Norwegian University of Science and Technology
Department of Mathematics

MA2501 Numerical Methods
Spring 2017

Exercise set 3

1 Write a MATLAB program for the solution of a linear system $\mathbf{Ax} = \mathbf{b}$ in the case where $\mathbf{A}$ is tridiagonal. More precisely, the program should take the three non-zero diagonals of $\mathbf{A}$ and the vector $\mathbf{b}$ as input and use Gaussian elimination without pivoting for the solution of the system.

Test your program on the matrix $\mathbf{A} \in \mathbb{R}^{200\times 200}$ with main diagonal $\mathbf{d} = [4, 4, \dots, 4]$ and lower and upper diagonals $\mathbf{a} = \mathbf{c} = [-1, -1, \dots, -1]$, and the right hand sides $\mathbf{b}_1 = [1, \dots, 1]^T$ and $\mathbf{b}_2 = [1, 2, 3, \dots, 200]^T$.

2 **Cf. Cheney and Kincaid, Exercise 8.1.19**

a) Prove that the product of two lower triangular matrices is lower triangular.

b) Prove that the product of two unit lower triangular matrices is unit lower triangular.

c) Prove that the inverse of an invertible lower triangular matrix is lower triangular.

d) Prove that the inverse of a unit lower triangular matrix is unit lower triangular.

e) Prove the previous statements for upper triangular matrices.

3 **a)** Assume that $\mathbf{A} \in \mathbb{R}^{n\times n}$ is invertible and has an LU factorization. Prove that the LU factorization is unique.

b) Find matrices $\mathbf{A} \in \mathbb{R}^{n\times n}$ that either do not have an LU factorization, or whose LU factorization is not unique.

4 Factor the following matrices into the LU decomposition using the LU Factorization Algorithm where $l_{ii} = 1$ for all i.

a)

$$\begin{bmatrix} 2 & -1 & 1 \\ 3 & 3 & 9 \\ 3 & 3 & 5 \end{bmatrix}$$

b)

$$\begin{bmatrix} 1.012 & -2.132 & 3.104 \\ -2.132 & 4.906 & -7.013 \\ 3.104 & -7.013 & 0.014 \end{bmatrix}$$

c)

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 1 & 1.5 & 0 & 0 \\ 0 & -3 & 0.5 & 0 \\ 2 & -2 & 1 & 1 \end{bmatrix}$$

d)

$$\begin{bmatrix} 2.1756 & 4.0231 & -2.1732 & 5.1967 \\ -4.0231 & 6.0000 & 0 & 1.1973 \\ -1.000 & -5.2107 & 1.1111 & 0 \\ 6.0235 & 7.0000 & 0 & -4.1561 \end{bmatrix}$$