Norwegian University of Science and Technology
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TMA4125 Matematikk 4N
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Exercise set 5

$\boxed{1}$ Suppose we have a long, thin gold bar of length $L = 1$m, where one end of the bar is held at constant temperature 0°C and the other end is insulated. This is modeled by the heat equation

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2},$$

with boundary conditions

$$\frac{\partial u(0,t)}{\partial x} = u(1,t) = 0$$

Here $c^2 \approx 1.27 \times 10^{-4}$ m^2/s is the thermal diffusivity of gold.

a) Compute all solutions of this equation of the form $u(x,t) = F(x)G(t)$ that satisfy the given boundary conditions.

b) The initial temperature distribution in the bar is given by

$$u(x,0) = 100\cos(\frac{\pi x}{2})°\text{C}$$

How long does it take for the insulated end of the bar to cool down to 20°C?

$\boxed{2}$ The steady state temperature of a plate of dimension $2\pi \times \pi$ (in metres) is modeled by the equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0,$$

for $0 \le x \le 2\pi$, $0 \le y \le \pi$. Three ends of the plate are insulated, i.e.

$$u_x(0,y) = u_x(2\pi,y) = u_y(x,0) = 0,$$

whilst the top end is held at the temperature

$$u(x,\pi) = 50(1+\cos x)°\text{C}$$

By writing $u(x,y) = F(x)G(y)$, solve this equation to find the steady state temperature $u(x,y)$ of the plate.

$\boxed{3}$ **a)** Find all the solutions of the form $u(x,t) = F(x)G(t)$ satisfying the PDE

$$\frac{\partial^2 u}{\partial t^2} - 2\frac{\partial u}{\partial t} + u = \frac{\partial^2 u}{\partial x^2},$$

for $t > 0$ in the region $0 < x < \pi$, with boundary conditions

$$u(0,t) = u(\pi,t) = 0$$

b) Find the solution of the above equation that satisfies in addition the initial conditions

$$u(x,0) = \sin x + \sin 2x$$

$$\frac{\partial u(x,0)}{\partial t} = -\sin 2x$$