MA0301 ELEMENTARY DISCRETE MATHEMATICS
SPRING 2017

1. Homework Set 3 – Solutions

Exercise 1. *Grimaldi's book (5. ed., Exercises 15.1): solve* **Exercise 1 c,d**

Solution 1. c) 1, d) 1

Exercise 2. *Grimaldi's book (5. ed., Exercises 15.1): solve* **Exercise 2 c,d**

Solution 2. c) $y = 1$, $w = 1$ and $y = 0$, $w = 1$, d) $y = 1$, $w = 1$ and $y = 0$, $w = 1$

Exercise 3. *Grimaldi's book (5. ed., Exercises 15.1): solve* **Exercise 11 b,c**

Solution 3.

$$
\begin{aligned}
b) \quad x + y + (x' + y + z)' &= x + y + xy'z' \\
&= x(1 + y'z') + y \\
&= x + y
\end{aligned}
$$

$$
\begin{aligned}
c) \quad yz + wx + z + (wz[xy + wz])) &= z(y + 1 + w[xy + wz]) + wx \\
&= z + wx
\end{aligned}
$$

Exercise 4. *Grimaldi's book (5. ed., Exercises 15.1): solve* **Exercise 10**

Solution 4. Pick one of the variables, say, x, and consider $x = 0$, then $x \cdot y \cdot z = 0$. Which then means that $0 + y + z = 0$. This implies that $y = z = 0$. For $x = 1$ it follows that $1 + y + z = 1$, which implies that $1 = x \cdot y \cdot z$, and this only can be true for $y = z = 1$.

Exercise 5. *Grimaldi's book (5. ed., Exercises 3.2): solve* **Exercise 16**

Solution 5. Done in the solutions for exercise set 2.

Exercise 6. *Let B be a Boolean algebra. For $x, y, z \in B$ find the dual expressions of*

$$i)\ x \cdot y' + x \cdot z' + y \cdot x'$$

$$ii)\ x \cdot y \cdot z' + x \cdot y' \cdot z$$

$$iii)\ x \cdot y \cdot (x + 0 + (z \cdot 1))$$

Solution 6. Note the parentheses.

$$i)\ (x + y') \cdot (x + z') \cdot (y + x')$$

$$ii)\ (x + y + z') \cdot (x + y' + z)$$

$$iii)\ (x + y + (x \cdot 1 \cdot (z + 0))$$

Exercise 7. *Let B be a Boolean algebra. Prove for $x, y \in B$ that $x \cdot y' = 0$ if and only if $x \cdot y = x$.*

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Solution 7. Let's start with assuming $x \cdot y' = 0$. Then $x \cdot y' + x \cdot y = 0 + x \cdot y = x \cdot y$. However, $x \cdot y' + x \cdot y = x \cdot (y + y') = 1 \cdot x = x$. Therefore, $x = x \cdot y$.

Now we assume $x \cdot y = x$. Then $(x \cdot y) \cdot y' = x \cdot 0 = 0 = x \cdot y'$.

Exercise 8. *Let B be a Boolean algebra. Let $x, y, z \in B$ and reduce the following expressions as much as possible.*

$$i)\ xyz'yx \qquad ii)\ xyz'yx'z'$$

Solution 8.

$$i)\ xyz'yx = (xx)(yy)z' = xyz'$$
$$ii)\ xyz'yx'z' = xx'yyz'z' = 0yz' = 0$$

2. Classroom Set 3 – Solutions

Exercise 1. *Grimaldi's book (5. ed., Exercises 15.1): solve* **Exercise 1 a,b**

Solution 1. a) 1, b) 1

Exercise 2. *Grimaldi's book (5. ed., Exercises 15.1): solve* **Exercise 2 a,b**

Solution 2. a) any values for y, w will result in 1, b) $y = 1$, $w = 1$ and $y = 0$, $w = 1$ and $y = 1$, $w = 0$

Exercise 3. *Grimaldi's book (5. ed., Exercises 15.1): solve* **Exercise 11 a**

Solution 3.

$$\begin{aligned} a)\quad xy + (x+y)z' + y &= y + (x+y)z' \\ &= xz' + y \end{aligned}$$

Exercise 4. *Grimaldi's book (5. ed., Exercises 15.1): solve* **Exercise 12**

Solution 4. This set of equations in a Boolean algebra has to hold simultaneously. Therefore, from $x + x'y = 0$ follows that $x = 0 = y$. From this and $x'y = x'z$ it follows that $z = 0$. Now, with $x = y = z = 0$ and $x'y + x'z' + zw = z'w$ follows that $w = 1$.

Exercise 5. *Let B be a Boolean algebra. For $x, y, z \in B$ find the dual expressions of*

$$i)\ (x + y') \cdot (z' + y)' \qquad ii)\ (1 + x) \cdot y + x \cdot y' \cdot z \qquad iii)\ (x \cdot y + 1) \cdot (0 + x) \cdot z$$

Solution 5. Note the parentheses. i) $(x \cdot y') + (z' \cdot y)'$, ii) $(0 \cdot x + y) \cdot (x + y' + z)$, iii) $((x + y) \cdot 0) + (1 \cdot x) + z$

Exercise 6. *Let B be a Boolean algebra. Prove for $x, y, z \in B$ that if $x \cdot y = x \cdot z$ and $x' \cdot y = x' \cdot z$, then $y = z$.*

Solution 6. Both $x \cdot y = x \cdot z$ and $x' \cdot y = x' \cdot z$ hold. Then their sum $x \cdot y + x' \cdot y = x \cdot z + x' \cdot z$ implies that $(x + x') \cdot y = (x + x') \cdot z$. With $x + x' = 1$ follows that $1 \cdot y = 1 \cdot z$ which implies that $y = z$.

Exercise 7. *Let B be a Boolean algebra. Let $x, y, z \in B$ and reduce the following expressions as much as possible.*

$$i)\ xyx'z \qquad ii)\ xyzy$$

Solution 7.

$$i)\ xyx'z = xx'yz = (xx')yz = 0yz = 0$$
$$ii)\ xyzy = xzyy = xzy$$