Norwegian University of Science
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MA0301 Elementary discrete mathematics Spring 2017

Exercise set 7

1 Homework Set 7

1 Grimaldi's book (5. ed., Exercises 3.1): solve **Ex. 6**

Consider the following six subsets of $\mathbb{Z}$.

$$A = \{2m + 1 \mid m \in \mathbb{Z}\}$$
$$B = \{2n + 3 \mid n \in \mathbb{Z}\}$$
$$C = \{2p - 3 \mid p \in \mathbb{Z}\}$$
$$D = \{3r + 1 \mid r \in \mathbb{Z}\}$$
$$E = \{3s + 2 \mid s \in \mathbb{Z}\}$$
$$F = \{3t - 2 \mid t \in \mathbb{Z}\}$$

Which of the following statements are true and which are false?

a) $A = B$
b) $A = C$
c) $B = C$
d) $D = E$
e) $D = F$
f) $E = F$

2 Grimaldi's book (5. ed., Exercises 3.2): solve **Ex. 13**

Prove or disprove each of the following for sets $A, B \subseteq \mathcal{U}$.

a) $\mathcal{P}(A \cup B) = \mathcal{P}(A) \cup \mathcal{P}(B)$
b) $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$

3 Grimaldi's book (5. ed., Exercises 5.1): solve **Ex. 2**

If $A = \{1, 2, 3\}$, and $B = \{2, 4, 5\}$, give examples of (a) three nonempty relations from A to B; (b) three nonempty relations on A.

4 Grimaldi's book (5. ed., Exercises 5.1): solve **Ex. 3**

For A, B as in Exercise 2, determine the following: (a) $|A \times B|$; (b) the number of relations from A to B; (c) the number of relations on A; (d) the number of relations from A to B that contain $(1, 2)$ and $(1, 5)$; (e) the number of relations from A to B that contain exactly five ordered pairs; and (f) the number of relations on A that contain at least seven elements.

5 Grimaldi's book (5. ed., Exercises 5.1): solve **Ex. 7**

a) If $A = \{1, 2, 3, 4, 5\}$ and $B = \{w, x, y, z\}$, how many elements are there in $\mathcal{P}(A \times B)$?

b) Generalize the result in part (a)

6 Grimaldi's book (5. ed., Exercises 5.2): solve **Ex. 1**

Determine whether or not each of the following relations is a function. If a relation is a function, find its range.

a) $\{(x, y) \in \mathbb{Z} \mid x, y \in \mathbb{Z}, y = x^2 + 7\}$, a relation from $\mathbb{Z}$ to $\mathbb{Z}$.
b) $\{(x, y) \in \mathbb{R} \mid x, y \in \mathbb{R}, y^2 = x\}$, a relation from $\mathbb{R}$ to $\mathbb{R}$.
c) $\{(x, y) \in \mathbb{R} \mid x, y \in \mathbb{R}, y = 3x + 1\}$, a relation from $\mathbb{R}$ to $\mathbb{R}$.
d) $\{(x, y) \in \mathbb{Q} \mid x, y \in \mathbb{Q}, x^2 + y^2 = 1\}$, a relation from $\mathbb{Q}$ to $\mathbb{Q}$.
e) $\mathcal{R}$ is a relation from A to B where $|A| = 5$, $|B| = 6$, and $|\mathcal{R}| = 6$.

7 Grimaldi's book (5. ed., Exercises 5.2): solve **Ex. 5**

Let $A, B, C \subseteq \mathbb{R}^2$ where $A = \{(x, y) \mid y = 2x + 1\}$, $B = \{(x, y) \mid y = 3x\}$, and $C = \{(x, y) \mid y = x - y = 7\}$. Determine each of the following:

a) $A \cap B$
b) $B \cap C$
c) $\overline{\overline{A} \cup \overline{C}}$
d) $\overline{B} \cup \overline{C}$

8 Grimaldi's book (5. ed., Exercises 5.2): solve **Ex. 15 c), d), f)**

For each of the following functions, determine whether it is one-to-one and determine its range.

c) $f : \mathbb{Z} \to \mathbb{Z}$, $f(x) = x^3 - x$

d) $f : \mathbb{R} \to \mathbb{R}$, $f(x) = e^x$

f) $f : [0, \pi] \to \mathbb{R}$, $f(x) = \sin x$

9 Grimaldi's book (5. ed., Exercises 5.3): solve **Ex. 2 b), d), f)**

For each of the following functions $f : \mathbb{Z} \to \mathbb{Z}$, determine whether the function is one-to-one and whether it is onto. If the function is not onto, determine the range $f(\mathbb{Z})$.

b) $f(x) = 2x - 3$

d) $f(x) = x^2$

f) $f(x) = x^3$

10 Grimaldi's book (5. ed., Exercises 5.3): solve **Ex. 3 b), d), f)**

For each of the following functions $g : \mathbb{R} \to \mathbb{R}$, determine whether the function is one-to-one and whether it is onto. If the function is not onto, determine the range $g(\mathbb{R})$.

b) $f(x) = 2x - 3$

d) $f(x) = x^2$

f) $f(x) = x^3$

11 Grimaldi's book (5. ed., Exercises 5.6): solve **Ex. 5**

If $\mathcal{U}$ is a given universe with (fixed) $S, T \in \mathcal{U}$, define $g : \mathcal{P}(\mathcal{U}) \to \mathcal{P}(\mathcal{U})$ by $g(A) = T \cap (S \cup A)$ for $A \subseteq \mathcal{U}$. Prove that $g^2 = g$.

12 Grimaldi's book (5. ed., Exercises 5.6): solve **Ex. 10 b), d)**

For each of the following functions $f : \mathbb{R} \to \mathbb{R}$, determine whether f is invertible, and, if so, determine f^{-1}.

b) $f : \{(x, y) \mid ax + by = c, b \neq 0\}$

d) $f : \{(x, y) \mid y = x^4 + x\}$

13 Grimaldi's book (5. ed., Exercises 5.6): solve **Ex. 14 b), c), f)**

Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2$. For each of the following subsets B of $\mathbb{R}$, find $f^{-1}(B)$.

b) $B = \{-1, 0, 1\}$

c) $B = [0, 1]$

f) $B = (0, 1] \cup (4, 9)$