## Ownership Structure and Economic Growth

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### Ownership Structure ⇒ Economic Growth?

- BlackRock, Vanguard, State Street, and Fidelity control 30% of votes of S&P 500 firms
- Top 10 chaebols account for half of stock market capitalization and exports in Korea



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Share

- Top 10 chaebols account for half of stock market capitalization and exports in Korea
- Firms maximize shareholder values ⇒
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Share

- Top 10 chaebols account for half of stock market capitalization and exports in Korea
- Firms maximize shareholder values ⇒
   Partially internalize externalities for commonly owned firm
- Ownership structure (common ownership, cross ownership, M&A, FDI, ...) ⇒
  Economic growth?
  - Business stealing effect
  - Technology spillover effect

### Quantitative Schumpeterian Growth Model with Ownership Structure

- Existing Schumpeterian growth models:
  - Lack of strategic interaction (e.g., monopolistic competition)
  - Very few firms in Markov perfect equilibrium
- This paper: Many oligopolists + Ownership structure

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- Quantify three inter-firm networks for publicly listed patenting firms in the US ( $\simeq$  1000 firms)
  - Ownership structure
  - Product market rivalry
  - Technology spillover

## Quantitative Schumpeterian Growth Model with Ownership Structure

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- ullet Quantify three inter-firm networks for publicly listed patenting firms in the US ( $\simeq$  1000 firms)
  - Ownership structure
  - Product market rivalry
  - Technology spillover
- Common ownership in the US:
  - Internalization of business stealing  $\Longrightarrow g \downarrow \downarrow$
  - Internalization of technology spillover  $\Longrightarrow g \uparrow$

#### Literature

- Competition & Innovation:
  - d'Aspremont and Jacquemin (1988); Aghion et al. (2001, 2005); Acemoglu and Akcigit (2012); Aghion et al. (2013); Bloom et al. (2013); Lopez and Vives (2019); Peters (2020); Akcigit and Ates (2021, 2023); Liu et al. (2022); Cavenaile et al. (2023); Anton et al. (2023, 2024); Kini et al. (2024); Hopenhayn and Okumura (2024) Quantitative Schumpeterian growth model with ownership structure
- Hedonic Demand / Empirical IO:
   Lancaster (1966); Rosen (1974); Berry et al. (1995); Nevo (2001); Pellegrino (2024); Ederer and Pellegrino (2024)
   Dynamic general equilibrium / R&D
- Oligopoly / Common Ownership / Market Power:
   Rubinstein and Yaari (1983); Rotemberg (1984); Neary (2003); Atkeson and Burstein (2008); Gutierrez and
   Philippon (2017); He and Huang (2017); Azar et al. (2018, 2022); Autor et al. (2020); Baqaee and Farhi (2020);
   De Loecker et al. (2020); Azar and Vives (2021); Edmond et al. (2023)

# Simple Static Partial Equilibrium Model

- $\bullet \:$  Firm  $i \in \{1, \dots, n\}$  chooses quantity  $q_i$  and R&D effort  $x_i$
- ullet Linear inverse demand:  $\mathbf{p}(\mathbf{q}) = \mathbf{b} oldsymbol{\Sigma} \mathbf{q}$
- $\bullet$  CRS production technology with marginal cost:  $\mathbf{m}(\mathbf{x}) = \mathbf{a} \Omega \mathbf{x}$
- Quadratic cost of R&D:  $c(x_i) = \frac{1}{2}x_i^2$

# Common Ownership Weights

- ullet  $\mathbf{K}=\left[\kappa_{ij}
  ight]$ : common ownership weights that firm i places on the value of firm j
- More overlapping ownership b/w firm i and j  $\Longrightarrow$  Higher  $\kappa_{ij}$

Proportional Influece

- Diagonal elements  $\kappa_{ii}$  are normalized to 1 for all firm i
- ullet  $\mathbf{K} = \mathbf{I}$ : dispersed ownership (each firm maximizes its own value)
- ullet  ${f K}=[1]$ : monopoly (maximizes total producer surplus)

### Cournot & R&D Game

• Firm *i*'s profit:

$$\begin{split} \pi_i(\mathbf{q},\mathbf{x}) &= [p_i - m_i]q_i - c(x_i) \\ &= \left[b_i - \sum_{j=1}^n \sigma_{ij}q_j - a_i + \sum_{j=1}^n \omega_{ij}x_j\right]q_i - \frac{1}{2}x_i^2 \end{split}$$

• Given  $\left\{q_j,x_j\right\}_{j\neq i}$ , firm i chooses  $q_i$  and  $x_i$  to maximize  $\sum_j \kappa_{ij}\pi_j(\mathbf{q},\mathbf{x})$ 

# Impact of Common Ownership on R&D

 $\bullet$  Comparative statics where  $\left\{q_k,x_k\right\}_{k \neq i}$  are held constant:

$$\frac{\partial x_i}{\partial \kappa_{ij}} = \frac{q_j}{2 - \omega_{ii}^2} (2\omega_{ij} - \omega_{ii}\sigma_{ij}) \qquad \forall \ j \neq i$$

- SOC:  $2 > \omega_{ii}^2$
- Internalize business stealing effect:  $\partial^2 x_i/\partial \kappa_{ij}\partial \sigma_{ij} < 0$
- Internalize technology spillover effect:  $\partial^2 x_i/\partial \kappa_{ij}\partial \omega_{ij}>0$

# Overview of Identification Strategy

Networks	Measurement
Common ownership ${f K}$	Institutional investor holdings in 13f filings (Backus et al., 2021)
Product market rivalry $\Sigma$	Product proximity (Hoberg and Phillips, 2016): Text analysis of business description in 10k filings
Technology spillover $\Omega$	Technology proximity (Jaffe, 1986; Bloom et al., 2013): Patent classification

# Schumpeterian Growth Model

Linear quadratic aggregator (final good):

$$Y_t = \mathbf{q}_t^T \mathbf{b}_t - \frac{1}{2} \mathbf{q}_t^T \mathbf{\Sigma} \mathbf{q}_t$$

Linear inverse demand:

$$\mathbf{p}_t = \mathbf{b}_t - \mathbf{\Sigma} \mathbf{q}_t$$

CRS production technology (intermediate good):

$$q_{i,t} = a_{i,t} l_{i,t}$$

- Each firm has knowledge capital  $z_{i,t}$
- Each firm allocates knowledge capital to improve labor productivity and product quality:

$$\zeta a_{i,t} + b_{i,t} = z_{i,t}$$

# Law of Motion of Knowledge Capital

$$d\mathbf{z}_t = \left(\underbrace{\Omega\mathbf{z}_t}_{\text{tech spillover}} + \underbrace{\mu\mathbf{x}_t}_{\text{R\&D}}\right)dt + \underbrace{\gamma\mathbf{z}_t dW_t}_{\text{shocks}}$$

