TMA 4275 Lifetime analysis
Exercise 4 - solution

Problem 1

a)

For the manual computations use the formulas derived on the lectures, see lecture slides. Or see the book, page 474, chapter 11.3.5

MINITAB: go to: Stat>Realibility/Survival>Distribution Analysis (Right censoring)>Non parametric distribution analysis. Choose the "veriable", the "Censor", the graphs that you want and the estimates that you want.

Minitab results:

```
Variable: time

Censoring Information  Count
Uncensored value           7
Right censored value       9

Censoring value: cens = 0


Nonparametric Estimates


Characteristics of Variable

            Standard   95.0% Normal CI
Mean(MTTF)     Error    Lower    Upper
   97.6610   8.40782  81.1820  114.140

Median = 109.2
IQR = 40  Q1 = 70  Q3 = 110


Kaplan-Meier Estimates

        Number  Number     Survival  Standard   95.0% Normal CI
 Time  at Risk  Failed  Probability     Error     Lower    Upper
 31.7       16       1     0.937500  0.060515  0.818892  1.00000
 57.5       14       1     0.870536  0.085566  0.702829  1.00000
 65.5       13       1     0.803571  0.101871  0.603907  1.00000
 70.0       11       1     0.730519  0.115880  0.503399  0.95764
 94.2        6       1     0.608766  0.147235  0.320190  0.89734
109.2        3       1     0.405844  0.192578  0.028398  0.78329
110.0        2       1     0.202922  0.172801  0.000000  0.54161
```

b)

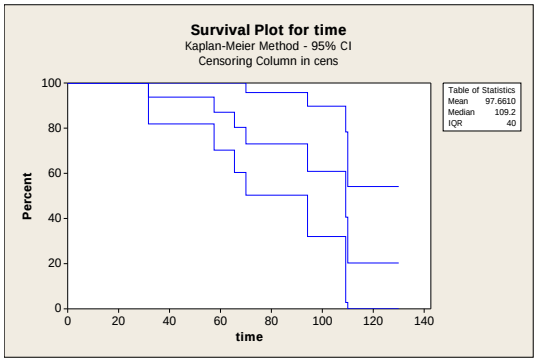

Figure 1: Kaplan Meier estimator from problem 1

The MTTF can be calculated (manualy) as the area under Kaplan-Meier plot. OR you can see the result from the MINITAB output.

c)

Quartiles can be estimated from the graph of the Kaplan Meier estimator by finding the value on th x-axis (time axis) corresponding to the value 1-α for the α quartile on the y-axis (percent axis).

d)

See page 479 eq. 11.20 on the book.

Problem 2

a)

For the manual computations use the formulas derived on the lectures, see lecture slides. Or the book, page 480, chapter 11.3.6

b)

How to run macros in Minitab:

1. Download macros from the course page.
2. Change the extension of the downloaded macro files from *.txt to *.mac
3. In Minitab in `Tools>Options` define the path to the folder with macro files
4. In Minitab `Editor>Enable Commands`
5. Type `%name_of_the_macro_without_extension parameters` i.e. type `%TMA4275nelson c1-c7` and press enter

The Minitab gives

Row	Time_1	Cum Haz Nelson	Survival Nelson	Survival KM
1	31.7	0.06250	0.939413	0.937500
2	57.5	0.13393	0.874653	0.870536
3	65.5	0.21085	0.809894	0.803571
4	70.0	0.30176	0.739515	0.730519
5	94.2	0.46843	0.625986	0.608766
6	109.2	0.80176	0.448539	0.405844
7	110.0	1.30176	0.272052	0.202922

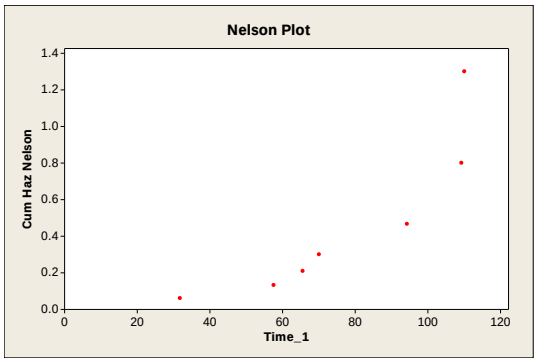

Figure 2: Nelson-plot from problem 2

Since the estimated cumulative failure rate is convex, the hazard rate of underlying distribution is increasing.

c)

The cumulative failure rate can be also estimated with use of the Kaplan-Meier estimates and formula $\hat{Z}(t) = -\log(\hat{R}_{KM}(t))$ (page 480, eq. 11.22).

Problem 3

a,b)

For the manuall computations do as in problems 1 and 2.
MINITAB gives:

```
Variable: C1

Censoring Information  Count
Uncensored value          41
Right censored value       7

Censoring value: C2 = 0


Nonparametric Estimates
```

Characteristics of Variable

Mean(MTTF)	Standard Error	95.0% Normal CI Lower	Upper
54.1946	0.729698	52.7644	55.6248

