Meld fra om feil! KH

Løsningsskisser/Kommentarer til Øving 4

Oppgavene 1, 2, 3 er rett fram. Siden de annenordens partiellderiverte er kont. i $\mathbb{R}^2$, vil

$$\frac{\partial^2 f}{\partial y\,\partial x} = \frac{\partial^2 f}{\partial x\,\partial y}$$

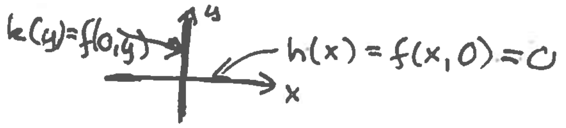
i Oppgave 1 a). Det ville kanskje vært naturlig å ta (den mer teoretiske) Oppgave 7 nå, og gjør det:

Oppgave 7 (2.5:4)

$$f(x,y) = \begin{cases} \dfrac{x^3y - xy^3}{x^2+y^2} & \text{når } (x,y) \neq (0,0) \\ 0 & \text{når } (x,y) = (0,0) \end{cases}$$

a) $f(x,0) = \dfrac{x^3 0 - x\cdot 0}{x^2+0^2} = \underline{0 \text{ når } x \neq 0}$, $f(x,0) = \underline{0 \text{ når } x = 0}$ (gitt)

Altså er $f(x,0) = 0$ for alle x. Tilsvarende for $f(0,y)$.



$h'(x) = 0$ alle x, $k'(y) = 0$ alle y

$\Rightarrow \underline{h'(0) = 0 = k'(0)}$

b) For $(x,y) \neq (0,0)$ får vi uten videre

$$\frac{\partial f}{\partial x} = \frac{(3x^2y - y^3)(x^2+y^2) - 2x(x^3y - xy^3)}{(x^2+y^2)^2} = \frac{y(x^4+4x^2y^2-y^4)}{(x^2+y^2)^2}$$

$$\frac{\partial f}{\partial y} = \text{"} x \leftrightarrow -y \text{"} = \frac{-x(y^4+4x^2y^2-x^4)}{(x^2+y^2)^2}$$

c) $$\frac{\partial^2 f}{\partial y\,\partial x}(0,0) \underset{\text{DEF}}{=} \lim_{h\to 0} \frac{\frac{\partial f}{\partial x}(0,h) - \frac{\partial f}{\partial x}(0,0)}{h} = \lim_{h\to 0} \frac{-\frac{h^5}{h^4} - 0}{h} = \underline{\underline{-1}}$$

$$\frac{\partial^2 f}{\partial x\,\partial y}(0,0) \underset{\text{DEF}}{=} \lim_{h\to 0} \frac{\frac{\partial f}{\partial y}(h,0) - \frac{\partial f}{\partial y}(0,0)}{h} = \lim_{h\to 0} \frac{\frac{h^5}{h^4} - 0}{h} = \underline{\underline{1}}$$

(Dette skyldes at $\dfrac{\partial^2 f}{\partial y\,\partial x} / \dfrac{\partial^2 f}{\partial x\,\partial y}$ ikke er kont. i $(0,0)$. Men hvem orker å vise det?)

TL går så over til derivasjon av vektorvaluerte funksjoner. Husk at $F: A \subset \mathbb{R} \to \mathbb{R}$ er deriverbar i a $\Leftrightarrow$ dersom

Tall eller 1x1-matrise

$$\lim_{h \to 0} \frac{F(a+h) - F(a) - F'(a)h}{h} = 0$$

Setning 6.1.7 i Kalkulus. Definisjon 2.6.2 en generalisering!

Kjerneregelen for vektorvaluerte funksjoner kan skrives på komponentform (2.7.2) og på matriseform (2.7.1). Øving 4 tar for seg (2.7.2) først.

Oppgave 4 (2.7:1)

$f = u^2 + v \quad , u = 2xy \; , v = x + y^2$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial f}{\partial v}\frac{\partial v}{\partial x} = 2u \cdot 2y + 1 \cdot 1 = \underline{\underline{8xy^2+1}}$$

$$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial u}\frac{\partial u}{\partial y} + \frac{\partial f}{\partial v}\frac{\partial v}{\partial y} = 2u \cdot 2x + 1 \cdot 2y = \underline{\underline{8x^2y+2y}}$$

Oppgave 5 (2.7:5)

$\mathbb{R}^2 \xrightarrow[G]{} \mathbb{R}^3 \xrightarrow[F]{} \mathbb{R}^2 \; , \; G(1,-2) = (1,2,3)$

$$F'(\underbrace{G(1,-2)}_{(1,2,3)})G'(1,2) = \begin{pmatrix} 2 & 1 & 4 \\ 0 & 2 & 2 \end{pmatrix}\begin{pmatrix} 1 & -2 \\ 3 & 1 \\ 2 & -1 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 13 & -7 \\ 10 & 0 \end{pmatrix}}}$$

Oppgave 6 (2.6:1a) b))

a) $\left.\begin{array}{l} F_1 = x^2y \\ F_2 = x + y^2 \end{array}\right\} \Rightarrow \begin{bmatrix} \frac{\partial F_1}{\partial x} & \frac{\partial F_1}{\partial y} \\ \frac{\partial F_2}{\partial x} & \frac{\partial F_2}{\partial y} \end{bmatrix} = \underline{\underline{\begin{bmatrix} 2xy & x^2 \\ 1 & 2y \end{bmatrix}}}$

b) $F(x,y,z) = \begin{bmatrix} e^{x^2y+z} \\ xyz^2 \end{bmatrix}; \begin{bmatrix} \frac{\partial F_1}{\partial x} & \frac{\partial F_1}{\partial y} & \frac{\partial F_1}{\partial z} \\ \frac{\partial F_2}{\partial x} & \frac{\partial F_2}{\partial y} & \frac{\partial F_2}{\partial z} \end{bmatrix} = \underline{\begin{bmatrix} 2xye^{x^2y+z}, & x^2e^{x^2y+z}, & e^{x^2y+z} \\ yz^2, & xz^2, & 2zxy \end{bmatrix}}$

Oppgave 8 (2.7: 8)

$T = f(x,y)$, $x = r\cos\theta$, $y = r\sin\theta$

a) $$\frac{\partial T}{\partial r} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial r} = \frac{\partial f}{\partial x}\cos\theta + \frac{\partial f}{\partial y}\sin\theta$$

$$\frac{\partial T}{\partial \theta} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial \theta} = -\frac{\partial f}{\partial x} r\sin\theta + \frac{\partial f}{\partial y} r\cos\theta$$

i flg. kjerneregelen.

b) $\underline{r = g(t)}$, $\underline{\theta = h(t)}$ $\Rightarrow$ $x = g(t)\cos h(t)$, $y = g(t)\sin h(t)$

$T = f(r\cos\theta, r\sin\theta) = \underline{f(g(t)\cos h(t), g(t)\sin h(t))}$

Ved kjerneregelen ($T = T(t)$):

$$T'(t) = \frac{\partial T}{\partial r}\frac{dr}{dt} + \frac{\partial T}{\partial \theta}\frac{d\theta}{dt}$$

$$= \left(\frac{\partial f}{\partial x}\cos\theta + \frac{\partial f}{\partial y}\sin\theta\right) g'(t) + \left(-\frac{\partial f}{\partial x} r\sin\theta + \frac{\partial f}{\partial y} r\cos\theta\right) h'(t)$$

der alle størrelsene på høyre side kan uttrykkes ved t.