University of Victoria | Mathematics & Statistics

Statistics 453/558 Midterm Test 1

February 18th, 2023

Name: **V#:**

Instructor: M. Miranda Duration: 50 minutes Total: 25 points

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1. (**13 marks**) An article in Communications of the ACM (Vol. 30, No. 5, 1987) studied different algorithms for estimating software development costs. Six algorithms were applied to several different software development projects and the percent error in estimating the development cost was observed. The goal is to identify which algorithm is better at estimating the development cost. The data from this experiment is show in the table below.

Table 1: Percent error in estimating the development cost of different algorithms.

Algorithm	Project 1	2	3	4	5	6
1 (SLIM)	1244	21	82	2221	905	839
2 (COCOMO-A)	281	129	396	1306	336	910
3 (COCOMO-R)	220	84	458	543	300	794
4 (COCOMO-C)	225	83	425	552	291	826
5 (FUNCTION POINTS)	19	11	-34	121	15	103
6 (ESTIMALS)	-20	35	-53	170	104	199

(a) Which design was implemented in this experiment? **(1 mark)**

Randomized complete block design

RCBD

(b) Based on your answer in (a), write the statistical model (equation of the model), listing each component. **(2 marks)**

$Y_{ij} = \mu + \tau_i + \beta_j + \epsilon_{ij}$

Y_{ij} is the response variable at the i-th treatment level and j-th block

μ is the overall mean

τ_i is the i-th treatment effect

β_j is the j-th block level

ϵ_{ij} is the error term

(c) Based on your answer in (a) and $\alpha = 0.05$, test the hypothesis that the algorithms do not differ in their mean cost estimation accuracy. Write the hypotheses, the test statistic and its distribution. **(4 marks)**

1) $H_0: \mu_1 = \dots = \mu_6$ vs H_1: At least one $\mu_i \neq \mu_j$

2) $F_0 = 5.377$.

3) $F_0 \sim F_{5,25}$

Since p-value $= 0.00172 < \alpha = 0.05$ we reject the hypothis of equality of the means.

(d) State the assumptions being checked and perform model adequacy checking. **(2 marks)**

We will check normality and constant variance of the error terms. The normality assumption is not violated but we seem to have an issue with the variance.

(e) Depending on your answer in part (c), perform pairwise comparisons to identify which algorithms are different from each other. **(2 marks)**

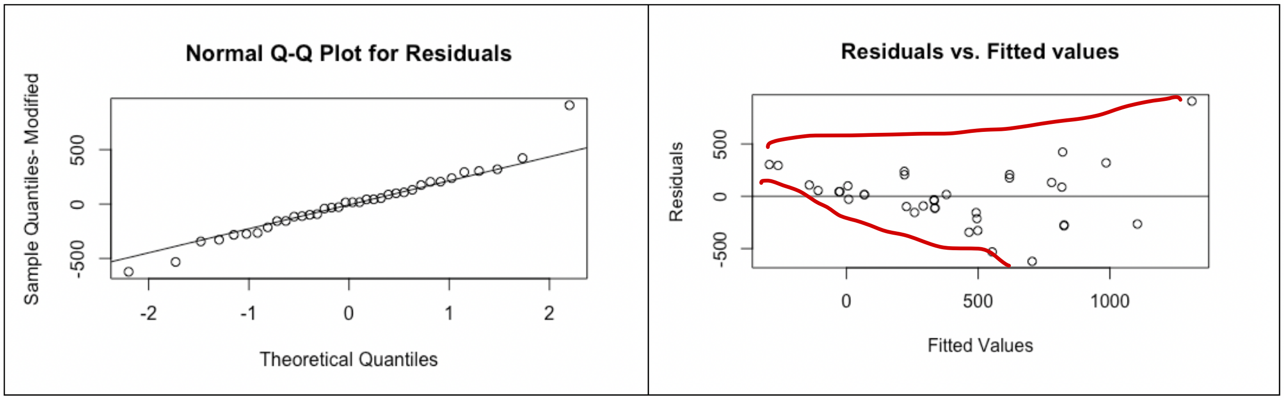
Based on the Tukey test, at $\alpha = 0.05$ we identify differences between algoritms 1 and 5 and 1 and 6.

(f) Is there an algorithm that produces lower error in estimating the development coast? If no, justify. If yes, which one? **(2 marks)**

Yes, algoritm 5 produces smaller error on average.

