TTK4115 Linear System Theory
Department of Engineering Cybernetics
NTNU

Solution to homework assignment 4

Problem 1: Input-output stability of discrete-time systems

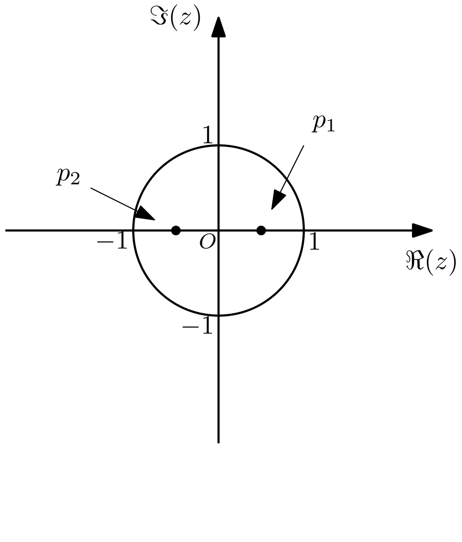
a) The discrete transfer function $\hat{g}(z) = \frac{\hat{y}(z)}{\hat{u}(z)}$ is given by

$$
\begin{aligned}
\hat{g}(z) &= \mathbf{C}(z\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D} \\
&= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} z+0.7 & 0.6 \\ -0.4 & z-0.7 \end{bmatrix}^{-1} \begin{bmatrix} 0.1 \\ 0.3 \end{bmatrix} - 1 \\
&= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{z-0.7}{z^2-0.25} & \frac{-0.6}{z^2-0.25} \\ \frac{0.4}{z^2-0.25} & \frac{z+0.7}{z^2-0.25} \end{bmatrix} \begin{bmatrix} 0.1 \\ 0.3 \end{bmatrix} - 1 \\
&= \frac{0.1z - 0.25}{z^2 - 0.25} - 1 \\
&= \frac{-z^2 + 0.1z}{z^2 - 0.25}.
\end{aligned}
$$

b) The system is BIBO stable if and only if every pole of the discrete transfer function $\hat{g}(z)$ has a magnitude less than 1. Therefore, the boundary of the region of the z-plane in which the poles of the discrete transfer function $\hat{g}(z)$ must be for the system to be BIBO stable is a circle of radius one centered around the origin. The poles of the discrete transfer function $\hat{g}(z)$ are equal to the roots of the denominator

$$d(z) = z^2 - 0.25 = (z - 0.5)(z + 0.5).$$

Hence, we obtain the poles $p_1 = 0.5$ and $p_2 = -0.5$. The boundary of the stability region and the poles are visualized as follows:



Because both poles are inside the stability region of the z-plane, the system is BIBO stable.

Problem 2: Stability of continuous-time systems

a) To find out if the system is marginally stable, asymptotically stable and/or unstable, we compute the eigenvalues of $\mathbf{A}$. The eigenvalues of $\mathbf{A}$ can be calculated from the characteristic polynomial of $\mathbf{A}$, which is given by

$$\det(\mathbf{A} - \lambda \mathbf{I}) = \begin{vmatrix} 1-\lambda & -2 \\ 0 & -1-\lambda \end{vmatrix} = \lambda^2 - 1 = (1-\lambda)(-1-\lambda).$$

The eigenvalues of $\mathbf{A}$ are equal to the roots the characteristic polynomial of $\mathbf{A}$. Hence, we obtain the eigenvalues $\lambda_1 = 1$ and $\lambda_2 = -1$. Because λ_1 has a positive real part, the system is not marginally stable and not asymptotically stable. Because the system is not marginally stable, it is unstable.

b) The transfer function $\hat{g}(s) = \frac{\hat{y}(s)}{\hat{u}(s)}$ is given by

$$\begin{aligned}
\hat{g}(s) &= \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + D \\
&= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} s-1 & 2 \\ 0 & s+1 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + 1 \\
&= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{s-1} & \frac{-2}{(s-1)(s+1)} \\ 0 & \frac{1}{s+1} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + 1 \\
&= \frac{1}{s-1} - \frac{2}{(s-1)(s+1)} + 1 \\
&= \frac{s+2}{s+1}.
\end{aligned}$$

c) To determine if the system is BIBO stable, we compute the poles of the transfer function. The denominator of the transfer function is given by

$$d(s) = s + 1.$$

The poles of the transfer function are equal to the roots of the denominator. Hence, we obtain that the pole of the system is given by $p = -1$. Because the pole p has a negative real part, we conclude that the system is BIBO stable.

