## Summary

### Jointly Distributed Random Variables

- If $X$ and $Y$ are discrete random variables, then their *joint probability mass function* $p(x, y)$ is defined by

$$p(x,y) = P\{X = x \text{ and } Y = y\}$$

- We say that $X$ and $Y$ are *jointly continuous* if there exists a nonnegative function $f(x, y)$ such that for every subset $C$ of the $xy$-plane $\mathbb{R}^2$, we have

$$P\{(X,Y) \in C\} = \iint_C f(x,y)\,dx\,dy$$

The function $f(x, y)$ is called the joint probability density function of $X$ and $Y$.

- If $X$ and $Y$ are jointly continuous, then they are individually continuous, and their individual probability density functions are given by

$$f_X(x) = \int_{-\infty}^{\infty} f(x,y)\,dy$$

and

$$f_Y(y) = \int_{-\infty}^{\infty} f(x,y)\,dx$$

- For any random variables $X$ and $Y$, the *joint* cumulative ~~*probability*~~ *distribution function* of $X$ and $Y$ is defined by

$$F(x,y) = P\{X \le x, Y \le y\}$$

### Independent Random Variables

- The random variables $X$ and $Y$ are *independent* if

$$P\{X \in A, Y \in B\} = P\{X \in A\} \cdot P\{Y \in B\} \quad \text{for } \underline{\text{all}} \text{ subsets } A, B \text{ of } \mathbb{R}.$$

- Discrete random variables $X$ and $Y$ are independent if and only if

$$p(x,y) = p_X(x)\, p_Y(y) \quad \text{for all } x, y$$

- Continuous random variables $X$ and $Y$ are independent if and only if

$$f(x,y) = f_X(x)\, f_Y(y) \quad \text{for all } x, y$$

- The random variables $X_1, X_2, \ldots, X_n$ are *independent* if

$$P\{X_1 \in A_1, X_2 \in A_2, \ldots, X_n \in A_n\} = \prod_{i=1}^{n} P\{X_i \in A_i\}$$

for $\underline{\text{all}}$ subsets $A_1, A_2, \ldots, A_n$ of $\mathbb{R}$.

**Sums of Independent Random Variables**

- If $X$ and $Y$ are independent discrete random variables, then the probability density function of $X+Y$ is given by

$$p_{X+Y}(n) = \sum_{\text{all } k} p_X(k)\, p_Y(n-k)$$

- If $X_1, X_2, \ldots, X_n$ are independent Poisson random variables with parameters $\lambda_1, \lambda_2, \ldots, \lambda_n$, respectively, then $X = X_1 + X_2 + \ldots + X_n$ is

a Poisson r.v. with parameter $\lambda = \sum_{i=1}^{n} \lambda_i$

- If $X$ and $Y$ are independent continuous random variables, then the probability ~~mass~~ density function of $X+Y$ is given by

$$f_{X+Y}(z) = \int_{-\infty}^{\infty} f_X(z-y)\, f_Y(y)\, dy$$

- If $X_1, X_2, \ldots, X_n$ are independent normal random variables with means $\mu_1, \mu_2, \ldots, \mu_n$ and variances $\sigma_1^2, \sigma_2^2, \ldots, \sigma_n^2$, respectively, then $X = X_1 + X_2 + \cdots + X_n$ is

a normal r.v. $\bar{w}$ mean $\mu = \sum_{i=1}^{n} \mu_i$ and

variance $\sigma^2 = \sum_{i=1}^{n} \sigma_i^2$.

**Conditional Distributions**

- If $X$ and $Y$ are discrete random variables and $p_Y(y) > 0$, then the *conditional probability mass function* of $X$ given that $Y = y$ is defined by

$$p_{X \mid Y}(x \mid y) = \frac{p(x,y)}{p_Y(y)}$$

- If $X$ and $Y$ are jointly continuous with joint density $f(x,y)$ and $f_Y(y) > 0$, then the *conditional probability density function* of $X$ given that $Y = y$ is defined by

$$f_{X \mid Y}(x \mid y) = \frac{f(x,y)}{f_Y(y)}$$