TDT4121 Introduction to algorithms

Assignment 7

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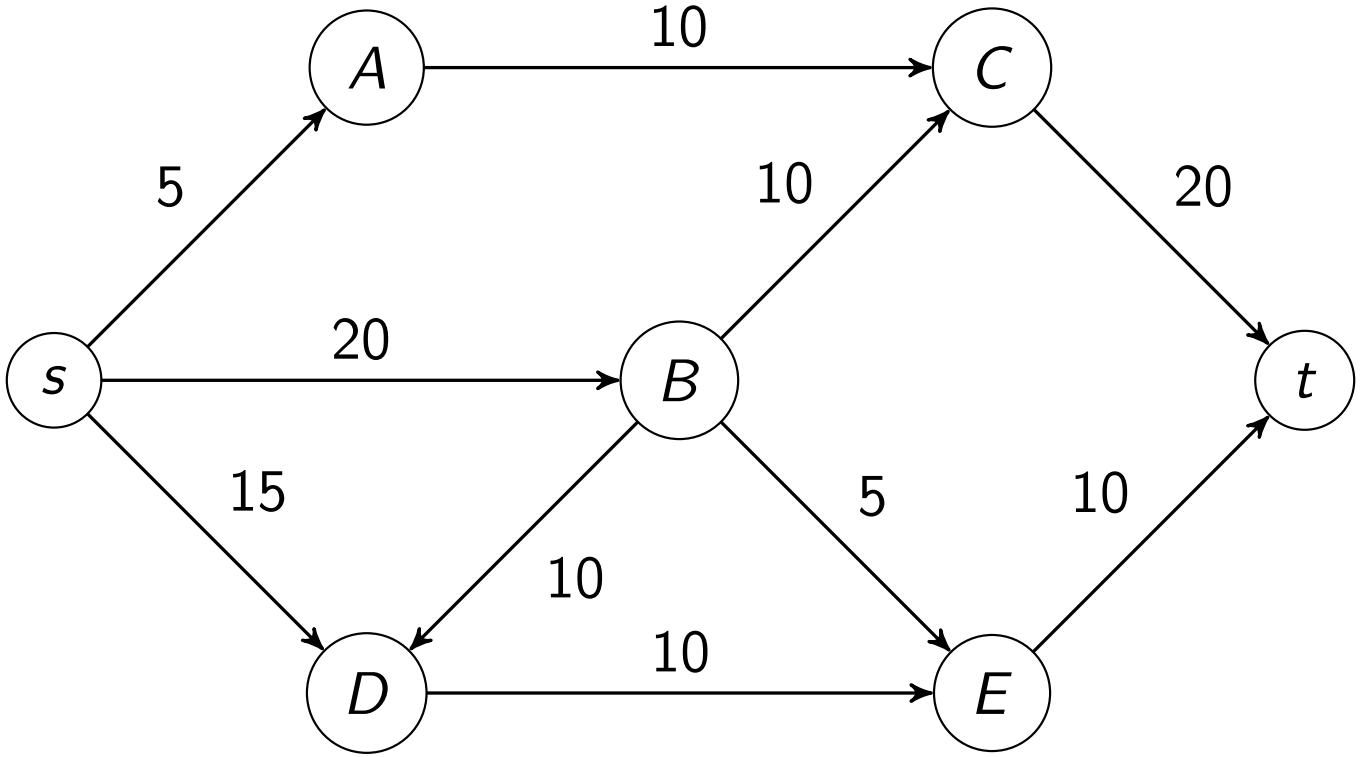
Graph flow

- We want to maximize the flow from s to t in a weighted graph.
- Note how the problem is bounded, either by the maximum amount of flow capacity out of s or into t.



