Norwegian University of Science and Technology
Department of Mathematical Sciences

TMA4145 Linear Methods
Fall 2017

Exercise set 2

Please justify your answers! The most important part is *how* you arrive at an answer, not the answer itself.

1 **a)** Determine the following numbers and decide in each case whether "supremum" can be replaced by "maximum":

1. $\sup_{x\in(0,\infty)} \frac{1}{x^2}$;
2. $\sup_{x\in\mathbb{R}} e^{-2|x|}$;
3. $\sup_{n\in\mathbb{N}} \frac{n^2+3}{n^2+1}$;
4. $\sup_{n\in\mathbb{N}}(-1)^n \frac{n+3}{n^2+1}$.

b) Determine the following numbers and decide in each case whether "infimum" can be replaced by "minimum":

1. $\inf_{x\in(0,\infty)} \frac{1}{x^2}$;
2. $\inf_{x\in\mathbb{R}} e^{-2|x|}$;
3. $\inf_{n\in\mathbb{N}} \frac{n^2+3}{n^2+1}$;
4. $\inf_{n\in\mathbb{N}}(-1)^n \frac{n+3}{n^2+1}$.

2 Let A be bounded above. Show that the supremum of A is unique.

3 Let $\{X_i\}_{i\in I}$ be a collection of subspaces of a vector space X. Show that the intersection $\cap_{i\in I} X_i$ is a subspace of X.

4 Let X be a vector space.

1. Prove that the additive inverse is unique (meaning for any $x \in X$ there exists a unique vector $y \in X$ such that $x + y = 0$; we denote the additive inverse of x by $-x$.)
2. Show that for every $x \in X$ we have $(-1)x = -x$. In words multiplication by the scalar -1 gives the additive inverse of a vector.

$\boxed{5}$

1. Let X be a vector space and T a linear mapping $T : X \to X$. Show that the range of T is a subspace of X.

2. Recall that $C^{(1)}(\mathbb{R})$ are the continuously differentiable functions, and $C(\mathbb{R})$ are the continuous functions. Let be the differentiation operator $Df(x) = f'(x)$. Determine the kernel and the range of the operator $T : C^{(1)}(\mathbb{R}) \to C(\mathbb{R})$ defined by $Tf = f' - 3f$ for $f \in C^{(1)}(\mathbb{R})$.