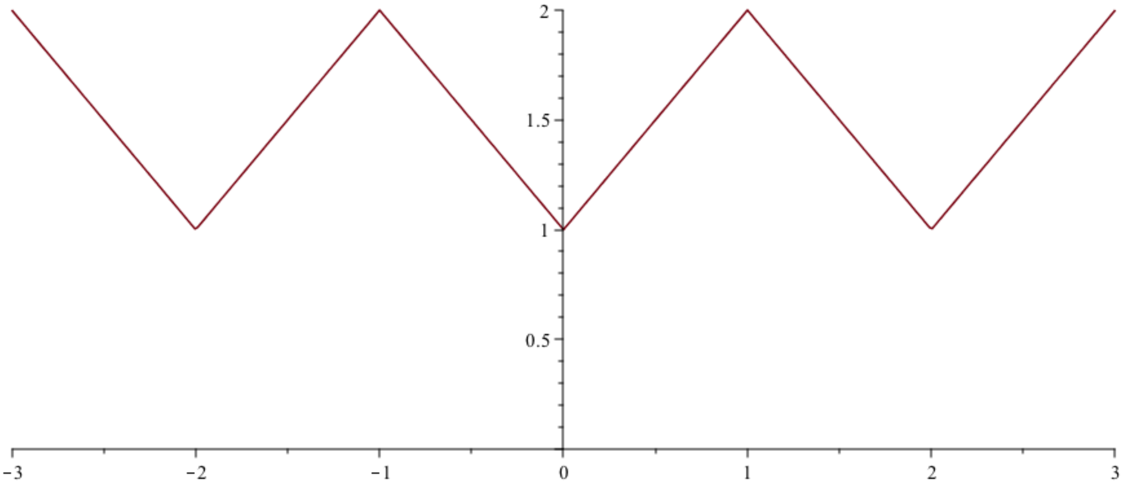
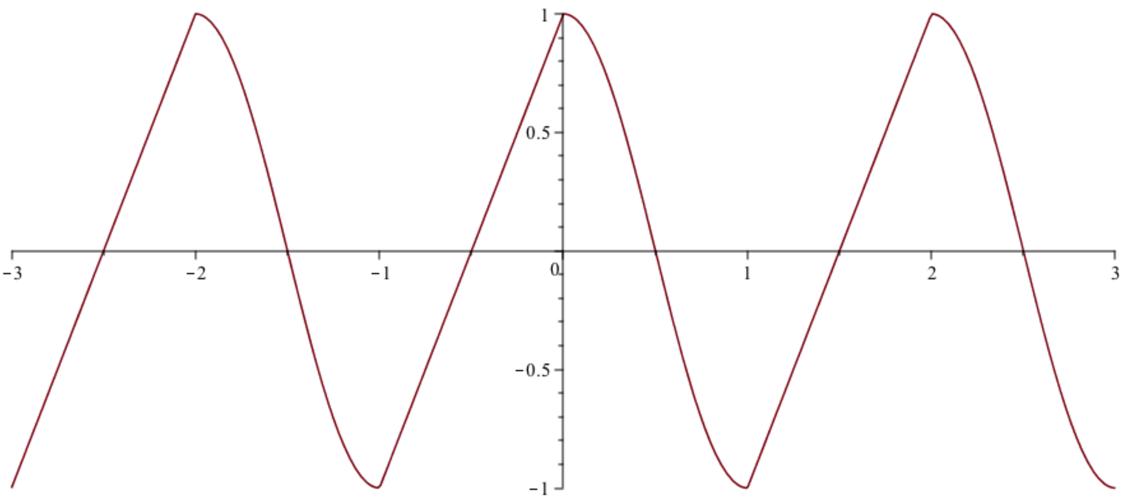
Norwegian University of Science and Technology Institutt for matematiske fag	TMA4999 Blodsuging Spring 2017 **Solutions to exercise set 1**

$\boxed{1}$ **a)**



b)



$\boxed{2}$ **a)** We first compute

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{0} -x \, dx = \frac{1}{2\pi}\left[\frac{-x^2}{2}\right]_{-\pi}^{0} = -\frac{-\pi^2}{4\pi} = \frac{\pi}{4}$$