d) The impulse response $g(t)$ of the system can be obtained from the inverse Laplace transform of the transfer function $\hat{g}(s)$:

$$\begin{aligned}
g(t) = \mathcal{L}^{-1}[\hat{g}(s)] &= \mathcal{L}^{-1}\left[\frac{s+2}{s+1}\right] = \mathcal{L}^{-1}\left[\frac{1}{s+1} + 1\right] \\
&= \mathcal{L}^{-1}\left[\frac{1}{s+1}\right] + \mathcal{L}^{-1}[1] = e^{-t} + \delta(t),
\end{aligned}$$

where $\delta(t)$ is the Dirac delta function.

e) The system is BIBO stable if and only if the impulse response $g(t)$ is absolutely integrable on the domain $[0, \infty)$. That is, the system is BIBO stable if and only if there exists a constant $M > 0$, such that

$$\int_0^\infty |g(t)| dt \leq M < \infty.$$

By computing this integral, we obtain the following:

$$\int_0^\infty |g(t)| dt = \int_0^\infty e^{-t} + \delta(t) dt = \int_0^\infty e^{-t} dt + \int_0^\infty \delta(t) dt = 1 + 1 = 2.$$

Hence, we obtain that the impulse response $g(t)$ is absolutely integrable on the domain $[0, \infty)$, with constant $M = 2$. Therefore, we conclude that the system is BIBO stable.

Problem 3: Internal stability

a) The eigenvalues of $\mathbf{A}$ can be calculated from the characteristic polynomial of $\mathbf{A}$, which is given by

$$\det(\mathbf{A} - \lambda \mathbf{I}) = \begin{vmatrix} -\lambda & 0 \\ -2 & -\lambda \end{vmatrix} = \lambda^2.$$

The eigenvalues of $\mathbf{A}$ are equal to the roots the characteristic polynomial of $\mathbf{A}$. Hence, we obtain the eigenvalue $\lambda_1 = 0$ with multiplicity 2. The corresponding eigenvectors can be obtained from the kernel of the matrix $(\mathbf{A} - \lambda_1 \mathbf{I})$:

$$\ker(\mathbf{A} - \lambda_1 \mathbf{I}) = \ker \left(\begin{bmatrix} 0 & 0 \\ -2 & 0 \end{bmatrix} \right) = \ker \left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \right) \implies \mathbf{q}_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

where $\mathbf{q}_1$ is the corresponding eigenvector. Note that $\mathbf{A}$ has only one eigenvector associated with λ_1.

b) Because there is only one eigenvector associated with the eigenvalue $\lambda_1 = 0$ and the (algebraic) multiplicity of the eigenvalue is two, the size of the corresponding Jordan block is 2×2. Therefore, the matrix $\mathbf{A}$ has an eigenvalue with a zero real part that is <u>not</u> a simple root of the minimal polynomial of $\mathbf{A}$. This implies that the system is neither marinally stable (or Lyapunov stable) nor asymptotically stable. Because the system is not marginally stable, it is unstable.

c) The eigenvalues of $\mathbf{A}$ can be calculated from the characteristic polynomial of $\mathbf{A}$, which is given by

$$\det(\mathbf{A} - \lambda \mathbf{I}) = \begin{vmatrix} -\lambda & 0 \\ 0 & -\lambda \end{vmatrix} = \lambda^2.$$

