TMA4125 Matematikk 4N
Spring 2017

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Exercise set 10

$\boxed{1}$ Solve the following system of linear equations by Gaussian elimination with partial pivoting

$$\begin{aligned} 0.139x_1 - 0.382x_2 + 2.32x_3 &= -6.04 \\ 12.1x_1 + 5.10x_2 + 0.504x_3 &= 5.12 \\ -1.58x_1 + 0.888x_2 + 1.85x_3 &= -7.02 \end{aligned}$$

In your calculations you should round all computations to three significant figures.

$\boxed{2}$ Solve the linear system

$$\begin{aligned} 2x_1 + x_2 + 3x_3 &= 7 \\ -4x_1 - x_2 + 2x_3 &= 14 \\ 2x_1 + x_3 &= 0 \end{aligned}$$

by first computing the relevant LU factorization.

$\boxed{3}$ Let A be the matrix

$$A = \begin{pmatrix} 4 & 2 & -2 \\ 2 & 10 & -13 \\ -2 & -13 & 21 \end{pmatrix}$$

Find a matrix L such that $A = LL^T$ (this is the Cholesky factorization).

$\boxed{4}$ Consider the system of linear equations

$$\begin{aligned} x_1 + 4x_2 + x_3 &= -2 \\ 4x_1 + x_2 + x_3 &= 1 \\ x_1 + x_2 + 2x_3 &= 0 \end{aligned}$$

a) Write down the Gauss-Seidel iterative scheme for this system of equations. Then find a matrix C so that the iteration is described by

$$x^{(m+1)} = Cx^{(m)} + v$$

for some constant vector v, which you need not find (see the section on Convergence and Matrix Norms from Chapter 20.3 for the second part). You should find that

$$C = \begin{pmatrix} 0 & -4 & -1 \\ 0 & 16 & 3 \\ 0 & -6 & -1 \end{pmatrix}$$

b) (Optional) Show that the matrix C has an eigenvalue greater than 1, and thus the Gauss-Seidel method does not converge.

c) A matrix is diagonally dominant if its diagonal entries are strictly greater in magnitude than all other entries on the same row. For any such matrix both the Gauss-Seidel and Jacobi iterations converge. Show that the system of linear equations can be written in terms of a diagonally dominant matrix by swapping the order of some of the equations.

d) Compute two steps of the Gauss-Seidel iteration starting from $x_0 = (0, 0, 0)^T$ for the system you obtain.

e) Compute two steps of the Jacobi iteration for this system from the same starting value $x_0 = (0, 0, 0)^T$.

$\boxed{5}$ (Optional) Write a program that solves systems of linear equations using the Gauss-Seidel iteration.