900***
	(0.255)	(0.0475)
Observations	740	740
R^2	0.976	0.555

Alternative Corporate Governance Models

1. Baseline proportional influence: $\kappa_{ij} = \frac{\sum_o s_{io} s_{jo}}{\sum_o s_{io}^2}$
2. Super-proportional influence: $\kappa_{ij}^{SP} = \frac{\sum_o \gamma_{io} s_{io} s_{jo}}{\sum_o \gamma_{io} s_{io}^2}$ where $\gamma_{io} = \sqrt{s_{io}}$

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3. Blockholder influence: $\kappa_{ij}^{BH} = \frac{\sum_o b_{io} s_{io} s_{jo}}{\sum_o s_{io}^2} \quad (i \neq j)$, where $b_{io} = 1$ if $s_{io} > 5\%$

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4. Structural estimation in the airline industry by Azar and Ribeiro (2021): $\kappa_{ij}^{AR} = \tau_i \kappa_{ij} \quad (i \neq j)$
   - 4.1 Uniform: $\tau = 0.29$
   - 4.2 Firm-specific: $\tau_i = \frac{\exp[\theta_0 + \log(\text{Investor HHI}_i)]}{1+\exp[\theta_0 + \log(\text{Investor HHI}_i)]}$ where $\theta_0 = 2.68$

Alternative Corporate Governance Models

	Ownership Structure					
	Dispersed Ownership	Baseline: Proportional Influence	Super Proportional Influence	Blockholder Influence	AR Uniform	AR Firm-Specific
Total R&D Expenditure	100.00	69.81	68.97	77.45	90.32	90.41
Growth Rate (%)	1.323	1.200	1.194	1.234	1.287	1.289
CE Welfare	100.00	99.41	99.37	99.59	99.86	99.86
Firm Value Share (%)	26.63	27.24	27.24	27.09	26.82	26.84

- Internalization of business-stealing > internalization of technology spillovers

Conclusion

- Quantitative Schumpeterian growth model with ownership structure
  - Utilizes micro data and computational capabilities
- Common ownership in the U.S.:
  1. Internalization of business-stealing effect $\Longrightarrow g \downarrow\downarrow$
  2. Internalization of technology spillover effect $\Longrightarrow g \uparrow$
- Potential applications:
  - M&A
  - Conglomerate (e.g. Korea)
  - Cross-shareholdings (e.g. Japan, Germany, AI companies AI)
  - FDI and international technology diffusion
  - Technology licensing

Share of Top 5 Shareholders in Largest Market Cap Firms

Back

Microsoft	
Vanguard	9.20%
Blackrock	7.75%
Steven Ballmer	4.48%
State Street	3.97%
Fidelity	2.66%

Nvidia	
Vanguard	8.93%
BlackRock	7.74%
Fidelity	4.12%
State Street	3.97%
Jensen Huang	3.80%

Apple	
Vanguard	9.29%
Blackrock	7.48%
State Street	3.96%
Fidelity	2.27%
Geode Capital	2.26%

Google	
Vanguard	7.36%
Blackrock	6.47%
State Street	3.39%
Fidelity	3.01%
Sergey Brin	2.99%

Amazon	
Jeffrey Bezos	8.58%
Vanguard	7.77%
Blackrock	6.50%
State Street	3.44%
Fidelity	3.10%

Meta	
Vanguard	7.55%
Blackrock	6.50%
Fidelity	5.38%
Accel IX LP	3.88%
State Street	3.40%

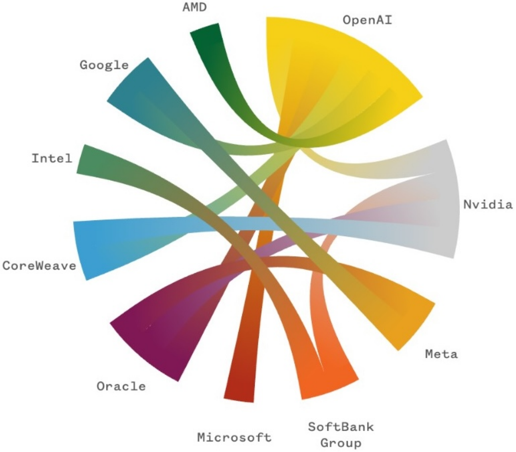
Merger Guidelines

Back

"For example, the partial owner may decide not to develop a new product feature to win market share from the firm in which it has acquired an interest, because doing so will reduce the value of its investment in its rival."

