MA0002

9.2.21 Vi har $A = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix}$.

a) skal finne matriseproduktet AB:

$$AB = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} -1\cdot 2+0\cdot(-1) & (-1)\cdot 3+0\cdot 1 \\ 1\cdot 2+2(-1) & 1\cdot 3+2\cdot 1 \end{bmatrix}$$

Her har vi brukt def. på matriseprodukt.

$$= \begin{bmatrix} -2 & -3 \\ 0 & 5 \end{bmatrix}$$

b) $$BA = \begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 2(-1)+3\cdot 1 & 2\cdot 0+3\cdot 2 \\ (-1)(-1)+1\cdot 1 & -1\cdot 0+1\cdot 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 6 \\ 2 & 2 \end{bmatrix}$$

(Legg merke til at $AB \neq BA$).

9.2.22

La A og B være matrisene i forrige oppg., og $C = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$. Vi skal finne ABC.

ABC = (AB)C, og vi vet fra forrige oppg. at $AB = \begin{bmatrix} -2 & -3 \\ 0 & 5 \end{bmatrix}$, så

$$ABC = (AB)C = \begin{bmatrix} -2 & -3 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -2\cdot 1 + (-3)\cdot 0 & -2\cdot 2 - 3(-1) \\ 0\cdot 1 + 5\cdot 0 & 0\cdot 2 + 5(-1) \end{bmatrix} = \underline{\begin{bmatrix} -2 & -1 \\ 0 & -5 \end{bmatrix}}$$

9.2.31) $A = \begin{bmatrix} 1 & 3 \\ 0 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 0 & -1 \\ 2 & 1 & 3 & 0 \end{bmatrix}$

a) $AB = \begin{bmatrix} 1 & 3 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 & -1 \\ 2 & 1 & 3 & 0 \end{bmatrix}$

$$= \begin{bmatrix} 1\cdot1+3\cdot2 & 1\cdot2+3\cdot1 & 1\cdot0+3\cdot3 & 1\cdot(-1)+3\cdot0 \\ 0\cdot1+(-2)(2) & 0\cdot2-2\cdot1 & 0\cdot0-2\cdot3 & 0\cdot(-1)-2\cdot0 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 5 & 9 & -1 \\ -4 & -2 & -6 & 0 \end{bmatrix}$$

b) Skal finne $B'A$

$B' = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 0 & 3 \\ -1 & 0 \end{bmatrix}$, så $B'A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 0 & 3 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 0 & -2 \end{bmatrix}$

$$= \begin{bmatrix} 1\cdot1+2\cdot0 & 1\cdot3+2(-2) \\ 2\cdot1+1\cdot0 & 2\cdot3+1(-2) \\ 0\cdot1+3\cdot0 & 0\cdot3+3(-2) \\ -1\cdot1+0\cdot0 & -1\cdot3+0(-2) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 \\ 2 & 4 \\ 0 & -6 \\ -1 & -3 \end{bmatrix}$$

9.2.32

$A = \begin{bmatrix} 1 & 4 & -2 \end{bmatrix}$, $B = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$

a) $AB = \begin{bmatrix} 1 & 4 & -2 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1\cdot(-1) + 4\cdot 2 + (-2)\cdot 3 \end{bmatrix}$

$= \begin{bmatrix} 1 \end{bmatrix} = \underline{1}$

b) $BA = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 1 & 4 & -2 \end{bmatrix} = \begin{bmatrix} -1\cdot 1 & -1\cdot 4 & -1(-2) \\ 2\cdot 1 & 2\cdot 4 & 2\cdot(-2) \\ 3\cdot 1 & 3\cdot 4 & 3(-2) \end{bmatrix}$

$= \underline{\begin{bmatrix} -1 & -4 & 2 \\ 2 & 8 & -4 \\ 3 & 12 & -6 \end{bmatrix}}$

9.2.34

$$A = \begin{bmatrix} 1 & -1 \\ 3 & 0 \\ 5 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 4 & 1 \\ 6 & 0 & 0 \end{bmatrix}$$

Skal vise at $(AB)' = B'A'$.

