Norwegian University of Science
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Department of Mathematical
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TMA4145 Linear Methods Fall 2017

Exercise set 1

Please justify your answers! The most important part is *how* you arrive at an answer, not the answer itself.

1 Based on your exposure to mathematics in school and at university answer the following questions.

a) State your favorite mathematical theorem, explain all the notions of the statement and explain in a few words your choice.

b) Give three applications of mathematics to real-world problems.

The answers should be given such that your fellow students in the course are able to understand them.

2 Let X, Y and Z be sets.

a) Show that $X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$.

b) Show that $X \backslash (Y \cup Z) = (X \backslash Y) \cap (X \backslash Z)$.

3 Show that the sets $\mathbb{Z}$ of integers and $\mathbb{Q}$ of rational numbers are countable.

4 Define functions on $\mathbb{R}$ with values in $\mathbb{R}$. (i) A function that is not left invertible; (ii) A function that is not right invertible. Show that the given functions have their respective properties.

5 Given the linear mapping $T : \mathbb{R}^2 \to \mathbb{R}^3$ given by $T = Ax$ with

$$A = \begin{pmatrix} -3 & -4 \\ 4 & 6 \\ 1 & 1 \end{pmatrix}.$$

a) Show that the matrix

$$A_l^{-1} = \frac{1}{9}\begin{pmatrix} -11 & -10 & 16 \\ 7 & 8 & -11 \end{pmatrix}$$

induces a left inverse T_l^{-1} of T.
This left inverse is not unique. Show that

$$\frac{1}{2}\begin{pmatrix} 0 & -1 & 6 \\ 0 & 1 & -4 \end{pmatrix}$$

gives another left inverse.

b) Turn this example into one for right inverses. Concretely, find a mapping $S : \mathbb{R}^3 \to \mathbb{R}^2$ that is based on the mapping T and give a right inverse for this mapping.