2023 Merger Guidelines by the U.S. Department of Justice and the Federal Trade Commission

Cross Shareholdings among AI Companies Back



Source: Wallstreetcn.com

Technology & Product Proximity: Example

Tesla vs. Ford	
Technology Proximity	0.11
Product Proximity	0.15

Apple vs. Intel	
Technology Proximity	0.57
Product Proximity	0.00

Empirical Literature: Common Ownership $\Longrightarrow$ R&D

- Anton et al. (2025):
  - Dependent variables: R&D, citation-weighted patents, market value of patents
    - $+$ Interaction term between common ownership and technology proximity
    - $-$ Interaction term between common ownership and product proximity
- Kini et al. (2024): DiD that exploits mergers between financial institutions
  - Dependent variables: Investments, new product development
    - $+$ Post (merger) $\times$ treatment (common owner) $\times$ technology proximity

R&D Externalities

1. Business-stealing effect
   - Innovators steal the business (profits) of other firms
2. Technology spillover effect
   - Innovation improves the productivity of other firms
3. Appropriability effect (market power)
   - Innovators cannot appropriate the entire consumer surplus

Generalized Hedonic-Linear Demand (Pellegrino, 2024)

- $i \in \{1, 2, \ldots, n\}$: firms / products
- 1 unit of product i provides
  - 1 unit of idiosyncratic characteristic $k \in \{1, 2, \ldots, n\}$
  - $\psi_{k,i}$ unit of shared characteristic $k \in \{n+1, n+2, \ldots, n+n_k\}$ where $\sum_k \psi_{k,i}^2 = 1$
- Aggregate each characteristic:

$$y_{k,t} = \begin{cases} q_{k,t} & k = 1, 2, \ldots, n \\ \sum_i \psi_{k,i} q_{i,t} & k = n+1, n+2, \ldots, n+n_k \end{cases}$$

- Linear-quadratic aggregator over characteristics:

$$Y_t = (1-\alpha) \sum_{k=1}^{n} \left(\underbrace{\hat{b}_{k,t} y_{k,t} - \frac{1}{2} y_{k,t}^2}_{\text{idiosyncratic characteristic}} \right) + \alpha \sum_{k=n+1}^{n+n_k} \left(\underbrace{\hat{b}_{k,t} y_{k,t} - \frac{1}{2} y_{k,t}^2}_{\text{shared characteristic}} \right)$$

Generalized Hedonic-Linear Demand (Pellegrino, 2024)

- Quality:

$$b_i = (1-\alpha)\,\hat{b}_i + \alpha \sum_{k=n+1}^{n+n_k} \psi_k \hat{b}_k$$

- Inverse demand:

$$\frac{\boldsymbol{p}}{P} = \boldsymbol{b} - \boldsymbol{\Sigma}\boldsymbol{q}$$

- Inverse cross-price elasticity of demand:

$$\frac{\partial \log p_i}{\partial \log q_j} = -\frac{q_j}{p_i} \cdot \sigma_{ij}$$

- Cross-price elasticity of demand:

$$\frac{\partial \log q_i}{\partial \log p_j} = -\frac{p_j}{q_i} (\boldsymbol{\Sigma}^{-1})_{ij}$$

