Практика 2

① Найти образ мн-ва при отображении $\underset{(x,y)}{\mathbb{R}^2} \to \underset{(u,v)}{\mathbb{R}^2}$

1) $x^2+y^2=1$

а) $\begin{cases} u = 2x \\ v = 3y \end{cases}$

б) $\begin{cases} u = ax + a_0 \\ v = by + b_0 \end{cases}$

1.а) $\begin{cases} x = \frac{u}{2} \\ y = \frac{v}{3} \end{cases}$ $\left(\frac{u}{2}\right)^2 + \left(\frac{v}{3}\right)^2 = 1$

$$\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2x \\ 3y \end{pmatrix}$$

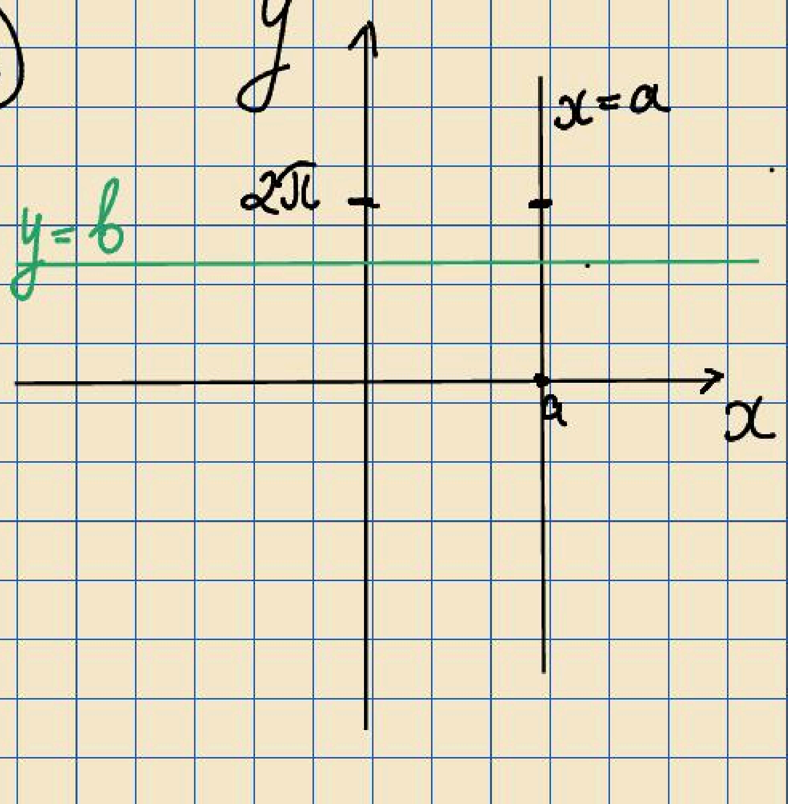
1.б) центр $(0,0) \to (a_0, b_0)$

