Norwegian University of Science and Technology
Institutt for matematiske fag

TMA4165 Differential Equations and Dynamical Systems
Spring 2017

Exercise set 12

You find solutions to the following exercises on the web page. Give it a try and ask if something is unclear:
J.S: 11.8, 11.9, 11.10, 12.1 (ii)
Exam 1992.3

These exercises will be presented / discussed in the exercise class:
J.S: 12.9, 12.19, 12.24
Exam 2005.3, Exam 2014.5, Exam 1995.6, Exam 2002.4

Exam 1992, 3 Give an example of an n-dimensional, dynamical system (n given and $n \geq 2$)

$$\dot{x} = f(x), \quad x \in \mathbb{R}^n$$

such that $f \in C^1(\mathbb{R}^n, \mathbb{R}^n)$, $f(0) = 0$, $\lim_{t\to\infty} x(t) = 0$ for all solutions, and not all eigenvalues of its linearisation at 0 have strictly negative real part.

Exam 2005, 3

a) Sketch an example of a phase diagram around an equilibrium point of index -2, 1, and 3, respectively.

b) Determine the index of the origo for the system

$$\dot{x} = 2xy$$
$$\dot{y} = 3x^2 - y^2.$$

Exam 2014, 5

a) State the Poincaré Bendixson theorem.

b) Let $\dot{x} = f(x)$ and $\dot{x} = g(x)$ be two systems in $\mathbb{R}^2$, where f and g are C^1 functions such that $\langle f(x), g(x) \rangle = 0$. Show that if $\dot{x} = f(x)$ has a periodic solution, then the system $\dot{x} = g(x)$ has at least one equilibriumpoint.

Exam 1995, 6 Let $f : E \mapsto \mathbb{R}^2$ be a C^1 vector field and $E \subset \mathbb{R}^2$ open, such that there exists an annulus A with $A \subset E$. f has no zeros inside A or on the boundary of A, and f points inwards along the boundary of A. Why must A contain at least one closed phase path? Show that if A contains 3 closed phase paths, then at least one of them must be a stable limit cycle.

$\boxed{\text{Exam 2002, 4}}$ Given the dynamical system $\dot{x} = f(x)$, where f belongs to C^1. Let

$$A = \{x \in \mathbb{R}^2 \mid 1 \leq \|x\| \leq 2\}.$$

Assume that $f(x) \neq 0$ for all x on the boundary of A. Sketch all possible phase diagrams in A under the assumptions that there are neither equilibrium points nor closed phase paths inside A, that the boundaries of A are closed phase paths, and that the boundaries of A either have the same or opposite orientation.