UNIT TEST PACKAGE DOCUMENTATION

Abstract. This `.pdf` file represents a documentation for Unit Tests package in Wolfram Mathematica, which related to the math. research https://arxiv.org/abs/1603.02468.

Contents

Installation and execution

Prior the unit test execution, one has to download the packages `main_definitions.m` and `unit_tests_package.m` from https://github.com/kolosovpetro/research_unit_tests. Package `unit_tests_package.m` is dependent on `main_definitions.m`. Open these packages in Mathematica and instal by clicking `File -> Install...`, click `Source` and choose corresponding file in dropped menu. Perform installation for two packages separately. Then recall the packages `main_definitions.m` and `unit_tests_package.m` in Mathematica notebook using the commands

```
Needs["MainDefinitions`"]
Needs["UnitTests`"]
```

Read also http://support.wolfram.com/kb/5648.

Each unit test is inside the package `unit_tests_package.m`, each unit test to be recalled by a single command without any inputs, for example, unit test 1 to be recalled by the

command `UnitTest1` to the Mathematica console. Any unit test conditions (iteration limits, etc) to be changed directly inside the package `unit_tests_package.m`.

Unit test 1

Command `UnitTest1` to Mathematica console verifies an identity

$$\mathbf{P}_{a,b}^{m}(n) = \mathbf{P}_{b}^{m}(n) - \mathbf{P}_{a}^{m}(n)$$

Unit test 2

Command `UnitTest2` to Mathematica console verifies an identity

$$\mathbf{P}_{a,b}^{m}(n) = \sum_{k} \mathbf{X}_{k}^{m}(a,b)(-1)^{m-k} n^{k}$$

Unit test 3

Command `UnitTest3` to Mathematica console verifies an identity

$$\mathbf{X}_{k}^{m}(a,b) = \mathbf{X}_{k}^{m}(b) - \mathbf{X}_{k}^{m}(a)$$

Unit test 4

Command `UnitTest4` to Mathematica console verifies an identity

$$\mathbf{X}_{t}^{m}(a,b) = (-1)^{m} \sum_{k=1}^{2m-t+1} \mathbf{H}_{m,t}(k)(b^{k} - a^{k})$$

Unit test 5

Command `UnitTest5` to Mathematica console verifies an identity

$$\mathbf{P}_{a,b}^{m}(n) = \sum_{k} (-1)^{2m-k} \sum_{\ell=1}^{2m-k+1} \mathbf{H}_{m,k}(\ell)(b^{\ell} - a^{\ell}) n^{k}$$

Unit test 6

Command `UnitTest6` to Mathematica console verifies an identity

$$\mathbf{Q}_{a,b}^{r}(n) = \mathbf{Q}_{b}^{r}(n) - \mathbf{Q}_{a}^{r}(n)$$

Unit test 7

Command `UnitTest7` to Mathematica console verifies an identity

$$(f_{t}^{r} * f_{t}^{r})[n] = \mathbf{Q}_{t,n-t+1}^{r}(n), \quad n \geq 1.$$

Unit test 8

Command `UnitTest8` to Mathematica console verifies an identity

$$n^{2m+1} + 1 = \sum_{r \geq 0} \mathbf{A}_{m,r}(f_{0}^{r} * f_{0}^{r})[n], \quad n > 0, \quad n \in \mathbb{N}$$

Unit test 9

Command `UnitTest9` to Mathematica console verifies an identity

$$n^{2m+1} - 1 = \sum_{r\geq 0} \mathbf{A}_{m,r}(f_1^r * f_1^r)[n], \quad n > 0, \quad n \in \mathbb{N}$$

Unit test 10

Command `UnitTest10` to Mathematica console verifies an identity

$$\mathbf{X}_t^m(a,b) = (-1)^m \sum_{j=t}^{m} \mathbf{A}_{m,j}(-1)^j \binom{j}{t} (S_{2j-t}(b) - S_{2j-t}(a))$$

Unit test 11

Command `UnitTest11` to Mathematica console verifies an identity

$$\mathbf{P}_{a,b}^m(n) = \sum_k \sum_{j\geq k} (-1)^{2m+j-k} \mathbf{A}_{m,j} \binom{j}{k} (S_{2j-k}(b) - S_{2j-k}(a)) n^k$$

Unit test 12

Command `UnitTest12` to Mathematica console verifies an identity

$$\mathbf{P}_{a+b}^m(a+b) \equiv \sum_k \binom{2m+1}{k} a^{2m+1-k} b^k$$

Unit test 13

Command `UnitTest13` to Mathematica console verifies an identity

$$n^{2m+1} = \mathbf{P}_n^m(n)$$

Unit test 14

Command `UnitTest14` to Mathematica console verifies an identity

$$n^s = n^{[s \text{ is even}]} \mathbf{P}_n^{\lfloor \frac{s-1}{2} \rfloor}(n)$$

Unit test 15

Command `UnitTest15` to Mathematica console verifies an identity

$$\mathbf{P}_{t,n-t+1}^m(n) = \sum_r \mathbf{A}_{m,r} \mathbf{Q}_{t,n-t+1}^r(n) \equiv \sum_r \mathbf{A}_{m,r}(f_t^r * f_t^r)[n], \quad n \geq 1.$$

Unit test 16

Command `UnitTest16` to Mathematica console verifies an identity

$$\mathbf{P}_n^m(n) - \mathbf{P}_{1,n+1}^m(n) = 1$$

Unit test 17

Command `UnitTest17` to Mathematica console verifies an identity

$$
\begin{aligned}
(a+b)^s &= \sum_k \binom{s}{k} a^{s-k} b^k \\
&\equiv (a+b)^{[s \text{ is even}]} \left(-1 + \mathbf{P}_{a+b+1}^{\lfloor \frac{s-1}{2} \rfloor} (a+b) \right) \\
&= (a+b)^{[s \text{ is even}]} \left(-1 + \sum_k \mathbf{A}_{\lfloor \frac{s-1}{2} \rfloor, k} \mathbf{Q}_{a+b+1}^k (a+b) \right) \\
&= (a+b)^{[s \text{ is even}]} \left(-1 + \sum_k \mathbf{A}_{\lfloor \frac{s-1}{2} \rfloor, k} (f_0^r * f_0^r)[a+b] \right), \quad s \geq 1, \quad a+b \geq 1.
\end{aligned}
$$

Unit test 18

Command `UnitTest18` to Mathematica console verifies an identity

$$
\begin{aligned}
(a+b)^s = \sum_k \binom{s}{k} a^{s-k} b^k &\equiv (a+b)^{[s \text{ is even}]} \left(1 + \mathbf{P}_{1,a+b}^{\lfloor \frac{s-1}{2} \rfloor} (a+b) \right) \\
&= (a+b)^{[s \text{ is even}]} \left(1 + \sum_k \mathbf{A}_{\lfloor \frac{s-1}{2} \rfloor, k} \mathbf{Q}_{1,a+b}^k (a+b) \right) \\
&= (a+b)^{[s \text{ is even}]} \left(1 + \sum_k \mathbf{A}_{\lfloor \frac{s-1}{2} \rfloor, k} (f_1^r * f_1^r)[a+b] \right), \quad s \geq 1, \quad a+b \geq 1.
\end{aligned}
$$