R output for Q1

```
> summary(res.aov)
                   Df  Sum Sq Mean Sq F value  Pr(>F)
factor(Algorithm)  5 2989130  597826   5.377 0.00172 **
factor(Project)    5 2287339  457468   4.115 0.00730 **
Residuals         25 2779574  111183
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```




```
    Algorithm count  mean    sd
1           1     6  885.0 813.0
2           2     6  560.0 452.0
3           3     6  400.0 253.0
4           4     6  400.0 264.0
5           5     6   39.2  59.8
6           6     6   72.5 102.0
```

```
$`factor(Algorithm)`
          diff        lwr        upr     p adj
2-1 -325.66667  -918.9482  267.61488 0.5494430
3-1 -485.50000 -1078.7815  107.78154 0.1557492
4-1 -485.00000 -1078.2815  108.28154 0.1565034
5-1 -846.16667 -1439.4482 -252.88512 0.0022068
6-1 -812.83333 -1406.1149 -219.55179 0.0033964
3-2 -159.83333  -753.1149  433.44821 0.9589482
4-2 -159.33333  -752.6149  433.94821 0.9594801
5-2 -520.50000 -1113.7815   72.78154 0.1098405
6-2 -487.16667 -1080.4482  106.11488 0.1532561
4-3    0.50000  -592.7815  593.78154 1.0000000
5-3 -360.66667  -953.9482  232.61488 0.4404428
6-3 -327.33333  -920.6149  265.94821 0.5441191
5-4 -361.16667  -954.4482  232.11488 0.4389450
6-4 -327.83333  -921.1149  265.44821 0.5425234
6-5   33.33333  -559.9482  626.61488 0.9999754
```

2. (**6 marks**) An industrial engineer is investigating the effect of four assembly methods (A, B, C, D) on the assembly time for a color television component. Four operators are selected for the study. Furthermore, the engineer knows that each assembly method produces such fatigue that the time required for the last assembly may be greater than the time required for the first, regardless of the method. That is, a trend develops in the required assembly time. To account for this source of variability, the engineer uses the Latin square design shown below.

Table 2: Assembly time for a color television component.

$Y_{ij} = \mu + T_i + B_j + \epsilon_{ij}$

row

	Operators			
Order of Assembly	1	2	3	4
1	C=10	D=14	A=7	B=8
2	B=7	C=18	D=11	A=8
3	A=5	B=10	C=11	D=9
4	D=10	A=10	B=12	C=14

assembly

$\alpha_i + \tau_j + \beta_k$ — column (ope)

(a) Write the statistical model for this data. **(2 marks)**

$Y_{ijk} = \mu + \cancel{T_i + B_j + \gamma_k} + \epsilon_{ijk}$

response

overall mean

Assembly method

Order (Block)

Operators (Block)

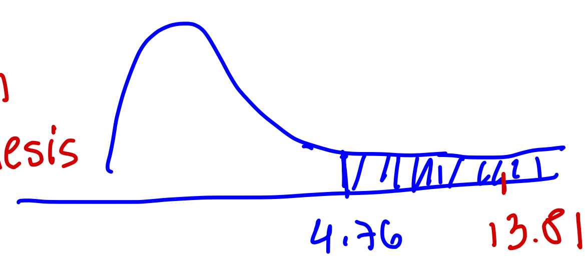
random error

(b) Complete the 6 empty spaces of the ANOVA table below and determine if there are any significant differences among the treatments for $\alpha = 0.05$. **(4 marks)**

	Df	SS	MS	F
Assembly Methods	3	72.5	24.167	13.81
Order	3	18.5	6.167	—
Operator	3	51.5	17.167	—
Residuals	6	10.5	1.750	—

$F_0 \sim F_{3,6}$

Based on the rejection region method, we reject the hypothesis of equality of treatments.



3. (**6 marks**) A product developer is investigating the tensile strength of a new synthetic fiber that will be used to make cloth for men's shirts. Strength is usually affected by the percentage of cotton used in the blend of materials for the fiber. The engineer conducts a completely randomized experiment with five levels of cotton content and replicated the experiment five times. The data are shown in the following table.

Table 3: Tensile strength of a new synthetic fiber.

Cotton Weight Percentage	Observations					Row Total
15 %	7	7	15	11	9	49
20 %	12	17	12	18	18	77
25 %	14	19	19	18	18	88
30 %	19	25	22	19	23	108
35 %	7	10	11	15	11	54
Total						376

```
> summary(res.aov)
                    Df  Sum Sq Mean Sq  F value    Pr(>F)
factor(WeightPerc)   4  475.8  118.94   14.76     9.13e-06 ***
Residuals           20  161.2    8.06
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Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The statistical model for this single-factor ANOVA model is:

$$Y_{ij} = \mu + \tau_i + \epsilon_{ij}, \quad i = 1, \ldots, 5; \, j = 1, \ldots, 5$$

where μ is the overall mean, τ_j is ith treatment effect, and $\epsilon_{ij} \sim N(0, \sigma^2)$.

(a) What are the values of the least squares estimators for μ and τ_3? **(4 marks)**

(b) What is the value of the unbiased estimator for σ^2? **(2 marks)**

a) $\hat{\mu} = \bar{y}_{..} = \frac{376}{25} = 15.04$

$\hat{\tau}_3 = \bar{y}_{3.} - \bar{y}_{..} = \frac{88}{5} - 15.04 = 2.56$

b) $\hat{\sigma}^2 = 8.06 = MSE$