MA0301 ELEMENTARY DISCRETE MATHEMATICS
SPRING 2017

1. Homework Set 1 – Solutions

Exercise 1. *Grimaldi's book (5. ed., Exercises 2.1, page 54): solve* **Exercise 3**

Solution 1. Since $p \to q$ is false p, q must have the truth values T, F, respectively (check truth table for $p \to q$).

a) False
b) False
c) True
d) False

Exercise 2. *Grimaldi's book (5. ed., Exercises 2.1, page 54): solve* **Exercise 8**

Solution 2.

a)

p	q	$\neg p \wedge q$	$\neg p \wedge q \to \neg p$
T	T	F	T
T	F	F	T
F	T	T	T
F	F	F	T

b)

p	q	r	$q \to r$	$p \to (q \to r)$
T	T	T	T	T
T	T	F	F	F
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	F	T
F	F	T	T	T
F	F	F	T	T

c)

p	q	r	$p \to q$	$(p \to q) \to r$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	T	T
F	T	F	T	F
F	F	T	T	T
F	F	F	T	F

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d)

p	q	$p \to q$	$q \to p$	$(p \to q) \to (q \to p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

e)

p	q	$p \to q$	$p \wedge (p \to q) \to q$
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	T

f)

p	q	$p \wedge q$	$(p \wedge q) \to p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

g)

p	q	$(\neg p \vee \neg q)$	$(\neg p \vee \neg q) \leftrightarrow q$
T	T	F	F
T	F	T	F
F	T	T	T
F	F	T	F

h)

p	q	r	$p \to q$	$q \to r$	$(p \to q) \wedge (q \to r)$	$p \to r$	$((p \to q) \wedge (q \to r)) \to (p \to r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

Exercise 3. *Grimaldi's book (5. ed., Exercises 2.1, page 54): solve* **Exercise 9**

Solution 3. Check truth tables: a), e), f), h)

Exercise 4. *Grimaldi's book (5. ed., Exercises 2.2, page 66): solve* **Exercise 13**

Solution 4. Both statements are true if and only if p, q and r all have the same truth value. There are several ways to argue, for example:

- Check the truth table.
- Assume that p, q and r all have the same truth value. Then both statements are true. Assume that not all have the same truth value, say p and q differ. (We can assume this without loss of generality.) Then both statements can be seen to be false by subtituting 1 and 0, and 0 and 1 for p and q, respectively.
- Both statements are just a bunch of implications that need to hold simultaneously, and one can check that all the implications in the second statement follow from the first, and vice versa.

Exercise 5. *Is the following argument valid?*

$$(p \to q) \wedge \neg p \implies \neg q$$

Detail all steps of your reasoning.

Solution 5. Check truth table:

p	q	$p \to q$	$((p \to q) \wedge \neg p) \to \neg q$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

Exercise 6. *Study EXAMPLE 2.18 in Grimaldi's book (5. ed., page 64) and solve* **Exercise 20** *of Exercises 2.2 (page 67).*

Solution 6. a) Comment: explain well the idea of circuits and the reasoning of associating a statement/proposition to such a circuit.

Possibilities for flow from T_1 to T_2:

(1) $(p \wedge (\neg r \vee q \vee \neg q)) \vee ((r \vee t \vee \neg r) \wedge \neg q)$

(2) $\Leftrightarrow (p \wedge (\neg r \vee T)) \vee ((r \vee \neg r \vee t) \wedge \neg q)$

(3) $\Leftrightarrow (p \wedge T) \vee ((T \vee t) \wedge \neg q)$

(4) $\Leftrightarrow p \vee (T \wedge \neg q)$

(5) $\Leftrightarrow p \vee \neg q$

b) Comment: explain well the idea of circuits and the reasoning of associating a statement/proposition to such a circuit.

Possibilities for flow from T_1 to T_2:

(6) $(p \vee (p \wedge q) \vee (p \wedge q \wedge \neg r)) \wedge ((p \wedge r \wedge t) \vee t)$

(7) $\Leftrightarrow (p \vee (p \wedge q) \vee (p \wedge q \wedge \neg r)) \wedge t$

(8) $\Leftrightarrow p \wedge t$

Exercise 7. *Grimaldi's book (5. ed., Exercises 2.3, page 85): solve* **Exercise 9**

Solution 7. a) Comment: insist on studying table 2.19 on page 78 in G's book.

(1) premise
(2) from (1) resolving negation
(3) from (2) and rule of conjunctive simplification
(4) premise
(5) from (3,4) rule and rule of disjunctive syllogism
(6) premise
(7) from (2) and rule of conjunctive simplification
(8) from (6,7) and modus tollens
(9) premise
(10) from (8,9) and rule of disjunctive syllogism
(11) from (5,10) and rule of conjunction

(12) from (11) and method of contradiction

b) direct proof:

(1) $p \to q$ premise
(2) $\neg q \to \neg p$ from (1) and $p \to q \Leftrightarrow \neg q \to \neg p$
(3) $p \vee r$ premise
(4) $\neg p \to r$ from (3) and $p \vee r \Leftrightarrow \neg p \to r$
(5) $\neg q \to r$ from (2,4) and law of syllogism
(6) $\neg r \vee s$ premise
(7) $r \to s$ from (6) and $\neg r \vee s \Leftrightarrow r \to s$
(8) $\Rightarrow \neg q \to s$ from (5,7) and law of syllogism

c) direct proof of 2.32:

