Norwegian University of Science
and Technology
Institutt for matematiske fag

TMA4125 Matematikk 4N Spring 2017

Exercise set 3

1 Find the Fourier transform $\mathcal{F}(f) = \hat{f}$ of the following functions (without access to formula tables)

a)

$$f(x) = \begin{cases} 1 & \text{for } a \le x \le b \\ 0 & \text{otherwise} \end{cases}$$

b)

$$f(x) = \begin{cases} e^{iax} & \text{for } -b \le x \le b \\ 0 & \text{otherwise} \end{cases}$$

c)

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{\sin w}{w} e^{iwx}\, dw$$

The remaining questions should be answered with the help of the table of Fourier transforms in Kreyszig, p.536

2 **a)** Show that for any functions f and g admitting Fourier transforms, and constants a and b, we have

$$\mathcal{F}(af + bg) = a\mathcal{F}(f) + b\mathcal{F}(g)$$

Use this to find the Fourier transform of

$$f(x) = \begin{cases} 1 - \cos x & \text{for } -\pi \le x \le \pi \\ 0 & \text{otherwise} \end{cases}$$

(Hint: express $\cos x$ in terms of complex exponentials)

b) Show that, for a continuous function f, for which $f(x) \to 0$ as $|x| \to \infty$, and f' exists and is absolutely integrable, we have

$$\mathcal{F}\{f'(x)\}(w) = iw\mathcal{F}\{f(x)\}(w)$$

Use this to find the Fourier transform of

$$f(x) = \frac{x\cos x - \sin x}{x^2}$$

3 Let f be the function

$$f(x) = \begin{cases} e^{-x} & \text{for } x \ge 0 \\ 0 & \text{otherwise} \end{cases}$$

a) Compute the convolution of f with itself, i.e. find

$$\big(f \star f\big)(x) = \int_{-\infty}^{\infty} f(u) f(x-u)\, du$$

b) Use this to find the Fourier transform of

$$g(x) = \begin{cases} xe^{-x} & \text{for } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

4 **a)** Let f be a function with Fourier transform $\hat{f}$. Let $g(x) = f(ax)$, where a is a nonzero real constant. Show that

$$\mathcal{F}\{g\}(w) = \frac{1}{|a|} \hat{f}\left(\frac{w}{a}\right)$$

b) By considering the function $f(x) = e^{-x^2}$, find a function that is equal to its own Fourier transform.

c) Evaluate the integral

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{x^2}{2}}\, dx$$

5 **a)** By computing the 3×3 discrete Fourier transformation matrix, find the discrete Fourier transform of the the vector $f = (1, 2, 3)^T$

b) Use the Fast Fourier transform algorithm together with the above answer to obtain the discrete Fourier transform of the vector $f = (1, 1, 2, 2, 3, 3)^T$.