$$AB = \begin{bmatrix} 1 & -1 \\ 3 & 0 \\ 5 & 2 \end{bmatrix} \begin{bmatrix} 2 & 4 & 1 \\ 6 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1\cdot 2 - 1\cdot 6 & 1\cdot 4 - 1\cdot 0 & 1\cdot 1 - 1\cdot 0 \\ 3\cdot 2 + 0\cdot 6 & 3\cdot 4 + 0\cdot 0 & 3\cdot 1 + 0\cdot 0 \\ 5\cdot 2 + 2\cdot 6 & 5\cdot 4 + 2\cdot 0 & 5\cdot 1 + 2\cdot 0 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 4 & 1 \\ 6 & 12 & 3 \\ 22 & 20 & 5 \end{bmatrix}$$

så $(AB)' = \left(\begin{bmatrix} -4 & 4 & 1 \\ 6 & 12 & 3 \\ 22 & 20 & 5 \end{bmatrix} \right)' = \begin{bmatrix} -4 & 6 & 22 \\ 4 & 12 & 20 \\ 1 & 3 & 5 \end{bmatrix}$. (*)

$B' = \begin{bmatrix} 2 & 6 \\ 4 & 0 \\ 1 & 0 \end{bmatrix}$, $A' = \begin{bmatrix} 1 & 3 & 5 \\ -1 & 0 & 2 \end{bmatrix}$, så

$$B'A' = \begin{bmatrix} 2 & 6 \\ 4 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 3 & 5 \\ -1 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 2\cdot 1 + 6(-1) & 2\cdot 3 + 6\cdot 0 & 2\cdot 5 + 6\cdot 2 \\ 4\cdot 1 + 0\cdot(-1) & 4\cdot 3 + 0\cdot 0 & 4\cdot 5 + 0\cdot 2 \\ 1\cdot 1 + 0\cdot(-1) & 1\cdot 3 + 0\cdot 0 & 1\cdot 5 + 0\cdot 2 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 6 & 22 \\ 4 & 12 & 20 \\ 1 & 3 & 5 \end{bmatrix},$$

og dette er det samme som (*).
Dermed er $(AB)' = B'A'$.

9.2.43 $A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$

Skal vise at B er invers av A:

$$AB = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$$

$$BA = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$$

Siden $AB = BA = I_2$, vet vi at B er den inverse av A., så $\underline{A^{-1} = B}$

9.2.45 $A = \begin{bmatrix} -1 & 1 \\ 2 & 3 \end{bmatrix}$ skal finne den inverse.

Sjekker at den eksisterer:

$a_{11}a_{22} - a_{21}a_{12} = (-1)\cdot 3 - 2\cdot 1 = -5 \neq 0$, så den eksisterer og

$$A^{-1} = \frac{1}{a_{11}a_{22}-a_{21}a_{12}}\begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix} = \frac{1}{-5}\begin{bmatrix} 3 & -1 \\ -2 & -1 \end{bmatrix}$$

$$= \underline{\frac{1}{5}\begin{bmatrix} -3 & 1 \\ 2 & 1 \end{bmatrix}}$$

9.2.46 $\quad B = \begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}$

Sjekker om den inverse eksisterer:

$b_{11}b_{22} - b_{21}b_{12} = 2 \cdot 2 - 3(-2) = 10 \neq 0,$

så den eksisterer. Da har vi

$$B^{-1} = \frac{1}{b_{11}b_{22} - b_{21}b_{12}} \begin{bmatrix} b_{22} & -b_{12} \\ -b_{21} & b_{11} \end{bmatrix} = \underline{\frac{1}{10}\begin{bmatrix} 2 & 2 \\ -3 & 2 \end{bmatrix}}$$

9.2.50 $\quad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Det er lett å se at $I_3 I_3 = I_3$,

så denne er dermed sin egen invers:

$\underline{(I_3)^{-1} = I_3}$

9.2.53 $\quad A = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$

$$\det A = a_{11}a_{22} - a_{12}a_{21}$$

$$= 2 \cdot 3 - (-1) \cdot 1 = 7 \neq 0 \text{, så}$$

matrisen A er invertibel.