The eigenvalues of $\mathbf{A}$ are equal to the roots the characteristic polynomial of $\mathbf{A}$. Hence, similar to a), we obtain the eigenvalue $\lambda_1 = 0$ with multiplicity 2. The corresponding eigenvectors can be obtained from the kernel of the matrix $(\mathbf{A} - \lambda_1 \mathbf{I})$:

$$\ker(\mathbf{A} - \lambda_1 \mathbf{I}) = \ker \left(\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \right) = \ker \left(\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \right) \implies \mathbf{q}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \mathbf{q}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

where $\mathbf{q}_1$ and $\mathbf{q}_2$ are the corresponding eigenvectors. Note that $\mathbf{A}$ has two eigenvectors associated with λ_1.

d) Because there are two eigenvector associated with the eigenvalue $\lambda_1 = 0$, which has an (algebraic) multiplicity of two, there are two Jordan blocks of size 1×1 associated with $\lambda_1 = 0$. Because the size of the Jordan blocks 1×1, the eigenvalue $\lambda_1 = 0$ (with multiplicity 2) is a simple root of the minimal polynomial of $\mathbf{A}$. Therefore, the eigenvalues of the matrix $\mathbf{A}$ have zero or negative real parts and those with zero real parts are simple root of the minimal polynomial of $\mathbf{A}$, which implies that the system is marginally stable (or Lyapunov stable). Because the eigenvalues of $\mathbf{A}$ do not have negative real parts, the system is not asymptotically stable. Moreover, because the system is marginally stable, it is not unstable.

e) To find the matrix $\mathbf{P}$, we solve the Lyapunov equation

$$\mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A} = -\mathbf{I}.$$

Note that $\mathbf{P}$ is a symmetric matrix, i.e. $\mathbf{P} = \mathbf{P}^T$. Let $\mathbf{P}$ be given by

$$\mathbf{P} = \begin{bmatrix} p_1 & p_2 \\ p_2 & p_3 \end{bmatrix},$$

where p_1, p_2 and p_3 are constant that are yet to be determined. Substituting the matrices $\mathbf{A}$ and $\mathbf{P}$ in the Lyapunov equation, we obtain

$$\begin{bmatrix} -4 & 1 \\ -2 & -2 \end{bmatrix} \begin{bmatrix} p_1 & p_2 \\ p_2 & p_3 \end{bmatrix} + \begin{bmatrix} p_1 & p_2 \\ p_2 & p_3 \end{bmatrix} \begin{bmatrix} -4 & -2 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}.$$

It follows that

$$\begin{bmatrix} -4p_1 + p_2 & -4p_2 + p_3 \\ -2p_1 - 2p_2 & -2p_2 - 2p_3 \end{bmatrix} + \begin{bmatrix} -4p_1 + p_2 & -2p_1 - 2p_2 \\ -4p_2 + p_3 & -2p_2 - 2p_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}.$$

From this, we obtain the equations

$$\begin{aligned} -8p_1 + 2p_2 &= -1, \\ -2p_1 - 6p_2 + p_3 &= 0, \\ -4p_2 - 4p_3 &= -1, \end{aligned}$$

which can be written in the following form:

$$\begin{bmatrix} -8 & 2 & 0 \\ -2 & -6 & 1 \\ 0 & -4 & -4 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix}.$$

Solving for p_1, p_2 and p_3 yields $p_1 = \frac{1}{8}$, $p_2 = 0$ and $p_3 = \frac{1}{4}$. Hence, we obtain the matrix

$$\mathbf{P} = \begin{bmatrix} \frac{1}{8} & 0 \\ 0 & \frac{1}{4} \end{bmatrix}.$$

The eigenvalues of $\mathbf{A}$ have negative real parts if and only if the matrix $\mathbf{P}$ is positive definite. The matrix $\mathbf{P}$ is positive definite if and only if all its leading principle minors are positive. The leading principle minors of $\mathbf{P}$ are

$$p_1 = \frac{1}{8} \qquad \text{and} \qquad \det(\mathbf{P}) = \begin{vmatrix} \frac{1}{8} & 0 \\ 0 & \frac{1}{4} \end{vmatrix} = \frac{1}{8} \cdot \frac{1}{4} = \frac{1}{32}.$$

Because all leading principle minors of $\mathbf{P}$ are positive, we conclude that the matrix $\mathbf{P}$ is positive definite and that the eigenvalues of $\mathbf{A}$ all have negative real parts. Because all eigenvalues of $\mathbf{A}$ have negative real parts, the system with system matrix $\mathbf{A}$ is asymptotically stable.

