Multiport waveguide couplers with periodic energy exchange

Jovana Petrovic

Institute Vinca, University of Belgrade, Mike Petrovica Alasa 12-14, 11000 Belgrade, Serbia (jovanap@vin.bg.ac.rs)

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In this Letter, a multiport directional optical coupler based on periodic energy exchange in a linear waveguide array is proposed. The periodic power transfer is achieved by choosing waveguide separations that render commensurate eigenvalues of the array coupling matrix. This is a general solution and offers a plethora of possibilities. Particularly interesting is an array that can be used to realize different couplers by simply choosing a different input waveguide. The proposed design principle is validated by full numerical simulations of realistic devices and the required fabrication precision is estimated. The proposed couplers are of interest for quantum optics, biosensing, and communications.

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Multiport optical couplers are of high interest as interferometric beam splitters in quantum optics [1,2] and biosensing [3]. They are constructed by concatenating or nesting single-mode two-port couplers or, less frequently, by using multimode interference [4]. On the other hand, waveguide arrays (WGAs), which are by construction suitable for application as multiport couplers, are used mostly as uniform or modulated optical lattices [5] and gratings [6]. Coupling between the waveguides of an array is effected via their evanescent fields that induce changes in polarization in the neighboring waveguides [7]. There is no fundamental reason why these arrays should not be used as couplers in communication systems; in fact, it is rather practical. In the most commonly used arrays of equivalent and equally spaced waveguides, evanescent-field coupling yields transfer of power between the waveguides that is quasi-periodic along the array. The quasi-periodicity is a direct consequence of the incommensurability of the eigenfrequencies of such an array. Such dynamics yield irrational power coupling ratios at any length of the array, which are inconvenient for applications. Exemptions are 2- and 3-guide arrays that support periodic power transfer and are, indeed, frequently used as couplers [8–10].

Periodic energy transfer in finite WGAs has been investigated to achieve diffractionless propagation. Applications in rerouting of an input beam [11,12] and construction of 1×2 couplers [12] have been proposed. The suggested approach relies on the condition of equidistance of eigenvalues. In [11], it is employed to induce harmonic oscillations and in [12,13], to induce full, intensity-only, and fractional revivals. The equidistance condition is relaxed only in the case of intensity revivals where eigenvalue differences are multiples of a constant inversely proportional to the revival length.

In this Letter, a new approach to achieving a periodic power transfer in WGAs is proposed. The basic assumption is commensurability of the system eigenvalues. This condition renders the general set of solutions. It contains the family of solutions given by the eigenvalue-equidistance condition. Moreover, some of the proposed WGAs have rational and round power coupling ratios suitable for construction of various multiport couplers. It is shown analytically and by numerical simulations of realistic waveguides that such WGAs can be designed by adjusting the ratios of inter-waveguide separations. Based on these findings, multifunctional couplers are designed that, unlike the previously proposed 1×2 couplers and routers, exploit multiple ports of the array. The simple geometry enables elimination of bend losses and, in principle, easier fabrication with respect to the nested 2-port couplers. On the other hand, when multiple input ports are used, the proposed WGAs require coherent input, while 2-port couplers do not. The coherence condition stems from the fact that the power coupling ratio of a WGA with more than 2 waveguides depends on the relative phases between its input ports.

In a discrete model of a WGA [14], coupling between neighboring waveguides is given by a symmetric tridiagonal coupling matrix C with non-zero side diagonals. An M-WGA is defined by coupling coefficients $\kappa_{n,n+1}$ for $n=\overline{1,M-1}$. Field distribution at the output of the array is obtained by solving a set of coupled differential equations: $i\frac{\partial\Psi(z)}{\partial z}=C\Psi(z)$, where $\Psi(z)=(\Psi_1(z), \Psi_2(z),\ldots,\Psi_M(z))$ is the vector of the complex field amplitudes.

Eigenvalues of a WGA with equal coupling between guides, $\kappa_{n,n+1}=\kappa$ for any n, can be calculated analytically and are given by the formula $\lambda_n = 2\kappa\cos(\pi n/(M+1))$, $n=\overline{1,M}$. For $M<4$ they are commensurate and the corresponding field powers $|\Psi_n(z)|^2$ evolve as sine functions. For $M>4$ their eigenspectra comprise incommensurate frequencies and dynamics become quasi-periodic [see an example in Fig. 1(a)]. For any WGA length, the ratios between the output amplitudes are irrational numbers, which renders these arrays impractical for construction of couplers.

