Least Squares Estimation - SLR

$$y_i = a + bx_i + \varepsilon_i$$

$$SSE = \sum_{i=1}^{N} (y_i - a - bx_i)^2$$

the LS estimators are the values of a & b that minimize SSE. To minimize SSE we take derivatives of SSE and set to 0, and solve the system

$$\frac{\partial SSE}{\partial a} = 0 \quad ; \quad \frac{\partial SSE}{\partial b} = 0$$

$$\frac{\partial SSE}{\partial a} = \sum_{i=1}^{N} \frac{\partial}{\partial a} (y_i - a - bx_i)^2$$

$$= -2 \sum_{i=1}^{N} (y_i - a - bx_i)$$

$$\rightarrow \sum_{i=1}^{N} (y_i - a - bx_i) = 0$$

$$\rightarrow \bar{y} - a - b\bar{x} = 0 \rightarrow a = \bar{y} - b\bar{x} \quad (1)$$

$$\frac{\partial SSE}{\partial b} = \sum_{i=1}^{N} \frac{\partial}{\partial b} (y_i - a - b x_i)^2$$

$$= -2 \sum_{i=1}^{N} (y_i - a - b x_i) x_i$$

Set $=0$ so that

$$\sum y_i x_i - a \sum x_i - b \sum x_i^2 = 0 \quad (2)$$

Now sub (1) into (2)

$$\sum y_i x_i - (\bar{y} - b\bar{x}) \sum x_i - b \sum x_i^2 = 0$$

Now solve for b,

$$\sum y_i x_i - \frac{1}{N} \left(\sum y_i\right) \sum x_i = b \sum x_i^2 - \frac{b \left(\sum x_i\right)^2}{N}$$

$$\rightarrow \hat{b} = \frac{\sum x_i y_i - \frac{1}{N} \left(\sum y_i\right)\left(\sum x_i\right)}{\sum x_i^2 - \frac{\left(\sum x_i\right)^2}{N}}$$

then (1) gives $\hat{a} = \bar{y} - \hat{b}\,\bar{x}$

so we have derived the estimators that minimize SSE.