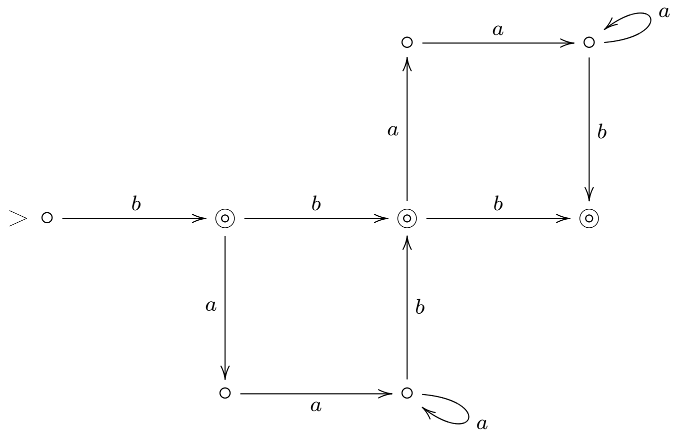
Norwegian University of Science and Technology
Institutt for matematiske fag

EXAM IN MA2301 THEORY OF COMPUTATION AND COMPUTATIONAL COMPLEXITY
Autumn 2008

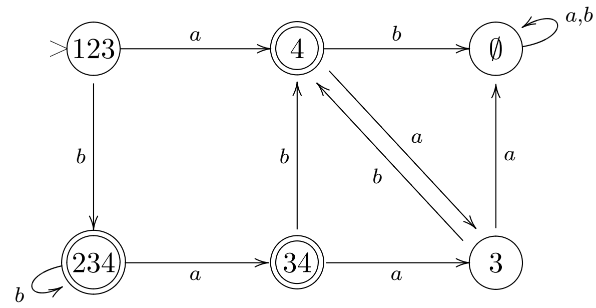
$\boxed{1}$ There are many possibilities. One of them is:



$\boxed{2}$ Using the standard procedure, we get the table:

State	Transition for a	Transition for b
$\{1,2,3\}$	$\{4\}$	$\{2,3,4\}$
$\{2,3,4\}$	$\{3,4\}$	$\{2,3,4\}$
$\{3,4\}$	$\{3\}$	$\{4\}$
$\{3\}$	$\emptyset$	$\{4\}$
$\{4\}$	$\{3\}$	$\emptyset$
$\emptyset$	$\emptyset$	$\emptyset$

The new initial state is $\{1,2,3\}$, the set of all states of the original automaton where we can get without reading a symbol. The final states of the deterministic automaton will be all those which contain 4, that is: $\{4\}$, $\{3,4\}$ and $\{2,3,4\}$. The diagram for the deterministic automaton is, therefore:



$\boxed{3}$ The language is

$$L = \{c^n b^m c^n a^k b^k \mid n, m, k \geq 0\}.$$

To see this, just note that the non-terminal symbol Z produces strings of the form b^m, the non-terminal Y produces $c^n b^m c^n$, and X produces $a^k b^k$. Finally, every string obtained using the rule $S \to X$ is also obtained using the rule $S \to YX$.

$\boxed{4}$ We use the Pumping Theorem. Assume for contradiction that

$$L = \{a^n b^{2n} c^{3n} \mid n > 0\}$$

is context-free. Then, according to the Pumping Theorem, there is $N > 0$ such that any string $w \in L$ of length at least N has a decomposition

$$w = uvxyz$$

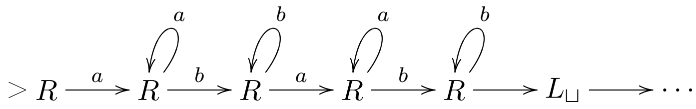
with the property that either v or y is non-empty and $uv^ixy^iz \in L$ for each $i \geq 0$.

Let us fix such an $N > 0$ and a decomposition $w = uvxyz$ for $w = a^N b^{2N} c^{3N}$. It is clear that neither of v and y can be composed of more than one symbol from $\{a, b, c\}$, since otherwise the "pumped" string $uv^ixy^iz \in L$ would not be of the form $a^*b^*c^*$. This means, however, that vy contains at most two symbols of a, b, c. Therefore, the number of occurences of one symbol in $uv^ixy^iz \in L$ remains constant for all $i \geq 0$, while the number of occurences of at least one the other two symbols changes. This is a contradiction, because uv^ixy^iz cannot be in L for $i \neq 1$ as the ratio between the numbers of a's, b's and c's cannot be $1{:}2{:}3$.

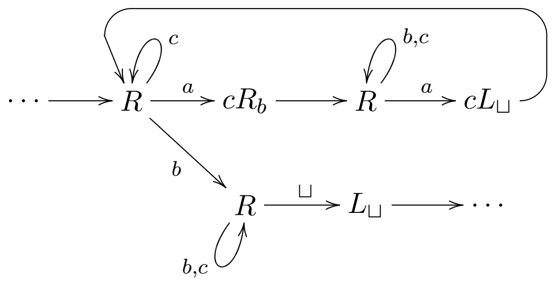
Therefore, the original assumption that L was context-free must have been wrong.

$\boxed{5}$ There are again many possibilities. The solution presented here is not the smallest possible such machine, but it should illustrate the main idea. It will have three parts which are run after each other. To keep the notation simple, the arrows to the rejecting state are not drawn in the diagrams—when a symbol is read for which an arrow is missing, one assumes that there is an arrow with this symbol to the rejecting state.

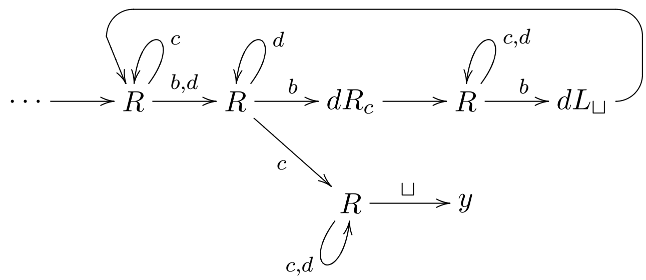
Part I. We first check whether the input string is of the form $aa^*bb^*aa^*bb^*$. This prevents troubles like the one with the machine on page 195 in [Lewis-Papadimitriou]:



Part II. Next, we compare the number of a's in the first and the second segment. This is done by replacing a's one by one with a new symbol c:



Part III. In a similar way, we compare the number of b's in the two segments by replacing them one by one with a new symbol d:



6 The formula is not satisfiable. To see this, one uses the purge algorithm. We start by checking out the possible truth assignments for x_1.

Case I. Assume $T(x_1) = \top$. Then we are left with the formula

$$\{(x_2), (\overline{x}_2 \vee x_3), (\overline{x}_3 \vee x_4), (\overline{x}_4), (x_2 \vee \overline{x}_4)\}.$$

It follows that $T(x_2) = \top$ and $T(x_4) = \bot$, and we must find a value for $T(x_3)$ to satisfy

$$\{(x_3), (\overline{x}_3)\},$$

which is impossible.

Case II. Assume $T(x_1) = \bot$. Then we get the following formula to inspect:

$$\{(\overline{x}_2 \vee x_3), (\overline{x}_3 \vee x_4), (\overline{x}_3), (x_2 \vee \overline{x}_4), (x_4)\}.$$

Therefore, $T(x_3) = \bot$ and $T(x_4) = \top$, and we must find a value for $T(x_2)$ to satisfy

$$\{(\overline{x}_2), (x_2)\},$$

which is again impossible.

$\boxed{7}$ The machine does not halt on $w = a$, which is very easy to check (the corresponding starting configuration is $\triangleright\underline{\sqcup}a$).