- $\mathbf{\Omega} = \left[\omega_{ij}
  ight]$ : technology spillover matrix
- $x_{i,t} = \sqrt{d_{i,t}}$ 
  - ullet  $d_{i,t}$ : R&D cost in terms of final good
  - Innovation elasticity is 0.5
- $\mu$ ,  $\gamma$ : positive scalars

# Market Clearing and Preference

Inelastic labor supply:

$$L = \sum_{i} l_{i,t}$$

Final good market clearing:

$$C_t + \underbrace{\sum_i d_{i,t}}_{\text{R\&D cost}} = Y_t$$

Risk neutral representative household:

$$\max \ \mathbf{E}_t \left[ \int_t^\infty \exp \left( -\rho s \right) C_s ds \right]$$

### Cournot-Nash Equilibrium

• Firm i's gross profit before subtracting dynamic R&D cost:

$$\pi_{i,t} = p_{i,t}q_{i,t} - w_tl_{i,t} = \left(b_{i,t} - \frac{w_t}{a_{i,t}} - \sum_i \sigma_{ij}q_{j,t}\right)q_{i,t} \quad \text{where} \quad \zeta a_{i,t} + b_{i,t} = z_{i,t}$$

• Given  $w_t$ ,  $z_{i,t}$ , and  $\left\{q_{j,t}\right\}_{i\neq i}$ , firm i chooses  $a_{i,t}$ ,  $b_{i,t}$ , and  $q_{i,t}$  to maximize  $\sum_j \kappa_{ij} \pi_{j,t}$ 

### Cournot-Nash Equilibrium

Firm *i*'s gross profit before subtracting dynamic R&D cost:

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- Given  $w_t$ ,  $z_{i,t}$ , and  $\left\{q_{j,t}\right\}_{i \neq j}$ , firm i chooses  $a_{i,t}$ ,  $b_{i,t}$ , and  $q_{i,t}$  to maximize  $\sum_{j} \kappa_{ij} \pi_{j,t}$
- Quantity is a linear function of knowledge capital:

$$\mathbf{q}_t(\mathbf{z}_t) = \left\{ \underbrace{2\frac{\zeta}{L}\mathbf{J}}_{\text{labor cost}} + \underbrace{\mathbf{\Sigma}}_{\text{substitutability}} + \underbrace{\mathbf{K} \circ \mathbf{\Sigma}}_{\text{ownership} \times \text{substitutability}} \right\}^{-1} \mathbf{z}_t$$

- J = [1]
- Ownership-weighted gross profits are expressed in quadratic form:  $\sum_i \kappa_{ij} \pi_{j,t} = \mathbf{z}_t^T \mathbf{Q}^i \mathbf{z}_t$

### Linear-Quadratic Differential Game

• Given other players' strategy  $\left\{x_{j,t}\right\}_{j\neq i,t\geq 0}$ , firm i chooses R&D effort  $\left\{x_{i,t}\right\}_{t\geq 0}$  to maximize

$$\max_{\left\{x_{i,t}\right\}_{t\geq0}}\quad V^{i}\left(\mathbf{z}_{0}\right)\equiv\mathbf{E}_{0}\left[\int_{0}^{\infty}\exp\left(-\rho t\right)\left\{\sum_{j}\kappa_{ij}\left(\pi_{j,t}-x_{j,t}^{2}\right)\right\}dt\right]$$

subject to  $d\mathbf{z}_t = (\mathbf{\Omega}\mathbf{z}_t + \mu\mathbf{x}_t)\,dt + \gamma\mathbf{z}_t dW_t$ 

Firm i's HJB equation:

$$\rho V^{i}\left(\mathbf{z}\right) = \max_{x_{i}} \left\{ \mathbf{z}^{T} \mathbf{Q}^{i} \mathbf{z} - \sum_{j} \kappa_{ij} x_{j}^{2} + V_{\mathbf{z}}^{i}\left(\mathbf{z}\right) \left[\mathbf{\Omega} \mathbf{z} + \mu \mathbf{x}\right] + \frac{\gamma^{2}}{2} \mathbf{z}^{T} V_{\mathbf{z}\mathbf{z}}^{i}\left(\mathbf{z}\right) \mathbf{z} \right\}$$

## HJB Equations ⇒ Riccati Equations

- Guess and verify  $V^i(\mathbf{z}) = \mathbf{z}^T \mathbf{X}^i \mathbf{z}$  (for any  $\mathbf{z}$ )
- ullet  $\mathbf{X}^i$  is the solution of stacked algebraic Riccati equations lacktriangle
- All public patenting firms in the US in our dataset  $\simeq$ 1000 firms  $\Longrightarrow$   $1000^3=1$  billion undetermined coefficients (20 seconds on my laptop)

Oligopolistic Schumpeterian	Computation time	# of firms	Productivity space
Cavenaile et al. (2023)	$O(2^n)$	4	6 grid
Our model	$O(n^4)$	≈1000	Continuous

#### **Balance Growth Path**

- R&D strategy:  $x_{i,t} = (\mu \mathbf{X}_i^i)^T \mathbf{z}_t$  where  $\mathbf{X}_i^i$  is the i th column of  $\mathbf{X}^i$
- The law of motion is rewritten as  $d\mathbf{z}_t = \mathbf{\Phi}\mathbf{z}_t dt + \gamma \mathbf{z}_t dW_t$  where

$$\mathbf{\Phi} \equiv \underbrace{\mathbf{\Omega}}_{\text{spillover}} + \underbrace{\mu^2 \left[ \mathbf{X}_1^1 \cdots \mathbf{X}_n^n \right]^T}_{\text{R&D}}$$

#### Theorem

Consider the deterministic economy ( $\gamma = 0$ ). If  $\Phi$  is irreducible, then:

- (i) There exists largest positive eigenvalue of  $\Phi$ , g, and associated positive eigenvector,  $\mathbf{z}^*$ .
- (ii) There exists a globally stable BGP such that the knowledge capital growth rate of all firms is g, and the knowledge capital distribution is a scalar multiple of  $\mathbf{z}^*$ .
  - Proof: Perron–Frobenius Theorem
  - " $\Phi$  is irreducible"  $\iff$  "All firms are directly or indirectly connected technologically"

### Intuition of the Existence of BGP

• On BGP,  $\mathbf{a}_t$ ,  $\mathbf{b}_t$ ,  $\mathbf{z}_t$ , and  $\mathbf{q}_t$  grow at the same rate

Technology Choice:  $\zeta a_{i,t} + b_{i,t} = z_{i,t}$ 

Linear Production Technology:  $q_{i,t} = a_{i,t} l_{i,t}$ 

Inelastic Labor Supply:  $L = \textstyle \sum_i l_{i,t}$ 

ullet Linear and quadratic term in  $old q_t$  of output grow at the same rate: Equilibrium Summary

$$Y_t = \mathbf{q}_t^T \mathbf{b}_t - \frac{1}{2} \mathbf{q}_t^T \mathbf{\Sigma} \mathbf{q}_t$$

