

UNIT TEST PACKAGE DOCUMENTATION

ABSTRACT. This .pdf file represents a documentation for Unit Tests package in Wolfram Mathematica, which related to the math. research <https://arxiv.org/abs/1603.02468>.

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INSTALLATION AND EXECUTION

Prior the unit test execution, one has to download the packages `main_definitions.m` and `unit_tests_package.m` from https://github.com/kolosovpetro/research_unit_tests. Package `unit_tests_package.m` is dependent on `main_definitions.m`. Open these packages in Mathematica and instal by clicking File -> Install..., click Source and choose corresponding file in dropped menu. Perform installation for two packages separately. Then recall the packages `main_definitions.m` and `unit_tests_package.m` in Mathematica notebook using the commands

```
Needs["MainDefinitions"]  
Needs["UnitTests"]
```

Read also <http://support.wolfram.com/kb/5648>.

Each unit test is inside the package `unit_tests_package.m`, each unit test to be recalled by a single command without any inputs, for example, unit test 1 to be recalled by the command `UnitTest1` to the Mathematica console. Any unit test conditions (iteration limits, etc) to be changed directly inside the package `unit_tests_package.m`.

UNIT TEST 1

Command `UnitTest1` to Mathematica console verifies an identity

$$\mathbf{P}_{a,b}^m(n) = \mathbf{P}_b^m(n) - \mathbf{P}_a^m(n)$$

UNIT TEST 2

Command `UnitTest2` to Mathematica console verifies an identity

$$\mathbf{P}_{a,b}^m(n) = \sum_k \mathbf{X}_k^m(a, b) (-1)^{m-k} n^k$$

UNIT TEST 3

Command `UnitTest3` to Mathematica console verifies an identity

$$\mathbf{X}_k^m(a, b) = \mathbf{X}_k^m(b) - \mathbf{X}_k^m(a)$$

UNIT TEST 4

Command `UnitTest4` to Mathematica console verifies an identity

$$\mathbf{X}_t^m(a, b) = (-1)^m \sum_{k=1}^{2m-t+1} \mathbf{H}_{m,t}(k) (b^k - a^k)$$

UNIT TEST 5

Command `UnitTest5` to Mathematica console verifies an identity

$$\mathbf{P}_{a,b}^m(n) = \sum_k (-1)^{2m-k} \sum_{\ell=1}^{2m-k+1} \mathbf{H}_{m,k}(\ell) (b^\ell - a^\ell) n^k$$

UNIT TEST 6

Command `UnitTest6` to Mathematica console verifies an identity

$$\mathbf{Q}_{a,b}^r(n) = \mathbf{Q}_b^r(n) - \mathbf{Q}_a^r(n)$$

UNIT TEST 7

Command `UnitTest7` to Mathematica console verifies an identity

$$(f_t^r * f_t^r)[n] = \mathbf{Q}_{t,n-t+1}^r(n), \quad n \geq 1.$$

UNIT TEST 8

Command `UnitTest8` to Mathematica console verifies an identity

$$n^{2m+1} + 1 = \sum_{r \geq 0} \mathbf{A}_{m,r}(f_0^r * f_0^r)[n], \quad n > 0, \quad n \in \mathbb{N}$$

UNIT TEST 9

Command `UnitTest9` to Mathematica console verifies an identity

$$n^{2m+1} - 1 = \sum_{r \geq 0} \mathbf{A}_{m,r}(f_1^r * f_1^r)[n], \quad n > 0, \quad n \in \mathbb{N}$$

UNIT TEST 10

Command `UnitTest10` to Mathematica console verifies an identity

$$\mathbf{X}_t^m(a, b) = (-1)^m \sum_{j=t}^m \mathbf{A}_{m,j}(-1)^j \binom{j}{t} (S_{2j-t}(b) - S_{2j-t}(a))$$

UNIT TEST 11

Command `UnitTest11` to Mathematica console verifies an identity

$$\mathbf{P}_{a,b}^m(n) = \sum_k \sum_{j \geq k} (-1)^{2m+j-k} \mathbf{A}_{m,j} \binom{j}{k} (S_{2j-k}(b) - S_{2j-k}(a)) n^k$$

UNIT TEST 12

Command `UnitTest12` to Mathematica console verifies an identity

$$\mathbf{P}_{a+b}^m(a+b) \equiv \sum_k \binom{2m+1}{k} a^{2m+1-k} b^k$$

UNIT TEST 13

Command `UnitTest13` to Mathematica console verifies an identity

$$n^{2m+1} = \mathbf{P}_n^m(n)$$

UNIT TEST 14

Command `UnitTest14` to Mathematica console verifies an identity

$$n^s = n^{[s \text{ is even}]} \mathbf{P}_n^{\lfloor \frac{s-1}{2} \rfloor}(n)$$

UNIT TEST 15

Command `UnitTest15` to Mathematica console verifies an identity

$$\mathbf{P}_{t,n-t+1}^m(n) = \sum_r \mathbf{A}_{m,r} \mathbf{Q}_{t,n-t+1}^r(n) \equiv \sum_r \mathbf{A}_{m,r}(f_t^r * f_t^r)[n], \quad n \geq 1.$$

UNIT TEST 16

Command `UnitTest16` to Mathematica console verifies an identity

$$\mathbf{P}_n^m(n) - \mathbf{P}_{1,n+1}^m(n) = 1$$

UNIT TEST 17

Command `UnitTest17` to Mathematica console verifies an identity

$$(a+b)^s = \sum_k \binom{s}{k} a^{s-k} b^k \equiv (a+b)^{[s \text{ is even}]} \left(-1 + \mathbf{P}_{a+b+1}^{\lfloor \frac{s-1}{2} \rfloor} (a+b) \right)$$

UNIT TEST 18

Command `UnitTest18` to Mathematica console verifies an identity

$$(a+b)^s = \sum_k \binom{s}{k} a^{s-k} b^k \equiv (a+b)^{[s \text{ is even}]} \left(1 + \mathbf{P}_{1,a+b}^{\lfloor \frac{s-1}{2} \rfloor} (a+b) \right)$$

UNIT TEST 19

Command `UnitTest19[a+b]` to Mathematica console verifies an identity

$$\begin{aligned} (a+b)^s &= \sum_k \binom{s}{k} a^{s-k} b^k \\ &= (a+b)^{[s \text{ is even}]} \left(-1 + \sum_k \mathbf{A}_{\lfloor \frac{s-1}{2} \rfloor, k} (f_0^r * f_0^r)[a+b] \right), \quad s \geq 1, \quad a+b \geq 1. \end{aligned}$$

UNIT TEST 20

Command `UnitTest20[a+b]` to Mathematica console verifies an identity

$$\begin{aligned} (a+b)^s &= \sum_k \binom{s}{k} a^{s-k} b^k \\ &= (a+b)^{[s \text{ is even}]} \left(1 + \sum_k \mathbf{A}_{\lfloor \frac{s-1}{2} \rfloor, k} (f_1^r * f_1^r)[a+b] \right), \quad s \geq 1, \quad a+b \geq 2. \end{aligned}$$

UNIT TEST 21

Command `UnitTest21` to Mathematica console verifies an identity

$$\mathbf{P}_{n+1}^m(n) = \sum_{r=0}^m \mathbf{A}_{m,r} \mathbf{C}_{n+1}^r(n) \equiv \sum_{r=0}^m \mathbf{A}_{m,r} \langle n \rangle^r * \langle n \rangle^r = n^{2m+1} + 1$$