Problem 4: Controllable decompositions

a) The transfer matrix of the system is given by

$$\begin{aligned}
\hat{\mathbf{G}}(s) &= \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} \\
&= \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} \frac{1}{s+4} & \frac{-4}{(s+2)(s+4)} & \frac{-10}{(s+2)(s-3)} \\ 0 & \frac{1}{s+2} & \frac{5}{(s+2)(s-3)} \\ 0 & 0 & \frac{1}{s-3} \end{bmatrix} \begin{bmatrix} 4 \\ -2 \\ -1 \end{bmatrix} \\
&= \begin{bmatrix} \frac{1}{s+4} & \frac{-4}{(s+2)(s+4)} + \frac{2}{s+2} & 0 \\ 0 & 0 & \frac{-1}{s-3} \end{bmatrix} \begin{bmatrix} 4 \\ -2 \\ -1 \end{bmatrix} \\
&= \begin{bmatrix} \frac{4}{s+4} + \frac{8}{(s+2)(s+4)} - \frac{4}{s+2} \\ \frac{1}{s-3} \end{bmatrix} \\
&= \begin{bmatrix} \frac{4s+8}{(s+2)(s+4)} + \frac{8}{(s+2)(s+4)} - \frac{4s+16}{(s+2)(s+4)} \\ \frac{1}{s-3} \end{bmatrix} \\
&= \begin{bmatrix} 0 \\ \frac{1}{s-3} \end{bmatrix}.
\end{aligned}$$

b) The controllability matrix is given by

$$\mathcal{C} = \begin{bmatrix} \mathbf{B} & \mathbf{AB} & \mathbf{A}^2\mathbf{B} \end{bmatrix} = \begin{bmatrix} 4 & 2 & 26 \\ -2 & -1 & -13 \\ -1 & -3 & -9 \end{bmatrix}.$$

Because the controllability matrix has two linearly independent columns (the third column is equal to six times the first column plus the second column), the column rank of the controllability matrix is two. Because the controllability matrix does not have full row rank, i.e. $\text{rank}(\mathcal{C}) = 2 < n$, we conclude that the system is <u>not</u> controllable.

c) The transformation matrix is given by

$$\mathbf{P} = \begin{bmatrix} 4 & 2 & 1 \\ -2 & -1 & 0 \\ -1 & -3 & 0 \end{bmatrix}.$$

From the similarity transformation $\mathbf{x}(t) = \mathbf{P}\hat{\mathbf{x}}(t)$, it follows that the controllable canonical decomposition of the system is given by

$$\begin{aligned}\dot{\hat{\mathbf{x}}}(t) &= \hat{\mathbf{A}}\hat{\mathbf{x}}(t) + \hat{\mathbf{B}}\mathbf{u}(t)\\ \mathbf{y}(t) &= \hat{\mathbf{C}}\hat{\mathbf{x}}(t),\end{aligned}$$

with

$$\hat{\mathbf{A}} = \mathbf{P}^{-1}\mathbf{A}\mathbf{P} = \frac{1}{5}\begin{bmatrix}0 & -3 & 1\\ 0 & 1 & -2\\ 5 & 10 & 0\end{bmatrix}\begin{bmatrix}-4 & -4 & -10\\ 0 & -2 & 5\\ 0 & 0 & 3\end{bmatrix}\begin{bmatrix}4 & 2 & 1\\ -2 & -1 & 0\\ -1 & -3 & 0\end{bmatrix} = \begin{bmatrix}0 & 6 & 0\\ 1 & 1 & 0\\ 0 & 0 & -4\end{bmatrix},$$

$$\hat{\mathbf{B}} = \mathbf{P}^{-1}\mathbf{B} = \frac{1}{5}\begin{bmatrix}0 & -3 & 1\\ 0 & 1 & -2\\ 5 & 10 & 0\end{bmatrix}\begin{bmatrix}4\\ -2\\ -1\end{bmatrix} = \begin{bmatrix}1\\ 0\\ 0\end{bmatrix},$$