Median = 54.8
IQR = 4.7 Q1 = 52.4 Q3 = 57.1

Kaplan-Meier Estimates

Time	Number at Risk	Number Failed	Survival Probability	Standard Error	95.0% Normal CI Lower	Upper
36.3	44	1	0.977273	0.0224675	0.933237	1.00000
41.7	42	1	0.954004	0.0317737	0.891729	1.00000
43.9	39	1	0.929543	0.0392618	0.852591	1.00000
49.9	38	1	0.905081	0.0452112	0.816469	0.99369
50.1	37	1	0.880619	0.0501722	0.782284	0.97896
50.8	36	1	0.856158	0.0544160	0.749504	0.96281
51.9	35	1	0.831696	0.0580998	0.717823	0.94557
52.1	34	1	0.807234	0.0613247	0.687040	0.92743
52.3	33	2	0.758311	0.0666551	0.627670	0.88895
52.4	31	1	0.733849	0.0688473	0.598911	0.86879
52.6	30	1	0.709388	0.0707647	0.570691	0.84808
52.7	29	1	0.684926	0.0724292	0.542968	0.82688
53.1	28	1	0.660465	0.0738578	0.515706	0.80522
53.6	27	2	0.611541	0.0760582	0.462470	0.76061
53.9	25	2	0.562618	0.0774423	0.410834	0.71440
54.1	23	1	0.538156	0.0778427	0.385587	0.69073
54.6	22	1	0.513695	0.0780533	0.360713	0.66668
54.8	21	2	0.464771	0.0779096	0.312071	0.61747
55.1	19	1	0.440310	0.0775543	0.288306	0.59231
55.4	18	1	0.415848	0.0770069	0.264917	0.56678
55.9	17	1	0.391386	0.0762634	0.241913	0.54086
56.0	16	1	0.366925	0.0753179	0.219304	0.51455
56.1	15	1	0.342463	0.0741627	0.197107	0.48782
56.5	14	1	0.318001	0.0727878	0.175340	0.46066
56.9	13	1	0.293540	0.0711806	0.154028	0.43305
57.1	12	2	0.244616	0.0671998	0.112907	0.37633
57.3	10	1	0.220155	0.0647792	0.093190	0.34712
57.7	9	1	0.195693	0.0620283	0.074120	0.31727
57.8	8	1	0.171232	0.0589010	0.055788	0.28668
58.1	7	1	0.146770	0.0553334	0.038318	0.25522
58.9	6	1	0.122308	0.0512336	0.021892	0.22272
59.0	5	1	0.097847	0.0464610	0.006785	0.18891

59.1	4	1	0.073385	0.0407800	0.000000	0.15331
59.6	3	1	0.048923	0.0337347	0.000000	0.11504
60.4	2	1	0.024462	0.0241597	0.000000	0.07181
60.7	1	1	0.000000	0.0000000	0.000000	0.00000

Data Display

Row	Time	Cum Haz Nelson	Survival Nelson	Survival KM
1	36.3	0.02273	0.977529	0.977273
2	41.7	0.04654	0.954529	0.954004
3	43.9	0.07218	0.930365	0.929543
4	49.9	0.09849	0.906201	0.905081
5	50.1	0.12552	0.882038	0.880619
6	50.8	0.15330	0.857874	0.856158
7	51.9	0.18187	0.833710	0.831696
8	52.1	0.21128	0.809546	0.807234
9	52.3	0.24253	0.784639	0.758311
10	52.4	0.27479	0.759732	0.733849
11	52.6	0.30812	0.734825	0.709388
12	52.7	0.34261	0.709918	0.684926
13	53.1	0.37832	0.685011	0.660465
14	53.6	0.41678	0.659165	0.611541
15	53.9	0.45845	0.632264	0.562618
16	54.1	0.50193	0.605363	0.538156
17	54.6	0.54738	0.578463	0.513695
18	54.8	0.59738	0.550251	0.464771
19	55.1	0.65001	0.522039	0.440310
20	55.4	0.70557	0.493828	0.415848
21	55.9	0.76439	0.465617	0.391386
22	56.0	0.82689	0.437407	0.366925
23	56.1	0.89356	0.409197	0.342463
24	56.5	0.96499	0.380988	0.318001
25	56.9	1.04191	0.352780	0.293540
26	57.1	1.13282	0.322124	0.244616
27	57.3	1.23282	0.291470	0.220155
28	57.7	1.34393	0.260819	0.195693
29	57.8	1.46893	0.230172	0.171232
30	58.1	1.61179	0.199531	0.146770
31	58.9	1.77845	0.168899	0.122308
32	59.0	1.97845	0.138283	0.097847
33	59.1	2.22845	0.107695	0.073385
34	59.6	2.56179	0.077167	0.048923
35	60.4	3.06179	0.046804	0.024462
36	60.7	4.06179	0.017218	0.000000

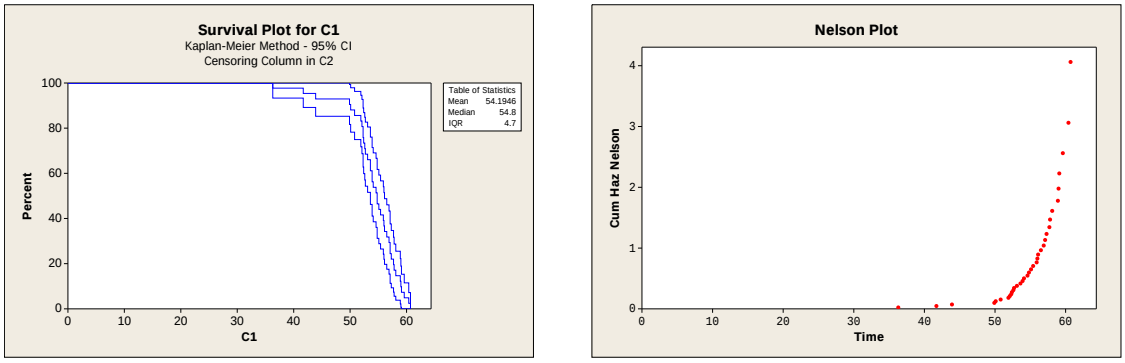

Figure 3: Kaplan Meier estimator and Nelson-plot from problem 3

c,d)

Note, that the first 4 values are censored and do not influence the analysis at all, they can be omitted. This happens when you have censored observations before you observe first failure time.
The estimated cumulative failure rate is convex, therefore the hazard rate of the underlying distribution is increasing.