1. Relation between P and Convolution

Previously we have established a relation between polynomial P and Binomial, Multinomial theorems. In this section a relation between P and convolution of the piecewise defined power function f_t^r is established. To show that P implicitly involves the discrete convolution of piecewise defined power function f_t^r let's refresh what P are

$$P = \sum \mathbf{A}\mathbf{Q}.$$

Meanwhile, the term Q is the power sum of the form

$$\mathbf{Q} = \sum k^r (n-k)^r$$

It could be noticed immediately that Q differs from the discrete convolution of f_t^r only in sense of boundary conditions of the summation. For instance, the discrete convolution of the piecewise defined power function f_t^r is

$$\begin{aligned}(f_t^r * f_t^r)[n] &= \sum_k f_{r,t}(k) f_{r,t}(n-k) = \sum_k k^r (n-k)^r [k \geq t][n-k \geq t] \\ &= \sum_k k^r (n-k)^r [t \leq k \leq n-t].\end{aligned}$$

It is now clear that discrete convolution $(f_t^k * f_t^k)[n]$ of piecewise defined power function f_t^r is a partial case of the power sum Q with $a =$ and $b =$, ie

$$(f_t^r * f_t^r)[n] = \mathbf{Q}_{t,n-t+1}^r(n), \quad n \geq 1.$$

Therefore, the polynomials $\mathbf{P}_{a,b}^m(n)$ are in relation with discrete convolution of piecewise defined power function f_t^r as follows

$$\mathbf{P}_{t,n-t+1}^m(n) = \sum_r \mathbf{A}_{m,r} \mathbf{Q}_{t,n-t+1}^r(n) \equiv \sum_r \mathbf{A}_{m,r} (f_t^r * f_t^r)[n], \quad n \geq 1.$$

Following this logic, we are able to find a relation between P and discrete convolution of f_t^r.

1.1. **Relation between Binomial theorem and Convolution.** As it is stated previously in (exp link), the polynomials P are able to be expressed in terms of convolution $(f_{r,t} * f_{r,t})[n]$ of $f_{r,t}(n)$. Consequently, by the equivalence between BT and P which is (4), the Binomial expansion could be expressed in terms of convolution $n_{\geq t}^r * n_{\geq t}^r$ as well,

$$\begin{aligned}(a+b)^{2m+1} = \sum_r \binom{2m+1}{r} a^{2m+1-r} b^r &\equiv -1 + \mathbf{P}_{a+b+1}^m(a+b) \\ &= -1 + \sum_r \mathbf{A}_{m,r} \mathbf{Q}_{a+b+1}^r(a+b) \\ &= -1 + \sum_r \mathbf{A}_{m,r} (f^r * f^r)[a+b].\end{aligned}$$