MA0301 ELEMENTARY DISCRETE MATHEMATICS
SPRING 2017

1. Homework Set 6 – Solutions

3.2.20)

Proof of Theorem 6(b):

$$\begin{aligned} x \in \overline{\bigcap_{i\in I} A_i} &\Leftrightarrow x \notin \bigcap_{i\in I} A_i \\ &\Leftrightarrow \exists i \in I \text{ such that } x \notin A_i \\ &\Leftrightarrow \exists i \in I \text{ such that } x \in \overline{A_i} \\ &\Leftrightarrow x \in \bigcup_{i\in I} \overline{A_i} \end{aligned}$$

4.2.15)

For the basis step we choose $n = 0$. Then the statement is

$$5F_2 = L_4 - L_0 \Leftrightarrow 5 \cdot 1 = 7 - 2,$$

which is true. For the inductive step we prove the statement for $n = k + 1$ assuming it holds for $n = k$ and $n = k - 1$:

$$\begin{aligned} 5F_{(k+1)+2} &= 5F_{k+2} + 5F_{k+1} \\ &= (L_k + L_{k+4}) + (L_{k-1} + L_{k+3}) \\ &= (L_k + L_{k-1}) + (L_{k+4} + L_{k+3}) \\ &= L_{k+1} + L_{k+5} \end{aligned}$$

5.1.1)

$$\begin{aligned} A \times B &= \{(1,2),(1,5),(2,2),(2,5),(3,2),(3,5),(4,2),(4,5)\} \\ B \times A &= \{(2,1),(5,1),(2,2),(5,2),(2,3),(5,3),(2,4),(5,4)\} \\ A \cup (B \times C) &= \{1,2,3,4,(2,3),(2,4),(2,7),(5,3),(5,4),(5,7)\} \\ (A \cup B) \times C &= \{(1,3),(1,4),(1,7),(2,3),(2,4),(2,7),(3,3),(3,4),(3,7),(4,3),(4,4),(4,7),(5,3),(5,4),(5,7)\} \end{aligned}$$

$(A \times C) \cup (B \times C) = (A \cup B) \times C$

5.1.5a)

Suppose $A \times B \nsubseteq C \times D$. Then there exists $(x, y) \in (A \times B) \setminus (C \times D)$, which means that $x \in A$ and $y \in B$, but either $x \notin C$ or $y \notin D$, implying $A \nsubseteq C$ or $B \nsubseteq D$.

For the opposite implication, suppose that either $A \nsubseteq C$ or $B \nsubseteq D$. We can assume $A \nsubseteq C$ without loss of generality. Then there is an $x \in A \setminus C$. Since B is nonempty, we can pick some $y \in B$ and get $(x, y) \in (A \times B) \setminus (C \times D)$, which means that $A \times B \nsubseteq C \times D$.

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b)

If we do not assume that A and B are empty, there are counterexamples to the result in a). Namely, we can choose A empty and $B \nsubseteq D$. Then the second condition ($A \subseteq C$ and $B \subseteq D$) fails, but $A \times B \subseteq C \times D$ holds, since $A \times B$ is empty.

5.1.9) Theorem 1b:

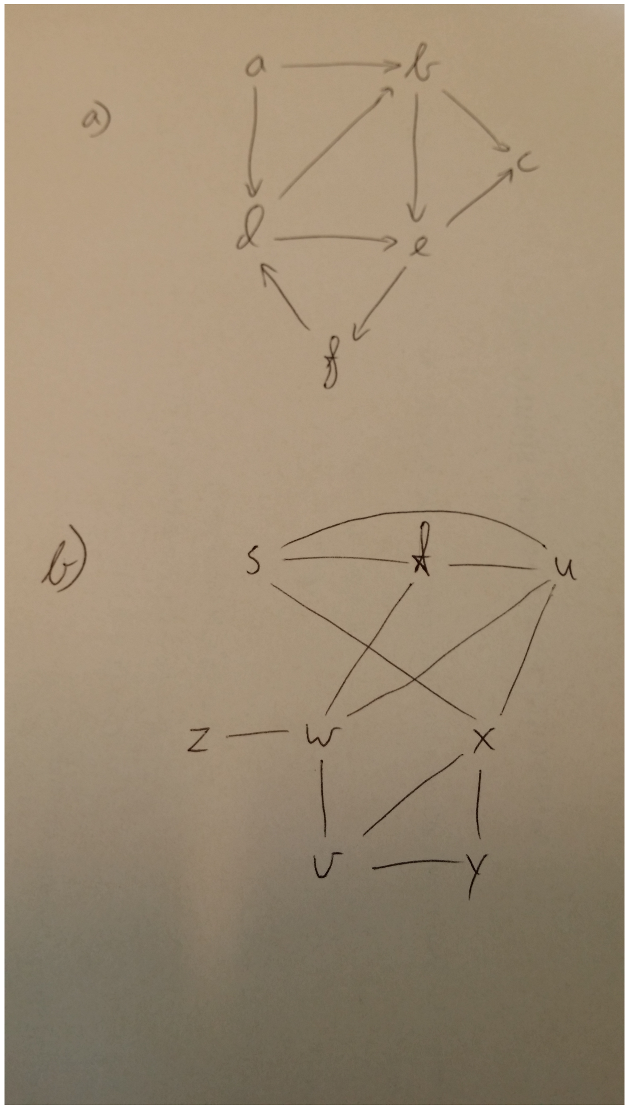
$$\begin{aligned}(x, y) \in A \times (B \cup C) &\Leftrightarrow x \in A \text{ and } y \in B \cup C \\ &\Leftrightarrow x \in A \text{ and } (y \in B \text{ or } y \in C) \\ &\Leftrightarrow (x \in A \text{ and } y \in B) \text{ or } (x \in A \text{ and } y \in C) \\ &\Leftrightarrow (x, y) \in A \times B \text{ or } (x, y) \in A \times C \\ &\Leftrightarrow (x, y) \in (A \times B) \cup (A \times C)\end{aligned}$$

Parts c) and d) are essentially the same as a) and b), just let C take on the role of A and flip the tuples.

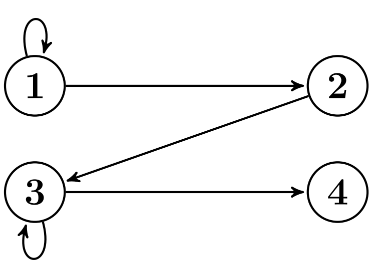
7.1.5) a) Reflexive, antisymmetric, transitive.
b) Reflexive, transitive. (Not antisymmetric since n and $-n$ divide each other.)
c) Reflexive, symmetric, transitive. (If $C = \mathcal{U}$, the relation is antisymmetric, too.)
d) Symmetric.
e) Symmetric.
f) Reflexive, symmetric, transitive.
g) Reflexive, symmetric.
h) Reflexive, transitive.

7.1.6) The relation in a) is a partial order. The ones in c) and f) are equivalence relations.

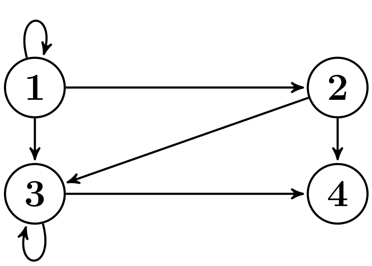
7.2.15)



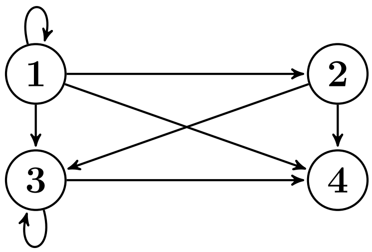
7.2.19) $\mathcal{R}$:



$\mathcal{R}^2$:



$\mathcal{R}^3 = \mathcal{R}^4$:



7.3.2)