Next by integrating by parts we find

$$
\begin{aligned}
a_n &= \frac{1}{\pi}\int_{-\pi}^{0} -x\cos nx \, dx = \frac{1}{\pi}\left(\left[\frac{-x\sin nx}{n}\right]_{-\pi}^{0} - \int_{-\pi}^{0}\frac{-\sin nx}{n}dx\right) \\
&= -\frac{1}{\pi}\left[\frac{\cos nx}{n^2}\right]_{-\pi}^{0} = \begin{cases} -\frac{2}{n^2\pi} & n \text{ odd} \\ 0 & n \text{ even} \end{cases}
\end{aligned}
$$

where we have used that the first term in the integration by parts vanishes. Finally, we have

$$
\begin{aligned}
b_n &= \frac{1}{\pi}\int_{-\pi}^{0} -x\sin nx \, dx = \frac{1}{\pi}\left(\left[\frac{x\cos nx}{n}\right]_{-\pi}^{0} - \int_{-\pi}^{0}\frac{\cos nx}{n}dx\right) \\
&= \frac{1}{\pi}\left(\frac{(-1)^n\pi}{n} - \left[\frac{\sin nx}{n^2}\right]_{-\pi}^{0}\right) = \frac{(-1)^n}{n}
\end{aligned}
$$

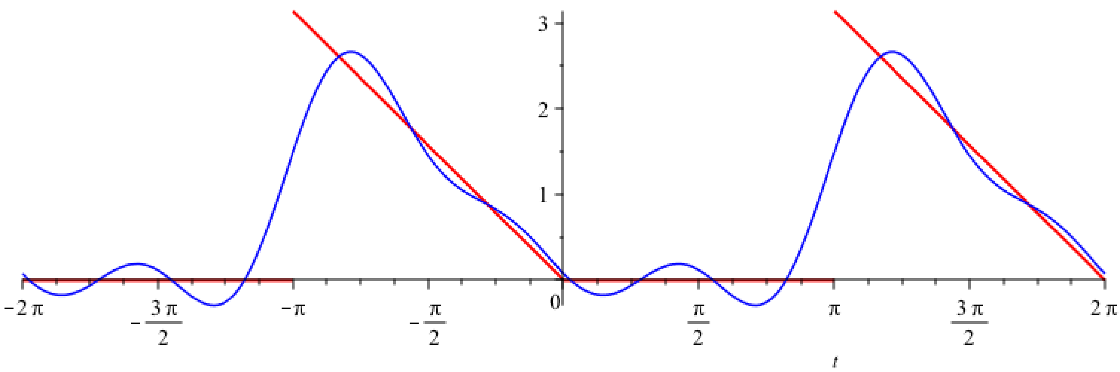
The Fourier series for f is therefore

$$
f(x) = \frac{\pi}{4} + \sum_{k=1}^{\infty}\frac{-2}{(2k-1)^2\pi}\cos(2k-1)x + \sum_{n=1}^{\infty}\frac{(-1)^n}{n}\sin nx
$$

b) The statement of this question was perhaps a little confusing. I had intended that the the third partial sum

$$
\frac{\pi}{4} - \frac{2}{\pi}\cos x - \sin x + \frac{1}{2}\sin 2x - \frac{2}{9\pi}\cos 3x - \frac{1}{3}\sin 3x
$$

be displayed alongside the exact function as shown below. Similar plots of the function together with either partial sums or individual terms in the series are also acceptable.



$\boxed{3}$ **a)** Even, as $f(-x) = (-x)^2 = x^2 = f(x)$

b) Odd, as $f(-x) = (-x)^3 = -x^3 = -f(x)$

c) Neither, eg $f(-1) = \frac{1}{2}$, whilst $f(1) = 2$

d) Neither in general, as $f(-x) = g(-x) - h(-x) = g(x) + h(x)$, unless one of g or h is zero, in which case it is either odd or even respectively (and both if $f(x) = 0$)

e) Neither, as $f(-x) = g(-x) + h(-x) = g(x) - h(x)$, except the same special cases as above

f) Odd, as $f(-x) = g(-x)h(-x) = -g(x)h(x)$

g) Even, as $f(-x) = g(h(-x)) = g(-h(x)) = g(h(x))$

$\boxed{4}$ **a)** Here we write

$$\begin{aligned}\int_{-\pi}^{\pi} \sin nx \cos mx \, dx &= \frac{1}{2}\int_{-\pi}^{\pi} \big(\sin(n+m)x + \sin(n-m)x\big)dx \\ &= -\frac{1}{2}\left[\frac{\cos(n+m)x}{n+m} + \frac{\cos(n-m)x}{n-m}\right]_{-\pi}^{\pi} = 0\end{aligned}$$

Alternatively, we could argue that the integral vanishes because it is the integral of an odd function from $-\pi$ to π.

b) Odd, because it is the product of and odd and an even function. We could also rewrite the function using the trigonometric identity (ie $f(x) = \frac{1}{2}(\sin 9x + \sin 7x)$) and obtain a sum of odd functions, which is again odd.

c) We write $f(x) = \frac{1}{2}(\sin 9x + \sin 7x)$. This function is its own Fourier series (any function of the form

$$a_0 + \sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx$$

is its own Fourier series, as can be seen by e.g. computing the coefficients as normal and invoking the orthogonality relations)