## Aggregation

Apply Ito's lemma:

$$\begin{array}{ll} \log Y_t &= \log \left(\mathbf{z}_t^T \mathbf{Q} \mathbf{z}_t\right) \\ d\mathbf{z}_t &= \mathbf{\Phi} \mathbf{z}_t dt + \gamma \mathbf{z}_t dW_t \end{array} \implies \quad \mathbf{E}_t \left[ \left. g_t \right| \mathbf{z}_t \right] \end{array}$$

Expected utility is expressed in quadratic form:

$$\mathbf{E}_t \left[ \left. \int_{-\infty}^{\infty} \exp \left( -\rho s \right) C_s ds \right| \mathbf{z}_t \right] = \mathbf{z}_t^T X \mathbf{z}_t$$

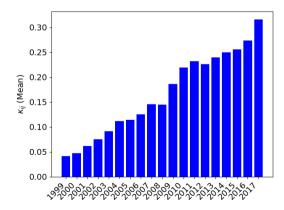
ullet Solve the equilibrium once  $\Longrightarrow$  Can compute expected growth and utility for any  $\mathbf{z}_t$ 



# Common Ownership ${f K}$

- Backus et al. (2021) construct a dataset on investors' holdings based on Form 13F
- Baseline: Rotemberg (1984) proportional influence assumption

Proportional Influece



## Product Market Rivalry ∑

- Hoberg and Phillips (2016) estimates product similarity using business descriptions in 10-K
- ullet Pellegrino (2024) estimates lpha to align with the cross-price elasticity of demand

$$\underbrace{\sigma_{ij}}_{\text{substitutability}} = \alpha \times \text{product similarity b/w } i \text{ and } j \quad (i \neq j)$$

# Technological Proximity $\widehat{\Omega}$

- Technological profile of firm i
  - ullet The vector of the share of patents held by firm i in each technology class
  - Baseline: group-level patent classifications ( $\approx 4000$ )
- Jaffe (1986) technology proximity measure  $\tilde{\omega}_{ij}$ :
  - $\bullet$  Cosine similarity of the technological profiles b/w firm i and j

# Distribution of Knowledge Capital

Variables	Identification
$\overline{\pi_{i,t}}$	Gross profit (before R&D cost) $=$ Revenue $-$ Cost of goods sold
$\mathbf{q}_t$	$\pi_{i,t} = \sum_i \kappa_{ij}  \sigma_{ij}  q_{i,t}  q_{j,t}$
$\zeta/L$	Matches sample firms' cost share
$\mathbf{z}_t$	$\mathbf{q}_t = \left\{ 2rac{\zeta}{L}\mathbf{J} + \mathbf{\Sigma} + \mathbf{K} \circ \mathbf{\Sigma}  ight\}^{-1} \mathbf{z}_t$

# Technology Spillover $oldsymbol{\Omega}=eta imes$ Technology Proximity $oldsymbol{\Omega}$

$$\boldsymbol{z}_{i,t+1} - \boldsymbol{z}_{i,t} = \beta \sum_{j \neq i} \tilde{\omega}_{ij,t} \boldsymbol{z}_{j,t} + \text{Year FE}_t + \epsilon_{i,t}$$

	(1)	(2)	(3)
	$\boldsymbol{z}_{i,t+1} - \boldsymbol{z}_{i,t}$	$z_{i,t+1}-z_{i,t} \\$	$z_{i,t+1}-z_{i,t} \\$
Σ ~ ~	0.000191***	0.000152***	0.000140***
$\sum_{j  eq i}  ilde{\omega}_{ij,t} z_{j,t}$	(0.000035)	(0.000035)	(0.000039)
DOD Even and it was		.037**	
√R&D Expenditure		(0.021)	
Year Fixed Effects	✓	✓	✓
IV			$\checkmark$
IV 1st Stage F-statistics			4176
No. observations	16,324	15,173	14,181

SEs clustered by years and 4-digit naics industries are reported in parentheses. \* p < 0.1, \*\* p < 0.05, \*\*\* p < 0.01.

• IV: Firm-specific tax price of R&D from federal and state-specific rules (Bloom et al., 2013)

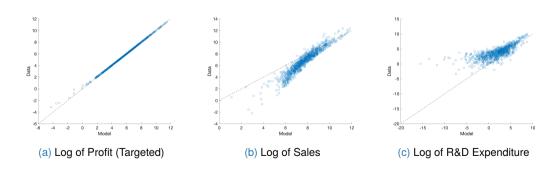
## Identification: Summary

Publicly available data + Compustat

Notation	Description	Value	Source
$oldsymbol{\Sigma}$	Product proximity		Form 10-K, Hoberg and Phillips (2016)
$\widetilde{\boldsymbol{\Omega}}$	Technological proximity		USPTO, Patent classification
$\mathbf{K}$	Common ownership weights		Form 13F, Backus et al. (2021)
$\alpha$	Product proximity $\rightarrow$ Substitutability	.12	Pellegrino (2024)
eta	Technological proximity $\rightarrow$ Spillover	.00014	Estimate the law of motion
$\gamma$	St.d. of idiosyncratic shocks	.027	Estimate the law of motion
$\zeta/L$	Labor augmentation efficiency	.0063	Compustat, Cost of goods sold
ho	Discount rate	.10	
$\mu$	R&D efficiency	.05	1.6% economic growth rate

#### Fit b/w Model and Data

Model (x-axis) vs Data (y-axis): firm-level profits (targeted), sales and R&D expenditures



# Exercise: Different Ownership Structures

Ownership Structure	Description
Baseline	Common ownership weights in 2017
Dispersed	$\mathbf{K}^D = \mathbf{I}$
Mean=1999	$\kappa_{ij,2017}^{M1999} = \mathrm{const}  imes \kappa_{ij,2017}$ and $\mathbf{E}\left[\kappa_{ij,2017}^{M1999} ight] = \mathbf{E}\left[\kappa_{ij,1999} ight]$ for $j  eq i$
Uniform	$\kappa^{U}_{ij,2017} = \mathbf{E}\left[\kappa_{ij,2017} ight]$ for $j  eq i$
Monopoly	$\mathbf{K}^M = [1]$

### **Total Output**

Total Output in 2017	Ownership (Baseline: 2017)				
(Social Optimum: 100)	Dispersed	Mean=1999	Uniform	Baseline	Monopoly
Baseline	91.30	91.02	90.78	89.08	89.17
Only Business Steal $oldsymbol{\Omega} = [0]$					
Only Tech Spill ${f \Sigma}={f I},\zeta/L=0$					

- ullet Inelastic labor supply  $\Longrightarrow$  Changes arise from product misallocation
- Common ownership exacerbates product misallocation