(1) $\neg p \leftrightarrow q$ premise
(2) $(\neg p \to q) \wedge (q \to \neg p)$ from (1) and $\neg p \leftrightarrow q \Leftrightarrow (\neg p \to q) \wedge (q \to \neg p)$
(3) $\neg p \to q$ from (2) and rule of conjunctive simplification
(4) $q \to r$ premise
(5) $\neg p \to r$ from (3,4) and law of syllogism
(6) $\neg r$ premise
(7) $\Rightarrow p$ from (5,6) and and modus tollens

Exercise 8. *Grimaldi's book (5. ed., Exercises 2.3, page 85): solve* **Exercises 11 c,d**

Solution 8. Comment: insist on detailed arguments.

c) $p = q = r = T,\ s = F$
d) $p = q = r = T,\ s = F$, or any combination with $p = s = F$.

2. Basic Set 1 – Solutions

Exercise 1. *Grimaldi's book (5. ed.): study and discuss* **The Laws of Logic** *(page 58-59) and the table 2.19 (page 78).*

Solution 1. Comment: students should have read and thought about the statements.

Exercise 2. *Grimaldi's book (5. ed., Exercises 2.2, page 66): solve* **Exercise 6**

Solution 2. a) $\neg(p \wedge (q \vee r) \wedge (\neg p \vee \neg q \vee r)) \Leftrightarrow \neg p \vee (\neg q \wedge \neg r) \vee (p \wedge q \wedge \neg r)) \Leftrightarrow \cdots \Leftrightarrow \neg p \vee \neg r$
b) $\neg((p \wedge q) \to r) \Leftrightarrow (p \wedge q) \wedge \neg r$
c) $\neg(p \to (\neg q \wedge r)) \Leftrightarrow \neg(\neg p \vee (\neg q \wedge r)) \Leftrightarrow p \wedge (q \vee \neg r)$
d) $\neg(p \vee q \vee (\neg p \wedge \neg q \wedge r)) \Leftrightarrow \neg p \wedge \neg q \wedge (p \vee q \vee \neg r) \Leftrightarrow \neg p \wedge \neg q \wedge \neg r$

Exercise 3. *Grimaldi's book (5. ed., Exercises 2.2, page 66): solve* **Exercise 18**

Solution 3. a) distributive law for $\vee$ over $\wedge$; inverse law; identity law
b) absorption law + commutativity of $\vee$; $p \to q \Leftrightarrow \neg p \vee q$; commutativity of $\wedge$; distributive law of $\wedge$ over $\vee$; inverse law; identity law; De Morgan's laws

Exercise 4. *Is the following argument valid?*

$$(p \to \neg q) \wedge (r \to q) \wedge r \implies \neg p$$

Detail all steps of your reasoning.

Solution 4. One may argue by looking at the 5th row of the truth table (see below). This is the only row where $r \to q$, $p \to \neg q$, r are true simultaneously, and here $\neg p$ is true as well.

p	q	r	$p \to \neg q$	$r \to q$	$\neg p$
T	T	T	F	T	F
T	T	F	F	T	F
T	F	T	T	F	F
T	F	F	T	T	F
F	T	T	T	T	T
F	T	F	T	T	T
F	F	T	T	F	T
F	F	F	T	T	T

Exercise 5. *Grimaldi's book (5. ed., Exercises 2.3, page 85): solve* **Exercises 8**

Solution 5.
(1) premise
(2) from (1) and rule of conjunctive simplification
(3) premise
(4) from (2,3) and rule of detachment
(5) from (1) and rule of conjunctive simplification
(6) from (4,5) and rule of conjunction
(7) premise
(8) from (7) and negation of implication
(9) from (8) and De Morgan's laws
(10) from (6,9) and rule of detachment
(11) premise
(12) from (11) and negation of implication
(13) from (12) and De Morgan's laws and double negation
(14) from (10, 13) and rule of detachment
(15) from (14) and rule of conjunctive simplification

Exercise 6. *Grimaldi's book (5. ed., Exercises 2.3, page 85): solve* **Exercises 10**

Solution 6.
a) rule of conjunctive simplification; rule of conjunction; rule of disjunctive amplification
b) rule of detachment; $(q \to r) \Leftrightarrow (\neg q \vee r)$; rule of detachment
c) modus tollens; rule of conjunction; De Morgan's laws
d) rule of detachment; modus tollens
e) rule of detachment; modus tollens; rule of detachment
f) rule of conjunctive simplification; rule of detachment; rule of conjunctive simplification; rule of disjunctive syllogism
g) rule of disjunctive syllogism; rule of detachment; law of syllogism; $(t \to r) \Leftrightarrow (\neg r \to \neg t)$
h) rule of disjunctive syllogism; rule of disjunctive syllogism

Exercise 7. *Grimaldi's book (5. ed., Exercises 2.3, page 85): solve* **Exercises 11 a,b**

Solution 7. a) Comment: discuss in detail the arguments.
a) $r = T$ and: $p = T, q = T$, or $p = F, q = T$, or $p = F, q = F$
b) $p = F, q = F, r = T$ or F, $p = F, q = T, r = T$