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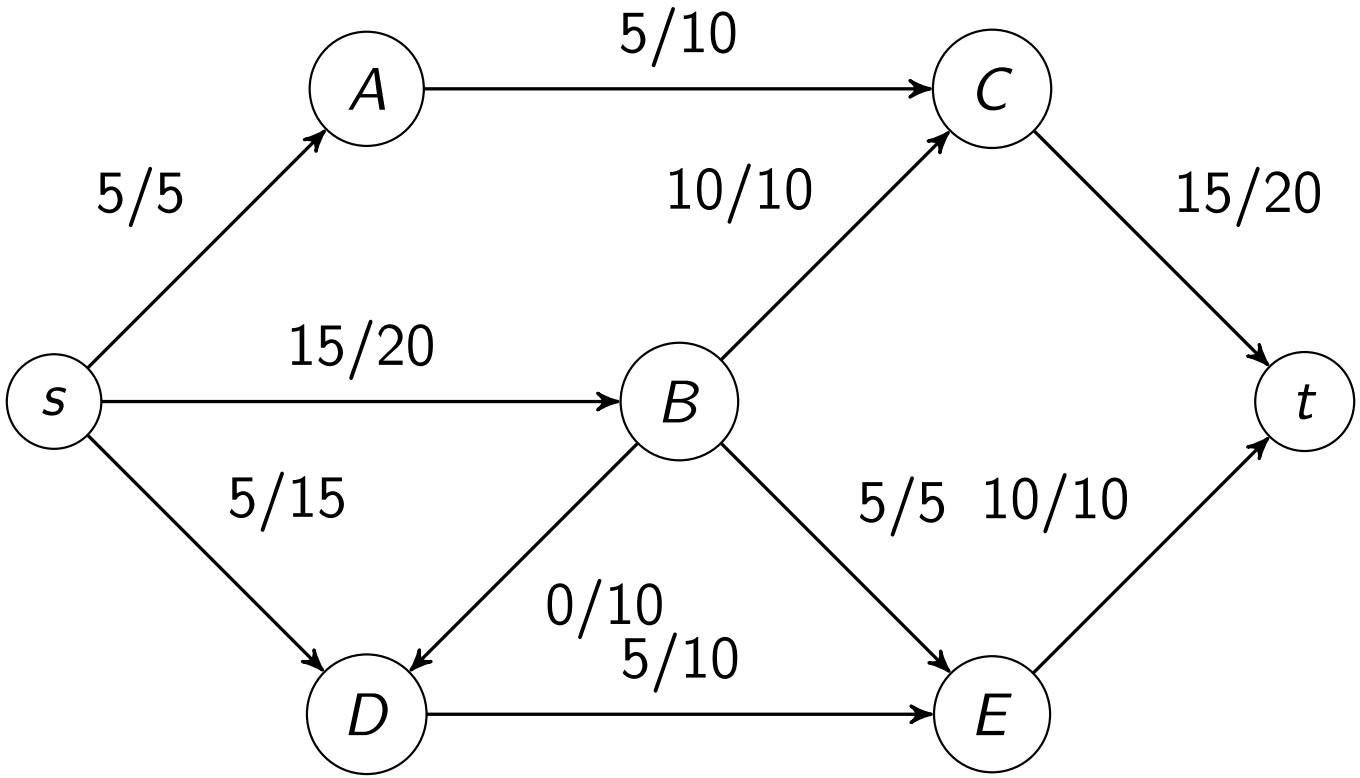
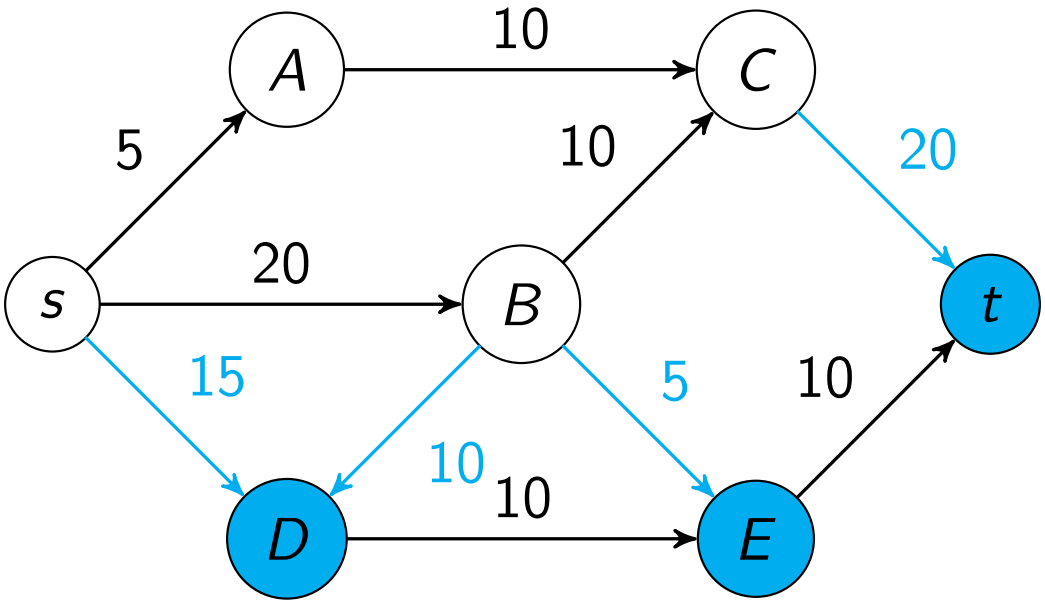


