

Prove the following equivalences by showing that the given sentences are tautologies.

A) $(p \rightarrow q) \Leftrightarrow (\neg p \vee q)$

| p | q | $p \rightarrow q$ | \Leftrightarrow | $\neg p \vee q$ | $\neg p$ |
|---|---|-------------------|-------------------|-----------------|----------|
| 0 | 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 0 |

B) $\neg(p \rightarrow q) \Leftrightarrow (p \wedge \neg q)$

| p | q | $\neg(p \rightarrow q)$ | \Leftrightarrow | $p \wedge \neg q$ | $\neg q$ |
|---|---|-------------------------|-------------------|-------------------|----------|
| 0 | 0 | 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 | 0 |

C) $\neg(p \wedge q) \Leftrightarrow (\neg p \vee \neg q)$

| p | q | $p \wedge q$ | $\neg(p \wedge q)$ | \Leftrightarrow | $(\neg p \vee \neg q)$ | $\neg p$ | $\neg q$ |
|---|---|--------------|--------------------|-------------------|------------------------|----------|----------|
| 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 | 1 | 1 | 0 | 1 |
| 0 | 1 | 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0 |

D) $\neg(p \vee q) \Leftrightarrow (\neg p \wedge \neg q)$ is not tautology

| p | q | $p \vee q$ | $\neg(p \vee q)$ | \Leftrightarrow | $(\neg p \vee \neg q)$ | $\neg p$ | $\neg q$ |
|---|---|------------|------------------|-------------------|------------------------|----------|----------|
| 0 | 0 | 0 | 1 | 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0 |