Методы оптимизации, лабораторная № 1

24 февраля 2020 г.

1 Логистическая регрессия

1.1 Вычисление ответа

$$a(x) = sign(\langle w, x \rangle)$$

$$p_+(x) = \sigma(\langle w, x \rangle)$$

1.2 Функционал качества

$$Q(w) = \frac{1}{L} \sum_{i=1}^{L} ln(1 + e^{-\langle w, x_i \rangle \cdot y_i})$$

$$Q^{l_2}(w) = Q(w) + \frac{\lambda}{2} ||w||^2 = Q(w) + \frac{\lambda}{2} \sum_{i=1}^{F+1} w_i^2$$

1.3 Градиент

$$\frac{\partial Q}{\partial w_j} = \frac{1}{L} \sum_{i=1}^{L} -\frac{e^{-\langle w, x_i \rangle \cdot y_i} \cdot y_i \cdot x_{i,j}}{1 + e^{-\langle w, x_i \rangle \cdot y_i}} = -\frac{1}{L} \sum_{i=1}^{L} \frac{y_i \cdot x_{i,j}}{1 + e^{\langle w, x_i \rangle \cdot y_i}}$$

$$\left(\frac{\partial Q}{\partial w} \right)_{F+1} = -\frac{1}{L} A_{F+1,L} \cdot b_L$$

$$A_{j,i} = y_i \cdot x_{i,j}$$

$$b_i = \frac{1}{1 + e^{\langle w, x_i \rangle \cdot y_i}}$$

$$\frac{\partial Q^{l_2}}{\partial w_j} = \frac{\partial Q}{\partial w_j} + \lambda \cdot w_j$$

$$\frac{\partial Q^{l_2}}{\partial w} = \frac{\partial Q}{\partial w} + \lambda \cdot w = -\frac{1}{L} A \cdot b + \lambda w$$

1.4 Гессиан

$$\frac{\partial^2 Q}{\partial w_j \partial w_k} = \frac{\partial}{\partial w_k}\frac{\partial Q}{\partial w_j} = \frac{1}{L}\sum_{i=1}^{L} \frac{x_{i,j} \cdot x_{i,k} \cdot y_i^2 \cdot e^{\langle w, x_i \rangle \cdot y_i}}{(e^{\langle w, x_i \rangle \cdot y_i} + 1)^2} = \frac{1}{L}\sum_{i=1}^{L} \frac{x_{i,j} \cdot x_{i,k} \cdot e^{\langle w, x_i \rangle \cdot y_i}}{(e^{\langle w, x_i \rangle \cdot y_i} + 1)^2}$$

$$\left(\frac{\partial^2 Q}{\partial^2 w}\right)_{F+1,F+1} = \frac{1}{L} A_{F+1,L} \cdot B_{L,F+1}$$

$$A_{j,i} = \frac{x_{i,j}}{e^{\langle w, x_i \rangle \cdot y_i} + 1}$$

$$B_{i,k} = \frac{x_{i,k} \cdot e^{\langle w, x_i \rangle \cdot y_i}}{e^{\langle w, x_i \rangle \cdot y_i} + 1}$$

$$\frac{\partial^2 Q^{l_2}}{\partial w_j \partial w_k} = \frac{\partial^2 Q}{\partial w_j \partial w_k} + \lambda \frac{\partial w_j}{\partial w_k}$$

$$\frac{\partial w_j}{\partial w_k} = \begin{cases} 0 & j \neq k \\ 1 & j = k \end{cases}$$

$$\frac{\partial^2 Q^{l_2}}{\partial^2 w} = \frac{\partial^2 Q}{\partial^2 w} + \lambda I_{F+1}$$