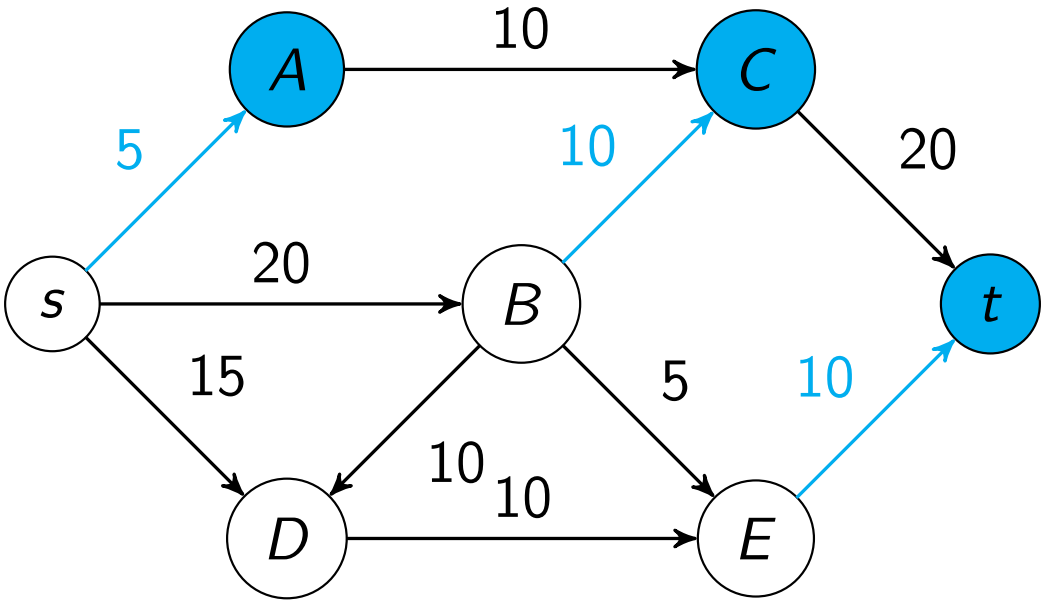
Maximum flow and minimum capacity cut

- We can divide the nodes of a graph in two disjoint sets A and B, such that $s \in A$ and $t \in B$
- This reveals a set of edges that start in A and end in B.
- The capacity of the cut, $c(A, B)$, is the sum of the capacities of these edges.



Maximum flow and minimum capacity cut

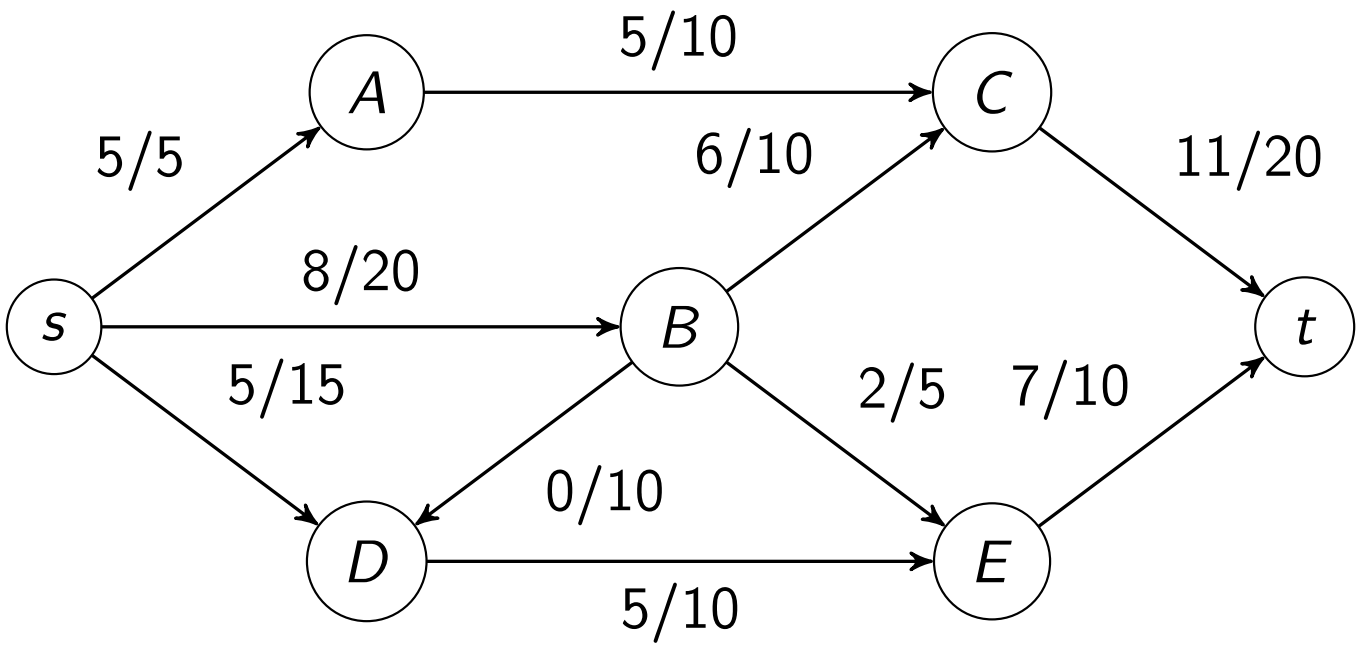
- The maximum flow of a graph is the same as its minimum capacity cut.



Residual graphs

Definition (Residual graph)

- For every edge $e = (u, v)$ for which $f(e) < c_e$, an edge $e = (u, v)$ with capacity $c_e - f(e)$.
- For every edge $e = (u, v)$ for which $f(e) > 0$, an edge $e' = (v, u)$ with capacity $f(e)$.



Ford–Fulkerson algorithm

function Ford-Fulkerson
    while there is a path p from s to t in G **do**
        Augment the flow with P
        Set the graph G to be its residual graph
    end while
end function

Hall's Theorem

Theorem (Hall)

A bipartite graph $G = (A \cup B, E)$ contains a matching that saturates every vertex in A if and only if $|N(S)| \geq |S|$ for all $S \subseteq A$.