MA0301 ELEMENTARY DISCRETE MATHEMATICS
SPRING 2017

1. Homework Set 12 – Solutions

Exercise 1. *Grimaldi's book (5. ed., Exercises 11.4, page 553): solve* **Ex. 1**

Solution 1. Drawing the edge $\{a, c\}$ in the exterior of the pentagon results in the vertex b being in the region formed by the edges $\{a, d\}$, $\{d, c\}$, $\{c, a\}$. The vertex e lies outside of this region. Therefore the edge $\{b, e\}$ must cross one of the edges $\{a, d\}$, $\{d, c\}$, $\{c, a\}$.

Exercise 2. *Grimaldi's book (5. ed., Exercises 11.4, page 553): solve* **Ex. 4**

Solution 2. Start with graph $G = (V, E)$ which is bipartite, i.e., $V = V_1 \cup V_2$ and edges $\{a, b\} \in E$ are such that $a \in V_1$ and $b \in V_2$. Assume that $H = (V_H, E_H)$ is a subgraph of G. Then $V_H = V_H \cap V = (V_H \cap V_1) \cup (V_H \cap V_2)$ and $(V_H \cap V_1) \cap (V_H \cap V_2) = \emptyset$. Any edge $\{x, y\} \in E_H$ is also an edge in G, with $x \in V_1$ and $y \in V_2$. Hence, H is bipartite.

Exercise 3. *Grimaldi's book (5. ed., Exercises 11.4, page 553): solve* **Ex. 5**

Solution 3. a) Choose $V_1 := \{a, d, e, h\}$ and $V_2 := \{b, c, f, g\}$.

b) Choose $V_1 := \{a, b, g, h\}$ and $V_2 := \{c, d, e, f\}$. What is the relation of this graph to the complete bipartite graph $K_{4,4}$?

c) This graph is not bipartite. Assume this graph $G = (V, E)$ was bipartite ($V = V_1 \cup V_2$) and consider the vertices a, b, c, d, e with $a \in V_1$. Then b and c must be in V_2. We also see that $d \in V_1$. Now, since there is an edge $\{d, e\}$ we must have $e \in V_2$. But the edge $\{c, e\}$ implies that $e \in V_1$, which shows that G is not bipartite.

Exercise 4. *Grimaldi's book (5. ed., Exercises 11.4, page 554): solve* **Ex. 16**

Solution 4. See Fig. 11.52(a) in Grimaldi's book. Consider the following mapping:

$$a \to s \quad b \to v \quad c \to z \quad d \to y \quad e \to t$$

and

$$f \to u \quad g \to r \quad h \to w \quad i \to x \quad j \to q.$$

Exercise 5. *Grimaldi's book (5. ed., Exercises 11.4, page 554): solve* **Ex. 21**

Solution 5. Let $G = (V, E)$ and suppose that $\deg(v) > 5$ for all vertices. Then $2|E| = \sum_{v \in V} \deg(v) \geq 6|V|$, which implies that $|E| \geq 3|V|$. This contradicts $|E| \leq 3|V| - 6$ in Cor. VII.37.

Exercise 6. *Grimaldi's book (5. ed., Exercises 11.5, page 563): solve* **Ex. 3**

Date: May 11, 2017.

Solution 6. a)
$$a \to g \to k \to i \to h \to b \to c \to d \to j \to f \to e \to a.$$

b)
$$a \to d \to b \to e \to g \to j \to i \to f \to h \to c \to a.$$

c)
$$a \to h \to e \to f \to g \to i \to d \to c \to b \to a.$$

d) No Hamilton cycle (why?). But there is a Hamilton path:
$$a \to c \to d \to b \to e \to f \to g.$$

e) No Hamilton cycle (why?). But there is a Hamilton path:
$$a \to b \to c \to d \to e \to j \to i \to h \to g \to f \to k \to l \to m \to n \to o.$$

f)
$$a \to b \to c \to d \to e \to j \to i \to h \to g \to l \to m \to n \to o \to t \to s \to r \to q \to p \to k \to f \to a.$$

Exercise 7. *Grimaldi's book (5. ed., Exercises 12.1, page 585): solve* **Ex. 3**

Solution 7. a) For $i = 1, \dots, 7$ we denote by $e_i := |E_i|$ and $v_i := |V_i|$ the number of edges respectively vertices of tree T_i. Recall that $v_i = e_1 + 1$, such that $\sum_{i=1}^{7} v_i = 7 + \sum_{i=1}^{7} e_i = 47$.

b) For $i = 1, \dots, N$ we denote by $e_i := |E_i|$ and $v_i := |V_i|$ the number of edges respectively vertices of tree T_i. Recall that $v_i = e_1 + 1$. We have $\sum_{i=1}^{N} v_i = 62 = N + \sum_{i=1}^{N} e_i = N + 51$. This implies that $N = 11$.

