MATHEMATICS

SECTION A

February 5, 2024

1 Matrix

1. If A is a square matrix satsifying $A'A = I$, write the value of $|A|$.

2. If $\mathbf{A} = \begin{pmatrix} 4 & 2 \\ -1 & 1 \end{pmatrix}$, show that$(\mathbf{A} - 2\mathbf{I})\,(\mathbf{A} - 3\mathbf{I}) = 0$.

3. Show that for the matrix $\mathbf{A} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{pmatrix}$, $A^3 - 6A^2 + 5A + 11I = 0$. Hence, find $\mathbf{A}^{-1}$

4. Using matrix method, solve the following system of equations :

$$3x + 2y + 3z = 8$$
$$2x + y - z = 1$$
$$4x - 3y + 2z = 4$$

2 Differentiation

5. Find the differential equation representing the family of curves $y = ae^{2x} + 5$, where is an arbitrary constant.

6. If $y = \cos(\sqrt{3})$, then find $\dfrac{dy}{dx}$.

7. Find the differential equation of the family of the curves $y = Ae^{2x} + Be^{-2x}$, where A and B are arbitrary constants.

8. If $x = ae^t\,(\sin t + \cos t)$ and $y = ae^t\,(\sin t - \cos t)$, then prove that $\dfrac{dy}{dx} = \dfrac{x + y}{x - y}$.

9. Differentiate $x^{\sin x} + (\sin x)^{\cos x}$ with respect to x.

3 Integration

10. Find :

$$\int \frac{x - 5}{(x - 3)^3}e^x dx$$

11. Find :

$$\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x\cos^2 x}dx$$

12. Find :

$$\int \frac{x-3}{(x-1)^3} e^x dx$$

13. Find :

$$\int \frac{2\cos x}{(1-\sin x)\left(2-\cos^2 x\right)} dx$$

14. Prove that

$$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$$

   and hence evalute

$$\int_0^{\frac{\pi}{2}} \frac{x}{\sin x + \cos x} dx$$

4   Vectors

15. Find the direction cosines of a line which makes equal angles with the coordinate axes.

16. A line passes through the point with position vector $2\hat{i} - \hat{j} + 4\hat{k}$ and is in the direction of the vector $\hat{i} + \hat{j} - 2\hat{k}$. Find the equation of the line in cartesian form.

17. Show that the points $A(-2\hat{i} + 3\hat{j} + 5\hat{k})$, $B(\hat{i} + 2\hat{j} + 3\hat{k})$ and $C(7\hat{i} - \hat{k})$ are collinear.

18. Find $\left|\vec{a} \times \vec{b}\right|$, if $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} + 5\hat{j} - 2\hat{k}$.

19. The scalar product of the vector $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ with a unit vector along to sum of the vectors $\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\vec{c} = \lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to 1. Find the value of λ and hence find the unit vector along $\vec{b} + \vec{c}$.

20. Using method of integration, find the area of the triangle whose vertices are $(1,0),(2,2),(3,1)$.

21. Find the vector and cartesian equations of the plane passing through the points having position vectors $\hat{i} + \hat{j} - 2\hat{k}$, $2\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} + \hat{k}$. Write the equation of a plane passing through a point $(2,3,7)$ and parallel to the plane obtained above. Hence, find the distance between the two parallel planes.

5   Probability

22. Find the probability distribution of X, the number of heads in a simultaneous toss of two coins.

23. If $P(notA) = 0.7$ and $P(B) = 0.7$ and $P(B \mid A) = 0.5$, then find $P(A \mid B)$.

24. A coin is tossed 5 times. What is the probability of getting

    (a) 3 heads.

    (b) at most 3 heads.

25. A bag contains 5 red and 4 black balls, a second bag contains 3 red and 6 black balls. One of the two bags is selected at random and two balls are drawn at random (without replacement) both of which are found to be red. Find the probability that the balls are drawn from the second bag.

6   Functions

26. Examine whether the opertion $*$ defined on $\mathbb{R}$, the set of all real numbers, by $a * b = \sqrt{a^2 + b^2}$ is a binary operation or not, and if it is a binary operation, find whether it is associative or not.

27. Check whether the relation R defined on the set $A = \{1, 2, 3, 4, 5, 6\}$ as $R = \{(a, b) : b = a + 1\}$ is reflexive, symmetric or transitive.

7   Intersection of Conics

28. Find the equation of the normal to the curve $x^2 = 4y$ which passes through the point $(1, 4)$.

8   Algebra

29. Solve for x:

$$\tan^{-1}(x + 1) + \tan^{-1}(x - 1) = \tan^{-1}\left(\frac{8}{31}\right)$$