

ИДЗ №1

ДУ 1-ого порядка и ДУ высших порядков, допускающие понижение

Задание №1. Найдите общий интеграл ДУ

1. $dy = \sqrt{4 - y^2} dx - x dy.$
2. $x^2 dy = 2x\sqrt{y^2 + 4} dx - dy.$
3. $2xy^2 dx - dy = x^2 dy - 8x dx.$
4. $y dy + \sqrt{y^2 + 4} dx = 2x^2 y dy.$
5. $y^2 dx - 2y dy = 2xy dy - 4 dx.$
6. $dy = \sqrt{y^2 + 4} dx - x dy.$
7. $\sqrt{x^2 + 4} dy - dx = y dx.$
8. $y^2 dx - 2xy dy = 4y dy - dx.$
9. $2x\sqrt{4 - y^2} dx - dy = x^2 dy.$
10. $x^2 dy = \sqrt{y^2 + 1} dx - 4 dy.$
11. $2x\sqrt{y^2 + 1} dx - x^2 dy = 4 dy.$
12. $4 dy - y dx = 2 dx - x^2 dy.$
13. $9 dx - x dy = dy - y^2 dx.$
14. $18x dx - x^2 dy = 4 dy - 2xy^2 dx.$
15. $9 dy = \sqrt{4 - y^2} dx - x^2 dy.$
16. $\sqrt{9 - y^2} dx - 4 dy = x^2 dy.$
17. $\sqrt{y^2 + 1} dx - 2 dy = x dy.$
18. $2x^2 y dy = \sqrt{y^2 + 1} dx + 8y dy.$
19. $4 dy = 2x\sqrt{9 - y^2} dx - x^2 dy.$
20. $\sqrt{y^2 + 4} dx - 9 dy = x^2 dy.$
21. $2 dx = \sqrt{9 - x^2} dy - y dx.$
22. $dy - 2xy dx = 4x dx - x^2 dy.$
23. $4 dx = \sqrt{9 - x^2} dy - y^2 dx.$
24. $y^2 dx - x dy = 2 dy - 4 dx.$
25. $2xy^2 dx = \sqrt{x^2 + 4} dy + x dx.$
26. $dx = 2y\sqrt{x^2 + 4} dy - y^2 dx.$
27. $8y dy - dx = y^2 dx - 2x^2 y dy.$
28. $dx = 2y\sqrt{4 - x^2} dy - y^2 dx.$
29. $-y^2 dx + \sqrt{x^2 + 1} dy = 4 dx.$
30. $\sqrt{9 - y^2} dx - 2 dy = x dy.$

Задание №2. Найдите общий интеграл ДУ

1. $(2y^2 - xy) dx = (x^2 - xy + y^2) dy.$
2. $(4x^2 + 4xy + 5y^2) dx = 4x(x + y) dy.$
3. $(6y^3 + 2x^2y) dx = (5xy^2 + x^3) dy.$
4. $(x + 3y) dx = (3x - y) dy.$
5. $(4x^2 + 4xy + 3y^2) dx = (4x^2 + 2xy) dy.$
6. $y(x - y) dx - x^2 dy = 0.$
7. $(6y^3 + 4x^2y) dx = (5xy^2 + 2x^3) dy.$
8. $xy' = y + (2x + y)(\ln(2x + y) - \ln x).$
9. $(2x^2 + 4xy) dy = (x^2 + 2xy + 5y^2) dx.$
10. $2x^2y' = y^2 + 5xy + 2x^2.$
11. $6y(x^2 + y^2) dx = (5xy^2 + 3x^3) dy.$
12. $y' = \frac{xy - y^2}{x^2 - 2xy}.$
13. $(4x^2 + 6xy + 3y^2) dx = (6x^2 + 2xy) dy.$
14. $y' = \frac{y}{x} + e^{\frac{y}{x}}.$
15. $(3x^2 + 2xy) dy = (x^2 + 3xy + 3y^2) dx.$
16. $2x^2 dy = (x^2 + y^2) dx.$
17. $x(3y^2 + x^2) dy = (4y^3 + 2x^2y) dx.$
18. $xy' = y + (x + 2y)(\ln(x + 2y) - \ln x).$
19. $(4x - y) dy = (x + 4y) dx.$
20. $(6x - y) dy = (x + 6y) dx.$
21. $2xy' = 2y + x \operatorname{tg} \frac{2y}{x}.$
22. $x(3y^2 + 2x^2) dy = 4y(y^2 + x^2) dx.$
23. $x^2y' - xy + y^2y' = 0.$
24. $(x^2 + 2xy) dx + xy dy = 0.$
25. $(3x^2 + 4xy) dy = (x^2 + 3xy + 5y^2) dx.$
26. $y^2 + x^2 \frac{dy}{dx} = xy \frac{dy}{dx}.$
27. $(x^2 + 2xy + 3y^2) dx = 2x(x + y) dy.$
28. $3x(y^2 + x^2) dy = (4y^3 + 6x^2y) dx.$
29. $xy' = \sqrt{4x^2 - y^2} + y.$
30. $(4x^2 + 6xy + 5y^2) dx = (6x^2 + 4xy) dy.$