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Only Business Steal $oldsymbol{\Omega} = [0]$	91.30	91.02	90.78	89.08	89.17
Only Tech Spill $\mathbf{\Sigma} = \mathbf{I}, \ \zeta/L = 0$	75.00	75.00	75.00	75.00	75.00

- Inelastic labor supply ⇒ Changes arise from product misallocation
- Common ownership exacerbates product misallocation

### Total R&D Expenditure

Total R&D in 2017 (Social Optimum: 100)	Ownership (Baseline: 2017)				
	Dispersed	Mean=1999	Uniform	Baseline	Monopoly
Baseline	26.16	25.90	25.56	22.36	19.36
Only Business Steal $oldsymbol{\Omega} = [0]$					
Only Tech Spill $\mathbf{\Sigma} = \mathbf{I}, \ \zeta/L = 0$					

- Internalization of business stealing > Internalization of technology spillover
- Network heterogeneity is important

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Only Business Steal $oldsymbol{\Omega} = [0]$	28.15	27.79	27.07	23.42	19.80
Only Tech Spill $\mathbf{\Sigma} = \mathbf{I}, \ \zeta/L = 0$					

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Only Business Steal $oldsymbol{\Omega} = [0]$	28.15	27.79	27.07	23.42	19.80
Only Tech Spill ${f \Sigma}={f I},\ \zeta/L=0$	18.27	18.34	18.75	18.86	19.84

- Network heterogeneity is important

## **Expected Growth Rate**

Expected Economic	Ownership (Baseline: 2017)				
Growth Rate in 2017 (%)	Dispersed	Mean=1999	Uniform	Baseline	Monopoly
Baseline	1.796	1.793	1.791	1.753	1.713
Only Business Steal ${f \Omega}=[0]$					
Only Tech Spill					
$\Sigma = I, \zeta/L = 0$					

- In baseline, the expected growth rate is
  - 0.043 pp (2.4%) lower compared to dispersed ownership, and
  - 0.040 pp (2.2%) lower compared to common ownership level in 1999.

## **Expected Growth Rate**

Expected Economic	Ownership (Baseline: 2017)				
Growth Rate in 2017 (%)	Dispersed	Mean=1999	Uniform	Baseline	Monopoly
Baseline	1.796	1.793	1.791	1.753	1.713
Only Business Steal $oldsymbol{\Omega} = [0]$	1.097	1.094	1.093	1.062	1.020
Only Tech Spill ${f \Sigma}={f I},\zeta/L=0$					

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Only Business Steal $oldsymbol{\Omega} = [0]$	1.097	1.094	1.093	1.062	1.020
Only Tech Spill ${f \Sigma}={f I},\zeta/L=0$	2.051	2.054	2.068	2.072	2.107

- In baseline, the expected growth rate is
  - 0.043 pp (2.4%) lower compared to dispersed ownership, and
  - 0.040 pp (2.2%) lower compared to common ownership level in 1999.

## **Expected Social Welfare**

Expected Social Welfare	Ownership (Baseline: 2017)				
(Social Optimum: 100)	Dispersed	Mean=1999	Uniform	Baseline	Monopoly
Baseline	87.72	87.42	87.16	85.25	85.18
Only Business Steal $oldsymbol{\Omega} = [0]$					
Only Tech Spill $\mathbf{\Sigma} = \mathbf{I}, \zeta/L = 0$					

- In baseline, consumption equivalent welfare loss is
  - 2.8% CE compared to dispersed ownership, and
  - 2.5% CE compared to common ownership level in 1999.

## **Expected Social Welfare**

Expected Social Welfare	Ownership (Baseline: 2017)				
(Social Optimum: 100)	Dispersed	Mean=1999	Uniform	Baseline	Monopoly
Baseline	87.72	87.42	87.16	85.25	85.18
Only Business Steal $oldsymbol{\Omega} = [0]$	88.83	88.53	88.30	86.44	86.41
Only Tech Spill $oldsymbol{\Sigma} = \mathbf{I}, \zeta/L = 0$					

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  - 2.8% CE compared to dispersed ownership, and
  - 2.5% CE compared to common ownership level in 1999.

## **Expected Social Welfare**

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(Social Optimum: 100)	Dispersed	Mean=1999	Uniform	Baseline	Monopoly
Baseline	87.72	87.42	87.16	85.25	85.18
Only Business Steal ${f \Omega}=[0]$	88.83	88.53	88.30	86.44	86.41
Only Tech Spill $\mathbf{\Sigma} = \mathbf{I},\ \zeta/L = 0$	68.81	68.82	68.88	68.89	69.02

- In baseline, consumption equivalent welfare loss is
  - 2.8% CE compared to dispersed ownership, and
  - 2.5% CE compared to common ownership level in 1999.

#### Firm Value Share

Firm Value	Ownership (Baseline: 2017)				
Share in 2017 (%)	Dispersed	Mean=1999	Uniform	Baseline	Monopoly
Baseline	28.74	29.63	33.43	33.34	40.92
Only Business Steal ${f \Omega}=[0]$					
Only Tech Spill ${f \Sigma}={f I},\zeta/L=0$					

- In baseline, firm value share is
  - 5.6% lower compared to dispersed ownership, and
  - 4.7% lower compared to common ownership level in 1999.

#### Firm Value Share

	Ownership	: 2017)		
Dispersed	Mean=1999	Uniform	Baseline	Monopoly
28.74	29.63	33.43	33.34	40.92
27.91	28.80	32.60	33.51	40.14
	28.74	Dispersed Mean=1999 28.74 29.63	Dispersed Mean=1999 Uniform 28.74 29.63 33.43	20.00

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Only Business Steal					
$\mathbf{\Omega} = [0]$	27.91	28.80	32.60	33.51	40.14
Only Tech Spill					
$\Sigma = I, \zeta/L = 0$	64.82	64.81	64.76	64.74	64.63

- In baseline, firm value share is
  - 5.6% lower compared to dispersed ownership, and
  - 4.7% lower compared to common ownership level in 1999.

