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Department of Mathematics

MA2501 Numerical Methods
Spring 2017

Exercise set 4

- 1 a) Compute the LDL^T and, if possible, Cholesky factorization of the following matrices:

$$\mathbf{A}_1 = \begin{bmatrix} -2 & -4 & 6 & -2 \\ -4 & -5 & 12 & -7 \\ 6 & 12 & -17 & 7 \\ -2 & -7 & 7 & 1 \end{bmatrix}, \quad \mathbf{A}_2 = \begin{bmatrix} 1 & 2 & -3 & 1 \\ 2 & 7 & -6 & -1 \\ -3 & -6 & 10 & -2 \\ 1 & -1 & -2 & 6 \end{bmatrix}.$$

- b) Use the previously computed factorizations to solve the linear systems $\mathbf{A}_j \mathbf{x} = \mathbf{b}$ with $\mathbf{b} = [-3, 3, 1, -3]^T$.
- 2 How many operations are required for the computation of the Cholesky factorization of a symmetric and positive definite matrix? Compare the result with the numerical complexity of the LU factorization.
- 3 Write a MATLAB program for the solution of a linear system $\mathbf{A}\mathbf{x} = \mathbf{b}$ with symmetric matrix $\mathbf{A} \in \mathbb{R}^{n \times n}$ using the LDL^T factorization. Include some error messages for the situation that \mathbf{A} is non-symmetric or the factorization fails.
- 4 Use the bisection method for finding a solution of the equation $x^2 - 2 = 0$ (i.e., for computing $\sqrt{2}$) starting with the interval $[1, 2]$. Stop the computations when the error is smaller than 10^{-3} . How many iterations would be needed to obtain a result with an error smaller than 10^{-12} ?
- 5 Implement the bisection method in MATLAB. Your program should take as an input the two initial points $a < b$, the desired accuracy $\varepsilon > 0$, and a reference to the function f a root of which is to be computed.
- Use your program for finding the solution of the equation $\tan(x) = 0$ in the interval $[2, 4]$ (which should be π) and a solution of the equation $3\cos(x)\sin(x^2) = 1$ in the interval $[0, 10]$. What happens if you apply the method to the equation $\tan(x) = 0$ with the starting interval $[1, 2]$?
- 6 Suppose $f(x)$ is $C[a, b]$ with a sign change on and unique zero in the interior of $[a, b]$. Let $h(x) = f(x)g(x)$ where $g(x)$ is a positive and continuous function on $[a, b]$. Let c_f^n and c_h^n be the n -th approximation from the Bisection method to the root of f and h

respectively, with initial interval $[a, b]$. Can we say anything about c_f^n compared to c_h^n ?