Norwegian University of Science and Technology
Department of Electronics and Telecommunications

TTT4120 Digital Signal Processing
Problem Set 3

Problem 1 (2.5 points)

Consider the two causal analog filters shown in Figure 1.

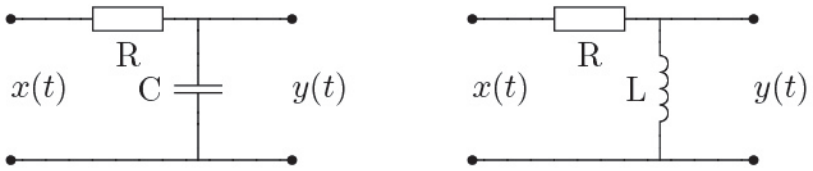

Figure 1: Analog filters

(a) Derive the differential equations and transfer functions for the two filters.

(b) Determine frequency responses and filter types (i.e. lowpass, highpass, bandpass, bandstop) for the filters.

(c) Derive the unit pulse responses for the filters.

Problem 2 (2.5 points)

Find the region of convergence and unit pulse response $h[n]$ for the following digital filters.

(a) A causal filter with transfer function

$$H(z) = \frac{1}{1 - \frac{2}{3}z^{-1}}$$

(b) A causal filters with transfer function

$$H(z) = \frac{1}{(1 + \frac{1}{2}z^{-1})(1 - z^{-1})}$$

(c) An anti-causal filter with transfer function

$$H(z) = \frac{z^{-1}}{(1 + \frac{3}{2}z^{-1})(1 - 3z^{-1})}$$

(d) Which of the three systems are stable? Justify your answer.

Problem 3 (3 points)

Consider a linear time-invariant (LTI) system with unit pulse response $h[n]$ and input signal $x[n]$.

$$h[n] = \begin{cases} \frac{1}{2^n} & n \geq 0 \\ 0 & n < 0, \end{cases}$$

$$x[n] = \begin{cases} 1 & n \geq 2 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Determine the z-transform of the unit pulse response $h[n]$ and input signal $x[n]$ and their respective ROCs.

(b) Derive an expression for the output signal $y[n]$ by performing the convolution in the time domain.

(c) Derive an expression for the output signal $y[n]$ using the z-transform.

Problem 4 (2 points)

A digital filter is given by the following difference equation

$$y[n] = x[n] - x[n-2] - \frac{1}{4}y[n-2].$$

(a) Find the transfer function of the filter.

(b) Find the poles and zeros of the filter and sketch them in the z-plane.

(c) Is the filter stable? Justify your answer based on the pole-zero plot.

(d) Determine the filter type (i.e. HP, LP, BP or BS) based on the pole-zero plot.