$\hat{\mathrm{Var}}[\hat{f}(t)]$

$$\hat{f}(t) = f_t(\hat{a},\hat{c}) = \hat{a} - \hat{a}\exp\{-\hat{c}t\}$$

- First Order Taylor Expansion about (a_0, c_0), where (a_0, c_0) are the unknown true values of the parameters.

$$- f_t(\hat{a},\hat{c}) \approx f_t(a_0, c_0) + \nabla f_t(a_0, c_0)' \left(\begin{pmatrix} \hat{a} \\ \hat{c} \end{pmatrix} - \begin{pmatrix} a_0 \\ c_0 \end{pmatrix} \right)$$

where $\nabla f_t = \begin{pmatrix} \frac{\partial f_t}{\partial a} \\ \frac{\partial f_t}{\partial c} \end{pmatrix}$

$$\frac{\partial f_t}{\partial a} = 1 - \exp\{-ct\}$$

$$\frac{\partial f_t}{\partial c} = t a \exp\{-ct\}$$

write

$$f_t(\hat{a},\hat{c}) \approx f_t(a_0,c_0) - \nabla f_t(a_0,c_0)'\begin{pmatrix} a_0 \\ c_0 \end{pmatrix}$$

$$+ \nabla f_t(a_0,c_0)'\begin{pmatrix} \hat{a} \\ \hat{c} \end{pmatrix}$$

then

$$\mathrm{Var}\left[f_t(\hat{a},\hat{c})\right] = \mathrm{Var}\left[\nabla f_t(a_0,c_0)'\begin{pmatrix} \hat{a} \\ \hat{c} \end{pmatrix}\right]$$

(Note $\mathrm{Var}[Ax] = A\,\mathrm{Var}[x]\,A'$ for constant matrix A and random vector x)

$$= \nabla f_t(a_0,c_0)'\,\mathrm{Var}\left[\begin{pmatrix} \hat{a} \\ \hat{c} \end{pmatrix}\right]\nabla f_t(a_0,c_0)$$

and we can estimate this quantity with

$$\nabla f_t(\hat{a},\hat{c})'\, C_{\hat{a},\hat{c}}\, \nabla f_t(\hat{a},\hat{c})$$

$$= \left(1-\exp\{-\hat{c}t\},\ t\hat{a}\exp\{-\hat{c}t\}\right)$$

$$\times\ C_{\hat{a},\hat{c}} \times \begin{pmatrix} 1-\exp\{-\hat{c}t\} \\ t\hat{a}\exp\{-\hat{c}t\} \end{pmatrix}$$