

# Методы оптимизации, лабораторная № 1

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## 1 Логистическая регрессия

### 1.1 Вычисление ответа

$$a(x) = \text{sign}(\langle w, x \rangle)$$

$$p_+(x) = \sigma(\langle w, x \rangle)$$

### 1.2 Функционал качества

$$Q(w) = \frac{1}{L} \sum_{i=1}^L \ln(1 + e^{-\langle w, x_i \rangle \cdot y_i})$$

$$Q^{l_2}(w) = Q(w) + \frac{\lambda}{2} \|w\|^2 = Q(w) + \frac{\lambda}{2} \sum_{i=1}^{F+1} w_i^2$$

### 1.3 Градиент

$$\frac{\partial Q}{\partial w_j} = \frac{1}{L} \sum_{i=1}^L -\frac{e^{-\langle w, x_i \rangle \cdot y_i} \cdot y_i \cdot x_{i,j}}{1 + e^{-\langle w, x_i \rangle \cdot y_i}} = -\frac{1}{L} \sum_{i=1}^L \frac{y_i \cdot x_{i,j}}{1 + e^{\langle w, x_i \rangle \cdot y_i}}$$

$$\left( \frac{\partial Q}{\partial w} \right)_{F+1} = -\frac{1}{L} A_{F+1,L} \cdot b_L$$

$$A_{j,i} = y_i \cdot x_{i,j}$$

$$b_i = \frac{1}{1 + e^{\langle w, x_i \rangle \cdot y_i}}$$

$$\frac{\partial Q^{l_2}}{\partial w_j} = \frac{\partial Q}{\partial w_j} + \lambda \cdot w_j$$

$$\frac{\partial Q^{l_2}}{\partial w} = \frac{\partial Q}{\partial w} + \lambda \cdot w = -\frac{1}{L} A \cdot b + \lambda w$$

## 1.4 Гессиан

$$\frac{\partial^2 Q}{\partial w_j \partial w_k} = \frac{\partial}{\partial w_k} \frac{\partial Q}{\partial w_j} = \frac{1}{L} \sum_{i=1}^L \frac{x_{i,j} \cdot x_{i,k} \cdot y_i^2 \cdot e^{\langle w, x_i \rangle \cdot y_i}}{(e^{\langle w, x_i \rangle \cdot y_i} + 1)^2} = \frac{1}{L} \sum_{i=1}^L \frac{x_{i,j} \cdot x_{i,k} \cdot e^{\langle w, x_i \rangle \cdot y_i}}{(e^{\langle w, x_i \rangle \cdot y_i} + 1)^2}$$

$$\left( \frac{\partial^2 Q}{\partial^2 w} \right)_{F+1, F+1} = \frac{1}{L} C_{F+1, L} \cdot D_{L, F+1}$$

$$C_{j,i} = \frac{x_{i,j}}{e^{\langle w, x_i \rangle \cdot y_i} + 1}$$

$$D_{i,k} = \frac{x_{i,k} \cdot e^{\langle w, x_i \rangle \cdot y_i}}{e^{\langle w, x_i \rangle \cdot y_i} + 1} = \frac{x_{i,k}}{1 + e^{-\langle w, x_i \rangle \cdot y_i}}$$

$$\frac{\partial^2 Q^{l_2}}{\partial w_j \partial w_k} = \frac{\partial^2 Q}{\partial w_j \partial w_k} + \lambda \frac{\partial w_j}{\partial w_k}$$

$$\frac{\partial w_j}{\partial w_k} = \begin{cases} 0 & j \neq k \\ 1 & j = k \end{cases}$$

$$\frac{\partial^2 Q^{l_2}}{\partial^2 w} = \frac{\partial^2 Q}{\partial^2 w} + \lambda I_{F+1}$$