Linear Regression :-

1. check if data is linear or sort of linear.

2. in 99.99% case real world data is sort of linear (not perfectly straight)

3. Linear Regression makes a best fit straight line on this sort of linear data.

4. This line is chosen to minimize the error (difference between predicted and actual values)

* Assumption of Linear Regression.

1. Linear Relationship :-
input and output should have a straight - line relationship.

2. No multicollinearity :-
input features should not be highly related to each other.

3. Normality of Residuals :-
The Prediction errors should follow a normal (bell - shaped) distribution.

4. Homoscedasticity :-
Error should have constant spread across all values of input.

5. No autocorrelation :-
Errors shouldn't follow a pattern they shoud be random.

* Simple Linear Regression Mathematical Formulation :-

(m, b)

Closed form solution (direct formula)

OLD (ordinary least square)

(Low dimension)

Non-closed form solution

[Gradient discent]

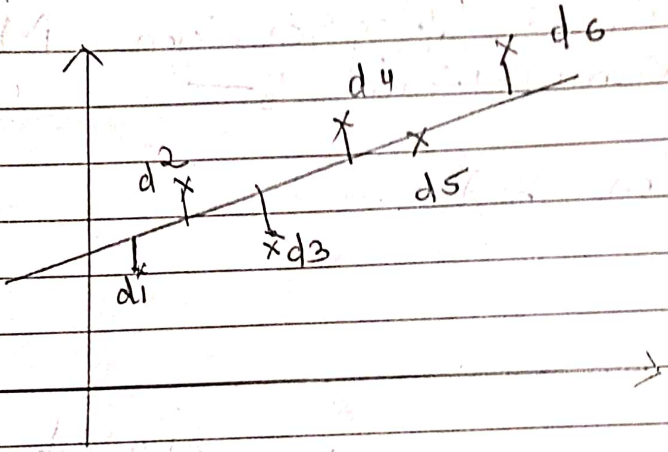
(High dimension)

Date
Page

$$\bar{Y} = mx + b$$

$$b = \bar{Y} - mx$$

$$m = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n} (x_i - \bar{x})^2}$$



$$E = d_1 + d_2 + d_3 + \ldots\ldots d_n$$

Positive and negative error can cancel out each other so here we use square of error.

why modulus is not use instead of square.?

→ Because modulus is not differentiable at origin. and square is differentiable at all points.

Date
Page

$$E = d_1^2 + d_2^2 + d_3^2 + \cdots + d_n^2$$

$$\boxed{E = \sum_{i=1}^{n} d_i^2} \quad \cdots\cdots 1 \text{ Error function}$$

$$d_i = y_i - \hat{y}_i \quad \cdots (2) \cdots \quad (\text{actual} - \text{Predicted})$$

Put (2) in (1)

$$E = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

$$\hat{y}_i = mx_i + b$$

$$E = \sum_{i=1}^{n} (y_i - mx_i - b)^2$$

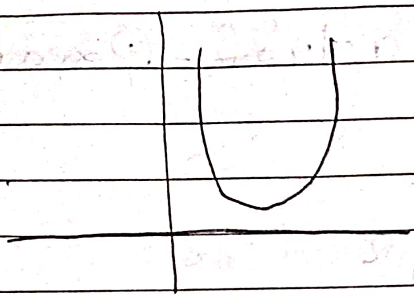
$$E(m, b) = \sum_{i=1}^{n} (y_i - mx_i - b)^2$$