$$\hat{\mathbf{C}} = \mathbf{C}\mathbf{P} = \begin{bmatrix}1 & 2 & 0\\ 0 & 0 & -1\end{bmatrix}\begin{bmatrix}4 & 2 & 1\\ -2 & -1 & 0\\ -1 & -3 & 0\end{bmatrix} = \begin{bmatrix}0 & 0 & 1\\ 1 & 3 & 0\end{bmatrix}.$$

d) From straightforward computation, we obtain

$$\begin{aligned}\hat{\mathbf{C}}_c(s\mathbf{I} - \hat{\mathbf{A}}_c)^{-1}\hat{\mathbf{B}}_c &= \begin{bmatrix}0 & 0\\ 1 & 3\end{bmatrix}\begin{bmatrix}s & -6\\ -1 & s-1\end{bmatrix}^{-1}\begin{bmatrix}1\\ 0\end{bmatrix}\\ &= \begin{bmatrix}0 & 0\\ 1 & 3\end{bmatrix}\begin{bmatrix}\frac{s-1}{s^2-s-6} & \frac{6}{s^2-s-6}\\ \frac{1}{s^2-s-6} & \frac{s}{s^2-s-6}\end{bmatrix}\begin{bmatrix}1\\ 0\end{bmatrix}\\ &= \begin{bmatrix}0\\ \frac{s+2}{s^2-s-6}\end{bmatrix}\\ &= \begin{bmatrix}0\\ \frac{s+2}{(s+2)(s-3)}\end{bmatrix}\\ &= \begin{bmatrix}0\\ \frac{1}{s-3}\end{bmatrix}.\end{aligned}$$

From a), we have

$$\hat{\mathbf{G}}(s) = \begin{bmatrix}0\\ \frac{1}{s-3}\end{bmatrix}.$$

Hence, it follows that

$$\hat{\mathbf{G}}(s) = \hat{\mathbf{C}}_c(s\mathbf{I} - \hat{\mathbf{A}}_c)^{-1}\hat{\mathbf{B}}_c.$$

Problem 5: Minimal realizations

a) The eigenvalues of $\mathbf{A}$ can be calculated from the characteristic polynomial of $\mathbf{A}$, which is given by

$$\det(\mathbf{A} - \lambda\mathbf{I}) = \begin{vmatrix}-5-\lambda & 2\\ 5 & 4-\lambda\end{vmatrix} = \lambda^2 + \lambda - 30 = (\lambda+6)(\lambda-5).$$

The eigenvalues of $\mathbf{A}$ are equal to the roots the characteristic polynomial of $\mathbf{A}$. Hence, we obtain the eigenvalues $\lambda_1 = -6$ and $\lambda_2 = 5$.

b) For $\lambda = \lambda_1 = -6$, we have

$$\operatorname{rank}\begin{bmatrix}\mathbf{A} - \lambda_1\mathbf{I} & \mathbf{B}\end{bmatrix} = \operatorname{rank}\begin{bmatrix}1 & 2 & 1 \\ 5 & 10 & -2\end{bmatrix} = 2.$$

Similarly, for $\lambda = \lambda_2 = 5$, we have

$$\operatorname{rank}\begin{bmatrix}\mathbf{A} - \lambda_2\mathbf{I} & \mathbf{B}\end{bmatrix} = \operatorname{rank}\begin{bmatrix}-10 & 2 & 1 \\ 5 & -1 & -2\end{bmatrix} = 2.$$

Because the matrix $\begin{bmatrix}\mathbf{A} - \lambda\mathbf{I} & \mathbf{B}\end{bmatrix}$ has full row rank for every eigenvalue λ of $\mathbf{A}$ (i.e. $\operatorname{rank}\begin{bmatrix}\mathbf{A} - \lambda\mathbf{I} & \mathbf{B}\end{bmatrix} = n = 2$ for every eigenvalue λ of $\mathbf{A}$), we conclude that the system is controllable.

