Norwegian University of Science and Technology
Department of Mathematics

MA2501 Numerical Methods
Spring 2017

Exercise set 8

1 Compute numerical approximations of the definite integral

$$\int_0^1 \sin(x^2)\,dx$$

using the composite trapezoid rule with $h = 1$, $h = 1/2$, and $h = 1/4$. In addition, provide estimates for the approximation error.

2 Compute numerical approximations of the definite integral

$$\int_0^2 e^{-x^2}\sin(x)\,dx$$

a) using the composite trapezoid rule with $h = 1$ and $h = 1/2$,

b) using the composite Simpson rule with $h = 1$ and $h = 1/2$,

(You may want to use Matlab for the computations.)

3 Compute numerical approximations of the definite integrals

$$\int_0^1 x^{7/2}\,dx \qquad \text{and} \qquad \int_0^1 x^{5/2}\,dx$$

using the trapezoid method and the Simpson method with $h = 1$, $h = 1/2$, $h = 1/4$.

a) Compute for all approximations the respective approximation errors and try to estimate numerically the convergence rate.

b) Which convergence rate would you usually expect for the trapezoid method and the Simpson method, and which have you actually observed? Try to explain possible discrepancies.

(You may want to use Matlab for the computations.)

4 Derive the formula for the closed Newton–Cotes rule with $n = 4$ (with nodes $x_j = a + j(b-a)/4$, $0 \le j \le 4$).

$\boxed{5}$ Construct for every $0 < \alpha < 1$ a quadrature rule of the form

$$Q(f, -1, 1) = 2\big(c_0 f(-\alpha) + c_1 f(0) + c_2 f(\alpha)\big)$$

that has degree of precision 2, i.e. integrates all polynomials up to degree 2 exactly.

It might be possible that polynomials of higher order than 2 are integrated exactly. What is the actual degree of precision of the different formulas?

$\boxed{6}$ Implement and use Romberg integration to approximate the integral in Exercise 2. Compute the Romberg table until $|R(n-1, n-1) - R(n, n)| < 10^{-6}$. Compare your result to the exact value of the integral.

$\boxed{7}$ Romberg integration is used to approximate $\int_2^3 f(x)dx$. If $f(2) = 0.51342$, $f(3) = 0.36788$, $R(2, 0) = 0.43687$, and $R(2, 2) = 0.43662$, find $f(2.5)$.