MA0301 ELEMENTARY DISCRETE MATHEMATICS
SPRING 2017

1. Homework Set 9 – Solutions

Exercise 1. *Find the number of distinct permutations of the sequence of letter:*
a) T H O S E, b) U N U S U A L, c) S O C I O L O G I C A L ,
d) S A N N S Y N L I G H E T S T E T T H E T S F U N K S J O N E N E

Solution 1. a) $\frac{5!}{1!1!1!1!1!} = 5! = 120$
b) $\frac{7!}{3!} = 840$
c) $\frac{12!}{3!2!2!2!}$
d) $\frac{33!}{5!6!2!5!5!}$

Exercise 2. *Consider the two permutations:*

$$a = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 6 & 4 & 5 & 1 & 2 \end{pmatrix} \qquad b = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 6 & 5 & 3 & 1 \end{pmatrix}$$

Calculate the permutations i) $a \circ b$, ii) $b \circ a$ and iii) find the inverses a^{-1} and b^{-1}.

Solution 2.

$$b \circ a = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 1 & 5 & 3 & 2 & 4 \end{pmatrix} \qquad a \circ b = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 2 & 1 & 4 & 3 \end{pmatrix}$$

$$a^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 6 & 1 & 3 & 4 & 2 \end{pmatrix} \qquad b^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 1 & 5 & 2 & 4 & 3 \end{pmatrix}$$

Exercise 3. *Grimaldi's book (5. ed., Exercises 6.1, page 317): solve* **Ex. 1**

Solution 3. a) $5^2 = 25$
b) $5^3 = 125$
c) $1 + 5 + 5^2 + 5^3 + 5^4 + 5^5$

Exercise 4. *Grimaldi's book (5. ed., Exercises 6.1, page 318): solve* **Ex. 5**

Solution 4. There are $5 + 5^2 + 5^3 + 5^4$ words with prefix xy in A.

Exercise 5. *Grimaldi's book (5. ed., Exercises 6.1, page 318): solve* **Ex. 18**

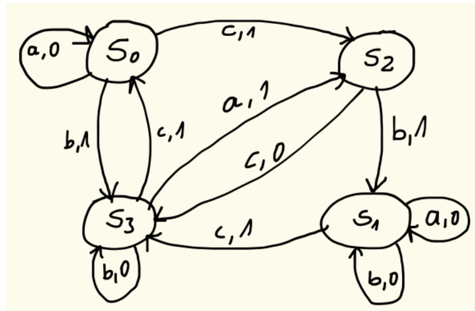
Solution 5. Should be clear.

Exercise 6. *Grimaldi's book (5. ed., Exercises 6.2, page 324): solve* **Ex. 3**

Solution 6. a) 010110

Date: March 13, 2017.

b)



(Missing an arrow $a, 1$ from s_2 to s_1.)

Exercise 7. *Grimaldi's book (5. ed., Exercises 6.3, page 332): solve* **Ex. 8**

Solution 7. The input sequences 110 or 111 (among others) bring you from s_2 to s_5.

2. Classroom Set 9 – Solutions

Exercise 8. *Grimaldi's book (5. ed., Exercises 6.1, page 318): solve* **Ex. 9**

Solution 8. a) $x \in AC$ implies that $x = ac$ for $a \in A$ and $c \in C$. This implies that $x \in BD$ since $A \subset B$ and $C \subset D$.

b) Assume that $A\emptyset \neq \emptyset$. Let $x \in A\emptyset$. This implies that $x = ae$ for $a \in A$ and $e \in \emptyset$. The latter is not possible, and therefore $A\emptyset = \emptyset$. An analog argument applies to $\emptyset A = \emptyset$.

Exercise 9. *Grimaldi's book (5. ed., Exercises 6.1, page 318): solve* **Ex. 17**

Solution 9. Induction implies that $A = A^n$ for all positive integers n. This implies that $A = A^+$. We can show that $A = A^2$ implies that $\lambda \in A$ (see Ex. 6.1, Ex. 15), and therefore $A = A^*$.

Exercise 10. *Grimaldi's book (5. ed., Exercises 6.2, page 325): solve* **Ex. 5**

Solution 10. a) 010000, s_2

b) starting at s_1: 100000, s_2; starting at s_2: 000000, s_2; starting at s_3: 110010, s_2

c)

	ν		ω	
	0	1	0	1
s_0	s_0	s_1	0	0
s_1	s_1	s_2	1	1
s_2	s_2	s_2	0	0
s_3	s_0	s_3	0	1
s_4	s_2	s_3	0	1

d) s_1

e) The unique input string is: $x = 101$

Exercise 11. *Grimaldi's book (5. ed., Exercises 6.2, page 325): solve* **Ex. 9**

Solution 11. b) $x = 1111$ or $x = 0000$

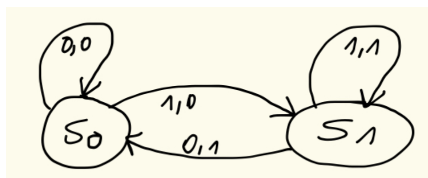
c) $A = \{111\}\{1\}^* \cup \{000\}\{0\}^* \subset \{0,1\}^*$

d) $A = \{11111\}\{1\}^* \cup \{00000\}\{0\}^* \subset \{0,1\}^*$

	ν		ω	
	0	1	0	1
s_0	s_4	s_1	0	0
s_1	s_3	s_2	0	0
s_2	s_3	s_2	0	1
s_3	s_3	s_3	0	0
s_4	s_5	s_3	0	0
s_5	s_5	s_3	1	0

Exercise 12. *Grimaldi's book (5. ed., Exercises 6.3, page 332): solve* **Ex. 5**

Solution 12. a)



b) 111 $\longrightarrow$ 011; 1010 $\longrightarrow$ 0101; 00011 $\longrightarrow$ 00001

c) The machine is a *unit delay*, i.e., for an input with n symbols it produces an output which starts with *zero* and then it copies the first $n-1$ symbols of the input.

d) It operates the same way with two states.

Exercise 13. *Grimaldi's book (5. ed., Exercises 6.3, page 332): solve* **Ex. 7**

Solution 13. a) transient states: s_0, s_1; sink state: s_4; submachines: $\{s_1, s_2, s_3, s_4, s_5\}$, $\{s_4\}$, $\{s_2, s_3, s_5\}$, $\{s_2, s_3, s_4, s_5\}$; strongly connected submachines: $\{s_4\}$, $\{s_2, s_3, s_5\}$.

b) transient states: s_2, s_3; sink state: s_4; submachine: $\{s_0, s_1, s_3, s_4\}$, $\{s_4\}$, $\{s_0, s_1\}$; strongly connected submachines: $\{s_4\}$, $\{s_0, s_1\}$.

c) transient states: none; sink state: s_6; submachine: $\{s_2, s_3, s_4, s_5, s_6\}$, $\{s_6\}$, $\{s_3, s_4, s_5, s_6\}$; strongly connected submachines: $\{s_6\}$.