EXAM IN COMPLEX ANALYSIS

Problem 1 Give geometric description of the set $\{z \in \mathbb{C}; \operatorname{Re} z = |z - 1|\}$

Problem 2 Find the image of the set $\{z \in \mathbb{C}; |z| > 1\}$ under the mapping
$$w = w(z) = 2\frac{2z - 1}{z - 2}.$$

Problem 3 Find analytic function $f(z)$ such that
$$\operatorname{Re} f(x, y) = x \cos x \cosh y + y \sin x \sinh y$$
and $f(0) = 0$.

Problem 4 Find the Taylor series entered at 0 for the function
$$f(z) = \frac{1}{1 + z + z^2}.$$
Hint: Multiply both numerator and denominator on $1 - z$.

Problem 5 Find $\operatorname{Res}_\infty z \cos^2 \frac{\pi}{z}$.

Problem 6 Evaluate the integral (taken in contrclockwise direction)
$$\int_{|z|=2} \frac{z}{z + 1} e^{1/z} dz$$

Problem 7 Evaluate the integral
$$\int_{-\infty}^{\infty} e^{ix} \frac{\sqrt{x + i}}{x^2 + 1} dx,$$
where the branch of the square root is defined by the relation $\sqrt{z + i}|_{z=0} = e^{i\pi/4}$.

Problem 8 Evaluate the integral
$$\int_0^{\infty} \frac{\log x}{(x + 1)^2} dx.$$

Problem 9 Find (any) conformal mapping of the unit disk onto the domain $D = \{z \in \mathbb{C}; |\operatorname{Re} z| < 1\} \setminus [0, +i\infty)$ (vertical strip without positive imaginary ray).