# The Ukranian fruit grower challenges

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## 1 Executive Summary

An Ukranian farmer Kuzma needs to maximize his earnings by growing and selling 10 types of fruit for his survival during the war. Due to the tightness of receiving resources to grow his fruits, he has to allocate his resources in the most optimal way. In detail, he should choose how many unit of 4 square foot of land for each type of fruit. The major resources consists of the amount of soil and fertilizers. He had used $ 800 to obtain those resources. Formulating the problem as a linear program leads to the following solution.

By focusing on growing 4 kinds of fruit, he can expect a return of 23% .Kuzma should build 6.5, 6.6, 3.1, 8.0 unit of 4 sqft land for oranges, grapefruits, limes, passion fruit, respectively. These values were determined by solving an linear program in Section 2.2 .

The resulting shadow prices of $0.10/kg, $3.31/kg, $3.53/kg ,$0.0/kg, $1.54/kg for oranges, grapefruits, limes, passion fruit indicates that the net earnings could be increased by $33.1 if the additional resource on potassium fertilizers can be purchase at a lower cost of $3.00/kg.

Up to $30 could be spent on this specific resource giving a new maximum of $643.04 with a solution of building 2.7, 7.9, 2.4, 12.15 unit of 4-sqft unit of land for oranges, grapefruits, limes, passion fruit.

This report in organized as follows. In Section 2, there is a detailed description of the problem. This consists of explanation on land unit, resource constraints, maximizing profit. Section 3 contains an in depth view on optimal resource allocation, strategic planning with extra funds, validation on amount of land unit, and the allowable changes in resource unit. In addition, section 4 discuss the necessity of restricting land unit within integer values.

# 2 Detailed problem description

The overall problem is described as follows. An Ukranian farmer named Kuzma needs to maximize his earnings by growing and selling 10 types of fruit for his survival.

Due to the Russia-Ukraine war, he has a major constraint of obtaining soil and fertilizers to grow his fruits. Thus, he has to optimize his amount of soil and fertilizers to maximize his profit. We can formulate this as a linear program. The problem of optimizing his resources is modelled as follows.

## 2.1 decision variables and constraints

Let $x_1$ represent the number of unit for a 4 square foot of land for oranges to grow. For example, $x_1 = 2.5$ implies that we build a total of 10 square foot land for oranges. (10 sqft = 2.5 unit*4 sqft)

The same idea goes for $x_2$, $x_3$, $x_4$, $x_5$, $x_6$, $x_7$, $x_8$, $x_9$, $x_{10}$. (each unit is 4 sqft) Each variable corresponds to different kind of fruits : grapefruits, mandarins, limes, pomelos, strawberries, blueberries, raspberries, kiwi, and passion fruit.

The detailed constraints are as follows:

1. Each fruit requires a certain amount of soil. Due to the difficulty of transporting soil, our supplier from Donetsk has allocated 2000 kg of soil to him.

2. Each fruit requires a specific kg of fertilizers. The fertilizers consists of 4 types, potassium, nitrogen, magnesium, calcium. Due to the tightness of importing fertilizers, Kuzma can only receive 50 kg, 44 kg , 55 kg, 45 kg of 4 types of fertilizers.

Written as a linear program the formulation is:

$$\begin{array}{rrcl} \text{Max} & cx & & \\ \text{ST} & Ax & \leq & b \\ & x & \geq & 0 \end{array}$$

where

$$c^T = \begin{bmatrix} 24 \\ 27 \\ 21 \\ 24 \\ 25 \\ 25 \\ 24 \\ 23 \\ 26 \\ 25 \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \end{bmatrix} \quad b = \begin{bmatrix} 2000 \\ 50 \\ 44 \\ 55 \\ 45 \end{bmatrix} \quad A = \begin{bmatrix} 96 & 75 & 82 & 84 & 96 & 92 & 97 & 88 & 90 & 77 \\ 1 & 2 & 2 & 2 & 3 & 3 & 4 & 3 & 3 & 3 \\ 2 & 3 & 3 & 1 & 2 & 3 & 3 & 4 & 1 & 1 \\ 1 & 2 & 1 & 2 & 1 & 1 & 2 & 2 & 3 & 1 \\ 2 & 1 & 1 & 3 & 1 & 2 & 1 & 2 & 3 & 2 \end{bmatrix}$$

Note that, $c^T$ refers to our objective function's coefficients. For example, the 2nd row of $c^T$ implies $27x_2$. Suppose $x_2 = 3$, then we produce a profit of \$27*3= \$81 by building 3 unit of 4 sqft land for grapefruits.