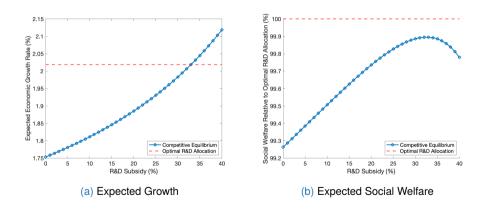
## When Common Ownership Affects only R&D Decisions

Common ownership only influence R&D decisions (d'Aspremont and Jacquemin, 1988)

	Ownership Structure			
	Dispersed	Common R&D	Baseline	
Output (Social Optimum: 100)	91.30	91.30	89.08	
R&D Expenditure (Social Optimum: 100)	26.17	19.76	22.36	
Expected Growth Rate (%)	1.796	1.726	1.753	
Expected Social Welfare (Social Optimum: 100)	87.72	87.49	85.25	
Firm Value Share (%)	28.74	29.04	34.34	

- Lowest R&D expenditure and expected growth rate
- Intermediate social welfare and firm value share

## Uniform R&D Subsidy Social Optimum



- Optimal rate is s = 33%, which increases g by 0.25 pp (14%)
- CE Welfare loss relative to optimal R&D allocation is reduced to 0.1% (Initially 0.7%)

#### Conclusion

- Quantitative Schumpeterian growth model with ownership structure
  - Utilize micro data and computational capability
- Common ownership in the US:
  - 1. Internalization of business stealing effect  $\Longrightarrow g \downarrow \downarrow$
  - 2. Internalization of technology spillover effect  $\Longrightarrow g \uparrow$
- Application:
  - Chaebols in Korea
  - Zaibatsu (pre-WWII) and cross-shareholding (late 20th century) in Japan
  - FDI / multinational companies and international technology diffusion

### Share of Top 5 Shareholders in Largest Market Cap Firms (Back)

Microsoft	
Vanguard	9.20%
Blackrock	7.75%
Steven Ballmer	4.48%
State Street	3.97%
Fidelity	2.66%

Nvidia	
Vanguard	8.93%
BlackRock	7.74%
Fidelity	4.12%
State Street	3.97%
Jensen Huang	3.80%

Apple	
Vanguard	9.29%
Blackrock	7.48%
State Street	3.96%
Fidelity	2.27%
Geode Capital	2.26%

Google	
Vanguard	7.36%
Blackrock	6.47%
State Street	3.39%
Fidelity	3.01%
Sergey Brin	2.99%

8.58%
7.77%
6.50%
3.44%
3.10%

Meta	
Vanguard	7.55%
Blackrock	6.50%
Fidelity	5.38%
Accel IX LP	3.88%
State Street	3.40%



# Equity Investments by Big tech in Al Startups (Back)

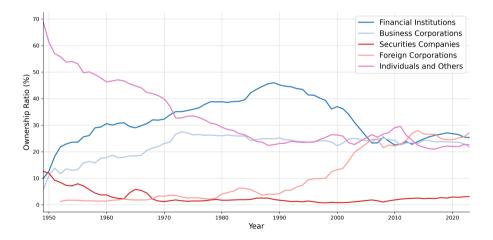
Shareholding percentage	Microsoft	Google	Amazon
OpenAl (ChatGPT)	49%	_	_
Anthropic (Claude)	_	14%	23%

## Technology & Product Proximity: Example

Tesla vs. Ford	
Technology Proximity	0.11
Product Proximity	0.15

Apple vs. Intel	
Technology Proximity	0.57
Product Proximity	0.00

# Ownership Ratio by Holder Types in Japan



# Rotemberg (1984) Proportional Influence

- $o \in \{1, 2, ..., n_o\}$ : owners
- $s_{io}$ : the proportion of shares in firm i owned by owner o where  $\sum_o s_{io} = 1$
- $\hat{V}_{i,t}$ : value of firm i
- $\tilde{V}_{o,t} \equiv \sum_i s_{io} \hat{V}_{i,t}$ : value of owner o
- Firms' objective:

$$\sum_{o} s_{io} \tilde{V}_{o,t} \propto \sum_{j} \kappa_{ij} \hat{V}_{j,t}$$

where

$$\kappa_{ij} \equiv \mathbf{s}_i^T \mathbf{s}_j / \mathbf{s}_i^T \mathbf{s}_i \quad \text{where} \quad \mathbf{s}_i \equiv \left[s_{i1},...,s_{io},...,s_{in_o}\right]^T$$

## **Total Surplus**

Total surplus for product i:

$$ts_i(\mathbf{q}, \mathbf{x}) = \pi_i(\mathbf{q}, \mathbf{x}) + cs_i(\mathbf{q}) = q_i \left[ b_i - \frac{1}{2} \sum_{i=1}^n \sigma_{ij} q_j - a_i + \sum_{i=1}^n \omega_{ij} x_j \right] - \frac{1}{2} x_i^2$$

#### **R&D** Externalities

- 1. Business stealing effect
  - Innovators steel the business (profits) of other firms
- 2. Technology spillover effect
  - Innovation improves the productivity of other firms
- 3. Appropriability effect (market power)
  - Innovators cannot appropriate the entire consumer surplus

#### R&D Allocation and Externalities

Firms maximize common owner weighted profits:

$$\mathbf{x}^* = (\mathbf{K} \circ \mathbf{\Omega})[\mathbf{\Sigma} + \mathbf{K} \circ \mathbf{\Sigma} - \mathbf{\Omega}(\mathbf{K} \circ \mathbf{\Omega})]^{-1}(\mathbf{b} - \mathbf{a})$$

Firms maximize common owner weighted total surplus (\*):

$$\mathbf{x}_{TS}^* = (\mathbf{K} \circ \mathbf{\Omega}) \left[ \frac{1}{2} (\mathbf{\Sigma} + \mathbf{K} \circ \mathbf{\Sigma}) - \mathbf{\Omega} (\mathbf{K} \circ \mathbf{\Omega}) \right]^{-1} (\mathbf{b} - \mathbf{a})$$

- $\mathbf{K} = [1]$  in  $(\star) \Longrightarrow$  Social Optimum
- Externalities: (i) Appropriability, (ii) Business stealing, (iii) Technology spillover

## Generalized Hedonic-Linear Demand (Pellegrino, 2024)

- $i \in \{1, 2, ..., n\}$ : firms / products
- 1 unit of product i provides
  - 1 unit of idiosyncratic characteristic  $k \in \{1, 2, ..., n\}$
  - $\psi_{k,i}$  unit of shared characteristic  $k \in \{n+1, n+2, ..., n+n_k\}$  where  $\sum_k \psi_{k,i}^2 = 1$
- Aggregate each characteristic:

$$y_{k,t} = \begin{cases} q_{k,t} & k = 1, 2, ..., n \\ \sum_{i} \psi_{k,i} q_{i,t} & k = n + 1, n + 2, ..., n + n_k \end{cases}$$

Linear quadratic aggregator over characteristics:

$$Y_t = (1-\alpha)\sum_{k=1}^n \left( \underbrace{\hat{b}_{k,t} y_{k,t} - \frac{1}{2} y_{k,t}^2}_{\text{idiosyncratic characteristic}} \right) + \alpha \sum_{k=n+1}^{n+n_k} \left( \underbrace{\hat{b}_{k,t} y_{k,t} - \frac{1}{2} y_{k,t}^2}_{\text{shared characteristic}} \right)$$

