TTK4150 Nonlinear Control Systems

Fall 2024 - Assignment 4 (Solution)

Department of Engineering Cybernetics,
Norwegian University of Science and Technology

Problem 1

a The function V_1 is given by

$$V_1(x,t) = x_1^2 + \left(1+e^t\right) x_2^2$$

Since $e^t \to \infty$ when $t \to \infty$, the term $(1+e^t)$ cannot be upper bounded uniformly in t. Hence, the function is not decrescent. Since $e^t \geq 0$, $\forall t \geq 0$ the function can be lower bounded by

$$\begin{aligned} V_1(x,t) &= x_1^2 + \left(1+e^t\right) x_2^2 \\ &\geq x_1^2 + x_2^2 \\ &= W_1(x) \end{aligned}$$

where W_1 is positive definite. This implies that V_1 is positive definite.

b The function V_2 is given by

$$V_2(x,t) = \frac{x_1^2 + x_2^2}{1+t}$$

Since $\frac{1}{1+t} \to 0$ as $t \to \infty$ the function V_2 cannot be lower bounded uniformly in t. Hence, it is not positive definite. The function can be upper bounded by

$$\begin{aligned} V_2(x,t) &= \frac{x_1^2 + x_2^2}{1+t} \\ &\leq x_1^2 + x_2^2 \\ &= W_2(x) \end{aligned}$$

where W_2 is positive definite. This implies that the V_2 is decrescent.

c The function V_3 is given by

$$V_3(x,t) = \left(1+\cos^4 t\right)\left(x_1^2 + x_2^2\right)$$

Since $1 \leq \left(1+\cos^4 t\right) \leq 2$, the function is both upper and lower bounded:

$$\begin{aligned} V_3(x,t) &= \left(1+\cos^4 t\right)\left(x_1^2 + x_2^2\right) \\ &\geq x_1^2 + x_2^2 \\ &= W_1(x) \end{aligned}$$

and

$$\begin{aligned} V_3(x,t) &= \left(1+\cos^4 t\right)\left(x_1^2 + x_2^2\right) \\ &\leq 2\left(x_1^2 + x_2^2\right) \\ &= W_2(x) \end{aligned}$$

Since W_1 and W_2 are both positive definite, we conclude that the function V_3 is positive definite and decrescent.

Problem 2 The function $V(x) = \frac{1}{2}\left(bx_1^2 + ax_2^2\right)$ is positive definite and time-invariant, and hence also decrescent. Differentiation along the system trajectories gives

$$\begin{aligned} \dot{V}(x) &= bx_1\dot{x}_1 + ax_2\dot{x}_2 \\ &= bx_1\left(-\phi(t)x_1 + a\phi(t)x_2\right) + ax_2\left(b\phi(t)x_1 - ab\phi(t)x_2 - c\psi(t)x_2^3\right) \\ &= -\phi(t)b\left(x_1^2 - 2ax_1x_2 + a^2x_2^2\right) - ac\psi(t)x_2^4 \\ &= -b\phi(t)\left(x_1 - ax_2\right)^2 - ac\psi(t)x_2^4 \\ &\leq -b\phi_0\left(x_1 - ax_2\right)^2 - ac\psi_0 x_2^4 = -W_3(x) \end{aligned}$$

Since W_3 is positive definite (and V is radially unbounded), by Theorem 4.9, the origin is globally uniformly asymptotically stable.