### 2.2 The optimal solution

The problem was solved on Excel producing the following results in its report:

The solution is $x_1 = 6.523510972$, $x_2 = 6.611285266$, $x_4 = 3.103448276$, $x_{10} = 8.015673981$, while 0 for other variables.

This means we should construct 6.5, 6.6, 3.1, 8.0 unit of 4 square foot for oranges, grapefruits, limes, and passion fruit, respectively. Also, we build 0 unit for other kind of fruits. This program produces the maximum profit, which is \$609.94.

## 3 Dual solution and sensitivity analysis

### 3.1 Dual variables

The dual variables have the value $y_1 = 0.109717868$ , $y_2 = 3.310344828$, $y_3 = 3.536050157$, $y_4 = 0.0$, $y_5 = 1.542319749$. These represent the value, or shadow prices, of the five resources soil, potassium, nitrogen, magnesium, and calcium fertilizers in \$/kg. In detail, for an additional increase in 1kg of the resource will increase the profit by the shadow price. For instance, if we had an extra 1kg of calcium fertilizers, we will increase in a profit of \$ 1.54

### 3.2 Future planning with extra funds

For future planning, if we had extra funds to spend on resources, we recommend doing the following. For example, let's assume Kuzma have an extra \$30 of fund to spend on soil or fertilizers. The maximum value of soil is \$0.1/kg and the cost to purchase more soil is \$0.3/kg. Buying more soil will produce a loss of \$0.2/kg.

In contrast, the value of potassium fertilizers at the maximum is \$3.31/kg and more potassium fertilizer can be purchase for \$3.00/kg. Thus, each kg of potassium fertilizer will increase the profit by \$0.31/kg.

Magnesium fertilizers with a shadow price of \$0.00/kg implies we can't purchase any more of it since we may fail to meet the constraints. Also, assume the cost of purchasing nitrogen and calcium fertilizers remain unchanged.

Thus, we recommend to purchase 10 kg of potassium fertilizers at \$3.00/kg which will produce a net increase profit of \$3.1. The new program formulation will be similar to program in page 2, except a change in the value 2nd row b vector. The value of 50 should be replaced to 60.

In conclusion, the new solution is $x_1 = 2.730407524$, $x_2 = 7.990595611$ , $x_4 = 2.413793103$, $x_{10} = 12.15360502$. The new maximum profit \$643.04 which is \$33.1 higher than the previous of \$609.94. Given there was \$30 extra fund to purchase more potassium fertilizers, the net increase in profit is \$3.1 as predicted by the shadow price of potassium fertilizers.

### 3.3 validation with complementary slackness

In this section, we confirm primal optimality using complementary slackness. By doing this, it will validate the correctness on expected profit based on the resources we have used. In detail, we plug in the shadow price into our resource minimizing function in the below.

$$\text{Min} \quad w^* = 2000y_1 + 50y_2 + 44y_3 + 45y_3$$

Then, the optimal $w^*$ will obtain a profit value of \$609.94. We can observe the value is equal to the primal optimal value $z^*$, which we obtained in our maximize profit function.

### 3.4 Sensitivity analysis

The changes to the $b$ vector allowed are as follows.

$$\begin{bmatrix} -173.41 \\ -19.37 \\ -6.00 \\ -21.03 \\ -2.64 \end{bmatrix} \leq \Delta b \leq \begin{bmatrix} +90.00 \\ +17.19 \\ +12.35 \\ +0.00 \\ +10.15 \end{bmatrix}$$

As an example we could make the following changes and still keep the same basis. We can increase an additional amount of potassium fertilizers resource (upperbound and 2nd row) up to 17.19 kg and decrease down to 19.37.

And, it's allowable changes to the to potassium fertilizers is between 30.63 kg and 67.19 kg. This also ensures the optimal number of unit to construct lands for each fruit is satisfying our 5 resources constraint. The same idea goes for other 4 resources. Please kindly refer to the given b vector.

# 4 Restriction to integer solutions

This problem does not require a restriction to integer solutions. The reason are as follows. It is reasonable to assume the amount of unit for a 4 sqft cultivation area $x_i$ to take in decimal values. For example, x1 = 2.5 implies that we build a total of 10 square foot land for oranges. In other words, we build 2 and a half unit of 4sqft land for oranges to grow.

# 5 Citations and references

# References

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