To obtain periodic energy exchange between waveguides, the coupling coefficients must be chosen so that the eigenvalues are commensurable. By imposing a condition that the ratio of eigenvalues is p/q, where p and q are integers, the ratios between the coupling coefficients are obtained. Here, for simplicity, the consideration is limited to arrays with the mirror symmetry, i.e., those with $\kappa_{n,n+1}=\kappa_{M-n,M-n+1}$. Analytical solutions are derived for 4- and 5-guide arrays. In a 4-guide array energy, the

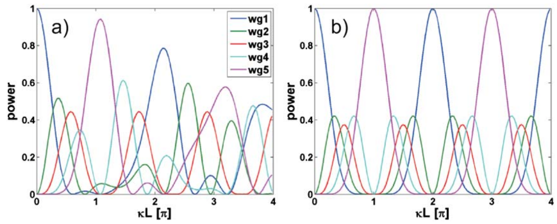

Fig. 1. 5-guide arrays with (a) equal coupling coefficients exhibiting quasi-periodic energy exchange and (b) coupling coefficients defined by $p = 2$, $q = 1$ exhibiting periodic energy exchange. Color codes are the same in (a) and (b).

exchange is periodic when $\kappa_{2,3}/\kappa_{1,2} = (p - q)/\sqrt{pq}$ and in a 5-guide array when $\kappa_{2,3}/\kappa_{1,2} = \sqrt{(p^2 - q^2)/2q^2}$ where $p > q$. Expressions for eigenvalues of arrays with $M \geq 6$ are cumbersome, but solutions can be easily found numerically. Figure 1(b) shows the solution for a 5-guide array with $\kappa_{1,2} = \kappa$ (used hereafter) and $p = 2$, $q = 1$. Oscillatory dynamics in all waveguides are apparent. Spectral analysis of the matrix C yields the oscillation period of $2\pi q$.

The coupling ratio at the output is determined by the product of the coupling strength κ and the array length L, and by the input state. The same array can perform functions of different couplers if different input ports are excited. For example, a 5-guide array from the above example with the length L satisfying $\kappa L = \pi/2$ can be used as a binomial coupler with the output power distribution $|\Psi(L)|^2 = \frac{1}{16}(1, 4, 6, 4, 1)$, an equal 1×4 coupler with the distribution $\frac{1}{4}(1, 1, 0, 1, 1)$, or a somewhat exotic $\frac{1}{8}(3, 0, 2, 0, 3)$ coupler, when the light is inserted into the end, next to the end, or the middle waveguide, respectively. Moreover, the same coupler can collect 99.6% of light from three input ports $|\Psi(0)|^2 = \frac{1}{3}(1, 0, 1, 0, 1)$ excited in phase into the middle output port.

Other useful couplers can be designed for different combinations of p and q. For instance, when the middle waveguide is used as the input of a 5-guide array with $p = 5$, $q = 2$, the array serves as an equal 1×5 coupler with the difference in output powers of less than 0.07%. An array with $p = 3n + 1$, $q = 2n + 1$, where $n \in N$, can be used as a 1×2 coupler. These examples are summarized in Table 1. In the same manner, a range of couplers can be designed for any M.

The corresponding WGAs are engineered by exploiting the fact that the coupling coefficient between two waveguides has a simple exponential dependence on the distance between them. A coupling coefficient between the neighboring waveguides n and $n + 1$ is given by $\kappa_{n,n+1} = Ae^{-\alpha L_{n,n+1}}$, where $L_{n,n+1}$ is the distance between the waveguides and A and α are constants that depend on the refractive index profile of the waveguides, but not on the distance between them. The periodicity is achieved if the waveguides are arranged so that

$$L_{n+1,n+2} = L_{n,n+1} + \frac{1}{2\alpha} \ln \frac{\kappa_{n,n+1}}{\kappa_{n+1,n+2}}, \tag{1}$$

where the ratio of the coupling coefficients is derived from the periodicity condition given above. Here, the consideration is limited to WGAs with identical waveguides. A more complicated solution would be to engineer the coupling coefficients by changing the refractive index profiles of the waveguides.

The finite-difference beam propagation method (BPM) was used to simulate light power transfer in realistic WGAs. The essential difference with respect to the discrete model is that the waveguide width is finite, hence the propagation dynamics depend on the field distributions across waveguides (mode profiles).

For convenience, the waveguide parameters were chosen based on the standard telecom single-mode fiber with the index contrast $\Delta n = 0.005$ and diameter $D = 8$ μm, and all simulations were performed at the wavelength of 1550 nm. The coupling coefficient of two waveguides κ was determined numerically by exploiting the fact that it is inversely proportional to a period of oscillation z_p between the waveguides, $\kappa z_p = 2\pi$. The array parameters A and α were found by an exponential fit of the calculated inverse period. For some regular waveguide profiles (e.g., rectangular), these parameters can be obtained analytically [7]. Distance $L_{1,2}$ was fixed and the distances between other waveguides were calculated by formula (1). The input mode to any waveguide was the fundamental mode.

