Dirichlet's drawer principle

Based on: *Matematyka Dyskretna* by Joanna Pomianowska

Imagine we have n=5 drawers and m=6 balls. If we put all the balls in the drawers, then at least one of the drawers will contain at least 2 balls. This is called the Dirichlet's drawer principle: if you put m objects in n containers, where m>n, than at least one container will have more than one object.

Example 1
If a group of students has 13 people, then at least two of them will have birthdays in the same month.

Example 2
If WSB has 1900 students, then at least 5 of them share the birthday.

Example 3
Let's consider a set A = 1,2,3...,100 . Let's select at random a 12-element subset of A and call it B. Then we divide B into two different 6-element subsets. Among these subsets there is a pair B_1 and B_2 such that the sum of the elements in B_1 equals the sum of the elements in B_2.

Proof:
The number of different 6-element subsets of a 12 element set is $\binom{12}{6}$ = 924.
The lowest possible sum of elements of a 6-element subset of B is 1+2+3+4+5+6=21.
The highest possible sum of elements of a 6-element subset of B is 95+96+97+98+99+100 = 585. All possible sum of elements in the subsets are between 21 and 585, so there are at most 585-21+1 =565 possible sums.Since the number of all subsets is larger than the number of all possible sums, there must be at least 2 subsets with the same sum of elements.

Example 4

20 students of computer science have to write chosen 3 out of 10 programs. How many students on average will write the same program?

In total, 60 programs will be written. Since there are 10 different programs, each of them
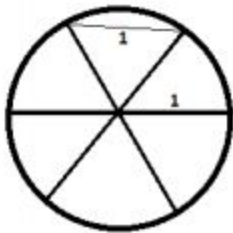
will be written 6 times on average.

Example 5

In every 5 consecutive natural numbers there are two numbers that have the same remainder from division by 4.

Example 6

On a circle with a radius of 1, 7 points where chosen. Prove that there must be a pair of points within a distance of 1.

Let's divide the circle into 6 sectors:

The distance between two points in a sector does not exceed 1. We have 6 sectors and 7 points.