Norwegian University of Science and Technology
Institutt for matematiske fag

MA0301 Elementary discrete mathematics
Spring 2017

Exercise set 8

1 Homework Set 8

$\boxed{1}$ Let $X := \{1, 2, 3, \ldots, 9, 10\}$. Decide whether the following sentences are statements (and determine its truth value) or propositional functions (and determine its truth set).

a) $\forall x \in X\ \exists y \in X\ (x + y < 14)$

b) $\forall y \in X\ (x + y < 14)$

c) $\forall x \in X\ \forall y \in X\ (x + y < 14)$

d) $\exists y \in X\ (x + y < 14)$

$\boxed{2}$ Which of the following sets are equal?

$A_1 := \{x \mid x^2 = 4x - 3\}$, $A_2 := \{x \mid x^2 = 3x - 2\}$, $A_3 := \{x \mid x \in \mathbb{N}, x < 3\}$, $A_4 := \{x \mid x \in \mathbb{N}, x \text{ odd}, x < 5\}$, $A_5 := \{1, 2\}$, $A_6 := \{1, 2, 1\}$, $A_7 := \{3, 1\}$, $A_8 := \{1, 1, 3\}$.

$\boxed{3}$ Grimaldi's book (5. ed., Exercises 7.3, page 364): solve **Ex. 5** Note: study in detail pages 360-364 in Grimaldi's book, in particular, *topological sorting* on page 360.

Topologically sort the Hasse diagram in part (a) of Example 38. [The subsets of $\{1, 2, 3\}$ sorted by inclusion.]

$\boxed{4}$ Grimaldi's book (5. ed., Exercises 7.3, page 365): solve **Ex. 6** Note: study in detail pages 344-347 in Grimaldi's book, in particular, the notion of *relation matrix* on page 346.

For $A = \{a, b, c, d, e\}$, the Hasse diagram for the poset $(A, \mathcal{R})$ is shown in Fig. 23. [The Hasse diagram in Fig. 23 contains the edges $(a, b), (a, c), (b, d), (c, d), (d, e)$.] (a) Determine the relation matrix for $\mathcal{R}$. (b) Construct the directed graph G (on A) that is associated with $\mathcal{R}$. (c) Topologically sort the poset $(A, \mathcal{R})$.

$\boxed{5}$ Grimaldi's book (5. ed., Exercises 7.3, page 366): solve **Ex. 26**

Given partial orders $(A, \mathcal{R})$ and $(B, \mathcal{S})$, a function $f : A \to B$ is called *order-preserving* if for all $x, y \in A$, $x\mathcal{R}y \implies f(x)\mathcal{S}f(y)$. How many such order-preserving functions are for each of the following, where $\mathcal{R}$, $\mathcal{S}$ both denote $\leq$ (the usual "less than or equal to" relation)?

(a) $A = \{1, 2, 3, 4\}$, $B = \{1, 2\}$;

(b) $A = \{1, \ldots, n\}$, $n \geq 1$, $B = \{1, 2\}$;

(c) $A = \{a_1, \ldots, a_n\} \subset \mathbb{Z}^+$, $n \geq 1$, $a_1 < \cdots < a_n$, $B = \{1, 2\}$;

(d) $A = \{1, 2\}$, $B = \{1, 2, 3, 4\}$;

(e) $A = \{1, 2\}$, $B = \{1, \ldots, n\}$, $n \geq 1$; and

(f) $A = \{1, 2\}$, $B = \{b_1, \ldots, b_n\} \subset \mathbb{Z}^+$, $n \geq 1$, $b_1 < \cdots < b_n$.

$\boxed{6}$ Grimaldi's book (5. ed., Exercises 1.3, page 24): solve **Ex. 13**

How many arrangements of the letters in MISSISSIPPI have no consecutive S's?

$\boxed{7}$ Grimaldi's book (5. ed., Exercises 1.3, page 25): solve **Ex. 20**

In the three parts of Fig. 8, eight points are equally spaced and marked on the circumference of a given circle [from A to H clockwise]. [In (a), the triangle ADG is marked, in (b) the triangle BDG and in (c) the quadrilateral ADFH.]

a) For parts (a) and (b) of Fig. 8 we have two different (though congruent) triangles. These two triangles (distinguished by their vertices) result from two selections from the vertices A, B, C, D, E, F, G, H. How many different (whether congruent or not) triangles can we inscribe in the circle in this way?

b) How many different quadrilaterals can we inscribe in the circle, using the marked vertices? [One such quadrilateral appears in part (c) of Fig. 8.]

c) How many different polygons of three or more sides can we inscribe in the given circle by using three or more of the marked vertices?

$\boxed{8}$ Grimaldi's book (5. ed., Exercises 1.3, page 25): solve **Ex. 23**

Determine the coefficient of x^9y^3 in the expansion of (a) $(x + y)^{12}$, (b) $(x + 2y)^{12}$, and (c) $(2x - 3y)^{12}$.

$\boxed{9}$ Grimaldi's book (5. ed., Exercises 1.4, page 34): solve **Ex. 7 a), b), c), f)**

Determine the number of integer solutions of

$$x_1 + x_2 + x_3 + x_4 = 32,$$

where

(a) $x_i \geq 0$, $1 \leq i \leq 4$
(b) $x_i > 0$, $1 \leq i \leq 4$
(c) $x_1, x_2 \geq 5$, $x_3, x_4 \geq 7$
(f) $x_1, x_2, x_3 > 0$, $0 < x_4 \leq 25$

$\boxed{10}$ Grimaldi's book (5. ed., Exercises 1.4, page 35): solve **Ex. 12**

Determine the number of integer solutions of

$$x_1 + x_2 + x_3 + x_4 + x_5 < 40,$$

where

(a) $x_i \geq 0$, $1 \leq i \leq 5$
(b) $x_i \geq -3$, $1 \leq i \leq 5$

$\boxed{11}$ Grimaldi's book (5. ed., Exercises 1.4, page 36): solve **Ex. 28 a), b)**

a) For $n \geq 4$, consider the strings made up of n bits - that is, a total of n 0's and 1's. In particular, consider those strings where there are (exactly) two occurences of 01. For example, if $n = 6$ we want to include strings such as 010010 and 100101, but not 101111 or 010101. How many such strings are there?

b) For $n \geq 6$, how many strings of n 0's and 1's contain (exactly) three occurences of 01?