Problem 3 The system is given by

$$\dot{x} = f(t,x) := \begin{bmatrix} x_2 \\ -x_1 - c(t)x_2 \end{bmatrix}$$

where $x := [x_1, x_2]^\top$. A continuously differentiable Lyapunov function candidate is taken as

$$V(x) = \frac{1}{2}\left(x_1^2 + x_2^2\right) \tag{1}$$

which is radially unbounded, positive definite, and time-invariant, and hence also decrescent. The derivative of V along the vector field f is

$$\begin{aligned} \dot{V}(t,x) &:= \langle \nabla V(x), f(t,x) \rangle \\ &= x_1 x_2 + x_2\left(-x_1 - c(t)\,x_2\right) \\ &= x_1 x_2 - x_1 x_2 - c(t)\,x_2^2 \\ &= -c(t)\,x_2^2 \\ &\leq -k_1 x_2^2 \\ &=: -W(x) \end{aligned}$$

Since W is positive semidefinite, by Theorem 4.8, we conclude that the origin is uniformly stable. Moreover, since $V : \mathbb{R}^2 \to \mathbb{R}$ is radially unbounded and $\langle \nabla V(x), f(t,x) \rangle \leq 0$ for all $(t,x) \in \mathbb{R}_{\geq 0} \times \mathbb{R}^2$, the origin ($x = 0$) is globally uniformly stable.
Since the original system is time-varying, the invariance principle does not apply, and we cannot use LaSalle's theorem to prove that the origin is asymptotically stable. However, in order to prove that $x_2(t) \to 0$ as $t \to \infty$ for every solution $t \mapsto x(t)$, we can apply either Lemma 8.2 or Theorem 8.4 in Khalil as described in Lecture 6:

Alternative 1: Barbalat's lemma. We have that

$$\begin{aligned} \dot{W}(t,x) := \langle \nabla W(x), f(t,x) \rangle &= -2k_1 x_2(-x_1 - c(t)x_2) \\ &= 2k_1 x_1 x_2 + 2k_1 c(t) x_2^2 \end{aligned}$$

Let $\|x\| \leq k$ where $k > 0$, it follows that $|x_i| \leq k$ and hence

$$|\langle \nabla W(x), f(t,x) \rangle| \leq 2k_1 k^2 + 2k_1 k_2 k^2$$

where we have used the fact that $c(t) \leq k_2$ for all $t \geq 0$. Hence, $W(x)$ is uniformly continuous. Consequently, all solutions approach the set

$$E = \{x \in \mathbb{R}^2 : W(x) = 0\} = \{x \in \mathbb{R}^2 : x_2 = 0\}.$$

That is, every solution $t \mapsto x(t)$ satisfies $x_2(t) \to 0$ as $t \to \infty$.

Alternative 2: LaSalle-Yoshizawa.
Let $\|x\| \leq k$, where $k > 0$, it follows that $|x_i| \leq k$ and hence

$$\begin{aligned} \|f(t,x)\|^2 &= x_2^2 + x_1^2 + 2x_1 c(t) x_2 + c(t)^2 x_2^2 \\ &\leq k^2 + k^2 + 2k^2 k_2 + k_2^2 k^2 \end{aligned}$$

where we have used that $c(t) \leq k_2$ for all $t \geq 0$. Consequently, all solutions approach the set

$$E = \{x \in \mathbb{R}^2 : W(x) = 0\} = \{x \in \mathbb{R}^2 : x_2 = 0\}.$$

That is, every solution $t \mapsto x(t)$ satisfies $x(t) \to 0$ as $t \to \infty$.

Problem 4 The system is given by

$$\begin{aligned} \dot{x}_1 &= h(t)\,x_2 - g(t)\,x_1^3 \\ \dot{x}_2 &= -h(t)\,x_1 - g(t)\,x_2^3 \end{aligned}$$

where $t \mapsto h(t)$ and $t \mapsto g(t)$ are bounded, continuously differentiable functions and $g(t) \geq k > 0,\ \forall t \geq 0$.

a It can be recognized that $x = 0$ is an equilibrium point. The stability properties are analyzed using the Lyapunov function candidate $V(x) = \frac{1}{2}\left(x_1^2 + x_2^2\right)$. The time-derivative along the trajectories of the system is found as

$$\begin{aligned}\dot{V}(x) &= x_1\left(h(t)\,x_2 - g(t)\,x_1^3\right) + x_2\left(-h(t)\,x_1 - g(t)\,x_2^3\right)\\ &= -g(t)\,x_1^4 + h(t)\,x_1x_2 - h(t)\,x_1x_2 - g(t)\,x_2^4\\ &= -g(t)\,x_1^4 - g(t)\,x_2^4\\ &= -g(t)\left(x_1^4 + x_2^4\right)\\ &\le -k\left(x_1^4 + x_2^4\right)\end{aligned}$$