Figure 2 shows the quasi-periodic energy exchange in a realistic 5-guide array with equally spaced waveguides and the periodic exchange in a WGA with waveguide spacing given by $p = 2$, $q = 1$. In the latter case, the regular oscillation pattern is apparent, as well as the possibility of the total power transfer between the end waveguides (analogous to the π pulse). The results match closely the power dynamics obtained by the discrete model (Fig. 1). The oscillation periods obtained by the two models agree, whereby the coupling strength in the discrete model is calibrated to match that in realistic waveguides.

A slow decay in the oscillation amplitude observed for the realistic WGA in Fig. 2(b) is a consequence of the fact that the field intensity distribution of the fundamental mode tails off outside the excited waveguide and

Table 1. Examples of Couplers Achievable by an Unequally Spaced 5-Guide Array; Output Powers are Normalized

p, q	κL	Input	Output	Coupler	Error
2, 1	$\pi/2$	1, 0, 0, 0, 0	1, 4, 6, 4, 1	binomial	0
2, 1	$\pi/2$	0, 1, 0, 0, 0	1, 1, 0, 1, 1	1×4	0
2, 1	$\pi/2$	0, 0, 1, 0, 0	3, 0, 2, 0, 3	1×3	0
2, 1	$\pi/2$	1, 0, 1, 0, 1	0, 0, 1, 0, 0	3×1	0
5, 2	$\pi/2$	0, 0, 1, 0, 0	1, 1, 1, 1, 1	1×5	0.07%
$3n + 1$, $2n + 1$	0.7π	0, 0, 1, 0, 0	1, 0, 0, 0, 1	1×2	0.04%

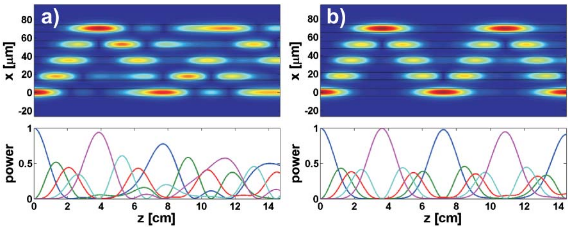

Fig. 2. BPM simulation of (a) quasi-periodic power transfer in an equally spaced WGA and (b) periodic power transfer in a WGA with $p = 2$, $q = 1$. Upper graphs, false color intensity distribution along the array (red is the highest intensity). Lower graphs, power in each waveguide (color coded as in Fig. 1).

exponentially decays with the distance from it. This ruins the ideal initial condition of a single waveguide excitation and the condition of coupling with the neighboring waveguides only. The stronger the mode confinement and/or the larger the distance between waveguides, the more accurate the discrete approximation.

The simulated field intensity profiles of some couplers listed in Table 1 are shown in Fig. 3. The arrays with lengths determined by the κL products in the table couple the light exactly as predicted, thus validating this simple method of WGA design. Cases (a) and (b) demonstrate two applications of a $\pi/2$ waveguide. The input ports in (b) are excited in phase.

In the ideal case of coupling to the nearest waveguide only, oscillatory dynamics are maintained indefinitely. However, in real WGAs, coupling to non-neighboring

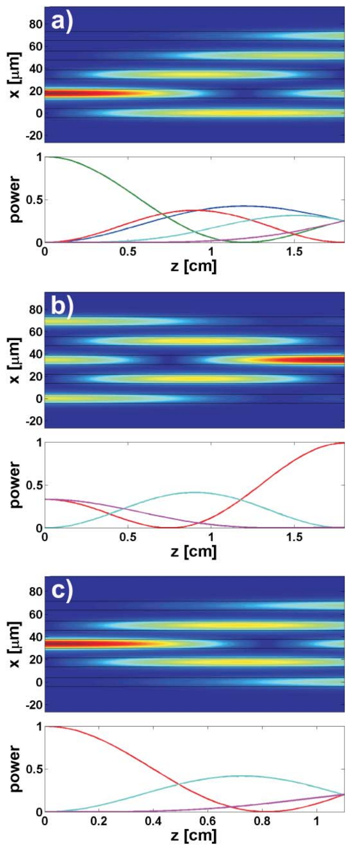

Fig. 3. Directional couplers: (a) 1×4, (b) 3×1, and (c) 1×5. Upper graphs, false color intensity distribution along the array (red the highest). Lower graphs, power in each waveguide (color coded as in Fig. 1).