# Generalized Hedonic-Linear Demand (Pellegrino, 2024)

Quality:

$$b_i = (1 - \alpha)\,\hat{b}_i + \alpha \sum_{k=n+1}^{n+n_k} \psi_k \hat{b}_k$$

Inverse demand:

$$\frac{\mathbf{p}}{P} = \mathbf{b} - \mathbf{\Sigma}\mathbf{q}$$

• Inverse cross price elasticity of demand:

$$\frac{\partial \log p_i}{\partial \log q_j} = -\frac{q_j}{p_i} \cdot \sigma_{ij}$$

Cross price elasticity of demand:

$$\frac{\partial \log q_i}{\partial \log p_j} = -\frac{p_j}{q_i} (\mathbf{\Sigma}^{-1})_{ij}$$

#### Static Profits

- Real gross profit:  $rac{\pi_{i,t}}{P_t} = \sum_j \kappa_{ij} \sigma_{ij} q_{i,t} q_{j,t}$
- Firms choose labor productivity and product quality:  $\zeta a_{i,t} = \sqrt{\zeta \frac{w_t}{P_t}}$ ,  $b_{i,t} = z_{i,t} \sqrt{\zeta \frac{w_t}{P_t}}$
- Labor market clearing:  $L=\sum_i rac{q_{i,t}}{a_{i,t}} \Longrightarrow \sqrt{\zeta rac{w_t}{P_t}} = rac{\zeta}{L} \sum_i q_{i,t}$
- $ullet \mathbf{q}_t = \mathbf{N}\mathbf{z}_t$  where  $N \equiv \left\{2rac{\zeta}{L}\mathbf{J} + \mathbf{\Sigma} + \mathbf{K} \circ \mathbf{\Sigma}
  ight\}^{-1}$
- $N_i$ : the i th row of N
- Ownership weighted profit:

$$\sum_{j} \kappa_{ij} \frac{\pi_{j,t}}{P_t} = \sum_{j} \kappa_{ij} \sum_{h} \kappa_{jh} \sigma_{jh} q_{j,t} q_{h,t} = \mathbf{z}_t^T \mathbf{Q}^i \mathbf{z}_t$$

where

$$\mathbf{Q}^{i} = \frac{1}{2} \sum_{j} \kappa_{ij} \sum_{h} \kappa_{jh} \sigma_{jh} \left( N_{j}^{T} N_{h} + N_{h}^{T} N_{j} \right)$$

Back

### Riccati Equations

•  $V^{i}(\mathbf{z}) = \mathbf{z}^{T}\mathbf{X}^{i}\mathbf{z}$  where  $\mathbf{X}^{i}$  is the solution of the stacked Riccati equation

$$0 = \mathbf{Q}^{i} - \mu^{2} \sum_{j} \kappa_{ij} \mathbf{X}_{j}^{j} \left( \mathbf{X}_{j}^{j} \right)^{T} + \left( \mathbf{\Phi} - \frac{1}{2} \left( \rho - \gamma^{2} \right) \mathbf{I} \right)^{T} \mathbf{X}^{i} + \mathbf{X}^{i} \left( \mathbf{\Phi} - \frac{1}{2} \left( \rho - \gamma^{2} \right) \mathbf{I} \right)$$

- $\mathbf{X}_{i}^{i} \equiv$  the *i* th column of  $\mathbf{X}^{i}$
- $\Phi \equiv \mathbf{\Omega} + \mu^2 \begin{bmatrix} \mathbf{X}_1^1 & \cdots & \mathbf{X}_n^n \end{bmatrix}^T$
- Algorithm: Given  $\left[ egin{array}{ccc} {f X}_{ au}^1 & \cdots & {f X}_{ au}^n \end{array} 
  ight]$ , update  $\left[ egin{array}{ccc} {f X}_{ au-\Delta}^1 & \cdots & {f X}_{ au-\Delta}^n \end{array} 
  ight]$  by