Setting $W_1 = W_2 = V$ and $W_3(x) = -k\left(x_1^4 + x_2^4\right)$, we conclude by Theorem 4.9 that the origin is uniformly asymptotically stable.

b The Lyapunov function does not satisfy the conditions of Theorem 4.10 (specifically, no exponent a exist such that both conditions hold). We also cannot use Theorem 4.7, since it only applies for time-invariant systems. However, we can apply Theorem 4.15, which states that the origin is an exponentially stable equilibrium of the original system if *and only if* it is an exponentially stable equilibrium of the system linearized about the origin. First, we have

$$\frac{\partial f}{\partial x}(t,x) = \begin{bmatrix} -3g(t)x_1^2 & h(t) \\ -h(t) & -3g(t)x_2^2 \end{bmatrix}$$

On a restricted domain $D = \left\{x \in \mathbb{R}^2 | \|x\| \le r\right\}$ for some $r > 0$, the Jacobian matrix $[\partial f/\partial x]$ is Lipschitz in x and bounded, since h and g are bounded. Hence, it fulfils the conditions of Theorem 4.15. We then have

$$\begin{aligned}A(t) &= \left.\frac{\partial f}{\partial x}(t,x)\right|_{x=0}\\ &= \begin{bmatrix} 0 & h(t) \\ -h(t) & 0 \end{bmatrix}\end{aligned}$$

Since $A(t)$ is time-varying, we cannot check stability by considering the eigenvalues (see p. 157 in Khalil). But we can examine the stability of the system using a Lyapunov function candidate. Consider again $V = \frac{1}{2}(x_1^2 + x_2^2)$. Inserting the dynamics of the linearized system gives

$$\dot{V} = x_1 h(t) x_2 - x_2 h(t) x_1 = 0$$

This means that any level curve of V is an invariant set for the linearized system. In other words, a solution starting in a set $\left\{x \in \mathbb{R}^2 \,|\, V(x) = c\right\}$ will remain in this set for all time. Since solutions of the system $\dot{x} = A(t)x$ starting at other points than the origin do not converge to the origin, the origin is not an asymptotically stable nor exponentially stable equilibrium point of the linearized system. Hence by Theorem 4.15 the origin of the original, nonlinear system is not exponentially stable.
(Notice that only the higher-order terms remained previously in $\dot{V}$ for the full system, in (**a**), suggesting that it is the higher-order terms which make the system converge to the origin, not the linear)

c Since V is a radially unbounded Lyapunov function for the system satisfying $\dot{V}(x) \le -k\left(x_1^4 + x_2^4\right)$ globally, we conclude by Theorem 4.9 that the origin is globally uniformly asymptotically stable.

d Since the system is not exponentially stable, it cannot be globally exponentially stable.

Problem 5

(1) The system is not input-to-state stable (ISS) since with $u(t) \equiv c < -1$ and $x(0) > 1$ we have $x(t) \to \infty$ for $t \to \infty$.

(2) Let $V(x) = \frac{1}{2}x^2$ which is positive definite and decrescent. Then

$$\begin{aligned}\dot{V}(x) &= -x^4 - ux^4 - x^6 \le -x^6 + |u|\,x^4\\ &\le -(1-\theta)\,x^6 - \theta x^6 + |u|\,x^4\\ &= -(1-\theta)\,x^6 - x^4\left(\theta x^2 - |u|\right)\end{aligned}$$

where $0 < \theta < 1$, and

$$\dot{V}(x) \leq -(1-\theta)\,x^6, \quad \forall \, \|x\| \geq \rho(|u|) > 0$$

where

$$\rho(|u|) = \sqrt{\frac{|u|}{\theta}}$$

By Theorem 4.19 in Khalil, the system is ISS.

