

TMA4230 Functional

Analysis

Spring 2017

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Exercise set 2

- 1 Suppose $(x_n)_n$ is a Schauder basis for a Banach space X. Show the following assertions.
 - a) X is separable.
 - b) The set $(x_n)_n$ is linearly independent.
- 2 Let X be a Banach space with a Schauder basis $(x_n)_n$. Define a norm on X by $||x||_a = \sup_{N \in \mathbb{N}} ||\sum_{n=1}^N a_n x_n||$ where (a_n) are the coefficients of x with respect to $(x_n)_n$.

Show that $||.||_a$ is a norm on X.

3 Show that ℓ^{∞} is not separable.