Norwegian University of Science
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Department of Mathematics

MA2501 Numerical Methods
Spring 2017

Exercise set 10

1 **a)** Construct the free cubic spline for the following data.

x	$f(x)$
0.1	−0.62049958
0.2	−0.28398668
0.3	0.00660095
0.4	0.24842440

b) The data in the previous task were constructed using the function

$$f(x) = x\cos x - 2x^2 + 3x - 1$$

Use the cubic spline to approximate $f(0.25))$ and $f'(0.25)$ and calculate the actual error in both cases.

2 Assume that S is a spline of degree k on $[a, b]$ with $k \geq 2$. Show that S' is a spline of degree $k - 1$ on $[a, b]$.

3 Find the linear function $f(x) = ax + b$ that best approximates the following points:

x	0	1	3	3	2	7
y	-1	1	4	3	3	6

4 Assume that you are given a number of points x_i, $1 \leq i \leq n$, and y_i, $1 \leq i \leq n$, and you want to find a parabola $f(x) = ax^2 + bx + c$ that best approximates these points.

a) Formulate this problem as a least squares problem and derive the corresponding normal equations.

b) Find the parabola that best approximates the following points:

x	-2	-1	0	0	2	3
y	5	1	-2	2	6	12

5 Let $A \in \mathbb{R}^{n\times m}$ with $m \leq n$ have full rank and let $b \in \mathbb{R}^n$. Show that the following procedure yields a solution of the least squares problem $\|Ax - b\|_2^2 \to \min$:

- Find an orthogonal matrix $Q \in \mathbb{R}^{n\times n}$ and an upper triangular matrix $R \in \mathbb{R}^{n\times m}$ of the form

$$R = \begin{pmatrix} \hat{R} \\ 0 \end{pmatrix} \qquad \text{with } \hat{R} \in \mathbb{R}^{m\times m} \text{ upper triangular}$$

  such that $A = QR$.
- Compute $y := Q^T b$ and write $y \in \mathbb{R}^n$ as

$$y = \begin{pmatrix} \hat{y} \\ z \end{pmatrix} \qquad \text{with } \hat{y} \in \mathbb{R}^m \text{ and } z \in \mathbb{R}^{n-m}.$$

- Solve the equation

$$\hat{R}x = \hat{y}.$$