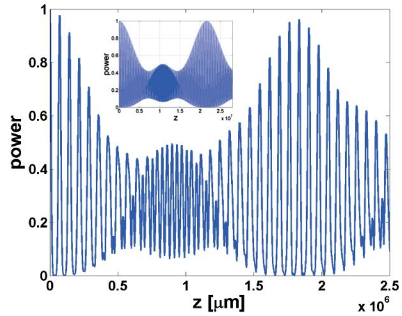

Fig. 4. Dephasing and revivals in an end waveguide of a 5-guide binomial coupler with $p = 2$, $q = 1$. Inset: discrete model of dephasing with the 2nd-order coupling coefficients $\kappa_{1,3} = \kappa_{3,5} = \sqrt{3/2}\kappa_{1,2}^2/A$ and $\kappa_{2,4} = \sqrt{3/2}\kappa_{1,3}$.

waveguides can induce dephasing which is, in the absence of decoherence, followed by revivals of oscillations. The result of a long BPM simulation in Fig. 4 shows the first revival of oscillations. This effect was also modeled analytically by activating other diagonals of the coupling matrix (inset in Fig. 4). For a given WGA, the k^{th}-order coupling coefficient was calculated as a 2-guide coupling coefficient $\kappa_{n,n+k} = Ae^{-\alpha L_{n,n+k}}$. It was found that the 2nd-order coupling introduces two orders of magnitude faster dephasing than the 3rd order coupling; hence, it is mainly responsible for the observed dephasing. The reduced contrast of the revived oscillations and the increased dephasing rate in realistic WGAs are due to the mode extension outside the waveguides.

Modern waveguide fabrication techniques such as optical lithography [15] or femtosecond laser writing [16] allow for precise control of the waveguide refractive index profile and geometry of WGA. Realization of the periodic energy exchange suggested here critically depends on the ability to precisely control waveguide separation. Hence, further statistical calculations have been performed to evaluate the impact of deviations from the design waveguide separations on the coupler output. A series of 1×4 couplers from Fig. 3(a) with all waveguide separations randomly distributed around their respective design values were simulated. The distribution of separations was normal with the standard deviation σ. The statistics were based on 50 simulations per deviation. It was found that the coupling ratio is highly sensitive to the waveguide separation, the maximum deviation from the optimal 1/4 being 2% for $\sigma = 0.1\%$ (18 nm resolution), 3.9% for $\sigma = 0.3\%$ (52 nm resolution), and 7% for $\sigma = 1\%$ (176 nm resolution). Therefore, fabrication of the proposed WGAs is challenging, but the reported laser-written WGAs [17,18] and the lithography reaching 32 nm/22 nm precision show that achieving the required accuracy is very near.

WGAs considered here can be used as directional 1xN couplers straightforwardly. Since their power coupling ratio depends on the relative phases of the input ports, their applications as beam combiners (couplers with multiple input ports) require nontrivial engineering of the phase of the input field. On the other hand, sensitivity to the input phase makes these WGAs useful as output couplers in multipath interferometers.

Similar periodic systems occur in nature. For example, coupling of atomic angular momenta, given by Clebsch–Gordan coupling coefficients, is perfectly periodic in atomic manifolds with any number of states. Indeed, the coupling coefficients used in Fig. 1(b) are equal to Clebsch–Gordan coefficients of $|F = 2\rangle$ hyperfine manifold of an atom [19]. The corresponding WGA power dynamics are identical to the Rabi population transfer between its m_F states [20]. This observation can be used to easily construct WGAs with a large number of waveguides, whereby the WGAs with $\kappa L = \pi/2$ could serve as binomial $\binom{M}{n}$ couplers.

In conclusion, a new general approach to building versatile multiport directional optical couplers is proposed. The approach is based on engineering of the coupling coefficients of a WGA to yield the eigenvalues of its coupling matrix commensurable. The suggested WGA design relies on choosing appropriate inter-waveguide separations and not on the waveguide refractive index profile. The design principle is proved by numerical simulation of a realistic 5-guide array, and a multifunctional 1xN coupler is demonstrated. The coupler output is highly sensitive to fabrication imperfections, but the required accuracy is nearly within the reach of the state-of-the-art fabrication techniques. The proposed WGAs can be of interest as directional 1xN couplers in optical circuits, input and output couplers in integrated multipath interferometers, simulators of atomic angular momenta, and for coherent transport studies. Applications of the presented approach to nonlinear or two-dimensional WGAs would be interesting extensions of this work.

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