Задание №3. Найдите общий интеграл ДУ

1. $y'x \ln x - y = 2y^{-1} \ln^6 x.$
2. $y' + \frac{1-2x}{x^2}y - 1 = 0.$
3. $e^{2y} dy = 3x dy + dx.$
4. $y' - 3y \operatorname{tg} x = 3y^{-2} \operatorname{tg}^8 x.$
5. $y' - y \operatorname{ctg} x = 2x \sin x.$
6. $y' - 2xy = 2xe^{x^2}.$
7. $y' \operatorname{tg} x - y = 3y^{-1} \sin^8 x.$
8. $y'x \ln x - y = 3x^3 \ln^2 x.$
9. $y' + xe^x y = e^{(1-x)e^x}.$
10. $y'x \ln x - y = 6y^2 \ln^{-7} x.$
11. $xy' = \frac{y}{x+1} + x.$
12. $\frac{dy}{dx} = \frac{1}{x \cos y + a \sin 2y}.$
13. $xy' - y = 2x^9 y^{-2}.$
14. $y' + \frac{y}{x} = \frac{\sin x}{x}.$
15. $xy' - 2y = x^{-1}y^2 \operatorname{ctg} x \operatorname{cosec} x.$
16. $y' - y \operatorname{th} x = \operatorname{ch}^2 x.$
17. $y = xy' + y' \ln y.$
18. $xy' - y = 2x^3 e^{4x} y^{-1}.$
19. $e^{-y} dx - (2y + xe^{-y}) dy = 0.$
20. $(y^2 + 1) dx = (1 - 4xy) dy.$
21. $e^{x^2} dy - xye^{x^2} dx + y^3 dx = 0.$
22. $x^2 y^2 y' + xy^3 = 1.$
23. $xy' + y = y^2 \ln x.$
24. $y' \operatorname{tg} x - y = y^{-1} \sin^4 x.$
25. $3y' - 2y = \frac{x^3}{y^2}.$
26. $y'x - 4y = x^2 \sqrt{y}.$
27. $xy' - y = x^6 y^{-2}.$
28. $y' + \frac{y}{x} = x^2 y^4.$
29. $y' - xy = -y^3 e^{-x^2}.$
30. $xy' - y = 3x^3 e^{6x} y^{-1}.$

