Norwegian University of Science and Technology
Department of Mathematical Sciences

TMA4230 Functional Analysis

Spring 2017

Exercise set 3

$\boxed{1}$ Show that the space of converging sequences c is isomorphic to the space of sequences converging to zero, c_0.

Hint: Let T be the mapping $T : c \to c_0$ defined by

$$T(a_1, a_2, \ldots) = (-a, a_1 - a, a_2 - a, \ldots),$$

where $a = \lim_n a_n$. Show that T is bijective, determine its inverse, and prove that these maps are linear and contiuous.

$\boxed{2}$ Show that the space of continuous linear functionals on ℓ^1 is isometrically isomorphic to ℓ^∞.

$\boxed{3}$ Let X be a normed space. Show the following assertions:

a) For $\varphi \in X^*$ the kernel of φ is a closed hyperplane, i.e. a closed subspace of codimension 1. (Note even more is true, which is not part of the problem: A linear functional on X is continuous if and only if $\ker\varphi$ is closed. Hence $\ker\varphi$ is either closed or dense in X.)

b) Suppose $\varphi, \eta \in X^*$ satisfy $\ker\varphi = \ker\eta$. Then $\varphi = c\eta$ for some constant c.

c) Given a hyperplane M in X. Then there exists a $\varphi \in X^*$ such that $M = \ker\varphi$.