Generating Trapdoor Primes

A short take on generating SNFS primes

Nikolai Rozanov

UCL - Computer Science

Table of contents - Introduction

Importance

Importance

- Look Up
- Benchmarking
- Deliberate Weakening

Method and Code

Algorithm

Step 1. Generating the prime q of the corresponding size.

Step 2. Generating the coefficients for polynomials f and g, according to the size suggestions in [1]

Step 3. Setting up a new polynomial G, which is the resultant of f and g and the variable is the leading coefficient of g.

Step 4. Finding roots of G-1 modulo q, if there are no roots then going back to Step 1.

Step 5. Then letting p=$|G|$, and checking if p is prime, (an addition is to check whether q divides p-1).

Main Script

Main Script

```
# Generating required Rings
X.<x>       = ZZ['x']
G.<x,g1>  = ZZ['x,g1']
# Main Loop
    #generating prime and Associated Field
    q = get_prime(bits_q)
    T.<g2>     = Integers(q)['g2']
        ##while loop
        f_poly,norm_f = get_f(X,degree_f,bits_q)
        g_poly,g0 = get_g(g1,x,bits_p,degree_f, norm_f)
        G_poly = get_G(f_poly,g_poly)
        temp = list(G_poly.coefficients())
        temp.reverse()
        T2 = T(temp)
    r     = T2.roots()
    root = r[0][0]
    rt    = int(root)
#     while(rt<int(2^(bits_p/degree_f)/norm_f)):
#         rt+=q
    p     = X([G_poly(1,rt)+1])
```

Helper Functions

Helper Functions

Prime Generation

```
def get_prime(bits_q):
    q = random_prime(2^bits_q -1,False,2^(bits_q -1))
    while(not is_prime(q)):
        q = random_prime(2^bits_q -1,False,2^(bits_q -1))
    return q
```

Poly f

```
def get_f(X,degree_f, bits_q):
    flag_irreducible = False
    while (not flag_irreducible):
        #f_vec = [ZZ.random_element(-int(2^(10)-1),int(2^(10)-1)) for _ in range(degree_f+1)]
        f_vec = [ZZ.random_element(-int(2^(bits_q/(2*(degree_f+1)))),int(2^(bits_q/(2*(degree_f+1))))) for _ in range(degree_f+1)]
        #f_vec = [ZZ.random_element(1,int(2^(bits_q/(2*(degree_f+1))))) for _ in range(degree_f+1)]

        norm_f = max(map(abs,f_vec))
        f_poly = X(list(f_vec))
        if f_poly.is_irreducible():
            flag_irreducible = True
    return f_poly,norm_f
```

Helper Functions

Poly g

```
def get_g(g1,x,bits_p ,degree_f ,norm_f):
    g0 = ZZ.random_element(-int(2^(bits_p/degree_f)/norm_f),int(2^(bits_p/degree_f)/norm_f))
    #g0 = ZZ.random_element(1,int(2^(bits_p/degree_f)/norm_f))
    g_poly = g1*x+g0
    return g_poly,g0
```

Poly G

```
def get_G(f_poly,g_poly):
    G_temp = f_poly.sylvester_matrix(g_poly,variable=x)
    G_poly = G_temp.determinant()-1
    return G_poly
```

Some Analysis

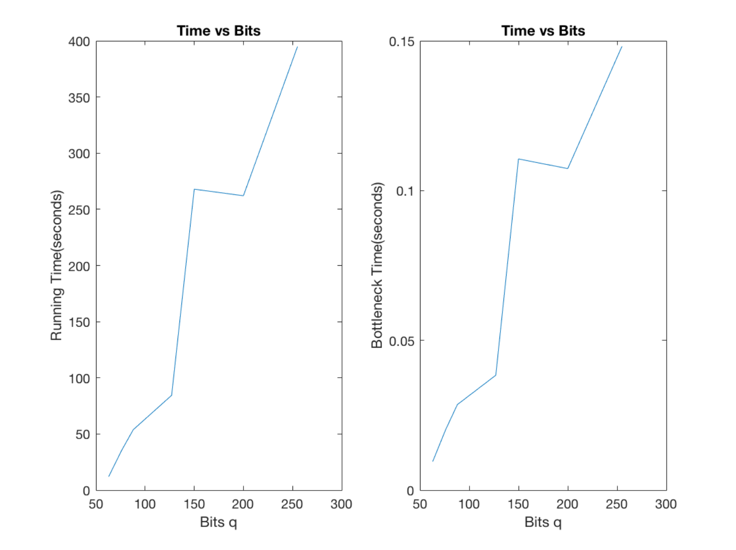
Plots

Running Time Data

Degree = 6/bits_q	Total Running Time (seconds)	Bit size P	Bottlenecks per iteration (seconds)
63	12.06977391	360	0.009566099405
76	35.19902706	455	0.02010813165
88	53.86601114	518	0.028564533
127	84.4335351	764	0.03836842561
150	267.9267499	900	0.1105723426
200	262.0755301	1209	0.1073431978
255	394.7975631	1543	0.1481677871

Plots

Running Time Plot



References I

J. Fried, P. Gaudry, N. Heninger, and E. Thomé.
A kilobit hidden snfs discrete logarithm computation.
arXiv preprint arXiv:1610.02874, 2016.