(3) The system is not ISS since with $u(t) \equiv 1$ and $x(0) > 1$ we have $x(t) \to \infty$ for $t \to \infty$.

(4) With $u(t) \equiv 0$, the origin of $\dot{x} = x - x^3$ is unstable. Hence, the system is not ISS.

Problem 6

(1) The system is given by

$$\begin{aligned} \dot{x}_1 &= -x_1 + x_1^2 x_2 \\ \dot{x}_2 &= -x_1^3 - x_2 + u \end{aligned}$$

Consider $V(x) = \frac{1}{2}\left(x_1^2 + x_2^2\right)$ which is a class $\mathcal{K}_\infty$ function. The derivative along the trajectories of the system is

$$\begin{aligned} \dot{V}(x) &= x_1 \dot{x}_1 + x_2 \dot{x}_2 \\ &= x_1\left(-x_1 + x_1^2 x_2\right) + x_2\left(-x_1^3 - x_2 + u\right) \\ &= -x_1^2 + x_1^3 x_2 - x_1^3 x_2 - x_2^2 + u x_2 \\ &= -x_1^2 - x_2^2 + u x_2 \\ &= -\|x\|_2^2 + u x_2 \end{aligned}$$

and upper bounded as

$$\begin{aligned} \dot{V}(x) &\leq -\|x\|_2^2 + |u x_2| \\ &\leq -\|x\|_2^2 + |u|\,\|x\|_2 \\ &= -\|x\|_2^2 + |u|\,\|x\|_2 + \theta\,\|x\|_2^2 - \theta\,\|x\|_2^2 \\ &= -(1-\theta)\,\|x\|_2^2 - \left(\theta\,\|x\|_2 - |u|\right)\|x\|_2 \\ &\leq -(1-\theta)\,\|x\|_2^2, \quad \forall \, \|x\|_2 \geq \frac{|u|}{\theta} \end{aligned}$$

where $\theta \in (0,1)$. Hence, by Theorem 4.19, the system is ISS with $\rho(|u|) = \frac{|u|}{\theta}$.

(2) The system is given by

$$\begin{aligned} \dot{x}_1 &= -x_1 + x_2 \\ \dot{x}_2 &= -x_1^3 - x_2 + u \end{aligned}$$

Consider the Lyapunov function candidate $V(x) = \frac{1}{4}x_1^4 + \frac{1}{2}x_2^2$. The derivative along the trajectories of the system is

$$\begin{aligned} \dot{V}(x) &= x_1^3 \dot{x}_1 + x_2 \dot{x}_2 \\ &= x_1^3\left(-x_1 + x_2\right) + x_2\left(-x_1^3 - x_2 + u\right) \\ &= -x_1^4 + x_1^3 x_2 - x_1^3 x_2 - x_2^2 + u x_2 \\ &= -x_1^4 - x_2^2 + u x_2 \end{aligned}$$

which is upper bounded as

$$\begin{aligned} \dot{V}(x) &= -x_1^4 - (1-\theta)\,x_2^2 + u x_2 - \theta x_2^2 \\ &\leq -x_1^4 - (1-\theta)\,x_2^2, \quad \forall \, |x_2| \geq \frac{|u|}{\theta} \end{aligned} \tag{2}$$

where $\theta \in (0,1)$. With $|x_2| \le \frac{|u|}{\theta}$, we have

$$\begin{aligned} \dot{V}(x) &\le -x_1^4 - x_2^2 + \frac{|u|^2}{\theta} \\ &= -(1-\theta)x_1^4 - x_2^2 - \left(\theta x_1^4 - \frac{|u|^2}{\theta}\right) \\ &\le -(1-\theta)x_1^4 - x_2^2, \quad \forall\, |x_1| \ge \sqrt{\frac{|u|}{\theta}} \end{aligned} \tag{3}$$

Combining (2) and (3) it follows that

$$\dot{V}(x) \le -(1-\theta)\left(x_1^4 + x_2^2\right), \quad \forall\, \|x\|_\infty \ge \rho(|u|)$$

where

$$\rho(r) = \max\left(\frac{r}{\theta}, \sqrt{\frac{r}{\theta}}\right)$$

Hence, the system is ISS.