Static Profits

- Firms choose labor productivity and product quality: $\zeta a_{i,t} = \sqrt{\zeta w_t}$, $b_{i,t} = z_{i,t} - \sqrt{\zeta w_t}$
- Labor market clearing: $L = \sum_i \frac{q_{i,t}}{a_{i,t}} \Longrightarrow \sqrt{\zeta w_t} = \frac{\zeta}{L} \sum_i q_{i,t}$
- $\boldsymbol{q}_t = \boldsymbol{N} \boldsymbol{z}_t$ where $\boldsymbol{N} \equiv \left\{2\frac{\zeta}{L}\boldsymbol{J} + \boldsymbol{\Sigma} + \boldsymbol{K} \circ \boldsymbol{\Sigma}\right\}^{-1}$
- $\boldsymbol{N}_i$: the ith row of $\boldsymbol{N}$
- Ownership weighted profit:

$$\sum_j \kappa_{ij} \frac{\pi_{j,t}}{P_t} = \sum_j \kappa_{ij} \sum_h \kappa_{jh} \sigma_{jh} q_{j,t} q_{h,t} = \boldsymbol{z}_t^T \boldsymbol{Q}^i \boldsymbol{z}_t$$

where

$$\boldsymbol{Q}^i = \frac{1}{2} \sum_j \kappa_{ij} \sum_h \kappa_{jh} \sigma_{jh} \left(\mathrm{N}_j^T \mathrm{N}_h + \mathrm{N}_h^T \mathrm{N}_j\right)$$

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Riccati Equations

- $V^i(z) = z^T X^i z$ where X^i is the solution of the stacked Riccati equation

$$0 = Q^i - \mu^2 \sum_j \kappa_{ij} X_j^j \left(X_j^j\right)^T + \left(\Phi - \frac{1}{2}\left(\rho - \gamma^2\right) I\right)^T X^i + X^i \left(\Phi - \frac{1}{2}\left(\rho - \gamma^2\right) I\right)$$

  - $X_i^i \equiv$ the ith column of X^i
  - $\Phi \equiv \Omega + \mu^2 \begin{bmatrix} X_1^1 & \cdots & X_n^n \end{bmatrix}^T$

- Algorithm: Given $\begin{bmatrix} X_\tau^1 & \cdots & X_\tau^n \end{bmatrix}$, update $\begin{bmatrix} X_{\tau-\Delta}^1 & \cdots & X_{\tau-\Delta}^n \end{bmatrix}$ by

$$-\frac{X_\tau^i - X_{\tau-\Delta}^i}{\Delta} = Q^i - \mu^2 \sum_j \kappa_{ij} X_{j,\tau}^j \left(X_{j,\tau}^j\right)^T + \left(\Phi_\tau - \frac{1}{2}\left(\rho - \gamma^2\right) I\right)^T X_\tau^i + X_\tau^i \left(\Phi_\tau - \frac{1}{2}\left(\rho - \gamma^2\right) I\right)$$

Summary of Equilibrium

Description	Expression
Production strategy	$\boldsymbol{q}_t = \boldsymbol{N}\boldsymbol{z}_t$
R&D strategy	$\boldsymbol{x}_t = \mu\tilde{\boldsymbol{X}}\boldsymbol{z}_t$
Law of motion	$d\boldsymbol{z}_t = \left(\boldsymbol{\Omega}\boldsymbol{z}_t + \mu\boldsymbol{x}_t\right)dt + \gamma\boldsymbol{z}_t dW_t$
Profit of final producers	$\Pi_t^F/P_t = \boldsymbol{q}_t^T\left(\frac{1}{2}\boldsymbol{\Sigma}\right)\boldsymbol{q}_t$
Total operating profit of firms	$\Pi_t/P_t = \boldsymbol{q}_t^T\left(\frac{1}{2}\boldsymbol{\Sigma}\circ\left(\boldsymbol{K}+\boldsymbol{K}^T\right)\right)\boldsymbol{q}_t$
Labor income	$w_t L/P_t = \boldsymbol{q}_t^T\left(\frac{\zeta}{L}\boldsymbol{J}\right)\boldsymbol{q}_t$
Output	$Y_t = \boldsymbol{q}_t^T\left(\frac{\zeta}{L}\boldsymbol{J} + \frac{1}{2}\boldsymbol{\Sigma} + \frac{1}{2}\boldsymbol{\Sigma}\circ\left(\boldsymbol{K}+\boldsymbol{K}^T\right)\right)\boldsymbol{q}_t$
Consumption	$C_t = Y_t - \boldsymbol{x}_t^T\boldsymbol{x}_t$

