1. Homework 5

- Combination of r elements out of total n elements, order doesn't matter, repetition not allowed

$$\binom{n+r-1}{r} \tag{1.1}$$

- Combination C of r elements out of total n elements, order doesn't matter, repetition allowed

$$\binom{n}{r} \tag{1.2}$$

- Permutation $\mathbf{P}$ or r elements out of n elements total, order matters, repetition allowed

$$n^r \tag{1.3}$$

- Permutation P or r elements out of n elements total, order matters, repetition not allowed

$$\frac{n!}{(n-r)!} = r!\binom{n}{r} \tag{1.4}$$

- Permutation with repetitions. The number of permutations of n objects with n_1 identical objects of type 1, n_2 identical objects of type 2,... ,and n_k identical objects of type k is

$$\frac{n!}{n_1!n_2!\cdots n_k!} \tag{1.5}$$

2. Exercises

Question 2.1. *In how many ways P can you arrange 7 people in a row?*

$$P = \binom{7}{1} = 7$$

Question 2.2. *How many words P (not necessarily meaningful) can you make from all the letters in the word COMBINATORICS?*

By the pattern (1.5), since we have doubled C, O, I, the P is

$$P = \frac{13!}{2! \cdot 2! \cdot 2!}$$

Question 2.3. *In a qualification round of a sports event there are 20 competitors. The first three gain qualification to the next round. How many possible outcomes P are there?*

$$P = \binom{20}{3}$$

Question 2.4. *How many ways P are there to fill a Duzy Lotek lottery ticket?*

Lottery ticket consists of 6 numbers over pull up to 49, therefore

$$P = \binom{49}{6} = 13,983,816$$

Question 2.5. *How many 4 digit numbers P have all different digits?*

Total digits from 0 to 9 is $n = 10$. We need to find 4 digit numbers, therefore $k = 4$. Now we use a pattern of (1.4), hence

$$P = 4!\binom{10}{4} = 24 \cdot 210 = 5040$$

Question 2.6. *In a sports event there are 30 competitors and the best three are awarded the gold, silver and bronze medals. How many possible outcomes P are there?*

Again, we use pattern (1.4) since order matters and repetitions are not allowed (since we can't clone teams by some moral laws), hence $n = 30$, $k = 3$ and P is

$$P = 3!\binom{30}{3} = 24360$$

Question 2.7. *We throw a coin 10 times and we number individual throws (e.g. throw 1, throw 2,...). How many different outcomes P are there?*

The pull n is $n = 10$. Can't explain why, but I think

$$P = \binom{10}{0} \cdot \binom{10}{1} \cdots \binom{10}{10}$$

Question 2.8. *In how many ways P can you write all the digits in a row such that 4 and 5 are neighbouring?*

All the digits are 0,1,...,9 so pull is $n = 10$. Assume, we just glued 4 and 5 so the pull becomes $n = 9$, thus the number of permutations P is

$$P = 9! = 362880$$

Question 2.9. *In a group of 15 girls and 10 boys we choose a 5 children delegation, in which there must be exactly 3 girls. In how many ways P can this be done?*

$$P = \binom{15}{3} \cdot \binom{10}{2}$$

Question 2.10. *On the shelf there are 2 novels, 4 textbooks and 3 albums. In how many ways P can we arrange them in such way that the books of each kind are next to each other?*

Pull is $n = 2 + 4 + 3 = 9$

$$P = \binom{9}{3}$$

Question 2.11. *In how many ways P can you put 5 letters into 8 mailboxes, if you can put only one letter in each mailbox?*

Assume that we have 8 letters

$$P = \frac{8!}{3!} = 6720$$

Question 2.12. *8 people want to register for 4 courses. How many possibilities P are there?*

$$P = \binom{8}{0} \cdot \binom{8}{1} \cdots \binom{8}{4}$$

Question 2.13. *In a hat there are 4 blue balls, 3 green balls and 2 white balls. In how many ways can we draw 3 balls (without putting them back into the hat, order of the draws matters) such that at least one ball is white? In how many ways can we draw 3 balls (without putting them back into the hat, order of the draws matters) such that every ball is in a different color?*

Denote

Question 2.14. *How many different registration plates P can be issued, if first two characters are letters and next three are digits.*

$$P = \binom{10}{2} \cdot \binom{29}{3}$$

Question 2.15. *In a tennis tournament there are 32 competitors. In how many ways P can we arrange them in first round pairs?*

Combination without repetition pattern $n = 32$, $k = 2$

$$P = 2!\binom{32}{2}$$

Or (I'm not sure)

$$P = \binom{32 + 2 - 1}{2}$$

Question 2.16. *In how many ways P can we deal 52 cards to 4 players?*

Permutation without repetition, $n = 52$, $k = 4$

$$P = 4!\binom{52}{4}$$

Question 2.17. *In how many ways P can we split a 17 people group into two 3-person groups, one 5-person group and three 2-person groups?*

$$P = \binom{17}{3}\frac{1}{2! \cdot 3!}$$