(3) With $u = 0$ the system is given by

$$\begin{aligned} \dot{x}_1 &= (x_1 - x_2)\left(x_1^2 - 1\right) \\ \dot{x}_2 &= (x_1 + x_2)\left(x_1^2 - 1\right) \end{aligned}$$

and it can be seen that it has an equilibrium set $\left\{x_1^2 = 1\right\}$. Hence, the origin is not globally asymptotically stable. It follows that the system is not ISS.

(4) The unforced system ($u = 0$) has equilibrium points $(-1,-1)$, $(0,0)$ and $(1,1)$. Hence, the origin is not globally asymptotically stable. Consequently, the system is not ISS.

Problem 7 The system is given by

$$\dot{x}_1 = -x_1 + x_2^2 \tag{4}$$

$$\dot{x}_2 = -x_2 \tag{5}$$

which can be viewed as a cascade. The system (5) is linear and its eigenvalue is negative. The origin of (5) is therefore GES. We now want to show that the system (4) is ISS when x_2 is viewed as input. Let $V = \frac{1}{2}x_1^2$. Then

$$\begin{aligned} \dot{V} &= x_1(-x_1 + x_2^2) \\ &= -(1-\theta)x_1^2 - \theta x_1^2 + x_1 x_2^2 \\ &\le -(1-\theta)x_1^2 - \theta x_1^2 + |x_1| x_2^2 \\ &\le -(1-\theta)x_1^2, \quad \forall\, |x_1| \ge \frac{x_2^2}{\theta} \end{aligned}$$

where $0 < \theta < 1$. Since $\rho(r) = \frac{r^2}{\theta}$ is a class $\mathcal{K}$ function, by Theorem 4.19, the system (4) is ISS with x_2 as input. Hence by Lemma 4.7, the origin of the cascade system (4)-(5) is globally asymptotically stable.
(Note: While it is tempting to use Lemma 4.6 to conclude that the system (4) is ISS, the function $f(t,x) = -x_1 + x_2^2$ is not globally Lipschitz in x_2, and hence, the lemma cannot be applied.)

Problem 8

a Let $\alpha(r) = r^{1/3}$, which is a class $\mathcal{K}_\infty$ function. We have

$$|y| \le |u|^{1/3} \Longrightarrow \|y_\tau\|_{\mathcal{L}_\infty} \le \left(\|u_\tau\|_{\mathcal{L}_\infty}\right)^{1/3} \Longrightarrow \|y_\tau\|_{\mathcal{L}_\infty} \le \alpha\left(\|u_\tau\|_{\mathcal{L}_\infty}\right).$$

Hence the system is $\mathcal{L}_\infty$ stable with zero bias.

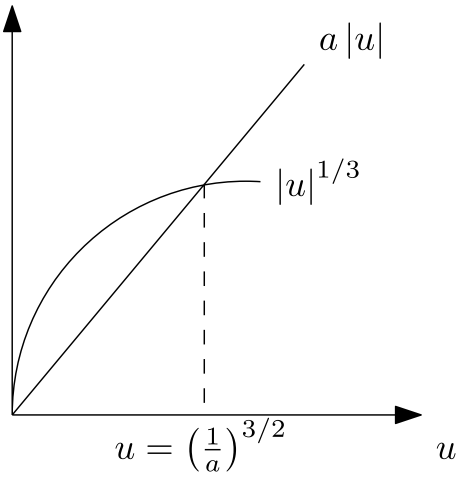

Figure 1: $y = a\,|u|$ and $y = |u|^{1/3}$.

