Norwegian University of Science and Technology
Institutt for matematiske fag

TMA4165 Differential Equations and Dynamical Systems Spring 2017

Exercise set 8

You find solutions to the following exercises on the web page. Give it a try and ask if something is unclear:
J.S.: 10.1 (i)–(iv), 10.2, 10.7
Ex 1996.4

These exercises will be presented / discussed in the exercise class:

E22 Find a strong Liapunov function at $(0,0)$ for the system

$$\dot{x} = x(y-b), \quad \dot{y} = y(x-a), \quad a, b > 0,$$

and confirm that all solutions starting in the domain $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 < 1$ approach the origin.

E23 Determine if the equilibrium point $(0,0)$ of the following systems is asymptotically stable, (Liapunov) stable or unstable.

a)

$$\begin{aligned}\dot{x} &= -x^3 - 2xy \\ \dot{y} &= x^2 - 3y^3.\end{aligned}$$

b)

$$\begin{aligned}\dot{x} &= -x - x^2 y \\ \dot{y} &= 2x^2 + y.\end{aligned}$$

E24 Show that $(0,0)$ is an asymptotically stable equilibrium point for the system

$$\begin{aligned}\dot{x} &= 2(x^2 + 2y^2)y - x^3 \\ \dot{y} &= -(x^2 + 2y^2)x - e^x y.\end{aligned}$$

Show that the domain of attraction of $(0,0)$ is all of $\mathbb{R}^2$.

Exam 1996, 4 Given $V \in C^1(\mathbb{R}^n, \mathbb{R})$.

a) Show that, if x_0 is a strict minimum for $V(x)$, then x_0 is an asymptotically stable equilibrium point for the system

$$\dot{x} = -\nabla V(x).$$

b) Let

$$V(x, y) = x^2(x-1)^2 + y^2.$$

Sketch the phase diagram of the system

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = -\nabla V(x, y).$$