$$-\frac{\mathbf{X}_{\tau}^{i}-\mathbf{X}_{\tau-\Delta}^{i}}{\Delta}=\mathbf{Q}^{i}-\mu^{2}\sum_{j}\kappa_{ij}\mathbf{X}_{j,\tau}^{j}\left(\mathbf{X}_{j,\tau}^{j}\right)^{T}+\left(\boldsymbol{\Phi}_{\tau}-\frac{1}{2}\left(\rho-\gamma^{2}\right)\mathbf{I}\right)^{T}\mathbf{X}_{\tau}^{i}+\mathbf{X}_{\tau}^{i}\left(\boldsymbol{\Phi}_{\tau}-\frac{1}{2}\left(\rho-\gamma^{2}\right)\boldsymbol{I}\right)^{T}\mathbf{X}_{\tau}^{i}+\mathbf{X}_{\tau}^{i}\left(\boldsymbol{\Phi}_{\tau}-\frac{1}{2}\left(\rho-\gamma^{2}\right)\boldsymbol{I}\right)^{T}\mathbf{X}_{\tau}^{i}+\mathbf{X}_{\tau}^{i}\left(\boldsymbol{\Phi}_{\tau}-\frac{1}{2}\left(\rho-\gamma^{2}\right)\boldsymbol{I}\right)^{T}\mathbf{X}_{\tau}^{i}+\mathbf{X}_{\tau}^{i}\left(\boldsymbol{\Phi}_{\tau}-\frac{1}{2}\left(\rho-\gamma^{2}\right)\boldsymbol{I}\right)^{T}\mathbf{X}_{\tau}^{i}+\mathbf{X}_{\tau}^{i}\left(\boldsymbol{\Phi}_{\tau}-\frac{1}{2}\left(\rho-\gamma^{2}\right)\boldsymbol{I}\right)^{T}\mathbf{X}_{\tau}^{i}+\mathbf{X}_{\tau}^{i}\left(\boldsymbol{\Phi}_{\tau}-\frac{1}{2}\left(\rho-\gamma^{2}\right)\boldsymbol{I}\right)^{T}\mathbf{X}_{\tau}^{i}+\mathbf{X}_{\tau}^{i}\left(\boldsymbol{\Phi}_{\tau}-\frac{1}{2}\left(\rho-\gamma^{2}\right)\boldsymbol{I}\right)^{T}\mathbf{X}_{\tau}^{i}+\mathbf{X}_{\tau}^{i}\left(\boldsymbol{\Phi}_{\tau}-\frac{1}{2}\left(\rho-\gamma^{2}\right)\boldsymbol{I}\right)^{T}\mathbf{X}_{\tau}^{i}+\mathbf{X}_{\tau}^{i}\left(\boldsymbol{\Phi}_{\tau}-\frac{1}{2}\left(\rho-\gamma^{2}\right)\boldsymbol{I}\right)^{T}\mathbf{X}_{\tau}^{i}+\mathbf{X}_{\tau}^{i}\left(\boldsymbol{\Phi}_{\tau}-\frac{1}{2}\left(\rho-\gamma^{2}\right)\boldsymbol{I}\right)^{T}\mathbf{X}_{\tau}^{i}+\mathbf{X}_{\tau}^{i}\left(\boldsymbol{\Phi}_{\tau}-\frac{1}{2}\left(\rho-\gamma^{2}\right)\boldsymbol{I}\right)^{T}\mathbf{X}_{\tau}^{i}+\mathbf{X}_{\tau}^{i}\left(\boldsymbol{\Phi}_{\tau}-\frac{1}{2}\left(\rho-\gamma^{2}\right)\boldsymbol{I}\right)^{T}\mathbf{X}_{\tau}^{i}+\mathbf{X}_{\tau}^{i}\left(\boldsymbol{\Phi}_{\tau}-\frac{1}{2}\left(\rho-\gamma^{2}\right)\boldsymbol{I}\right)^{T}\mathbf{X}_{\tau}^{i}+\mathbf{X}_{\tau}^{i}\left(\boldsymbol{\Phi}_{\tau}-\frac{1}{2}\left(\rho-\gamma^{2}\right)\boldsymbol{I}\right)^{T}\mathbf{X}_{\tau}^{i}+\mathbf{X}_{\tau}^{i}\left(\boldsymbol{\Phi}_{\tau}-\frac{1}{2}\left(\rho-\gamma^{2}\right)\boldsymbol{I}\right)^{T}\mathbf{X}_{\tau}^{i}+\mathbf{X}_{\tau}^{i}\left(\boldsymbol{\Phi}_{\tau}-\frac{1}{2}\left(\rho-\gamma^{2}\right)\boldsymbol{I}\right)^{T}\mathbf{X}_{\tau}^{i}+\mathbf{X}_{\tau}^{i}\left(\boldsymbol{\Phi}_{\tau}-\frac{1}{2}\left(\rho-\gamma^{2}\right)\boldsymbol{I}\right)^{T}\mathbf{X}_{\tau}^{i}+\mathbf{X}_{\tau}^{i}\left(\boldsymbol{\Phi}_{\tau}-\frac{1}{2}\left(\rho-\gamma^{2}\right)\boldsymbol{I}\right)^{T}\mathbf{X}_{\tau}^{i}+\mathbf{X}_{\tau}^{i}\left(\boldsymbol{\Phi}_{\tau}-\frac{1}{2}\left(\rho-\gamma^{2}\right)\boldsymbol{I}\right)^{T}\mathbf{X}_{\tau}^{i}+\mathbf{X}_{\tau}^{i}\left(\boldsymbol{\Phi}_{\tau}-\frac{1}{2}\left(\rho-\gamma^{2}\right)\boldsymbol{I}\right)^{T}\mathbf{X}_{\tau}^{i}+\mathbf{X}_{\tau}^{i}\left(\boldsymbol{\Phi}_{\tau}-\frac{1}{2}\left(\rho-\gamma^{2}\right)\boldsymbol{I}\right)^{T}\mathbf{X}_{\tau}^{i}+\mathbf{X}_{\tau}^{i}\left(\boldsymbol{\Phi}_{\tau}-\frac{1}{2}\left(\rho-\gamma^{2}\right)\boldsymbol{I}\right)^{T}\mathbf{X}_{\tau}^{i}+\mathbf{X}_{\tau}^{i}\left(\boldsymbol{\Phi}_{\tau}-\frac{1}{2}\left(\rho-\gamma^{2}\right)\boldsymbol{I}\right)^{T}\mathbf{X}_{\tau}^{i}+\mathbf{X}_{\tau}^{i}\left(\boldsymbol{\Phi}_{\tau}-\frac{1}{2}\left(\rho-\gamma^{2}\right)\boldsymbol{I}\right)^{T}\mathbf{X}_{\tau}^{i}+\mathbf{X}_{\tau}^{i}\left(\boldsymbol{\Phi}_{\tau}-\frac{1}{2}\left(\rho-\gamma^{2}\right)\boldsymbol{I}\right)^{T}\mathbf{X}_{\tau}^{i}+\mathbf{X}_{\tau}^{i}\left(\boldsymbol{\Phi}_{\tau}-\frac{1}{2}\left(\rho-\gamma^{2}\right)\boldsymbol{I}\right)^{T}\mathbf{X}_{\tau}^{i}+\mathbf{X}_{\tau}^{i}\left(\boldsymbol{\Phi}_{\tau}-\frac{1}{2}\left(\rho-\gamma^{2}\right)\boldsymbol{I}\right)^{T}\mathbf{X}_{\tau}^{i}+\mathbf{X}_{\tau}^{i}\left(\boldsymbol{\Phi}_{\tau}-\frac{1}{2}\left(\rho-\gamma^{2}\right)\boldsymbol{I}\right)^{T}\mathbf{X}_{\tau}^{i}+\mathbf{X}_{\tau}^{i}\left(\boldsymbol{\Phi}_{\tau}-\frac{1}{2}\left(\rho-\gamma^{2}\right)\boldsymbol{I}\right)^{T}\mathbf{X}_{\tau}^{i}+\mathbf{X}_{\tau}^{i}\left(\boldsymbol{\Phi}_{\tau}-\frac{1}{2}\left(\rho-\gamma^{2}$$

# Summary of Equilibrium

Description	Expression
Production strategy	$\mathbf{q}_t = N\mathbf{z}_t$
R&D strategy	$\mathbf{x}_t = \mu  ilde{X} \mathbf{z}_t$
Law of motion	$d\mathbf{z}_t = (\Omega \mathbf{z}_t + \mu \mathbf{x}_t) dt + \gamma \mathbf{z}_t dW_t$
Profit of final producers	$\Pi_t^F/P_t = \mathbf{q}_t^T\left(rac{1}{2}\Sigma ight)\mathbf{q}_t$
Total operating profit of firms	$\Pi_t/P_t = \mathbf{q}_t^T \left( \frac{1}{2} \Sigma \circ (K + K^T) \right) \mathbf{q}_t$
Labor income	$w_t L/P_t = \mathbf{q}_t^T \left( rac{\zeta}{L} J  ight) \mathbf{q}_t$
Output	$Y_t = \mathbf{q}_t^T \left( \frac{\zeta}{L} J + \frac{1}{2} \Sigma + \frac{1}{2} \Sigma \circ (K + K^T) \right) \mathbf{q}_t$
Consumption	$C_t = Y_t - \mathbf{x}_t^T \mathbf{x}_t$

# Output and Expected Utility

• Output:  $Y_t = \mathbf{q}_t^T Q \mathbf{q}_t$  where

$$Q = \frac{\zeta}{L}J + \frac{1}{2}\Sigma + \frac{1}{2}\Sigma \circ \left(K + K^T\right)$$

