Norwegian University of Science and Technology
Department of Electronics and Telecommunications

TTT4120 Digital Signal Processing
Problem Set 5

The main topics for this problem set are correlation, energy density spectra, and simple filter design. Relevant chapters from the textbook are 2.5.1, 2.6.1, 2.6.2, 2.6.4, 4.2.5, and 4.4.7. The maximum score for each problem is given in parentheses.

Problem 1 (4 points)

(a) Derive the energy density spectrum $S_{xx}(f)$ of the signal

$$x[n] = \begin{cases} a^n & n \geq 0, \quad |a| < 1 \\ 0 & n < 0. \end{cases}$$

(b) Derive the autocorrelation $r_{xx}(l)$ of the signal given in 1a.

Use $r_{xx}(l)$ to verify the expression for $S_{xx}(f)$ found in 1a.

(c) Plot $x[n]$, $r_{xx}(l)$ and $S_{xx}(f)$ for $a = 0.5$, $a = 0.9$, and $a = -0.9$. Let $n \in [0, 50]$, $l \in [-50, 50]$ and $f \in [-0.5, 0.5]$ in your plots.

Compare the plots for the three different values of a. Write down your observations and explain them.

What properties of the autocorrelation function can you see from the plots?

(d) Find the energy of the signal $x[n]$.

(e) At this point, the signal $x[n]$ is first passed through the filter $h_1[n]$ and then the result is fed to the filter $h_2[n]$, where the first filter is given in terms of its impulse response and the second one in terms of its frequency response as follows. The final result is denoted by $y[n]$.

$$h_1[n] = \delta[n] - a\delta[n-1]$$

$$H_2(f) = \begin{cases} \cos(2\pi f) & |f| \leq \frac{1}{4} \\ 0 & \frac{1}{4} < |f| \leq \frac{1}{2} \end{cases}$$

Find the energy density spectrum of the output signal, $S_{yy}(f)$. Compare the result to $S_{xx}(f)$ and comment. Find the total energy in the output signal.

Problem 2 (3 points)

Suppose we have a radar that emits a signal $x[n]$. When the signal hits an object, it is reflected and the radar will receive a delayed version of the signal. By measuring the delay, we can determine the distance of the object.

However, the emitted signal will be attenuated and contaminated by noise as it travels through the air, so the received signal is given by

$$y[n] = \begin{cases} \alpha x[n-D] + w[n], & \text{if an object is hit,} \\ w[n], & \text{if no object is hit,} \end{cases}$$

where α is the attenuation factor, D is the delay, and $w[n]$ is the noise.

In this problem, you are given a emitted signal $x[n]$ and a received signal $y[n]$ from a radar system. These can be found in a file called `signals.mat` on the course homepage under Problem sets. Download the file and use the Matlab command `load` or the Python command `scipy.io.loadmat` to load the signals vectors x and y.

(a) Plot the signals $x[n]$ and $y[n]$.

Can you determine reliably whether an object has been hit by the emitted signal from these two plots only? Explain.

(b) Find the crosscorrelation function $r_{yx}(l)$ by using the Matlab function `xcorr` or the Python function `scipy.signal.correlate`. Plot the result. Notice that the output of these functions is a vector of length $2L-1$ that correspond to the values of $r_{yx}(l)$ for $l \in [-L+1, L-1]$.

(c) Find the crosscorrelation function $r_{yx}(l)$ by using the Matlab functions `conv` and `fliplr` or the Python functions `np.convolve` and `np.flip`. Plot the result using `stem` or `plt.stem` and verify that the result is the same as in 2(a).

(d) Based on the plot of $r_{yx}(l)$, find out whether an object has been hit by the emitted signal, and if so, determine the value of the delay D.

Is this result more reliable than that of the direct comparison of the signals $x[n]$ and $y[n]$ in 2(a)?

Problem 3 (3 points)

When listening to a live music performance in a concert hall or another confined space, the sound received by a listener consists of not only the direct sound from the artists/instruments, but also delayed reflections from surfaces and objects in the surrounding environment. These echoes are possibly not present in a musical recording done in a studio (where echoes are often unwanted and therefore physically avoided/damped). In order to create an illusion of listening to the recording in a more spacious/hall-like environment, the sound engineer may add multiple closely spaced echoes to the recording. Alternatively, it can be done during playback in the listeners' home equipment.

Consider the simple filter structure in Figure 1. The output signal $y[n]$ is composed of the input signal $x[n]$ and a delayed (by R samples) and scaled (by the factor α) replica of the input.

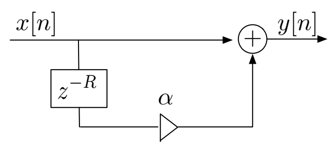

Figure 1: Filter structure of single-echo-generating filter

(a) Find the transfer function $H(z)$ for the filter in Figure 1.

(b) Given a sample frequency of 22,050Hz for the input signal. What is the delay in seconds for a given delay R?

(c) Implement the filter in Matlab or Python, plot the unit sample response and the frequency response. (Hint: You can use the Matlab functions `impz` and `freqz` or the Python functions `lti`, `dimpulse` and `freqz` from `scipy.signal`.)

(d) Load the sound file `piano.wav`. The sample frequency is 22,050Hz (Available on It's learning). Pass it through the filter and observe the effect for different values of R and α. Comment!

A filter giving multiple echoes is shown in Figure 2.

(e) Implement the filter in Figure 2 in Matlab or Python. Vary the parameters R, α and N (where R is the spacing between echoes and N is the number of echoes, α as above).

(f) Notice the difference in impulse response compared to the single-echo case.

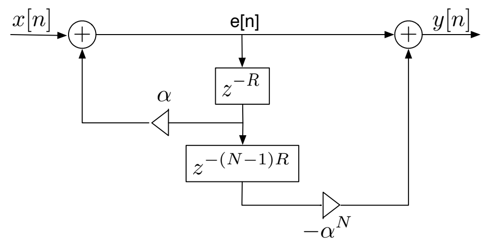

Figure 2: Filter structure for generating N echoes.

(g) How does the filtered signal sound compared to the original signal?

(h) Which of the two filter structures gives the most "natural" sound? Why?