Probability

1. 3 Blue marbels
4 red marbels
6 green marbels
2 yellow marbels

2 marbles are drawn at random,

$$15C_2 = \frac{15!}{2!(15-2)!} = \frac{15 \times 14 \times 13!}{2 \times 1 (13!)}$$

$$= 105$$

Probability that atleast one is green = ?

Non-green marbels = 3 + 4 + 2 = 9

$$9C_2 \Rightarrow \frac{9!}{2!(9-2)!} = \frac{9 \times 8}{2 \times 1} = 36$$

$$P(\text{non-green}) = \frac{36}{105} = \frac{12}{35}$$

$$P(\text{at least one green}) = 1 - P(\text{non green})$$

$$= 1 - \frac{12}{35}$$

$$= \frac{23}{35}$$

3 Blue marbels
4 Red marbels
6 green marbels
2 Yellow marbels

P(Either blue or Yellow) = ?

$N = 3+4+6+2 = 15$

$$15C_2 = \frac{15!}{2!(15-2)!} = \frac{15 \times 14}{2 \times 1} = 105$$

No of blue / Yellow marbles $= 3+2 = 5$

$$5C_2 = \frac{5!}{2!(5-2)!} = \frac{5 \times 4 \times 3!}{2 \times 1 (3!)}$$

$$P(\text{Either blue or Yellow}) = \frac{10}{105} = \frac{2}{21}$$

3.

3 Blue marbles

4 red

6 green

2 Yellow

$3+4+6+2=15$

$$15C_4 = \frac{15!}{4!(11)} = \frac{15\times14\times13\times12}{4\times3\times2\times1}$$

$$= 1365$$

P(non-blue marbles) =)

$$4+6+2=12$$

$$12C_4 = \frac{12!}{4!(12-4)!} = \frac{12!}{4!8!} = 495$$

$$=) \frac{495}{1365} = \frac{33}{91}$$

4. $10! = 10\times10 = 100$

Treat the specific pair of books as a single Unit.

=) So now we are arranging in 9 units

=) the pair & the other 8 book

9! ways.

2 books within the pair can be arranged in 2!

Favourable arrangements = $\frac{9! \times 2!}{10 \times 9!} = \frac{2!}{10} = \frac{2}{10}$

$\frac{1}{5}$ //

(5.) Probability that a leap year has 53 Sundays and 52 Mondays.

A leap year - 366 days

For A leap year = $\frac{366}{7}$ = 52 weeks & 2 extra days

2 extra days combination = {(S, M), (M, T), (T, W), (W, T), (T, F), (F, S), (S, S)}

7 ways.

53 Sundays & 52 Monday is only 1 combination

(S, M)

$\frac{1}{7}$ //

(6) 20 consecutive Integer, 2 are chosen at random
P(Their sum is odd) = ?

$$^{20}C_2 = \frac{20 \times 19 \times 18!}{2! \times 18!} \Rightarrow 190$$

No of odd Integer = 10

$\Rightarrow$ ways to chose one odd $= {}^{10}C_1 = 10$

No of even Integer = 10

$\Rightarrow$ ways to chose one even $= {}^{10}C_1 = 10$

$$10 \times 10 = 100$$

$$P(\text{their sum is odd}) = \frac{100}{190} = \frac{10}{19}$$

(7) 3 Blue marbels, 4 red, 6 green & 2 Yellow

$$3+4+6+2 = 15 \text{ marbles}$$

$$^{15}C_3 = \frac{15!}{3!(15-3)!} = \frac{15 \times 14 \times 13 \times 12!}{3 \times 2 \times 1 \times 12!}$$

$$= 455$$

$C(2,1) = \frac{2!}{1!(2-1)!} = 2$

$C(4,2) = \frac{4!}{2!(4-2)!} = 6$

P(one yellow & 2 are red) $2 \times 6 = 12$

$\Rightarrow \frac{12}{455}$

(8) 10 persons working on a project.

4 are graduates

If 3 are selected, P(at least one graduate among them)

P(at least 1 graduate) = 1 − P(no graduate)

Non-graduates are 6

$^6C_3 = \frac{6!}{3!(6-3)!} = \frac{6!}{3! \times 3!} = 20$

total ways $= {}^{10}C_3 = \frac{10!}{3!(10-3)!}$

$= \frac{10!}{3!(7)!} \Rightarrow 120$

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P(at least 1 graduate) $= 1 - \left(\frac{20}{120}\right) = 1 - \frac{1}{6} = \frac{5}{6}$

(9) 5 couples :) Out of them 5 peoples are chosen at random.

Probability that there are at least 2 couple

Given
5 couples =) 10 people, choose 5 randomly

P(at least 2 couples) = ?

Total =) $^{10}C_5 = \frac{10!}{5! \times 5!} = 252$

Now lets calculate the no of ways to choose exactly 2 couples & 1 additional person not forming a couple.

*) Choose 2 couples from 5 : $\binom{5}{2}$ ways

Choose 1 person from the remaining 3 couples (not forming a couple) : $\binom{3}{1} \times \binom{2}{1}$ ways.

