Norwegian University of Science and Technology
Department of Mathematical Sciences

TMA4145 Linear Methods
Fall 2017

Exercise set 10

Please justify your answers! The most important part is *how* you arrive at an answer, not the answer itself.

1 Let M be the plane of $\mathbb{R}^3$ given by $x_1 + x_2 + x_3 = 0$. Find the linear mapping that is the orthogonal projection of $\mathbb{R}^3$ onto this plane.

2 Let $A \subset \mathbb{R}$ be a set.

Prove that if A is bounded from below, then there is a sequence $(a_n) \subset A$ such that $a_n \to \inf A$ as $n \to \infty$. (In other words, prove that $\inf A \in \bar{A}$.)

Similarly, if A is bounded from above, prove that there is a sequence $(a_n) \subset A$ such that $a_n \to \sup A$ as $n \to \infty$. (In other words, prove that $\sup A \in \bar{A}$.)

3 Let T be a bounded linear operator on a Hilbert spae X. Show that the operator norm of T can be expressed in terms of the innerproduct of X:

$$\|T\| = \sup\{\langle Tx, y\rangle : x, y \in X \quad \text{with} \quad \|x\| = \|y\| = 1\}.$$

4 Let $M = \{x \in \ell^2 : x = (x_1, 0, x_3, 0, x_5, ...)\}$ be the subspace of odd sequences in ℓ^2. Determine the orthogonal complement $M^\perp$. You must prove that the space you find really is the orthogonal complement of M.

5 Let c_f be the subspace of ℓ^2 that consists of all sequences with finitely many non-zero terms.

a) Show that best approximation fails for c_f.

b) Why does this not contradict the best approximation theorem from class?