TMA4125 Matematikk 4N
Spring 2017

Exercise set 9

1 (Adapted from Question 5, Chapter 19.3 of Kreyszig)

a) Find the linear polynomial $p_1(x)$ that interpolates the function $f(x) = e^{-x}$ at the points $x_0 = 0$ and $x_1 = 0.5$. Evaluate this polynomial at $x = 0.25$ to obtain an estimate of $e^{-0.25}$.

b) Now find the quadratic polynomial $p_2(x)$ interpolating the same function at the points $x_0 = 0$, $x_1 = 0.5$ and $x_2 = 1$. Use this polynomial to estimate $e^{-0.25}$.

c) Give a crude estimate for the error $\epsilon_1(x) = e^{-x} - p_1(x)$ at $x = 0.25$ using only the two values $p_1(0.25)$ and $p_2(0.25)$

d) Now use the theorem on the error of polynomial interpolation (given in Kreyszig 19.3) to find upper and lower bounds for the error $\epsilon_1(0.25)$. How does this compare to the estimate obtained previously? By evaluating $e^{-0.25}$ using a calculator, state the true value of the error.

2 **a)** Use Newton's divided difference interpolation to find the polynomial of lowest degree interpolating the data

i	0	1	2	3
x_i	-2	0	1	2
$f(x_i)$	-1	1	-1	3

Display your calculations in the form of a difference table (see example 4 from Kreyszig, 19.3)

b) Suppose an extra point is added at $x_4 = 2.5$, where $f(x_i) = 9.125$. By updating your difference table, show that the divided difference $f[x_0, x_1, x_2, x_3, x_4] = 0$. Conclude that the interpolating polynomials of lowest degree for the data are the same, before and after the new data point is added.

3 **a)** Use the rectangular rule and the trapezoid rule with $n = 4$ to derive approximations of the integral

$$\int_0^1 \tan(x)dx$$

b) The second derivative of $\tan x$ is given by $2\tan x \sec^2 x$, which is increasing on $[0, 1]$. Use this to find an upper bound for the error of the approximation obtained from the trapezoid rule (see Kreyszig 19.5 on Error Bounds)

4 **a)** Use Simpson's rule with $2m = 4$ to find an approximation of the integral

$$\int_1^2 \cos(e^{1-x^2})dx$$

b) Find an error estimate for the above approximation by implementing Simpson's rule for the same integral with $2m = 2$ and comparing the results (see Error Estimation for Simpson's Rule by Halving h, Kreyszig 19.5)

5 (Optional) Write a program that computes divided differences $f[x_0, \ldots, x_k]$ for a given set of data points $(x_i, f(x_i))$. Try using this as the basis of a program that computes interpolating polynomials.

6 (Optional) Write a program that evaluates approximate integrals of a function using Simpson's rule. As a more challenging but rewarding exercise, you could try implementing the adaptive Simpson's rule outlined in the textbook, i.e. that successively halves the subintervals of the integration until a desired tolerance is achieved.