Assignment 1: Ping Pong

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1 Research

The first thing I did was to figure out the size of an (average) Ping Pong ball: After a quick look at Google, I saw that the average size is 40mm in diameter.

The next question was how large an average-sized school bus is?
Well, I quickly found the length and width of school busses, but nothing about the in-cabin room.
Luckily, after some research, I found the homepage form a guy who changed an old school bus into a mobile home. He wrote down the interior dimensions of his bus and these are the dimensions I used for my calculations!
His homepage: http://www.vonslatt.com/bus-main.shtml

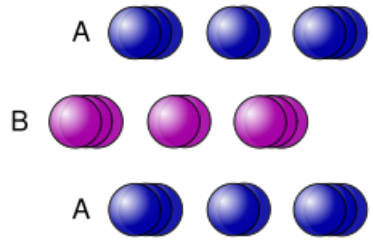
2 Calculations

First of all, I calculated the Volume of the bus:
The dimensions are $35'x7.5'x6.5'$. I am from Europe, so I'm used to the metric system ;) That's why I converted these dimensions into meters:
$35feet = 10.668m$
$7.5feet = 2.286m$
$6.5feet = 1.9812m$

To get the Volume, I used the standard-Volume-formula:
$V = G * h$
$V = 10.668 * 2.286 * 1.9812 \approx 48m^3$

Next, let's concentrate on the ping pong balls:
How many spheres will fit in $1m^3$?

Obviously the spheres won't lay straight on top of each other, more like:



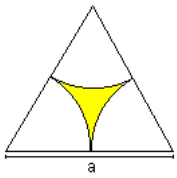
But there will be a space between the spheres. So the next step is to calculate this free space.

After some research time, I found the "Theorie of sphere packing".
This seemed to be exactly what I was looking for!
It says that the average density in a three-dimensional Euclidean space is:

$$\frac{\pi}{\sqrt{18}} \simeq 0.74048$$

But I didn't want to rely on that, so I made my own calculations.

I took a look at 3 Spheres on top of each other and saw that they formed a triangle:



Where a is 40mm (as the diameter is 40mm)

So I calculated the yellow area:
$\frac{\sqrt{3}}{4} - \frac{\pi}{8} * a^3 = 64.5mm^3$

So every 3 spheres, I will loose $64.5mm^3$.
How many spheres does it take until I loose one ball? The answer is 25 spheres.

So: after 25 spheres, I will loose one sphere.
Let's keep that in mind for later!

Now how many spheres are there in $1m^3$?
In each m are 25 spheres. I simply calculated this by taking a square box of 40mm for each sphere.
But we'll have to take a look at $1m^3$. As the spheres won't sit on top of each other, I will have to take two different rows, one with 25 and one with 24 spheres.

So in each m^3 there are 61.250 spheres, as there are 2500 rows in $1m^3$.
But, as we calculated before, we loose 1 sphere every 25 spheres!
So we loose 2450 spheres, therefore we've got 58.800 spheres in $1m^3$.

As we've got a $48m^3$ big bus, we'll just have to multiply it by 48.

So my answer is:
It takes **2 322 400** to fill one average-sized school bus.

3 Sources

$http://en.wikipedia.org/wiki/Close-packing_of_equal_spheres$
$http://www.vonslatt.com/bus-main.shtml$