Norwegian University of Science and Technology
Department of Mathematical Sciences

TMA4145 Linear Methods
Fall 2017

Exercise set 11

Please justify your answers! The most important part is *how* you arrive at an answer, not the answer itself.

1 Let $L^2[-1,1]$ be the closure of $C[-1,1]$ with respect to the innerproduct

$$\langle f,g\rangle = \int_{-1}^{1} f(t)\overline{g(t)}dt.$$

Apply Gram-Schmidt to the monomial basis $\{1, x, x^2, x^3, ...\}$ up to degree 3.

2 Consider the exponential basis $\{e^{2\pi int} : n \in \mathbb{Z}\}$ in $(L^2[0,1], \langle .,. \rangle)$. Verify Parseval's relation for this particular case directly. Try to explain how Fourier series and some of their properties fit into this problem.

3 We define the cyclic shift matrix T_1 and the modulation matrix M_1 by

$$T_1 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \qquad M_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{2\pi i/3} & 0 \\ 0 & 0 & e^{4\pi i/3} \end{pmatrix}.$$

a) Show that

1. $T_1 M_1 = e^{2\pi i/3} M_1 T_1$.
2. $T_1^3 = I_3$ and $M_1^3 = I_3$.
3. M_1 and T_1 are unitary matrices.

b) Show that $\{\frac{1}{\sqrt{3}} M_1^k T_1^l : k,l \in \{1,2,3\}\}$ is an orthonormal basis of the space of complex 3×3 matrices $M_3(\mathbb{C})$ with respect to the innerproduct $\langle A, B\rangle = \operatorname{tr}(AB^*)$.

4 Let $\{e_n : n \in \mathbb{N}\}$ be the standard basis in the ℓ^p-spaces.

a) Show that $\sum_{n=0}^{\infty} \alpha_n e_n$ converges in ℓ^p for $1 \leq p < \infty$ if and only if $(\alpha_n)_{n\in\mathbb{N}} \in \ell^p$.

b) Show that $\sum_{n=0}^{\infty} \alpha_n e_n$ converges in ℓ^∞ if and only if $(\alpha_n)_{n\in\mathbb{N}}$ converges to zero.