TMA4125 Matematikk 4N

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**Exercise set 4**

$\boxed{1}$ **a)** State whether the following partial differential equations are linear and homogeneous:

$$u_{tt} = u_{xx}$$

$$v_{xx} + v_{yy} - \frac{2y}{x^3} = 0$$

**b)** Verify that $u(x,t) = x^2 + t^2$ is a solution of the first equation above, and that $v(x,y) = \frac{y}{x}$ is a solution of the second equation.

$\boxed{2}$ **a)** Find the general solution $y = y(t)$ of the ordinary differential equation

$$y'' - y' - 6y = 0$$

Use the above to find solutions $u = u(x,t)$ of the partial differential equation

$$u_{tt} - u_t - 6u = 0$$

**b)** Do the same for the ordinary differential equation

$$y'' + 2y' + 2y = 0$$

and the partial differential equation

$$u_{tt} + 2u_t + 2u = 0$$

$\boxed{3}$ **a)** Use the method of separation of variables to find the most general solution of the form $u(x,t) = F(x)G(t)$ to the wave equation

$$\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2}$$

for $0 < x < 2$ and $t > 0$, with boundary conditions

$$u(0,t) = u(2,t) = 0$$

**b)** Solve the above equation subject to the inital conditions

$$u(x,0) = \sin \pi x$$

$$u_t(x,0) = \cos \pi x$$

$\boxed{4}$ **a)** Write down d'Alembert's solution to the wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

with initial conditions

$$u(x,0) = f(x)$$
$$u_t(x,0) = 0$$

**b)** Suppose the above wave equation models the vertical oscillations $u = u(x,t)$ in the region $0 \leq x \leq L$ of a uniform string fixed at two points $x = 0$ and $x = L$. What boundary conditions do we require in addition to the above initial conditions? Briefly discuss the physical meaning of the initial conditions and the constant $c$.

**c)** What conditions must the function $f(x)$ satisfy for the solution obtained in the first part of this question to satisfy the boundary conditions of the string model?