b The two curves $|y| = |u|^{1/3}$ and $|y| = a\,|u|$ intersect at the point $|u| = (1/a)^{3/2}$. See Figure 1. Therefore, for $|u| \le (1/a)^{3/2}$ we have

$$|y| \le |u|^{1/3} \le (1/a)^{3/2 \cdot 1/3} = (1/a)^{1/2}$$

while for $|u| > (1/a)^{3/2}$ we have

$$|y| \le a\,|u|$$

Thus

$$|y| \le a\,|u| + (1/a)^{1/2}, \quad \forall\,|u| \ge 0.$$

Setting $\gamma = a$ and $\beta = (1/a)^{1/2}$ we obtain

$$\|y_\tau\|_{\mathcal{L}_\infty} \le \gamma\,\|u_\tau\|_{\mathcal{L}_\infty} + \beta.$$

c The example shows that a nonzero bias term may be used to achieve finite-gain stability in situations where it is not possible to have finite-gain stability with zero bias.

Problem 9 For the norm $|y(t)|$ of the system output we have

$$\begin{aligned} |y(t)| = |h(u)| &= |h(u) - h(0) + h(0)| \\ &\le |h(u) - h(0)| + |h(0)| \\ &\le L|u - 0| + |h(0)| = L|u| + |h(0)| \end{aligned}$$

using the triangle inequality and the property that the system is globally Lipschitz. We can now look at each of the two cases:

(1) $h(0) = 0 \Rightarrow |h(u)| \le L\,|u|,\ \forall\,u$. For $p = \infty$ we have

$$\sup_{t \ge 0} |y(t)| \le L \sup_{t \ge 0} |u(t)|$$

which shows that the system is finite-gain $\mathcal{L}_\infty$ stable with zero bias. For $p \in [1, \infty)$ we have

$$\left(\int_0^\tau |y(t)|^p\,dt\right)^{\frac{1}{p}} \le \left(L^p \int_0^\tau |u(t)|^p\,dt\right)^{\frac{1}{p}} \quad \Rightarrow \quad \|y_\tau\|_{\mathcal{L}_p} \le L\,\|u_\tau\|_{\mathcal{L}_p}.$$

Hence for each $p \in [1, \infty)$ the system is finite gain $\mathcal{L}_p$ stable with zero bias.

(2) Now let $|h(0)| = k > 0$. Then $|h(u)| \le L\,|u| + k$. For $p = \infty$ we have

$$\sup_{t \ge 0} |y(t)| \le L \sup_{t \ge 0} |u(t)| + k$$

which shows that the system is finite gain $\mathcal{L}_\infty$ stable. For $p \in [1, \infty)$ the integral

$$\int_0^\tau (L\,|u(t)| + k)^p \, dt$$

diverges as $\tau \to \infty$. The system is not $\mathcal{L}_p$ stable for $p \in [1, \infty)$, as can be seen by letting $u(t) \equiv 0$.

Problem 10 The closed-loop transfer functions are given by

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} \frac{s-1}{s+2} & \frac{-1}{s+2} \\ \frac{1}{s+2} & \frac{s+1}{(s-1)(s+2)} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}, \quad \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} = \begin{bmatrix} \frac{s+1}{s+2} & \frac{-(s+1)}{(s-1)(s+2)} \\ \frac{s-1}{s+2} & \frac{s+1}{s+2} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

The closed-loop transfer function from (u_1, u_2) to (y_1, y_2) (or (e_1, e_2)) has four components. Due to pole-zero cancellation of the unstable pole $s = 1$, three of these components do not contain the unstable pole; thus, each component by itself is input-output stable. If we restrict our attention to any one of these components, we miss the unstable hidden mode. By studying all four components we will be sure that unstable hidden modes must appear in at least one component.

