Norwegian University of Science
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MA0301 Elementary discrete mathematics Spring 2017

Exercise set 11

1 Homework Set 11

1 Grimaldi's book (5. ed., Exercises 11.1, page 519): solve **Ex. 7**

Seven towns a, b, c, d, e, f, and g are connected by a system of highways as follows: (1) I-22 goes from a to c, passing through b; (2) I-33 goes from c to d and then passes through b as continues to f; (3) I-44 goes from d through e to a; (4) I-55 goes from f to b, passing through g; and (5) I-66 goes from g to d.

a) Using vertices for towns and directed edges for segments of highways between towns, draw a directed graph that models this situation.

b) List the paths from g to a.

c) What is the smallest number of highway segments that would have to be closed down in order for travel from b to d to be disrupted?

d) Is it possible to leave town c and return there, visiting each of the other towns only once?

e) What is the answer to part (d) if we are not required to return to c?

f) Is it possible to start at some town and drive over each of these highways exactly once? (You are allowed to visit a town more than once, and you need not return to the town from which you started.)

2 Grimaldi's book (5. ed., Exercises 11.1, page 520): solve **Ex. 15**

For the undirected graph in Fig. 12 (two vertices v and w with a loop at v and an edge between v and w), find and solve a recurrence relation for the number of closed v-v walks of length $n \geq 1$, if we allow such a walk, in this case, to contain or consist of one or more loops.

3 Grimaldi's book (5. ed., Exercises 11.1, page 520): solve **Ex. 16**

See the book.

4 Grimaldi's book (5. ed., Exercises 11.2, page 529): solve **Ex. 6**

Find all (loop-free) nonisomorphic undirected graphs with four vertices. How many of these graphs are connected?

5 Grimaldi's book (5. ed., Exercises 11.2, page 529): solve **Ex. 12**

a) Let G be an undirected graph with n vertices. If G is isomorphic to its own complement $\overline{G}$, how many edges must g have? (Such a graph is called *self-complementary*.)

b) Find an example of a self-complementary graph on four vertices and one on five vertices.

c) If G is a self-complementary graph on n vertices, where $n > 1$, prove that $n = 4k$ or $n = 4k + 1$, for some $k \in \mathbb{Z}^+$.