Norwegian University of Science
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MA0301 Elementary discrete mathematics Spring 2017

Exercise set 7

1 Basic exercises

1 Grimaldi's book (5. ed., Exercises 5.2): solve **Ex. 12**

For $n, k \in \mathbb{Z}^+$, prove that $\lceil n/k \rceil = \lfloor (n-1)/k \rfloor + 1$.

2 Grimaldi's book (5. ed., Exercises 5.2): solve **Ex. 14**

Let $a_1, a_2, a_3, \ldots$ be the integer sequence defined recursively by (1) $a_1 = 1$; and (2) For all $n \in \mathbb{Z}^+$ where $n \geq 2$, $a_n = 2a_{\lfloor n/2 \rfloor}$.

a) Determine a_n for all $2 \leq n \leq 8$.

b) Prove that $a_n \leq n$ for all $n \in \mathbb{Z}^+$.

3 Grimaldi's book (5. ed., Exercises 5.2): solve **Ex. 15 a), b), e)**

For each of the following functions, determine whether it is one-to-one and determine its range.

a) $f : \mathbb{Z} \to \mathbb{Z}$, $f(x) = 2x + 1$

b) $f : \mathbb{Q} \to \mathbb{Q}$, $f(x) = 2x + 1$

e) $f : [-\pi/2, \pi/2] \to \mathbb{R}$, $f(x) = \sin x$

4 Grimaldi's book (5. ed., Exercises 5.2): solve **Ex. 22**

For $n \in \mathbb{Z}^+$ define $X_n = \{1, 2, 3, \ldots, n\}$. Given $m, n \in \mathbb{Z}^+$, $f : X_m \to X_n$ is called *monotone increasing* if for all $i, j \in X_m$, $1 \leq i < j \leq m \implies f(i) \leq f(j)$. (a) How many monotone increasing functions are there with domain X_7 and codomain X_5? (b) Answer part (a) for the domain X_6 and codomain X_9. (c) Generalize the results in part (a) and (b). (d) Determine the number of monotone increasing functions $f : X_{10} \to X_8$ where $f(4) = 4$. (e) How many monotone increasing functions $f : X_7 \to X_{12}$ satisfy $f(5) = 9$? (f) Generalize the results in parts (d) and (e).

5 Grimaldi's book (5. ed., Exercises 5.3): solve **Ex. 2 a), c), e)**

For each of the following functions $f : \mathbb{Z} \to \mathbb{Z}$, determine whether the function is one-to-one and whether it is onto. If the function is not onto, determine the range $f(\mathbb{Z})$.

a) $f(x) = x + 7$
c) $f(x) = -x + 5$
e) $f(x) = x^2 + x$

6 Grimaldi's book (5. ed., Exercises 5.3): solve **Ex. 3 a), c), e)**

For each of the following functions $g : \mathbb{R} \to \mathbb{R}$, determine whether the function is one-to-one and whether it is onto. If the function is not onto, determine the range $g(\mathbb{R})$.

a) $f(x) = x + 7$
c) $f(x) = -x + 5$
e) $f(x) = x^2 + x$

7 Grimaldi's book (5. ed., Exercises 5.6): solve **Ex. 10 a), c)**

For each of the following functions $f : \mathbb{R} \to \mathbb{R}$, determine whether f is invertible, and, if so, determine f^{-1}.

a) $f : \{(x, y) \mid 2x + 3y = 7\}$
c) $f : \{(x, y) \mid y = x^3\}$

8 Grimaldi's book (5. ed., Exercises 5.6): solve **Ex. 14 a), d), e)**

Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2$. For each of the following subsets B of $\mathbb{R}$, find $f^{-1}(B)$.

a) $B = \{0, 1\}$
d) $B = [0, 1)$
e) $B = [0, 4]$

9 Grimaldi's book (5. ed., Exercises 5.6): solve **Ex. 15**

Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{6, 7, 8, 9, 10, 11, 12\}$. How many functions $f : A \to B$ are such that $f^{-1}(\{6, 7, 8\}) = \{1, 2\}$?