Norwegian University of Science and Technology
Department of Mathematical Sciences

TMA4230 Functional Analysis
Spring 2017

Exercise set 8

1 Let X be a Banach space and T a compact operator on X. Show that if (x_n) is a sequence in X such that (x_n) converges weakly to 0, then $\lim_{n\to\infty} \|Tx_n\| = 0$.

2 Let X be a Banach space and $T : X \to X$ an invertible, bounded, linear operator. Show that λ is in the spectrum of T if and only if $1/\lambda$ is in the spectrum of T^{-1}.

3 Let $\ell^1(\mathbb{Z})$ be the space of absolutely convergent series $(x_n)_{n\in\mathbb{Z}}$. Define the convolution $x * y = ((x * y)_n)$ between $x = (x_n)_{n\in\mathbb{Z}}$ and $y = (y_n)_{n\in\mathbb{Z}}$ by

$$(x * y)_n = \sum_{k\in\mathbb{Z}} x_k y_{n-k}.$$

a) Show that convolution is commutative and associative.

b) Show that $(\ell^1(\mathbb{Z}), *)$ is a Banach algebra, i.e.

$$\|x * y\|_1 \le \|x\|_1 \|y\|_1, \quad textforall \quad x, y \in \ell^1(\mathbb{Z}).$$