



Statistics 453/558 Midterm Test 2

March 23rd, 2023

Name:

V#:

Instructor: M. Miranda

Duration: 50 minutes

Total: 25 points

1. [5 marks] Consider the 2^4 design in four blocks with ABD, and ABC as the independent effects chosen to be confounded with blocks. Generate the design by writing down the block number for each label on your table. Find the other effect confounded with blocks.

Runs	A	B	C	D	ABC	ABD	Block	(1)
(1)	-	-	-	-	-	-	1	-
a	+	-	-	-	+	+	2	+
b	-	+	-	-	+	+	2	
ab	+	+	-	-	-	-	1	(0,0)
c	-	-	+	-	+	-	3	(1,0)
ac	+	-	+	-	-	+	4	(0,-1)
bc	-	+	+	-	-	+	4	
abc	+	+	+	-	+	-	3	
d	-	-	-	+	-	+	4	
ad	+	-	-	+	+	-	3	
bd	-	+	-	+	+	-	3	
cd	-	-	+	+	+	+	2	
abd	+	+	-	+	-	+	4	
acd	+	-	+	+	-	-	1	
bcd	-	+	+	+	-	-	1	
abcd	+	+	+	+	+	+	2	

- The other effect that is confounded with blocks is the generalized interaction CD.
 $(ABD \times ABC = CD)$

2. [12 marks] One-quarter fraction of the 2^k design.

- Generate a 2^{5-2}_{III} design using the defining relations (design generators) $I = -ABD$ and $I = -ACE$ by completing the table below.
- Find all aliases for the main factor A.
- Suppose factors B and E are not significant. Project the 2^{5-2}_{III} into the subset of significant factors. Write the run labels for the projected model in the last column of the table below. Based on these new labels, is the projected design a 2^3 design?
- Use the **contrast notation** to write down the estimated main effect of D for your model in (c).
- Use the **contrast notation** to write down the Mean Square [redacted] for factor D for your model in (c).

a)

A	B	C	D = -AB	E = -AC	Run label 2^{5-2}_{III}	Run label projected model
-	-	-	-	-	(1)	(1)
+	-	-	+	+	ade	ad
-	+	-	+	-	bd	d
+	+	-	-	+	abe	a
-	-	+	-	+	ce	c
+	-	+	+	-	acd	acd
-	+	+	+	+	bcd	cd
+	+	+	-	-	abc	ac

b) $A = -BD = -CE = ABCDE$

c) Yes, the new projected design is a 2^3 design on factors A, C, and D

d) Contrast $D = -(1) + ad + d - a - c + acd + cd - ac$
 $D = \frac{\text{Contrast}}{2^{(5-2)-1} \cdot n} = \frac{-(1) + ad + d - a - c + acd + cd - ac}{4}$

e) $MSD = \frac{(\text{Contrast})^2}{2^{(5-2)} \cdot n} = \frac{(-(1) + ad + d - a - c + acd + cd - ac)^2}{8}$

3. [8 marks] One half fraction fractional design.

- (a) Construct a 2^{5-1} design with the highest resolution using the generators $E = \pm ABCD$ and writing down the runs of **both fractions**.

Hint: Use the same table and add columns for both generators and columns for the labels for both generators. Make sure your table heading clearly distinguishes between both designs.

- (b) What is the alias structure for all main effects of the model in (a) for both fractions?

a)

	Runs						
generator $E = + ABCD$	A	B	C	D	$E = + ABCD$	$E = - ABCD$	generator $E = - ABCD$
e	-	-	-	-	+	-	(1)
a	+	-	-	-	-	+	ae
b	-	+	-	-	-	+	be
abe	+	+	-	-	+	-	ab
c	-	-	+	-	-	+	ce
ace	+	-	+	-	+	-	ac
bce	-	+	+	-	+	-	bc
abc	+	+	+	-	-	+	abce
d	-	-	-	+	-	+	de
ade	+	-	-	+	+	-	ad
bde	-	+	-	+	+	-	bd
cde	-	-	+	+	+	-	cd
acd	+	-	+	+	-	+	acde
bcd	-	+	+	+	-	+	bcede
abd	+	+	-	+	-	+	abde
abcde	+	+	+	+	+	-	abcd

b) Principal fraction: $A = BCDE$, $B = ACDE$, $C = ABDE$
 $D = ABCE$, $E = ABCD$

Also correct

$$[A] = A + BCDE$$

$$[D] = D + ABCE$$

$$[B] = B + ACDE$$

$$[E] = E + ABCD$$

$$[C] = C + ABDE$$

Alternate fraction:

$$A = -BCDE, \quad B = -ACDE, \quad C = -ABDE$$
$$D = -ABCE, \quad E = -ABCD$$

Also correct

$$[A] = A - BCDE$$

$$[B] = B - ACDE$$

$$[C] = C - ABDE$$

$$[D] = D - ABCE$$

$$[E] = E - ABCD$$