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TMA4125 Matematikk
4N
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Exercise set 13

$\boxed{1}$ We consider the heat equation $u_t = u_{xx}$ on a bar of length 1, with boundary conditions $u(0,t) = u(1,t) = 0$ for $t > 0$ and initial condition $u(x,0) = x(1-x)$.

a) Formulate an explicit method for the numerical solution of the equation.
Use step sizes $h = 0.25$ in the x-direction and $k = 0.25$ in the t-direction, and perform three time-steps.

b) Formulate the Crank-Nicolson method for the numerical solution of the equation.
Use step sizes $h = 0.25$ in the x-direction and $k = 0.25$ in the t-direction, and perform three time-steps.

c) Comment on the results you obtain. Which solution is more accurate? Why?

$\boxed{2}$ A parabolic evolution equation is given by

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial x}, \quad 0 < x < 1, t > 0$$

with boundary conditions $u(0,t) = 0, u(1,t) = 1$, and initial condition $u(x,0) = x$. This may be solved by the methods outlined in 21.6 of Kreyszig.

a) Formulate an explicit method for its numerical solution (Hint: use the usual method, i.e. approximation (4) on p.934 of Kreyszig, but with an additional finite difference approximation in the right hand side to account for the extra u_x appearing on the right hand side of the equation. You may wish to re-read pages 920–921 of Kreyszig).
Perform two time steps with step size $h = \frac{1}{4}$, $k = \frac{1}{16}$.

b) In a similar manner, modify the Crank-Nicolson scheme to obtain a numerical method for the given equation.
Using step sizes $h = k = \frac{1}{4}$, write down a system of linear equations for the first time step.