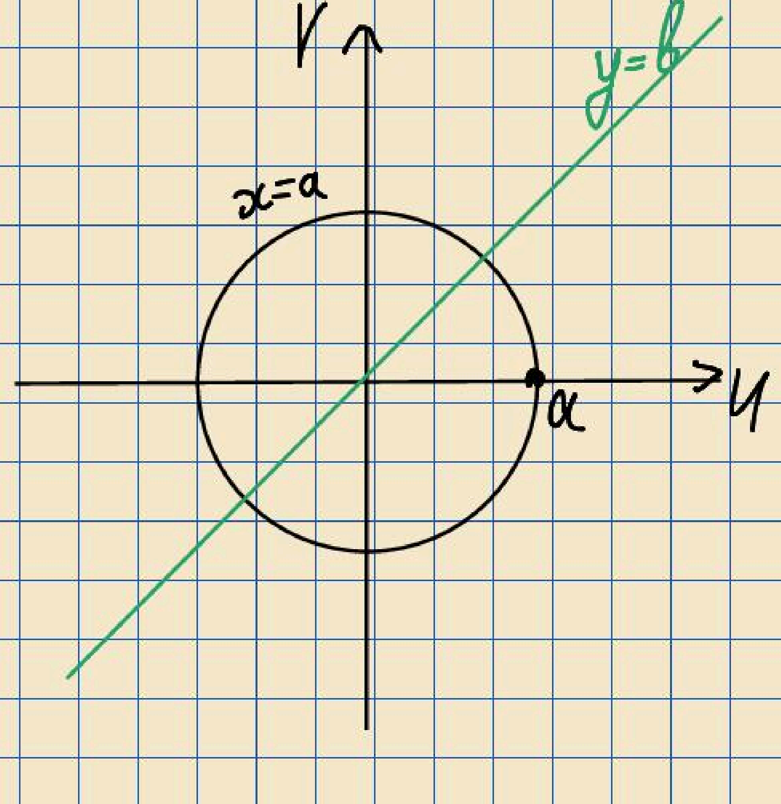
2) $x = a$
$y = b$

а) $\begin{cases} u = x\cos y \\ v = x\sin y \end{cases}$

б) $\begin{cases} u = x^2 - y^2 \\ v = 2xy \end{cases}$

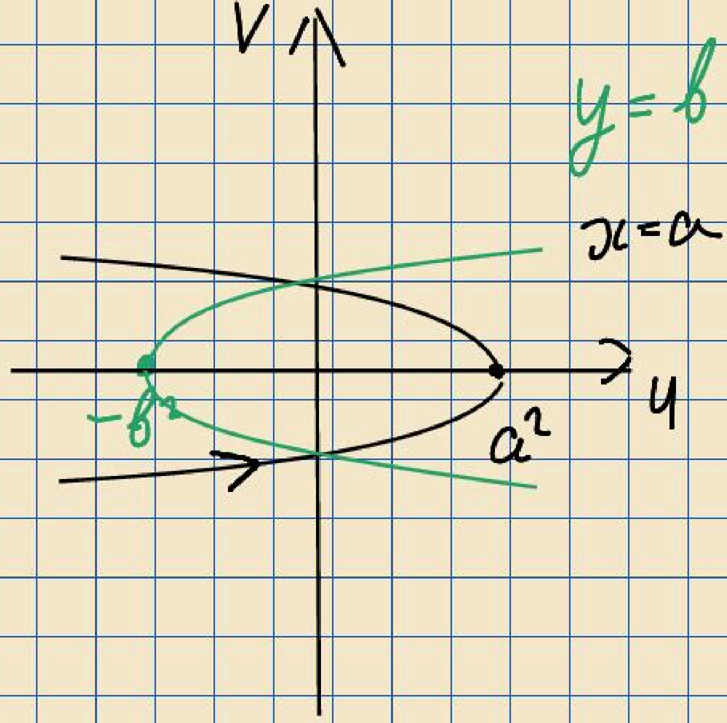
2.а)





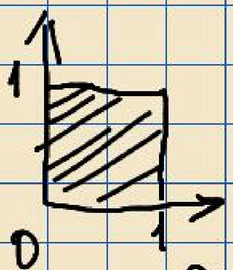
2.б) $\begin{cases} u = x^2 - y^2 \\ v = 2xy \end{cases}$ $\begin{cases} u = a^2 - y^2 \\ v = 2ay \end{cases}$

$u = a^2 - \frac{v^2}{4a^2}$



$\begin{cases} u = x^2 - в^2 \\ v = 2вx \end{cases}$

$u = \frac{v^2}{4в^2} - в^2$

3) 