Number of ways for exactly 2 couples:

$$\binom{5}{2} \times \binom{3}{1} \times \binom{2}{1} = 10 \times 3 \times 2 = 60$$

*) calculate the no of ways to choose exactly 3 couples.

Choose 3 couples from 5: $\binom{5}{3}$ ways

Number of ways for exactly 3 couples: $\binom{5}{3} = 10$

Total ways for atleast 2 couples = 60 + 10 = 70.

Probability ~~that there~~ are atleast 2 couples is $\frac{5}{18}$.

(10) The probability of a lottery ticket being a prized ticket is 0.2.

4 tickets are purchased

P(winning a prize) ⇒ ?
(atleast one ticket)

P(win) = 0.2

P(lose) = 1 − P(win) = 1 − 0.2 = 0.8

P(lose on all 4) = $(P(\text{lose}))^4 = (0.8)^4 = 0.4096$

P(at least one win) = 1 − P(lose on all 4)

= 1 − 0.4096

= 0.5904.

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(11) 39 red balls, 26 green balls

- Place 1 red ball in Box 1.
- Place the remaining 38 red balls and 26 green balls in Box 2.

↳ Box 1 contains 1 red ball and 0 green balls

↳ Box 2 contains 38 red balls and 26 green balls

Box 1 $P(R_1) = \frac{1}{1} = 1$

Box 2 $P(R_2) = \frac{38}{38+26} = \frac{38}{64}$

Probability of choosing either box is $\frac{1}{2}$.

Overall probability $P(R) = \frac{1}{2} \times P(R_1) + \frac{1}{2} \times P(R_2)$

$= \frac{1}{2} \times 1 + \frac{1}{2} \times \frac{38}{64}$

$\Rightarrow \frac{1}{2} + \frac{19}{64}$

$\Rightarrow \frac{32+19}{64} = \frac{51}{64}$

chandras

(12)

6 Red Balls

8 Blue Balls

7 green balls

=) 5 are drawn with replacement then Probability atleast three are red.

$6+8+7=21$

$P(R) = \frac{6}{21} = \frac{2}{7}$

$P(\text{Not Red}) = 1 - \frac{2}{7} = \frac{5}{7}$

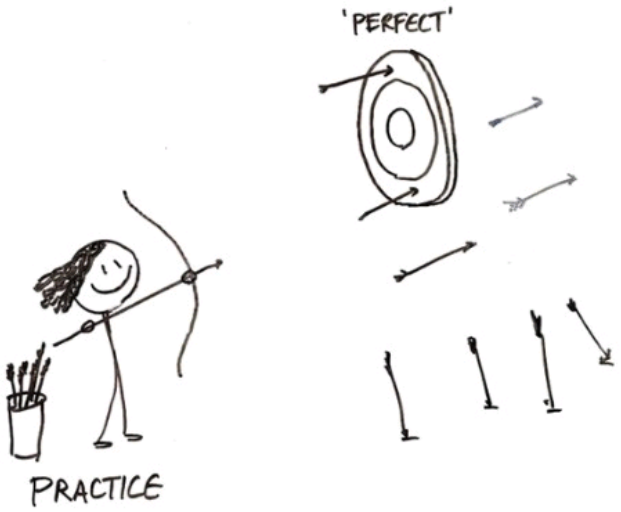
Probability atleast three are red is

$$\frac{{}^6C_3 \times {}^{15}C_2 + {}^6C_4 \times {}^{15}C_1 + {}^6C_5 \times {}^{15}C_0}{{}^{21}C_5}$$

$= 0.1145$

- Practice:

1. A box contains 3 blue marbles, 4 red, 6 green marbles and 2 yellow marbles. If two marbles are drawn at random, what is the probability that at least one is green?
2. A box contains 3 blue marbles, 4 red, 6 green marbles and 2 yellow marbles. If two marbles are picked at random, what is the probability that they are either blue or yellow?
3. A box contains 3 blue marbles, 4 red, 6 green marbles and 2 yellow marbles. If four marbles are picked at random, what is the probability that none is blue?
4. 10 books are placed at random in a shelf. The probability that a pair of books will always be together is?
5. What is the probability that a leap year has 53 Sundays and 52 Mondays?
6. Out of 20 consecutive integers, two are chosen at random. The probability that their sum is odd is?
7. A box contains 3 blue marbles, 4 red, 6 green marbles and 2 yellow marbles. If three marbles are drawn what is the probability that one is yellow and two are red?



- Practice:

8. Out of 10 persons working on a project, 4 are graduates. If 3 are selected, what is the probability that there is at least one graduate among them?
9. In a party there are 5 couples. Out of them 5 people are chosen at random. Find the probability that there are at the least two couples?
10. The probability of a lottery ticket being a prized ticket is 0.2. When 4 tickets are purchased, the probability of winning a prize on atleast one ticket is?
11. There are two boxes, one containing 39 red balls & the other containing 26 green balls. You are allowed to move the balls between the boxes so that when you choose a box random & a ball at random from the chosen box, the probability of getting a red ball is maximized. This maximum probability is
12. There are 6 red balls, 8 blue balls and 7 green balls in a bag. If 5 are drawn with replacement, what is the probability at least three are red?