

1 Spectral approximation in \mathbf{R}^2

In this assignment we consider the model problems described in the note “The Poisson problem: Mixed Dirichlet/Neumann boundary conditions along curved boundaries”.

Assume that we are solving the two-dimensional Laplace problem in a deformed square using a spectral Galerkin method. Both the solution and the geometry (x and y) are approximated as a polynomial of degree N in each spatial direction on the reference domain. Gauss-Lobatto quadrature is used for the integration of the bilinear and linear forms.

1.1 Model problem 1

We shall first consider model problem 1 from the note. Implement the Laplace problem you obtain by setting $f = 0$, with the described mixed boundary conditions, and with $g = 1$, $L_x = 3$, $L_y = 4$. Provide a plot of the solution and a plot of the discrete L^2 norm of the error versus N in a **semi-log** plot.

1.2 Model problem 2

We next consider model problem 2 from section 2 in the note. The aim is to reproduce the experiments reported in section 3. The physical domain is given in Figure 2. In this experiment $f = 0$ and $g = 1$. Read carefully the note and in particular sections 2 and 3, to perform this task. The Gordon-Hall algorithm is used to create the grid on the deformed domain. Reproduce Figures 3 to 5, and optionally also Figure 6.