Exercise 8. *Grimaldi's book (5. ed., Exercises 12.1, page 585): solve* **Ex. 5**

Solution 8. Paths.

Exercise 9. *Grimaldi's book (5. ed., Exercises 12.1, page 586): solve* **Ex. 21 a,b,c**

Solution 9. a) $3, 4, 6, 3, 8, 4,\ 3, 4, 6, 6, 8, 4$

b) The statement holds for pendant vertices as they do not appear in the sequence. When removing an edge $\{x, y\}$ and y is pendant vertex (of the tree or resulting subtree), then x is placed in the sequence and $\deg(x)$ is lowered by one. In this process x may become pendant in a subtree and therefore does not appear again in the sequence, or x is left and appears again in the sequence. Hence, x is listed $\deg(x) - 1$ times.

c)
$$
\begin{array}{cccccccc}
 & & & & & & \bullet_3 & \bullet_4\\
 & & & & & & | & /\\
\bullet_1 & - & \bullet_2 & - & \bullet_6 & - & \bullet_5 & \\
 & & & & & & | & \backslash\\
 & & & & & & \bullet_8 & \bullet_7
\end{array}
$$

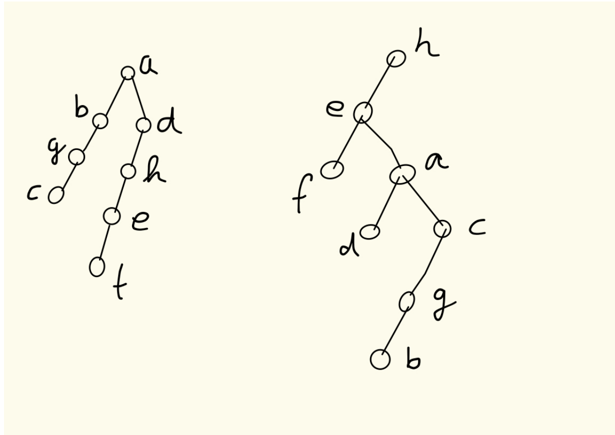
Exercise 10. *Grimaldi's book (5. ed., Exercises 12.2, page 604): solve* **Ex. 6**

Solution 10. Preorder: $1, 2, 5, 9, 14, 15, 10, 16, 17, 3, 6, 4, 7, 8, 11, 12, 13$

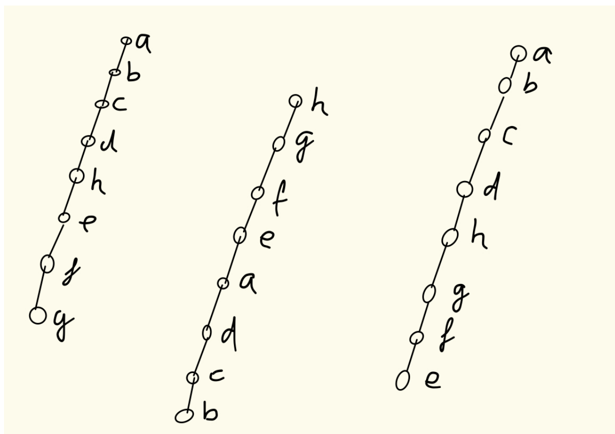
Postorder: $14, 15, 9, 16, 17, 10, 5, 2, 6, 3, 7, 11, 12, 13, 8, 4, 1$

Exercise 11. *Grimaldi's book (5. ed., Exercises 12.2, page 604): solve* **Ex. 7**

Solution 11. a) i) + iii) and ii)



b)



Exercise 12. *Grimaldi's book (5. ed., Exercises 12.2, page 604): solve* **Ex. 8**

Solution 12.