c) For $\lambda = \lambda_1 = -6$, we have

$$\operatorname{rank}\begin{bmatrix}\mathbf{A} - \lambda_1\mathbf{I} \\ \mathbf{C}\end{bmatrix} = \operatorname{rank}\begin{bmatrix}1 & 2 \\ 5 & 10 \\ -5 & 1 \\ 10 & -2\end{bmatrix} = 2.$$

For $\lambda = \lambda_2 = 5$, we have

$$\operatorname{rank}\begin{bmatrix}\mathbf{A} - \lambda_2\mathbf{I} \\ \mathbf{C}\end{bmatrix} = \operatorname{rank}\begin{bmatrix}-10 & 2 \\ 5 & -1 \\ -5 & 1 \\ 10 & -2\end{bmatrix} = 1.$$

Because the matrix $\begin{bmatrix}\mathbf{A} - \lambda\mathbf{I} \\ \mathbf{C}\end{bmatrix}$ does not have full row rank for every eigenvalue λ of $\mathbf{A}$ (i.e. $\operatorname{rank}\begin{bmatrix}\mathbf{A} - \lambda\mathbf{I} \\ \mathbf{C}\end{bmatrix}$ is not $n = 2$ for every eigenvalue λ of $\mathbf{A}$), we conclude that the system is <u>not</u> observable.

d) The system is a minimal realization if and only if it is controllable and observable. From b) and c), we have that the system is controllable, but not observable. Therefore, the system is <u>not</u> a minimal realization.

e) The transfer matrix of the system is given by

$$
\begin{aligned}
\hat{\mathbf{G}}(s) &= \mathbf{C}(s\mathbf{I}-\mathbf{A})^{-1}\mathbf{B}+\mathbf{D} \\
&= \begin{bmatrix} -5 & 1 \\ 10 & -2 \end{bmatrix} \begin{bmatrix} s+5 & -2 \\ -5 & s-4 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ -2 \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \end{bmatrix} \\
&= \begin{bmatrix} -5 & 1 \\ 10 & -2 \end{bmatrix} \begin{bmatrix} \frac{s-4}{s^2+s-30} & \frac{2}{s^2+s-30} \\ \frac{5}{s^2+s-30} & \frac{s+5}{s^2+s-30} \end{bmatrix} \begin{bmatrix} 1 \\ -2 \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \end{bmatrix} \\
&= \begin{bmatrix} \frac{-5s+25}{s^2+s-30} & \frac{s-5}{s^2+s-30} \\ \frac{10s-50}{s^2+s-30} & \frac{-2s+10}{s^2+s-30} \end{bmatrix} \begin{bmatrix} 1 \\ -2 \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \end{bmatrix} \\
&= \begin{bmatrix} \frac{-5(s-5)}{(s-5)(s+6)} & \frac{s-5}{(s-5)(s+6)} \\ \frac{10(s-5)}{(s-5)(s+6)} & \frac{-2(s-5)}{(s-5)(s+6)} \end{bmatrix} \begin{bmatrix} 1 \\ -2 \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \end{bmatrix} \\
&= \begin{bmatrix} \frac{-5}{s+6} & \frac{1}{s+6} \\ \frac{10}{s+6} & \frac{-2}{s+6} \end{bmatrix} \begin{bmatrix} 1 \\ -2 \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \end{bmatrix} \\
&= \begin{bmatrix} \frac{-7}{s+6} \\ \frac{14}{s+6} \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \end{bmatrix} \\
&= \begin{bmatrix} \frac{s-1}{s+6} \\ \frac{-2s+2}{s+6} \end{bmatrix}.
\end{aligned}
$$

f) The least common denominator of the transfer function $\hat{\mathbf{G}}(s)$ is $d(s) = s+6$. Therefore, the degree of $\hat{\mathbf{G}}(s)$ is one (i.e. $\deg(\hat{\mathbf{G}}(s)) = 1$). The system is a minimal realization if and only if $\dim(\mathbf{A}) = \deg(\hat{\mathbf{G}}(s))$. Because $\mathbf{A}$ is a matrix of size 2×2, we have $\dim(\mathbf{A}) = 2$. Hence, because $\dim(\mathbf{A}) \neq \deg(\hat{\mathbf{G}}(s))$, we conclude that the system is <u>not</u> a minimal realization.