MA2501 Numerical Methods
Spring 2017

Exercise set 7

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1 **Cf. Cheney and Kincaid, Exercise 4.1.9**

Consider the data points

x_i	0	1	2	4	6
$f(x_i)$	1	9	23	93	259

a) Find the interpolation polynomial through these data points using Newton interpolation, and compute an approximation of f at $x = 3$.

b) Do the same using only the first four interpolation points.

2 Suppose we have the nodes

$$x_0 = -2,\ x_1 = -1,\ x_2 = 1 \text{ and } x_3 = 4,$$

and know the divided differences

$$f[x_3] = 11,\ f[x_2, x_3] = 5,\ f[x_2, x_0, x_1] = -2, \text{ and } f[x_0, x_2, x_1, x_3] = 0.6.$$

What is f(-1)?

3 Use the interpolation error formula to find a bound for the error, and compare the bound to the actual error for the case $n = 2$ for Exercise 6 in Exercise set 6.

4 Given the function $f(x) = e^x \sin x$ on the interval $[-4, 2]$.

a) Show by induction

$$f^{(m)}(x) = \frac{d^m}{dx^m} f(x) = 2^{m/2} e^x \sin(x + m\pi/4).$$

b) Let $p_n(x)$ be the polynomial that interpolates $f(x)$ in $n+1$ uniformly distributed nodes (including the endpoints). Find an upper bound for the interpolation error on this interval, expressed using n. What must n be to guarantee an error less than 10^{-5}? Use trial and error or calculate it using MATLAB.

c) Use MATLAB to verify that the value of n found in **b)**, indeed results in an error less than 10^{-5}.