TMA4230 Functional Analysis
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Exercise set 1

1 Let $c = (c_n)_n$ a sequence of scalars. We denote by $(\delta_n)_n$ the standard basis. Show the following assertions.

a) If $1 \leq p < \infty$, then $\sum_n c_n \delta_n$ converges in ℓ^p if and only if $c \in \ell^p$.

b) The series $\sum_n c_n \delta_n$ converges in ℓ^∞ if and only if $c \in c_0$.

2 Suppose that $\|.\|_a$ and $\|.\|_b$ are tow norms on a vector space X. We denote by $B_r^a(x)$ and $B_r^b(x)$ the open balls of radius r at $x \in X$ w.r.t. the norms $\|.\|_a$ and $\|.\|_b$, respectively.

a) Show that $\|.\|_a$ and $\|.\|_b$ are equivalent norms if and only if there exists some $r > 0$ such that

$$B_{1/r}^a(0) \subseteq B_1^b(0) \subseteq B_r^a(0).$$

3 Let s be the vector space of real-valued sequences $(a_n)_{n\in\mathbb{N}}$. Show that the $\|.\|_1$-norm and $\|.\|_\infty$-norm are not equivalent on s.