TMA4145 Linear Methods

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Exercise set 7

Please justify your answers! The most important part is *how* you arrive at an answer, not the answer itself.

$\boxed{1}$ Suppose A is a closed subspace of a Banach space $(X, \|.\|)$. Show that $(A, \|.\|)$ is a complete subspace of X, i.e. $(A, \|.\|)$ is a Banach space.

$\boxed{2}$ Let $(X, \|.\|)$ be a normed space and $A \subseteq X$. Show that

$$\overline{A} = \cap_{n\in\mathbb{N}}(A + B_{1/n}(0)),$$

where $A + B_{1/n}(0) = \{x \in X : x = a + y \ \ a \in A,\ y \in B_{1/n}(0)\}$.

$\boxed{3}$ Suppose $(X, \|.\|)$ is a normed space.

a) Show that $B_r(x) = \{y \in X : \ \|x - y\| < r\}$ is an open set in X.

b) Show that singletons are closed sets, i.e. for any $x \in X$ we have that $\{x\}$ is closed.

$\boxed{4}$ Consider the integral equation

$$f(x) = \sin x + \lambda \int_0^3 e^{-(x-y)} f(y)dy$$

for some scalar λ.

a) Determine for which λ there exists a continuous function f on $[0, 3]$ that solves this integral equation.

b) Pick one of the values of λ found in **a)**. Use the method of iteration, as described in Banach's fixed point theorem, to find approximations f_1 and f_2 to a potential solution by starting with $f_0(x) = 1$ on $[0, 3]$.

$\boxed{5}$ Let A be a non-empty subset of a normed space $(X, \|.\|)$.

a) Show that the closure of the linear span of A is a closed subspace of X, denoted by $\overline{\text{span}(A)}$.

b) We define the *closed linear span* of A, denoted by $\overline{span}(A)$, as the intersection of all the closed linear subspaces containing A. Show that $\overline{span}(A) = \overline{\text{span}(A)}$.