MA0301 ELEMENTARY DISCRETE MATHEMATICS
SPRING 2017

1. Homework Set 10 – Solutions

Exercise 1. *Let $\Sigma := \{a, b, c, d\}$ be an alphabet. Find regular languages corresponding to the following regular expressions. Note, if the set is infinite, then list the first ten elements.*

a) $a(b \vee c \vee d)a$

b) $a^*b^*c^*$

c) $a(bc)^*d$

Solution 1. a) $\{aba, aca, ada\}$

b) $\{\lambda, c, a, b, aa, bb, cc, ab, ac, bc, abc, \ldots\}$

c) $\{ad, abcd, abcbcd, abcbcbcd, abcbcbcbcd, abcbcbcbcbcd, abcbcbcbcbcbcd,$
$abcbcbcbcbcbcbcd, abcbcbcbcbcbcbcbcd, \ldots\}$

Exercise 2. *Let $\Sigma := \{a, b, c, d\}$ be an alphabet. Find regular expressions that correspond to the following regular languages.*

a) $\{ab, ac, ad\}$

b) $\{ab, ac, bb, bc\}$

c) $\{a, ab, abb, abbb, abbb, \ldots\}$

Solution 2. a) $a(b \vee c \vee d)$

b) $(a \vee b)(b \vee c)$

c) ab^*

Exercise 3. *Let $A := \{a, b, c\}$ be an alphabet.*

a) Give a regular expression for the language $L_1 \subset A^*$ where all elements have exactly two b's.

b) Give a regular expression for the language $L_2 \subset A^*$ where all elements have exactly two b's and two c's.

c) Give a regular expression for the language $L_3 \subset A^*$ where all elements have one or more a's, followed by one or more b's and then one or more c's.

Solution 3. a) $(a \vee c)^*b(a \vee c)^*b(a \vee c)^*$

b) $a^*ba^*ba^*ca^*ca^* \vee a^*ba^*ca^*ba^*ca^*a^* \vee a^*ca^*ba^*ba^*ca^*$
$\vee a^*ca^*ca^*ba^*ba^* \vee a^*ca^*ba^*ca^*ba^* \vee a^*ba^*ca^*ca^*ba^*$

c) $aa^*bb^*cc^*$

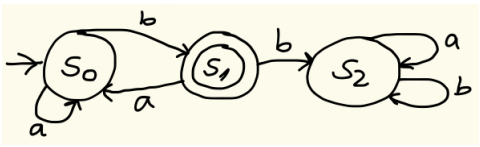
Exercise 4. *a) Draw the state diagram $D(M)$ of the automaton M with states $S := \{s_0, s_1, s_2\}$, accepting states $Y := \{s_1\}$, input alphabet $I := \{a, b\}$, described in the state table $T(M)$:*

b) Write a regular expression for the language accepted by M.

Date: March 16, 2017.

	ν	
	a	b
s_0	s_0	s_1
s_1	s_0	s_2
s_2	s_2	s_2

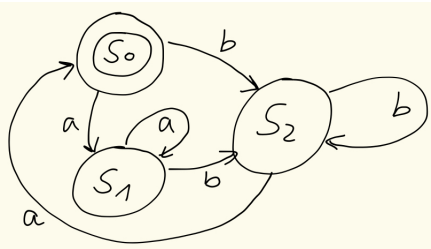
Solution 4. a)



b) $a^*b(aa^*b)^*$

Exercise 5. *Find an automaton M that accepts the regular language given by the regular expression $(a^*(ba)^*bb^*a)^*$.*

Solution 5.



2. Classroom Set 10 – Solutions

Exercise 1. *Find regular languages corresponding to the following regular expressions. Note, if the set is infinite, then list the first ten elements.*

a) $(a \vee b)(c \vee d)$

b) $(ab^*\lambda) \vee (cd)^*$

Solution 1. a) $\{ac, ad, bc, bd\}$

b) $\{a, ab, abb, abbb, \lambda, cd, cdcd, cdcdcd, cdcdcdcd, abbbb, \ldots\}$

Exercise 2. *Let $\Sigma := \{a, b, c, d\}$ be an alphabet. Find regular expressions that correspond to the following regular languages.*

a) $\{ab, abab, ababab, abababab, \ldots\}$

b) $\{ab, abb, aab, aabb\}$

Solution 2. a) $a(ba)^*b$

b) $(a \vee aa)(b \vee bb)$

Exercise 3. *Let $A := \{a, b, c\}$ be an alphabet.*

a) Give a regular expression for the language $L_1 \subset A^*$ where all elements have two or more b's.

b) Give a regular expression for the language $L_2 \subset A^*$ where all elements begin and end with a and contain at least one b and one c.

Solution 3. a) $(a \vee b \vee c)^*b(a \vee b \vee c)^*b(a \vee b \vee c)^*$

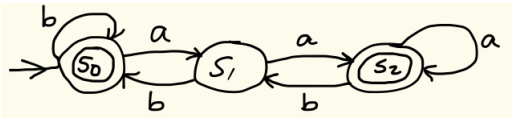
b) $a((a \vee b \vee c)^*b(a \vee b \vee c)^*c(a \vee b \vee c)^* \vee (a \vee b \vee c)^*c(a \vee b \vee c)^*b(a \vee b \vee c)^*)a$

Exercise 4. *a) Draw the state diagram $D(M)$ of the automaton M with states $S := \{s_0, s_1, s_2\}$, accepting states $Y := \{s_0, s_2\}$, input alphabet $I := \{a, b\}$, described in the state table $T(M)$:*

	ν	
	a	b
s_0	s_1	s_0
s_1	s_2	s_0
s_2	s_2	s_1

b) Which of the following input words are accepted by M?
1) abba
2) aabbb
3) babab
4) aaabbb
5) bbaab

Solution 4. a)



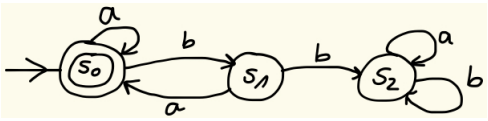
b) 2), 3) 4)

Exercise 5. *a) Draw the state diagram $D(M)$ of the automaton M with states $S := \{s_0, s_1, s_2\}$, accepting states $Y := \{s_0\}$, input alphabet $I := \{a, b\}$, described in the state table $T(M)$:*

	ν	
	a	b
s_0	s_0	s_1
s_1	s_0	s_2
s_2	s_2	s_2

b) Write a regular expression for the language accepted by M.

Solution 5. a)



b) $(a \vee ba)^*$

Exercise 6. *Find an automaton M that accepts the regular language given by the regular expression $aa^*bb^*cc^*$.*

Solution 6.