Expected utility:

$$V(\mathbf{z}_t) \equiv \mathbf{E}_t \left[ \left. \int_t^\infty \exp\left(-\rho s\right) C_s ds \right| \mathbf{z}_t \right] = \mathbf{z}_t^T X \mathbf{z}_t$$

where X is the solution of the Lyapunov equation (obtained from households' HJB equation):

$$0 = Q - \mu^2 \tilde{X}^T \tilde{X} + X \left(\Phi - \frac{1}{2} \left(\rho - \gamma^2\right) I\right) + \left(\Phi - \frac{1}{2} \left(\rho - \gamma^2\right) I\right)^T X$$



## Social Optimum

- Static optimal allocation:  $\mathbf{q}_t^* = N^*\mathbf{z}_t$  where  $N^* \equiv \left\{2\frac{\zeta}{L}J + \Sigma\right\}^{-1}$
- $\bullet$  Optimal output:  $Y_t^* = \mathbf{z}_t^T Q^* \mathbf{z}_t$  where  $Q^* = \frac{1}{2} N^*$
- Optimal expected utility:

$$V^*\left(\mathbf{z}_t\right) \equiv \mathbf{E}_t \left[ \left. \int_t^\infty \exp\left(-\rho s\right) C_s ds \right| \mathbf{z}_t \right] = \mathbf{z}_t^T X^* \mathbf{z}_t,$$

where  $X^*$  is the solution of the Riccati equation (obtained from planner's HJB equation):

$$0 = Q^* - \mu^2 \left(X^*\right)^2 + X^* \left(\Phi^* - \frac{1}{2} \left(\rho - \gamma^2\right) I\right) + \left(\Phi^* - \frac{1}{2} \left(\rho - \gamma^2\right) I\right) X^*$$

- Optimal R&D:  $\mathbf{x}_t^* = \mu X^* \mathbf{z}_t$
- Optimal technology transition matrix:  $\Phi^* = \Omega + \mu^2 X^*$

# Stochastic Process of Output

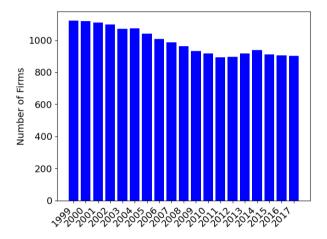
Applying It's lemma,

$$d\log Y_t = \left[\frac{\mathbf{z}_t^T \left(Q\Phi + \Phi^T Q\right)\mathbf{z}_t}{Y_t} + \gamma^2 \left\{\frac{\sum_i z_{i,t}^2 Q_{ii}}{Y_t} - \frac{2\mathbf{z}_t^T Q \operatorname{diag}\left(\mathbf{z}_t^2\right) Q\mathbf{z}_t}{Y_t^2}\right\}\right] dt + \frac{2\gamma \mathbf{z}_t^T Q \operatorname{diag}\left(\mathbf{z}_t\right)}{Y_t} dW_t$$

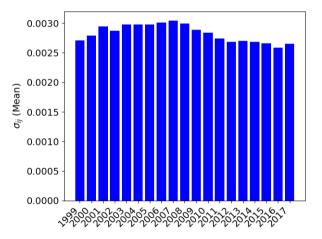
where 
$$Y_t = \mathbf{z}_t^T Q \mathbf{z}_t$$
 and  $\Phi = \Omega + \mu^2 \widetilde{X}$ 

Description	Expression
Tech Spillover	$\mathbf{z}_t^T (Q\Omega + \Omega Q)  \mathbf{z}_t / Y_t$
R&D	$\mu^2 \mathbf{z}_t^T ig(Q \widetilde{X} + \widetilde{X}^T Qig)  \mathbf{z}_t / Y_t$
Ito	$\gamma^2 \left\{ \sum_i z_{i,t}^2 Q_{ii}/Y_t - 2\mathbf{z}_t^T Q \mathrm{diag}\left(\mathbf{z}_t^2 ight) Q \mathbf{z}_t/Y_t^2  ight\}$
Total	$\mathbf{E}\left[d\log Y_{t} ight]$

# Number of Sample Firms

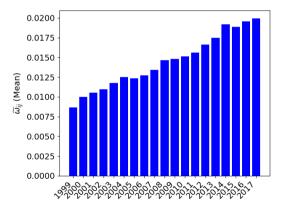


## Trend of Product Substitutability

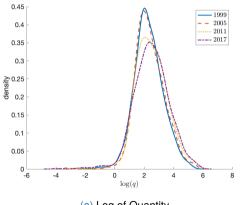


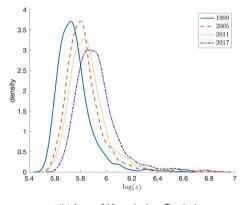
# **Technological Proximity**

- Merge USPTO data with Compustat firms using DISCERN 2 dataset (Arora et al., 2024)
- Jaffe measure, Group-level patent classification, Stack for 5 years



# Distributions of Estimated Knowledge Capital and Quantity





(a) Log of Quantity

(b) Log of Knowledge Capital

# First Stage Back

	(1)
Dependent Variable:	$z_{i,t}$
User cost of R&D	-39.495***
	(4.7044)
Year Fixed Effects	✓
No. observations	12,947

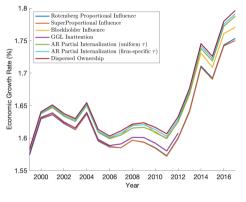
SEs clustered by years and 4-digit naics industries are reported in parentheses.

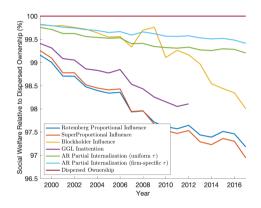
• IV: User cost of R&D, driven by state-level tax variations (Wilson, 2009; Bloom et al., 2013)

# Alternative Corporate Governance Models: Ederer and Pellegrino (2024)

- 1. Super-proportional influence:  $\tilde{\kappa}_{ij}=\frac{\sum_{z=1}^Z s_{iz}\gamma_{iz}s_{jz}}{\sum_{z=1}^Z s_{iz}\gamma_{iz}s_{iz}}$  where  $\gamma_{iz}=\sqrt{s_{iz}}$
- 2. Blockholder influence:  $\tilde{\kappa}_{ij}=\frac{\sum_{z=1}^Z s_{iz}b_{iz}s_{jz}}{\sum_{z=1}^Z s_{iz}s_{jz}} \quad (i\neq j),$  where  $b_{iz}=1$  if  $s_{iz}>5\%$
- 3. Rational investor inattention
  - Gilje et al. (2020) (GGL) estimate the probability that an investor votes against Institutional Shareholders Service recommendations
  - Utilize the estimate to capture the investor's level of attention
- 4. Governance frictions and entrenchment
  - Azar and Ribeiro (2021) (AR) estimate an objective function where the manager of firm i discounts other firms' profit by  $\tau_i$

### Alternative Corporate Governance Models





(a) Expected Growth

(b) Expected Social Welfare

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