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Example: Symmetric Equilibrium

Assumption

- Symmetric product substitutability, technology spillover, and ownership structure: $\sigma_{ij} = \sigma, \omega_{ij} = \omega, \kappa_{ij} = \kappa \quad \forall i \neq j$

- R&D strategy: $x^*_{i,t} = \mu \left(\tilde{x}_1 z_{i,t} + \tilde{x}_2 \sum_{j \neq i} z_j \right)$
  - $\tilde{x}_1$: market size effect (> 0)
  - $\tilde{x}_2$: strategic substitutability (< 0) / complementarity (> 0)

- Growth rate: $g = \underbrace{(n-1)\omega}_{\text{Tech Spillover}} + \underbrace{\mu^2 (\tilde{x}_1 + (n-1)\tilde{x}_2)}_{\text{R\&D}}$

- Stability (irreducibility) requires $\omega + \mu^2 \tilde{x}_2 > 0$
  - Tech spillover (ω) must be strong relative to strategic substitutability $(\tilde{x}_2 < 0)$

Output and Expected Utility

- Output: $Y_t = \boldsymbol{q}_t^T \boldsymbol{Q} \boldsymbol{q}_t$ where

$$\boldsymbol{Q} = \frac{\zeta}{L}\boldsymbol{J} + \frac{1}{2}\boldsymbol{\Sigma} + \frac{1}{2}\boldsymbol{\Sigma} \circ \left(\boldsymbol{K} + \boldsymbol{K}^T\right)$$

- Expected utility:

$$V\left(\boldsymbol{z}_t\right) \equiv \boldsymbol{E}_t \left[\int_t^{\infty} \exp\left(-\rho s\right) C_s ds \middle| \boldsymbol{z}_t \right] = \boldsymbol{z}_t^T \boldsymbol{X} \boldsymbol{z}_t$$

where $\boldsymbol{X}$ is the solution of the Lyapunov equation (obtained from households' HJB equation):

$$0 = \boldsymbol{Q} - \mu^2 \tilde{\boldsymbol{X}}^T \tilde{\boldsymbol{X}} + \boldsymbol{X}\left(\boldsymbol{\Phi} - \frac{1}{2}\left(\rho - \gamma^2\right)\boldsymbol{I}\right) + \left(\boldsymbol{\Phi} - \frac{1}{2}\left(\rho - \gamma^2\right)\boldsymbol{I}\right)^T \boldsymbol{X}$$

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Social Optimum

- Static optimal allocation: $\boldsymbol{q}_t^* = \boldsymbol{N}^* \boldsymbol{z}_t$ where $\boldsymbol{N}^* \equiv \left\{2\frac{\zeta}{L}\boldsymbol{J} + \boldsymbol{\Sigma}\right\}^{-1}$
- Optimal output: $Y_t^* = \boldsymbol{z}_t^T \boldsymbol{Q}^* \boldsymbol{z}_t$ where $\boldsymbol{Q}^* = \frac{1}{2}\boldsymbol{N}^*$
- Optimal expected utility:

$$V^*(\boldsymbol{z}_t) \equiv \boldsymbol{E}_t \left[\int_t^\infty \exp(-\rho s)\, C_s ds \middle| \boldsymbol{z}_t \right] = \boldsymbol{z}_t^T \boldsymbol{X}^* \boldsymbol{z}_t,$$

where $\boldsymbol{X}^*$ is the solution of the Riccati equation (obtained from planner's HJB equation):

$$0 = \boldsymbol{Q}^* - \mu^2 (\boldsymbol{X}^*)^2 + \boldsymbol{X}^* \left(\boldsymbol{\Phi}^* - \frac{1}{2}(\rho - \gamma^2) \boldsymbol{I} \right) + \left(\boldsymbol{\Phi}^* - \frac{1}{2}(\rho - \gamma^2) \boldsymbol{I} \right) \boldsymbol{X}^*$$