Let $b = 0$

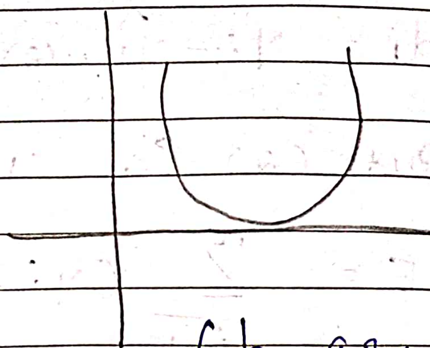
$$E(m) = \sum_{i=1}^{n} (y_i - mx_i)$$

Let $m = 1$

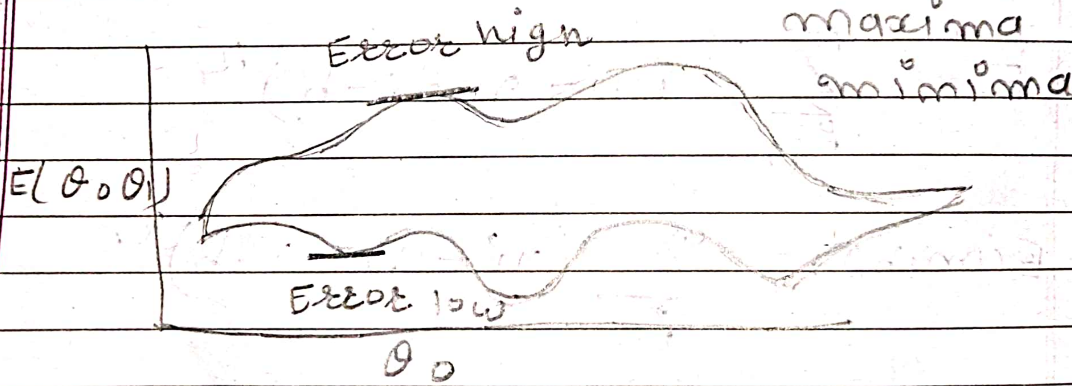
$$E(m, b) = \sum_{i=1}^{n} (y_i - x_i - b)^2$$



(m graph)



(b graph)



$$\frac{\partial E}{\partial b} = \sum_{i=1}^{n} \frac{\partial}{\partial b} (y_i - m x_i - b)^2 = 0$$

$$\frac{\partial E}{\partial b} = \sum_{i=1}^{n} 2 (y_i - m x_i - b)(-1) = 0$$

$$\frac{\partial E}{\partial b} = \sum_{i=1}^{n} -2\,(y_i - mx_i - b) = 0$$

divide both side by -2

$$\sum_{i=1}^{n} (y_i - mx_i - b) = 0$$

$$\sum_{i=1}^{n} y_i - \sum_{i=1}^{n} mx_i - \sum_{i=1}^{n} b = 0$$

divede both side by n

$$\frac{\sum_{i=1}^{n} y_i}{n} - \frac{\sum_{i=1}^{n} mx_i}{n} - \frac{\sum_{i=1}^{n} b}{n} = 0$$

$$\bar{y} - m\bar{x} - \frac{\cancel{n}b}{\cancel{n}} = 0$$

$$\left[\frac{y_i}{n} = \bar{y}, \quad \frac{x_i}{n} = \bar{x} \right] \text{ : mean}$$

$$\left[\frac{b + b + b + \dots + b}{n} \right] \text{ n times } \frac{nb}{n}$$

$$\bar{y} - m\bar{x} - b = 0$$

$$b = \bar{y} - m\bar{x} \quad \dots\dots (3)$$

Date
Page

$$E = \sum (y_i - mx_i - \bar{y} + m\bar{x})^2 = 0$$

$$\frac{\partial E}{\partial m} = \sum \frac{\partial}{\partial m} (y_i - mx_i - \bar{y} + m\bar{x})^2$$

$$\frac{\partial E}{\partial m} = \sum -2 (y_i - mx_i - \bar{y} + m\bar{x})(-x_i$$

$$\sum (y_i - mx_i - \bar{y} + m\bar{x})(x_i - \bar{x}) =$$

$$\sum \left[(y_i - \bar{y}) - m(x_i - \bar{x}) \right] (x_i - \bar{x}$$

$$\sum \left[(y_i - \bar{y})(x_i - \bar{x}) - m(x_i - \bar{x})^2 \right]$$

$$\sum (y_i - \bar{y})(x_i - \bar{x}) - m \sum (x_i - \bar{x})^2$$

$$m = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n} (x_i - \bar{x})^2} \ldots$$