$\begin{cases} u = x - xy \\ v = xy \end{cases}$

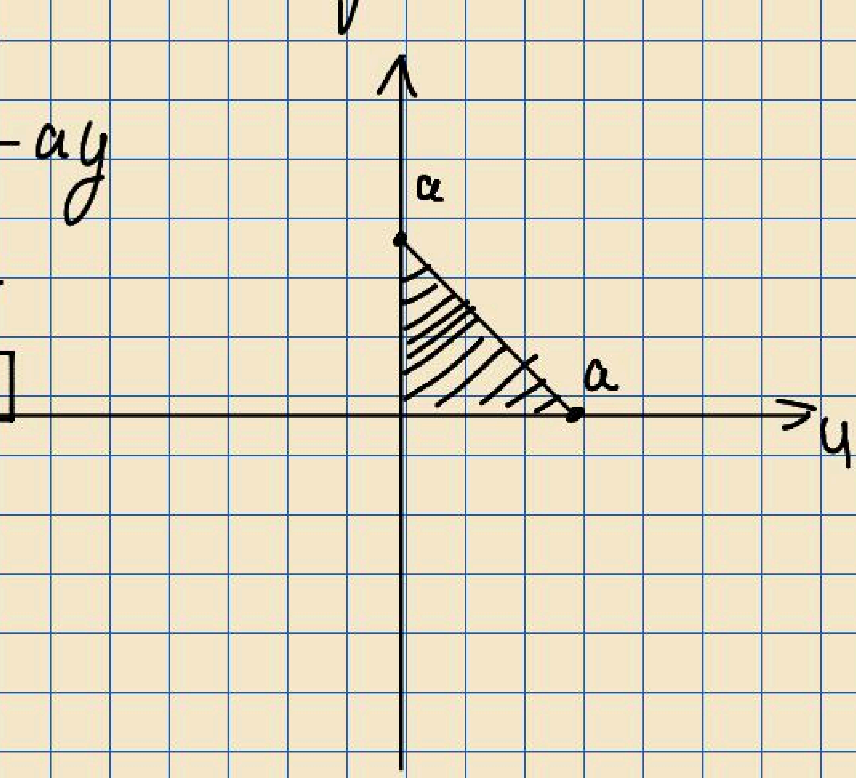
отображаем куски границы

1 $x = a$

$\begin{cases} u = a - ay \\ v = ay \end{cases}$

$y \in [0,1]$

$v = a - u$



② Найти мн-во значений $\mathbb{R}^2 \mapsto \mathbb{R}$

1) $u = x - 2y - 3 \; : \mathbb{R}$

2) $u = x^2 + 2x + y^2 - 4y$

3) $u = x^2 + y^2 - 2xy + 2x - 2y - 3$

4) $u = e^{2y} - e^{2xy} + 2$

2) $u = x^2 + 2x + y^2 - 4y = (x+1)^2 + (y-2)^2 - 5 \; : \; [-5, +\infty)$

3) $u = (x-y)^2 + 2(x-y) - 3 = t^2 - 4 \; : \; [-4; +\infty)$

$x - y + 1 = t$

4) $u = -(e^{xy})^2 + e^{xy} + 2 = -\left(t-\frac{1}{2}\right)^2 + \frac{9}{4} : \left(-\infty; \frac{9}{4}\right]$

$t = e^{xy} \in (0, +\infty)$

$t_0 = \frac{-b}{2a} = \frac{1}{2}$

$u_0 = -\frac{1}{4} + \frac{1}{2} + 2 = \frac{9}{4}$

③ Явл. ли нормой?

на $\mathbb{R}$: 1) $\|x\| = |\operatorname{arctg} x|$ – однородность

2) $\|x\| = 2\sqrt{x^2}$ +

3) $\|x\| = 2\sqrt{|x|}$ – однородность

на $\mathbb{R}^2$ 4) $\|x\| = |x_1| + |x_2|$ +

5) $\|x\| = |x_1 + x_2|$ – 0 при $x_1 = -x_2$

6) $\|x\| = [|x_1|] + [|x_2|]$ – однородность

на $C[a,b]$ 7) $\|f\| = |f(a)|$ – $f(a)=0$, а f не 0

на $\mathbb{R}[a,b]$ 8) $\|f\| = \int_a^b |f| dx$ –

на $C^1[a,b]$ 9) $\|f\| = \max|f| + \max|f'|$

$\max|f| + \max|f'| \le \max|f| + \max|g| + \max|f'| + \max|g'|$

④ $x^m \in \mathbb{R}^4$:

$x^m = \left(\sqrt{m+1} - \sqrt{m}, \frac{m-1}{m+1}, \frac{2m^2-1}{m^2}, \left(1-\frac{1}{m}\right)^m\right)$

Найти $\lim_{m\to\infty} x^m = \left(0, 1, 2, \frac{1}{e}\right)$

(5) $x^m \in \mathbb{R}^2 \quad \lim x^m - ?$

$$x^m = \left(m \sin \frac{\pi m}{2};\ m \cos \frac{\pi m}{2}\right)$$

$m = 2k \quad x^{2k} = (0, 2k) \to \infty$

$m = 2k+1 \quad x^{2k+1} = \left((-1)^k (2k+1), 0\right) \to \infty$

$\|x^m\| = m$