- Optimal R&D: $\boldsymbol{x}_t^* = \mu \boldsymbol{X}^* \boldsymbol{z}_t$
- Optimal technology transition matrix: $\boldsymbol{\Phi}^* = \boldsymbol{\Omega} + \mu^2 \boldsymbol{X}^*$

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Property of BGP

- On the BGP, a_t, b_t, z_t, and q_t grow at the same rate

Knowledge Capital:	$\zeta a_{i,t} + b_{i,t} = z_{i,t}$
Linear Production Technology:	$q_{i,t} = a_{i,t} l_{i,t}$
Inelastic Labor Supply:	$L = \sum_i l_{i,t}$

- The linear and quadratic terms in q_t of output grow at the same rate:

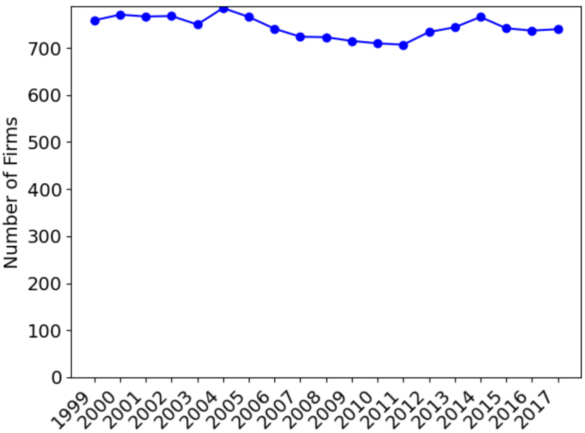
$$Y_t = \boldsymbol{q}_t^T \boldsymbol{b}_t - \frac{1}{2} \boldsymbol{q}_t^T \boldsymbol{\Sigma} \boldsymbol{q}_t$$

Growth Decomposition

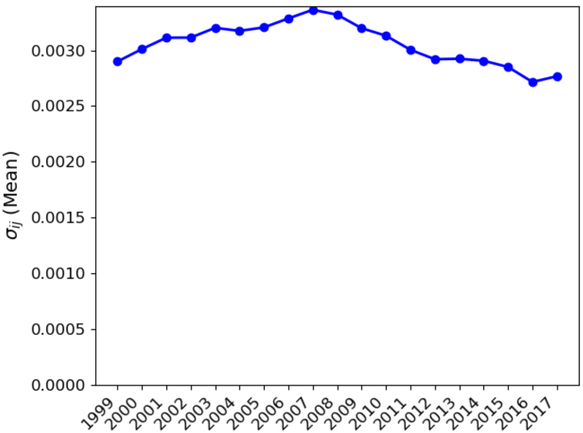
- Aggregate output: $Y_t = \boldsymbol{z}_t^T \boldsymbol{Q} \boldsymbol{z}_t$
- $d\boldsymbol{z}_t/dt = \boldsymbol{\Phi}\boldsymbol{z}_t$ where $\boldsymbol{\Phi} = \boldsymbol{\Omega} + \mu^2 \widetilde{\boldsymbol{X}} - \delta \boldsymbol{I}$

$$\frac{d\log Y_t}{dt} = \underbrace{\frac{\boldsymbol{z}_t^T \left(\boldsymbol{Q}\boldsymbol{\Omega} + \boldsymbol{\Omega}\boldsymbol{Q}\right)\boldsymbol{z}_t}{Y_t}}_{\text{Tech Spillover}} + \underbrace{\frac{\mu^2 \boldsymbol{z}_t^T \left(\boldsymbol{Q}\widetilde{\boldsymbol{X}} + \widetilde{\boldsymbol{X}}^T \boldsymbol{Q}\right)\boldsymbol{z}_t}{Y_t}}_{\text{R\&D}} - \underbrace{2\delta}_{\text{Depreciation}}$$

