Proof as a Video Game

Formalizing Introductory Analysis in the Lean Theorem Prover

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Lean Prover

Why Lean?

- **Removes error from proofs**
- **“All” of math is documented into library**
- **Lean is another way to approach proving theorems**
- **Makes math proofs more intuitive**

Why Are We Researching This?

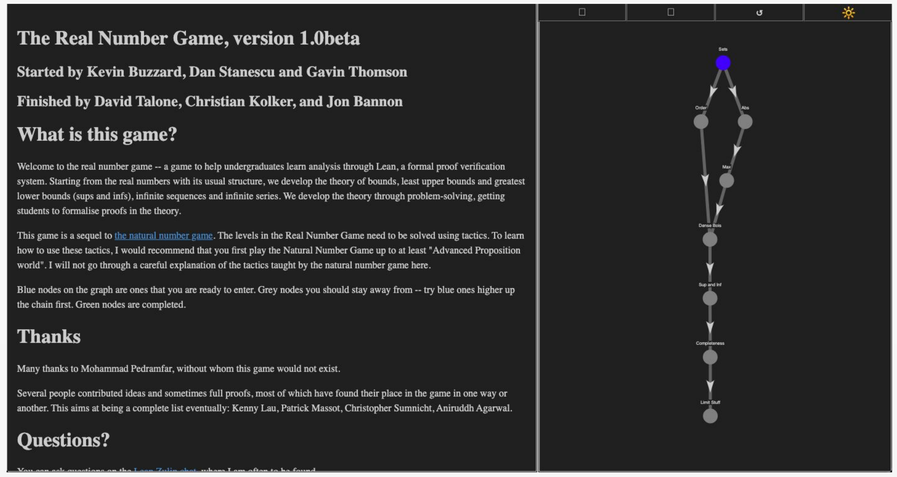
Computers are never wrong. Humans are wrong all the time. So, what if we use computers to see if we are wrong or right?

Our goal for this research project was to make a game which could be used to assist in teaching real analysis.

We were working to convince a computer that real analysis proofs were correct, which is often more difficult than doing the proof “normally”.

The benefit of doing this on Lean is that you forced to complete each and every step in the proof regardless of how big or small it is. Since you must complete every step, you will consequently understand the proof better.

The Real Number Game



Starting from raw Lean code, we managed to produce a prototype for a “playable” game.

Curry-Howard Correspondence

type theory		*logic*	
τ	type	ϕ	proposition
τ	inhabited type	ϕ	theorem
e	well-typed program	π	proof
$\rightarrow$	function space	$\rightarrow$	implication
$*$	product	$\wedge$	conjunction
$+$	sum	$\vee$	disjunction
$\forall$	type quantifier	$\forall$	2nd order quantifier
B	inhabited type	$\top$	truth
void	uninhabited type	$\bot$	falsity

Above shows a way of relating mathematical logic (Right) to type theory (Left). A type is referring to a data type which is commonly used in Computer Science. We get the above table by the Curry-Howard Isomorphism, and we use this to translate mathematical logic to type theory. Computers read type theory, so it is important to be able to translate between type theory and mathematical logic. Lean uses type theory to read mathematical proofs and solve them.

The Future of Mathematics

Presently, Lean can be used to check proofs for correctness. In the future, Lean can potentially be used by AI to actually prove theorems rather than just assisting humans proof them.

This means that computers can discover its own theorems and prove them.

An Actual Level

Weirdly Satisfing