9.2.67

$$A = \begin{bmatrix} 2 & -1 & -1 \\ 2 & 1 & 1 \\ -1 & 1 & -1 \end{bmatrix}$$

Skal finne A^{-1} dersom den eksisterer.

$$\left[\begin{array}{ccc|ccc} 2 & -1 & -1 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 1 & 0 \\ -1 & 1 & -1 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} (R_1) \\ (R_2) \\ (R_3) \end{array}$$

$$\begin{array}{r} (R_1)+2(R_3) \\ (R_2)+2(R_3) \\ (R_3) \end{array} \left[\begin{array}{ccc|ccc} 0 & 1 & -3 & 1 & 0 & 2 \\ 0 & 3 & -1 & 0 & 1 & 2 \\ -1 & 1 & -1 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} (R_4) \\ (R_5) \\ (R_6) \end{array}$$

$$\begin{array}{r} (R_4) \\ \dfrac{(R_5)-3(R_4)}{8} \\ (R_3) \end{array} \left[\begin{array}{ccc|ccc} 0 & 1 & -3 & 1 & 0 & 2 \\ 0 & 0 & 1 & -\frac{3}{8} & \frac{1}{8} & -\frac{1}{2} \\ -1 & 1 & -1 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} (R_7) \\ (R_8) \\ (R_9) \end{array}$$

$$\begin{array}{r} -(R_9) \\ (R_7)+3(R_8) \\ (R_8) \end{array} \left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -\frac{1}{8} & \frac{3}{8} & \frac{1}{2} \\ 0 & 0 & 1 & -\frac{3}{8} & \frac{1}{8} & -\frac{1}{2} \end{array}\right] \begin{array}{l} (R_{10}) \\ (R_{11}) \\ (R_{12}) \end{array}$$

$$\begin{array}{r} (R_{10})+(R_{11})-(R_8) \\ (R_{11}) \\ (R_{12}) \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 1 & 0 & -\frac{1}{8} & \frac{3}{8} & \frac{1}{2} \\ 0 & 0 & 1 & -\frac{3}{8} & \frac{1}{8} & -\frac{1}{2} \end{array}\right],$$

sai $\quad A^{-1} = \begin{bmatrix} 1/4 & 1/4 & 0 \\ -1/8 & 3/8 & 1/2 \\ -3/8 & 1/8 & -1/2 \end{bmatrix}$

9.2.68 $\quad A = \begin{bmatrix} -1 & 3 & -1 \\ 2 & -2 & 3 \\ -1 & 1 & 2 \end{bmatrix}$

$$\left[\begin{array}{ccc|ccc} -1 & 3 & -1 & 1 & 0 & 0 \\ 2 & -2 & 3 & 0 & 1 & 0 \\ -1 & 1 & 2 & 0 & 0 & 1 \end{array}\right] \begin{array}{c} (R_1) \\ (R_2) \\ (R_3) \end{array}$$

$$\begin{array}{r} -(R_1) \\ (R_2)+2(R_3) \\ (R_3)-(R_1) \end{array} \left[\begin{array}{ccc|ccc} 1 & -3 & 1 & -1 & 0 & 0 \\ 0 & 0 & 7 & 0 & 1 & 2 \\ 0 & -2 & 3 & -1 & 0 & 1 \end{array}\right] \begin{array}{c} (R_4) \\ (R_5) \\ (R_6) \end{array}$$

$$\begin{array}{r} (R_4) \\ (R_6) \\ \frac{(R_5)}{7} \end{array} \left[\begin{array}{ccc|ccc} 1 & -3 & 1 & -1 & 0 & 0 \\ 0 & -2 & 3 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1/7 & 2/7 \end{array}\right] \begin{array}{c} (R_4) \\ (R_6) \\ (R_7) \end{array}$$

$$\begin{array}{r} (R_4) \\ -\frac{1}{2}(R_6)+\frac{3}{2}(R_7) \\ (R_7) \end{array} \left[\begin{array}{ccc|ccc} 1 & -3 & 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 1/2 & 3/14 & -\frac{1}{14} \\ 0 & 0 & 1 & 0 & 1/7 & 2/7 \end{array}\right] \begin{array}{c} (R_4) \\ (R_8) \\ (R_7) \end{array}$$

$$\begin{array}{r} (R_4)+3(R_8)-(R_7) \\ (R_8) \\ (R_7) \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & 1/2 & -1/2 \\ 0 & 1 & 0 & 1/2 & 3/14 & -1/14 \\ 0 & 0 & 1 & 0 & 1/7 & 2/7 \end{array}\right]$$

so $$A^{-1} = \begin{bmatrix} 1/2 & 1/2 & -1/2 \\ 1/2 & 3/14 & -1/14 \\ 0 & 1/7 & 2/7 \end{bmatrix}$$