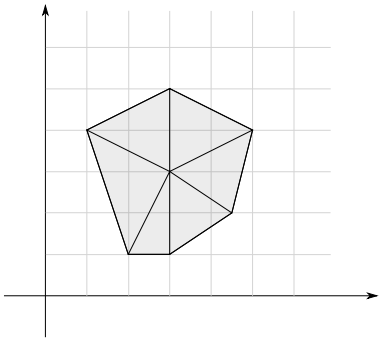
Norwegian University of Science
and Technology
Institute of mathematics

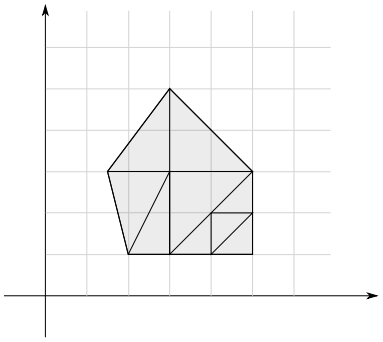
TMA4220 Finite Element Method Autumn 2017

Exercise set 4

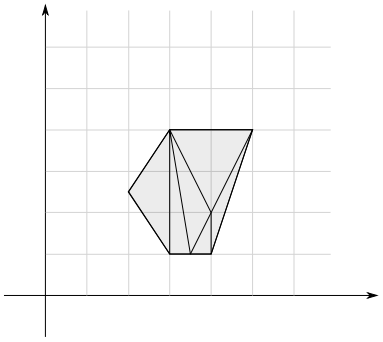
1 Which of the following meshes constitute a Delaunay triangulation? What changes would need to be done to make it a proper Delaunay mesh?



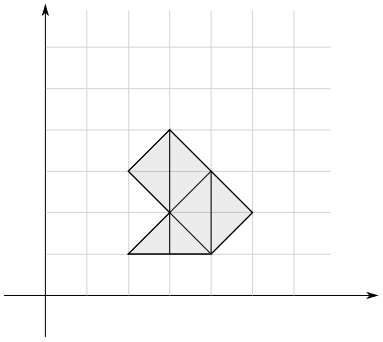
(a)



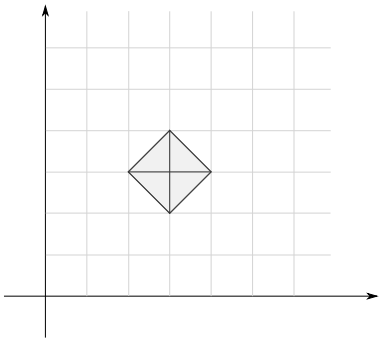
(b)



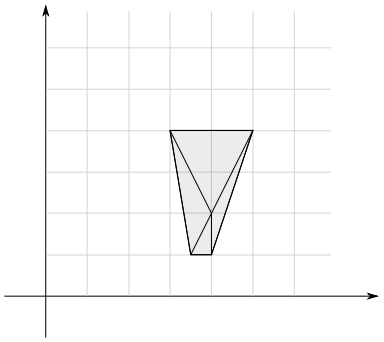
(c)



(d)



(e)



(f)

2 A family of meshes $\mathcal{T}_h$ is said to be quasi-uniform if there exist some $\tau > 0$

$$\frac{\min_{K\in\mathcal{T}_h} h_K}{\max_{K\in\mathcal{T}_h} h_K} \geq \tau, \quad \forall h > 0$$

Let $x_i, i = \{0, 1, ..., n\}$ be the nodes in a 1D-partitioning of some interval $I = [x_0, x_n]$. Let the element size be $h_i = x_i - x_{i-1}$. Are the following family of meshes quasi-uniform?

(Hint: Consider the case $\lim_{h\to 0} \mathcal{T}_h$ by letting $\lim_{n\to\infty} \mathcal{T}_h$).

a)

$$x_i = 0.5\frac{i}{n}$$

b)

$$x_i = 0.5\left(\frac{i}{n}\right)^2$$

c)

$$x_i = \sin\left(\frac{i}{n}\cdot\frac{\pi}{2}\right)$$

d)

$$x_i = \sin\left(\frac{i}{n}\cdot\frac{\pi}{4}\right)$$

3 Consider the linear system

$$\begin{pmatrix} 1 & 1 & -1 \\ 1 & 5 & -3 \\ -1 & -3 & 6 \end{pmatrix} x = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

a) Solve the above system (exactly) using the conjugate gradient method.

b) What are the associated Krylov subspaces?

4 We examine the numerical approximation of the diffusion-transport equation

$$0.01u_{xx} + 2u_x = 0, \quad u(0) = 0, \quad u(1) = 1$$

We find an approximate solution using linear finite elements on a uniform grid of element size h.

a) Show that the discretization results in a linear system of the form

$$(A + C)u = f,$$

and write this relation in the form

$$\alpha u_{n+1} + \beta u_n + \gamma u_{n-1} = \delta, \quad n = 2, \ldots, M - 1$$

where you should specify the coefficients $\alpha, \beta, \gamma, \delta$, as functions of h.

b) Find and plot the exact solution to the equation. Contrast this to the behaviour of the approximate solution - how small must h be to ensure that u is monotone?

c) Explain how the method can be modified to eliminate unwanted oscillations from the numerical solution, regardless of the element size h.