Norwegian University of Science and Technology
Department of Mathematical Sciences

TMA4145 Linear Methods
Fall 2017

Exercise set 13

Please justify your answers! The most important part is *how* you arrive at an answer, not the answer itself.

1 Given the matrix

$$A = \begin{pmatrix} 1 & 2 \\ 2 & 2 \\ 2 & 1 \end{pmatrix}.$$

a) Compute the singular value decomposition of A.

b) Use the result of a) to find:

1. Bases for the following vector spaces: $\ker(A), \ker(A^*), \operatorname{ran}(A), \operatorname{ran}(A^*)$.
2. The pseudo-inverse of A.
3. Find the minimal norm solution of $Ax = b$ for $b = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$.

(Exam 2016)

2 Let U be a $n \times n$ matrix with columns $u_1, ..., u_n$. Show that the following statements are equivalent:

1. U is unitary.
2. $\{u_1, ..., u_n\}$ is an orthonormal basis of $\mathbb{C}^n$.

3 Let T be the shift operator on ℓ^2 defined by $T(x_1, x_2, ...) = (0, x_1, x_2, ...)$.

1. Show that T has no eigenvalues.
2. Does T^* have any eigenvalues?

4 Let X be a finite dimensional vector space and T a linear transformation on X. Show that $X = \ker(T) \oplus \operatorname{ran}(T^*)$, where $\oplus$ denotes the direct sum of the vector spaces.

Hint: Use that we know that $\ker(T)^{\perp} = \operatorname{ran}(T^*)$.