Задание №4. Найдите общий интеграл ДУ

1. $(x^3 + xy^2) dx + (x^2y + y^3) dy = 0.$
2. $\frac{(2x-y) dx + (2y+x) dy}{x^2+y^2} = 0.$
3. $4(x^3 - xy^3) dx + 6(y^5 - x^2y^2) dy = 0.$
4. $(x - y + 2) dx - (x - y - 3) dy = 0.$
5. $(\sin xy + xy \cos xy) dx + x^2 \cos xy dy = 0.$
6. $\frac{3x^2+y^2}{y^2} dx - \frac{2x^3+5y}{y^3} dy = 0.$
7. $e^x(2xy + x^2y + \frac{y^3}{3}) dx + e^x(x^2 + y^2) dy = 0.$
8. $(2x + 3x^2y) dx + (x^3 - 3y^2) dy = 0.$
9. $(3x^2 + 2xy + 2xy^6) dx + (x^2 + 6x^2y^5) dy = 0.$
10. $(x - 2y)y' + x^2 + y = 0.$
11. $(e^x + y + \sin y) dx + (e^y + x + x \cos y) dy = 0.$
12. $3x^2(1 + y^5) dx + y^2(3 + 5x^3y^2) dy = 0.$
13. $(x + y - 1) dx + (e^y + x) dy = 0.$
14. $(3x^2 + y^2 + 6x^5y^2) dx + (2xy + 2x^6y) dy = 0.$
15. $2xy dx + (x^2 - y^2) dy = 0.$
16. $(2 - 9xy^2)x dx + (4y^2 - 6x^3)y dy = 0.$
17. $(y^2 + 6x^5y^2) dx + (2xy + 3y^2 + 2x^6y) dy = 0.$
18. $(y^3 - 2xy) dx + (3xy^2 - x^2) dy = 0.$
19. $x(x + 2y) dx + (x^2 - y^2) dy = 0.$
20. $2(xy + xy^6) dx + (x^2 + 3y^2 + 6x^2y^5) dy = 0.$
21. $(x^3 + y) dx + (x - y) dy = 0.$
22. $(2xy + y^2 + 4x^3y^4) dx + (x^2 + 2xy + 4x^4y^3) dy = 0.$
23. $(x - y) dx + \left(\frac{1}{y^2} - x\right) dy = 0.$
24. $\left(\frac{x}{\sin y} + 2\right) dx + \frac{(x^2+y) \cos y}{\cos 2y-1} dy = 0.$
25. $(2x^3 - xy^2) dx + (2y^3 - x^2y) dy = 0.$
26. $(2x + e^{x/y}) dx + e^{x/y} \left(1 - \frac{x}{y}\right) dy = 0.$
27. $(2xy - 1) dx + (x^2 + 1) dy = 0.$
28. $(\sin y + y^2 \sin x) dx + (x \cos y - 2y \cos x) dy = 0.$
29. $(2x + 1)y' + 4x + 2y = 0.$
30. $(3x^2 \sin y + y \sin x) dx + (x^3 \cos y - \cos x) dy = 0.$

Задание №5. Найдите общий интеграл ДУ

1. $y' = \frac{x+2y-3}{2x-2}$.

2. $y' = \frac{x+y-2}{2x-2}$.

3. $y' = \frac{3y-x-4}{3x+3}$.

4. $y' = \frac{2y-2}{x+y-2}$.

5. $y' = \frac{x+y-2}{3x-y-2}$.

6. $y' = \frac{2x+y-3}{x-1}$.

7. $y' = \frac{x+y-8}{3x-y-8}$.

8. $y' = \frac{x+3y+4}{3x-6}$.

9. $y' = \frac{3y+3}{2x+y-1}$.

10. $y' = \frac{x+2y-3}{4x-y-3}$.

11. $y' = \frac{x-2y+3}{-2x-2}$.

12. $y' = \frac{x+8y-9}{10x-y-9}$.

13. $y' = \frac{2x+3y-5}{5x-5}$.

14. $y' = \frac{4y-8}{3x+2y-7}$.

15. $y' = \frac{x+3y-4}{5x-y-4}$.

16. $y' = \frac{y-2x+3}{x-1}$.

17. $y' = \frac{x+2y-3}{x-1}$.

18. $y' = \frac{3x+2y-1}{x+1}$.

19. $y' = \frac{5y+5}{4x+3y-1}$.

20. $y' = \frac{x+4y-5}{6x-y-5}$.

21. $y' = \frac{x+y+2}{x+1}$.

22. $y' = \frac{2x+y-3}{4x-4}$.

23. $y' = \frac{2x+y-3}{2x-2}$.

24. $y' = \frac{y}{2x+2y-2}$.

25. $y' = \frac{x+5y-6}{7x-y-6}$.

26. $y' = \frac{x+y-4}{x-2}$.

27. $y' = \frac{2x+y-1}{2x-2}$.

28. $y' = \frac{3y-2x+1}{3x+3}$.

29. $y' = \frac{6y-6}{5x+4y-9}$.

30. $y' = \frac{x+6y-7}{8x-y-7}$.