Problem 11 Using $V(x) = a \int_0^x h(\sigma)$, we have

$$\dot{V}(x) = ah(x)\dot{x} = h(x)\left[-x + \frac{1}{k}h(x) + u\right] = \frac{1}{k}h(x)\,[h(x) - kx] + h(x)u$$

Under the sector condition $h \in [0, k]$ (see Definition 6.2), we have

$$h(x)\,[h(x) - kx] \le 0$$

which leads to

$$\dot{V} = \frac{1}{k}h(x)\,[h(x) - kx] + h(x)u \le yu$$

Thus, by Definition 6.3 the system is passive.

Problem 12

a We have

$$\begin{aligned} \dot{V}(\tilde{x}) &= \tilde{x}_1\dot{\tilde{x}}_1 + m\tilde{x}_2\dot{\tilde{x}}_2 \\ &= \tilde{x}_1\tilde{x}_2 + \tilde{x}_2\left[-f_3\left[(\tilde{x}_1 + x_{1d})^3 - x_{1d}^3\right] - f_1\tilde{x}_1 + \tilde{u}\right] \end{aligned}$$

Selecting $\tilde{u}$ as

$$\tilde{u} = f_3\left[(\tilde{x}_1 + x_{1d})^3 - x_{1d}^3\right] + f_1\tilde{x}_1 - \tilde{x}_1 + v$$

yields

$$\dot{V}(\tilde{x}) = \tilde{x}_2 v$$

which shows that the system is passive from the input v to the output $y = \tilde{x}_2$.

b The zero state observability is checked as follows:

$$y = 0 \implies \tilde{x}_2 = 0 \implies \dot{\tilde{x}}_2 = 0 \implies -f_3\left[(\tilde{x}_1 + x_{1d})^3 - x_{1d}^3\right] - f_1\tilde{x}_1 = 0$$

Since the only real solution is $\tilde{x}_1 = 0$, the system is zero state observable.

c Take the storage function V as a Lyapunov function candidate, which is radially unbounded. In order to stabilize the origin we choose v such that $\dot{V}$ becomes negative semidefinite. Let $v = -k\tilde{x}_2$. This gives

$$\dot{V} = -k\tilde{x}_2^2\ .$$

Let $S = \{\tilde{x} \in \mathbb{R}^2 \,|\, \dot{V} = 0\}$. Since $\tilde{x}_2$ is the output of the system, and we have shown that the system is zero-state observable, no solution can stay identically in S other than the trivial solution. Hence by Corollary 4.2 in Khalil, the origin is a globally asymptotically stable equilibrium of the system.

(Notice that this choice of v stabilizes the system by reintroducing an artificial damping to the system.)

Problem 13 Let the input to the system be denoted $\tilde{u}$ and the output be denoted $\tilde{y}$. From the block diagram we have the following relations

$$\begin{aligned} \tilde{y} &= h(t,u) - K_1 u \\ \tilde{u} + \tilde{y} &= K u \end{aligned}$$

From the sector condition we have that

$$\begin{gathered} (h(t,u) - K_1 u)^T (h(t,u) - K_2 u) \leq 0 \\ K = K_2 - K_1 = K^T > 0 \end{gathered} \tag{6}$$

Evaluating the block diagram it can be seen that

$$h(t,u) - K_1 u = \tilde{y} \tag{7}$$

and

$$\begin{aligned} h(t,u) - K_2 u &= h(t,u) - K_2 u - K_1 u + K_1 u \\ &= \tilde{y} - (K_2 - K_1) u \\ &= \tilde{y} - K u \\ &= \tilde{y} - \tilde{u} - \tilde{y} \\ &= -\tilde{u} \end{aligned} \tag{8}$$

Using (7), (8) and the sector condition (6) we have

$$\begin{aligned} (h(t,u) - K_1 u)^T (h(t,u) - K_2 u) &= \tilde{y}^T (-\tilde{u}) \\ &= -\tilde{u}^T \tilde{y} \\ &\leq 0 \\ &\Rightarrow \tilde{u}^T \tilde{y} \geq 0 \end{aligned}$$

which implies that the system is passive from $\tilde{u}$ to $\tilde{y}$, which corresponds to being in sector $[0, \infty]$.