Combinatorics

Combinatorics basically deals with counting of elements in different sets.

Let us consider the following example: in how many ways can we select k=2 elements out of the set: {a,b,c,d,e} of n=5 elements?

The answer to this question depends on our decisions regarding the rules of this selection. There are two decisions to make:

1. R - do we allow for repetitive selection of the same element?
2. O - does the order of selecting the elements matter?

R - no O - no	R - no O - yes	R - yes O - no	R - yes O - yes
Combinations without repetitions	**Variations without repetitions**	**Combinations with repetitions**	**Variations with repetitions**
ab, ac, ad, ae bc, bd, be cd, ce de	ab, ac, ad, ae, Ba, bc, bd, be ca, cb, cd, ce da, db, dc,de Ea, eb, ec, ed	aa, ab, ac, ad, ae bb, bc, bd, be cc, cd, ce dd, de ee	aa, ab, ac, ad, ae ba, bb, bc, bd, be ca, cb, cc, cd, ce da, db, dc, dd, de ea, eb, ec, ed, ee
10	20	15	25
$\binom{n}{k} = \frac{n!}{k!(n-k)!}$	$\frac{n!}{(n-k)!}$	$\binom{n+k-1}{k}$	n^k
How many 3-student groups can be selected from a 30 student group?	How many 3-letter words can be made out of the 26 letters of the English alphabet?	Number of possible lottery numbers draws, when the balls are returned to the machine after being drawn.	k people are registering for one of n courses. How many possible outcomes there are?

Note the special case, when we want to count all the variations without repetitions, when k=n, i.e. we are selecting all the elements in the set. In this case we have:

$$\frac{n!}{(n-k)!} = \frac{n!}{(n-n)!} = \frac{n!}{0!} = n!$$

In this case we are counting all possible orderings of our set, so called **permutations.** Example: in how many ways can we arrange 7 people standing in a row? Answer: 7! = 720.

Let us also consider the following situation - we want to know the number of possible outcomes of shuffling a deck of 52 cards. This is counted as 52! = 8.0658175e+67. And what if we wanted to count all possible deals in bridge? In this case the order in which each player got his cards does not matter. For this case we have a special scheme of counting called **permutations with repetitions.** In it, the set of n-elements is divided into subsets of n_1, n_2,....,n_m elements. Within the subsets the order of the elements does not matter. The formula for permutations with repetitions is:

$$\frac{n!}{n_1!n_2!n_3!n_4!...n_m!}$$

In the case of bridge dealings:

$$\frac{52!}{13!13!13!13!}$$