TDT4121 Introduction to algorithms

Assignment 5

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Greedy algorithms

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- Greedy algorithms can be very varied, but there are some common patterns that help us work with them.

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- Greedy algorithms can be very varied, but there are some common patterns that help us work with them.

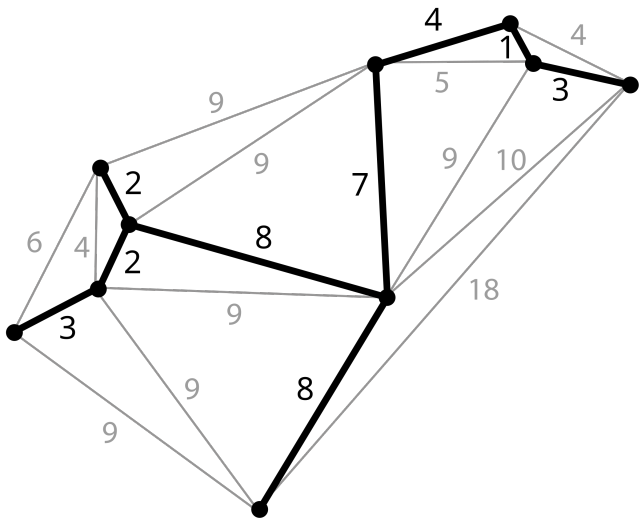
- Greedy algorithm stays ahead. Show that after each step of the greedy algorithm, its solution is at least as good as the solution of any other algorithm.
- Exchange argument. Gradually transform any solution to the one found by the greedy algorithm without hurting its quality.
- Structural. Discover a simple "structural" bound asserting that every possible solution must have a certain value. Then show that our algorithm always achieves this bound.

Tips seen in class slides, taken from Kevin Wayne.

Minimum spanning tree

Suppose we have a weighted graph, and we want to reduce its edges to the least possible, in the sense that

- We should have no redundant edges; the graph should be a tree
- The total cost of the edges must be minimized.
- The graph should remain connected.



Minimum spanning tree

- There exist different optimal algorithms to solve this problem.

Kruskal's algorithm

- Iterate over the sorted edges in order of increasing weight.
- If an edge can be added to the result graph without creating a cycle, add it. Otherwise skip it.

The algorithm stops naturally when result graph is fully connected.

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Reverse-delete algorithm

- Iterate over the sorted edges in order of decreasing weight.
- If an edge can be removed without making the graph disconnected, remove it. Otherwise skip it.

The algorithm stops naturally when result graph is a tree.

- These symmetric algorithms find an optimal answer.

Minimum spanning tree

- We can use the following results to prove the previous algorithms work.

Cut property

Take a weighted graph G where all the edge costs are distinct, and take a non-empty proper subgraph S of G. Since G is connected, there exists at least an edge between S and $G \setminus S$. Let e be the least weighted of such edges.
Then any minimum spanning tree must contain e.

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Then any minimum spanning tree must contain e.

Cycle property

Take a weighted graph G where all the edge costs are distinct, and let C be any cycle in G, where e is the most expensive edge in said cycle.
Then e cannot belong to any minimum spanning tree of G.

Interval partitioning problem

- Suppose we are in charge of organising classrooms for lectures.
- You are given a list on n lectures with their starting and finishing times s_i and f_i,
- You would like to minimise the number of classrooms needed to hold the lectures, while ensuring that they never overlap.

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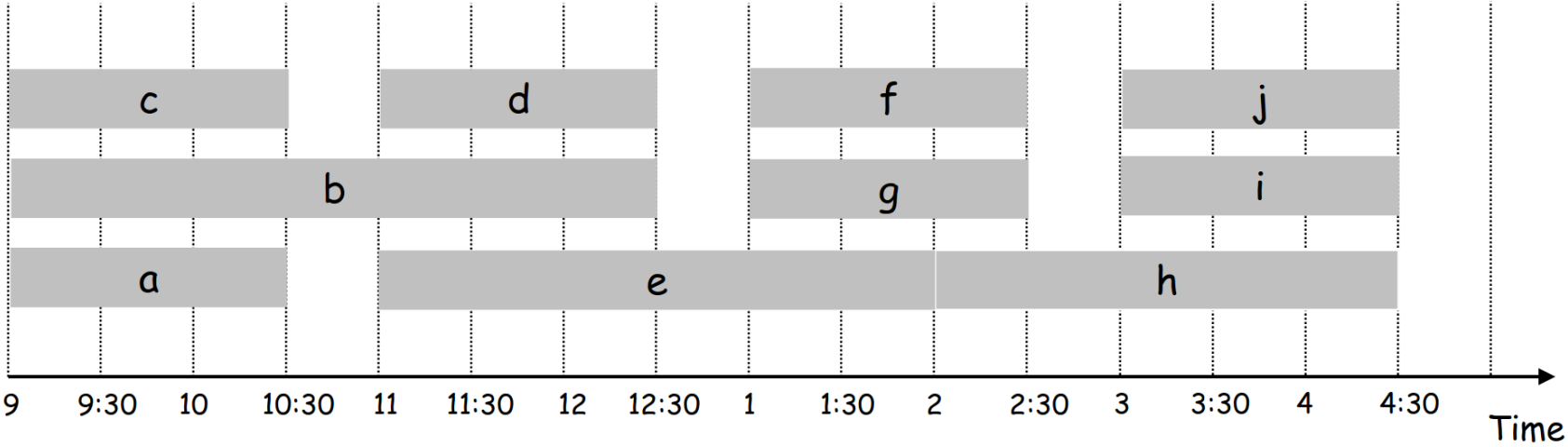

Figure seen in class slides, taken from Kevin Wayne.

Interval partitioning problem

```
function ScheduleClasses(List lectures; Integer n)
    Sort lectures by starting time
    d ← 0
    for j in 1, ..., n do
        for i in 1, ..., d do
            if lecture j fits in classroom i then
                schedule lecture j in classroom i
        if lecture j has no been scheduled then
            d ← d + 1
            schedule lecture j in classroom d
```

- Note how the algorithm avoids overlapping lectures in the same classroom.
- In the recitation lectures we saw how the number of classrooms allocated is at most to the depth of the problem.

Interval partitioning problem

- Note how if there are d lectures that request the same time instant, then we know that we need at least d classrooms to hold them without overlapping.
- The maximal of these values is called the *depth* of the problem. It is a lower bound to how many classrooms we need.

Interval partitioning problem

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- The maximal of these values is called the *depth* of the problem. It is a lower bound to how many classrooms we need.

- In fact, we proved with the previous algorithm that the optimal answer for a problem of depth d requires exactly d classrooms.
- This hints at an intrinsically local structure in the problem, where assigning a classroom to a lecture does not have broad consequences on the overall schedule.
- When designing another algorithm to solve this problem, we can use this result to check that our solution achieves this bound.