Number of Sample Firms

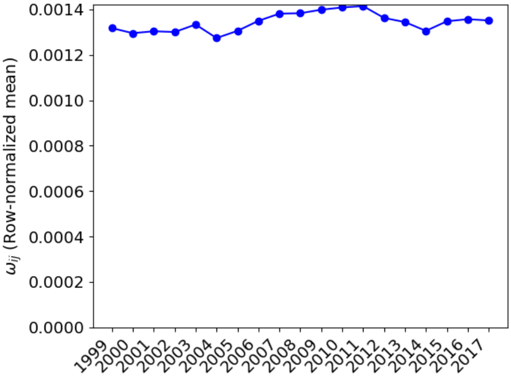


Trend of Product Substitutability



Technological Proximity

- Merge USPTO data with Compustat firms using DISCERN 2 dataset (Arora et al., 2024)
- Jaffe measure, group-level patent classification, stacked over 5 years



Correlation Across Networks

	K	Σ	Ω
K	1.0000	-0.0035	0.0115
Σ	-0.0035	1.0000	0.2542
Ω	0.0115	0.2542	1.0000

- K: Ownership network
- Σ: Product substitutability network
- Ω: Technological proximity network

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Microeconometric Estimates vs. GHL (Pellegrino, 2024) (1/2)

Market	Firm i	Firm j	Micro Estimate	GHL
Auto	Ford	Ford	-4.320	-5.197
Auto	Ford	General Motors	0.034	0.056
Auto	Ford	Toyota	0.007	0.017
Auto	General Motors	Ford	0.065	0.052
Auto	General Motors	General Motors	-6.433	-4.685
Auto	General Motors	Toyota	0.008	0.005
Auto	Toyota	Ford	0.018	0.025
Auto	Toyota	General Motors	0.008	0.008
Auto	Toyota	Toyota	-3.085	-4.851

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Microeconometric Estimates vs. GHL (Pellegrino, 2024) (2/2)

Market	Firm i	Firm j	Micro Estimate	GHL
Cereals	Kellogg's	Kellogg's	-3.231	-1.770
Cereals	Kellogg's	Quaker Oats	0.033	0.023
Cereals	Quaker Oats	Kellogg's	0.046	0.031
Cereals	Quaker Oats	Quaker Oats	-3.031	-1.941
Computers	Apple	Apple	-11.979	-8.945
Computers	Apple	Dell	0.018	0.025
Computers	Dell	Apple	0.027	0.047
Computers	Dell	Dell	-5.570	-5.110

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First Stage Back

	R&D
	(1)
State tax credit component of R&D user cost	-1.16^{***}
	(0.29)
Federal tax credit component of R&D user cost	-34.29^{***}
	(3.64)
Firm fixed effects	✓
Year fixed effects	✓
No. of observations	16197

SEs clustered by years and 4-digit NAICS industries are reported in parentheses.

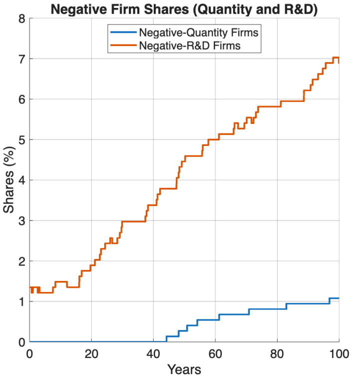
- IV: User cost of R&D, driven by federal and state-specific rules variations (Wilson, 2009; Bloom et al., 2013)

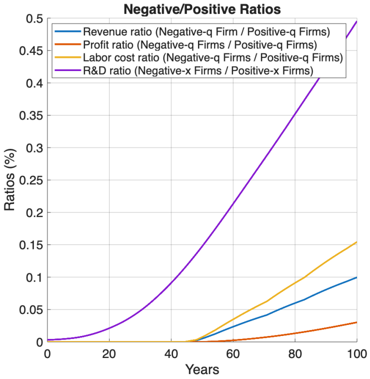
Negative R&D and Output

- Issue with the model: negative output and R&D
  - Inada condition is not satisfied
  - Non-negativity constraint makes model intractable

Negative R&D and Quantity

- Firms with negative values are negligible along the transition path
- The weight on values 100 years and beyond is 0.005% when $\rho = 0.1$





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