Задание №6. Решите ДУ

1. $x^2 y'' = (y')^2$.
2. $x(y'' + 1) + y' = 0$.
3. $y'' \operatorname{tg} x - y' + \frac{1}{\sin x} = 0$.
4. $x^3 y'' + x^2 y' = 1$.
5. $xy'' = y'(1 + \ln \frac{y'}{x})$.
6. $y'' + y' \operatorname{tg} x = \sin 2x$.
7. $xy'' - y' + \frac{1}{x} = 0$.
8. $y'' x \ln x = y'$.
9. $xy'' - y' = x^2 e^x$.
10. $y'' + y' \operatorname{tg} x = \operatorname{tg} x$.
11. $y'' + 2x \cdot (y')^2 = 0$.
12. $xy'' = y' \cdot \ln \frac{y'}{x}$.
13. $xy'' + y' + x = 0$.
14. $y'' = \frac{y'}{x} + x$.
15. $xy'' + 3y' - x^2 = 0$.
16. $y'' \operatorname{tg} x = y' + 1$.
17. $y'' = y' + x$.
18. $xy'' + y' = \sqrt{x}$.
19. $y'' \operatorname{tg} 5x = 5y'$.
20. $(1 + x^2)y'' + 2xy' = x^3$.
21. $y'' - 2y' \operatorname{ctg} x = \sin^3 x$.
22. $x^2 y'' + xy' = 1$.
23. $xy'' = y'(\ln y' - \ln x)$.
24. $-xy'' + 2y' = 2/x^2$.
25. $(1 - x^2)y'' - xy' = 0$.
26. $(1 + x^2)y'' = 2xy'$.
27. $y'' - 2y' \operatorname{tg} x = \sin x$.
28. $xy'' - y' + \frac{1}{x} = 0$.
29. $y'' x \ln x = y'$.
30. $xy'' - y' = x^2 e^x$.

Задание №7. Найдите решение задачи Коши

1. $4y^3y'' = y^4 - 1, \quad y(0) = \sqrt{2}, \quad y'(0) = 1/(2\sqrt{2}).$
2. $y'' = 128y^3, \quad y(0) = 1, \quad y'(0) = 8.$
3. $y''y^3 + 64 = 0, \quad y(0) = 4, \quad y'(0) = 2.$
4. $y'' + 2\sin y \cos^3 y = 0, \quad y(0) = 0, \quad y'(0) = 1.$
5. $y'' = 32\sin^3 y \cos y, \quad y(1) = \pi/2, \quad y'(1) = 4.$
6. $y'' = 98y^3, \quad y(1) = 1, \quad y'(1) = 7.$
7. $y''y^3 + 49 = 0, \quad y(3) = -7, \quad y'(3) = -1.$
8. $4y^3y'' = 16y^4 - 1, \quad y(0) = \sqrt{2}/2, \quad y'(0) = 1/\sqrt{2}.$
9. $y'' + 8\sin y \cos^3 y = 0, \quad y(0) = 0, \quad y'(0) = 2.$
10. $y'' = 72y^3, \quad y(2) = 1, \quad y'(2) = 6.$
11. $y''y^3 + 36 = 0, \quad y(0) = 3, \quad y'(0) = 2.$
12. $y'' = 18\sin^3 y \cos y, \quad y(1) = \pi/2, \quad y'(1) = 3.$
13. $4y^3y'' = y^4 - 16, \quad y(0) = 2\sqrt{2}, \quad y'(0) = 1/\sqrt{2}.$
14. $y'' = 50y^3, \quad y(3) = 1, \quad y'(3) = 5.$
15. $y''y^3 + 25 = 0, \quad y(2) = -5, \quad y'(2) = -1.$
16. $y'' + 18\sin y \cos^3 y = 0, \quad y(0) = 0, \quad y'(0) = 3.$
17. $y'' = 8\sin^3 y \cos y, \quad y(1) = \pi/2, \quad y'(1) = 2.$
18. $y'' = 32y^3, \quad y(4) = 1, \quad y'(4) = 4.$
19. $y''y^3 + 16 = 0, \quad y(1) = 2, \quad y'(1) = 2.$
20. $y'' + 32\sin y \cos^3 y = 0, \quad y(0) = 0, \quad y'(0) = 4.$
21. $y'' = 50\sin^3 y \cos y, \quad y(1) = \pi/2, \quad y'(1) = 5.$
22. $y'' = 18y^3, \quad y(1) = 1, \quad y'(1) = 3.$

23. $y''y^3 + 9 = 0$, $y(1) = 1$, $y'(1) = 3$.
24. $y^3y'' = 4(y^4 - 1)$, $y(0) = \sqrt{2}$, $y'(0) = \sqrt{2}$.
25. $y'' + 50\sin y \cos^3 y = 0$, $y(0) = 0$, $y'(0) = 5$.
26. $y'' = 8y^3$, $y(0) = 1$, $y'(0) = 2$.
27. $y''y^3 + 4 = 0$, $y(0) = -1$, $y'(0) = -2$.
28. $y'' = 2\sin^3 y \cos y$, $y(1) = \pi/2$, $y'(1) = 1$.
29. $y^3y'' = y^4 - 16$, $y(0) = 2\sqrt{2}$, $y'(0) = \sqrt{2}$.
30. $y'' = 2y^3$, $y(-1) = 1$, $y'(-1) = 1$.