Norwegian University of Science and Technology
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TMA4125 Matematikk 4N
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Exercise set 1

1 Sketch the graph of the following functions f and g, which are periodic with period 2, and are defined as follows for $|x| \leq 1$:

$$f(x) = |x| + 1$$
$$g(x) = \begin{cases} 2x + 1 & \text{for } -1 \leq x < 0 \\ \cos \pi x & \text{for } 0 \leq x < 1 \end{cases}$$

2 **a)** Find the Fourier series of the 2π-periodic function f given for $|x| \leq \pi$ by

$$f(x) = \begin{cases} -x & \text{for } -\pi \leq x < 0 \\ 0 & \text{for } 0 \leq x < \pi \end{cases}$$

Compute the values of the Fourier series at the following points: $x = -\pi, x = \frac{-\pi}{2}, x = 0, x = \frac{\pi}{2}$.

b) Sketch the graph of f together with the first three terms in its Fourier series.

3 Which of the following functions $f : \mathbb{R} \to \mathbb{R}$ are odd, even, or neither?

a) $f(x) = x^2$

b) $f(x) = x^3$

c) $f(x) = 2^x$

(In the remaining examples, g is an even function and h is an odd function)

d) $f(x) = g(x) - h(x)$

e) $f(x) = g(x) + h(x)$

f) $f(x) = g(x)h(x)$

g) $f(x) = g(h(x))$

4 **a)** Use the identity $\sin a \cos b = \frac{1}{2}\big(\sin(a + b) + \sin(a - b)\big)$ to prove the following orthogonality relation (for any positive integers m, n):

$$\int_{-\pi}^{\pi} \sin nx \cos mx \, dx = 0$$

b) Let $f(x) = \sin(8x)\cos(x)$. Is this function odd, even, or neither?

c) Calculate the Fourier series of the above function (hint: you may find it helpful to use the trigonometric identity given in part a)