MAIN_DEFINITIONS.M PACKAGE DOCUMENTATION

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1. Introduction

This file represents a documentation for `main_definitions.m` Mathematica package. To get started proceed to GitHub repository https://github.com/KolosovPetro/arXiv1603.02468-Mathematica-Implementations, pull it, and find the package `main_definitions.m`. This package doesn't have any dependencies on other Mathematica packages. To get started simply install it to your Mathematica by clicking `File -> Install...`, click `Source` and choose corresponding file in dropped menu. Then recall the package `main_definitions.m` in Mathematica notebook using the command

```
Needs["MainDefinitions`"]
```

Example of a Mathematica notebook where `main_definitions.m` recalled is available in github repository as well. Read also http://support.wolfram.com/kb/5648.

2. Functions in package `main_definitions.m`

We now set the following notation, which remains fixed for the remainder of this paper:

- `A[m,r]` is a real coefficient defined recursively

$$\texttt{A[m,r]} := \begin{cases} (2r+1)\binom{2r}{r} & \text{if } r = m \\ (2r+1)\binom{2r}{r} \sum_{d=2r+1}^{m} \texttt{A[m,d]} \binom{d}{2r+1} \frac{(-1)^{d-1}}{d-r} B_{2d-2r} & \text{if } 0 \le r < m \\ 0 & \text{if } r < 0 \text{ or } r > m \end{cases}$$

  where B_t are Bernoulli numbers.

- `L[m,n,k]` is polynomial of degree $2m$ in n, k

$$\texttt{L[m,n,k]} := \sum_{r=0}^{\texttt{m}} \texttt{A[m,r]}\texttt{k}^{r}\texttt{(n-k)}^{r}$$

- `P[m,n,b]` is polynomial of degree $2m+1$ in b, n

$$\texttt{P[m,n,b]} := \sum_{\texttt{k}=0}^{\texttt{b}-1} \texttt{L[m,n,k]}$$

- `ConvolveSum[n, r, b]` is a convolutional power sum

$$\texttt{ConvolveSum[n, r, b]} := \sum_{k=0}^{b-1} k^r (n-k)^r$$

- `H[m,t,b]` is a real coefficient defined as

$$\texttt{H[m,t,b]} := \sum_{j=\texttt{t}}^{\texttt{m}} \binom{j}{\texttt{t}} \texttt{A[m,j]} \frac{(-1)^j}{2j-\texttt{t}+1} \binom{2j-\texttt{t}+1}{\texttt{b}} B_{2j-\texttt{t}+1-\texttt{b}}$$

- `X[m,t,j]` is polynomial of degree $2m-t$ in b

$$\texttt{X[m,t,j]} := (-1)^m \sum_{k=1}^{2m-t+1} \texttt{H[m,t,k]} \cdot \texttt{j}^k$$

- `S[p, n]` is a common power sum

$$\texttt{S[p, n]} := \sum_{k=0}^{n-1} k^p$$

- `MacaulayPow[x,n,a]` is powered Macaulay bracket

$$\texttt{MacaulayPow[x, n, a]} := \begin{cases} (x-a)^n, & x \geq a \\ 0, & \text{otherwise} \end{cases} \qquad a \in \mathbb{Z}$$

- `MacaulayPowStrict[x,n,a]` is powered Macaulay bracket

$$\texttt{MacaulayPowStrict[x,n,a]} := \begin{cases} (x-a)^n, & x > a \\ 0, & \text{otherwise} \end{cases} \qquad a \in \mathbb{Z}$$

3. Examples of tests

For example, we can verify the identity $n^{2m+1} = \texttt{P[m,n,n]}$ as follows

$$\texttt{P[m,n,n]} = n^{2m+1}$$

All results of the manuscript `https://arxiv.org/abs/1603.02468` can be verified similarly.