Appendix: Single pendulum with lateral and roll inputs

Xiaojing Qi
School of Vehicle and Mobility, Tsinghua University, China, 100084

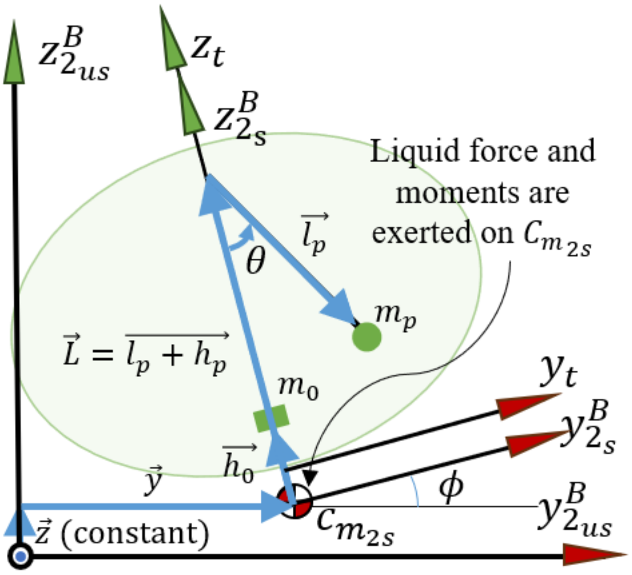

Fig. 2 (b)

Position vectors of lumped masses m_0 and m_p are

$$\overrightarrow{R_{p0}} = \vec{y} + \overrightarrow{h_0} = (y - h_0 \sin\phi)\vec{j} + (h_0 \cos\phi)\vec{k} \tag{1}$$

$$\overrightarrow{R_p} = \vec{y} + \vec{L} + \overrightarrow{h_p} = \big(y - L\sin\phi + l_p \sin(\phi+\theta)\big)\vec{j} + \big(L\cos\phi - l_p\cos(\phi+\theta)\big)\vec{k} \tag{2}$$

Then, differentiate it to obtain the velocity vector

$$\overrightarrow{\dot{R}_{p0}} = (\dot{y} - h_0\dot{\phi}\cos\phi)\vec{j} + (-h_0\dot{\phi}\sin\phi)\vec{k} \tag{3}$$

$$\begin{aligned}\vec{\dot{R}}_p &= \left[\dot{y} - \big(L\cos\phi + l_p\cos(\phi+\theta)\big)\dot{\phi} + l_p\dot{\theta}\cos(\phi+\theta)\right]\vec{j} \\ &\quad + \left[\big(-L\sin\phi + l_p\sin(\phi+\theta)\big)\dot{\phi} + l_p\dot{\theta}\sin(\phi+\theta)\right]\vec{k}\end{aligned} \tag{4}$$

Differentiate again to obtain the acceleration vector

$$\overrightarrow{\ddot{R}_{p0}} = [\ddot{y} - \ddot{\phi}h_0\cos\phi + \dot{\phi}^2 h_0\sin\phi]\vec{j} + [-\ddot{\phi}h_0\sin\phi - \dot{\phi}^2 h_0\cos\phi]\vec{k} \tag{5}$$

$$\begin{aligned}\vec{\ddot{R}}_p &= \begin{bmatrix}\ddot{y} + \ddot{\phi}[-L\cos\phi + L_p\cos(\phi+\theta)] + \ddot{\theta}l_p\cos(\phi+\theta) \\ +\dot{\phi}^2[L\sin\phi - l_p\sin(\phi+\theta)] + \dot{\theta}^2[-l_p\sin(\phi+\theta)] \\ -2\dot{\phi}\dot{\theta}l_p\sin(\phi+\theta)\end{bmatrix}\vec{j} \\ &\quad + \begin{bmatrix}\ddot{\phi}[-L\sin\phi + L_p\cos(\phi+\theta)] + \ddot{\theta}l_p\sin(\phi+\theta) \\ +\dot{\phi}^2[L\cos\phi - l_p\cos(\phi+\theta)] + \dot{\theta}^2[-l_p\cos(\phi+\theta)] \\ -2\dot{\phi}\dot{\theta}l_p\cos(\phi+\theta)\end{bmatrix}\vec{k}\end{aligned} \tag{6}$$

We select plane $y^B_{2_{us}} o^B_{2_{us}} z^B_{2_{us}}$ (coordination definition please refer to Fig.2.(b))as zero potential energy surface, the kinetic and potential energy T and U are

$$T = \frac{1}{2} m_p \left(\left(\overrightarrow{\dot{R}_p} \vec{j} \right)^2 + \left(\overrightarrow{\dot{R}_p} \vec{k} \right)^2 \right) + \frac{1}{2} m_0 \left(\left(\overrightarrow{\dot{R}_{p0}} \vec{j} \right)^2 + \left(\overrightarrow{\dot{R}_{p0}} \vec{k} \right)^2 \right) \tag{7}$$

$$U = m_p g \left[L \cos\phi - l_p \cos(\phi + \theta) \right] \tag{8}$$

According to the Euler-Lagrange equation

$$\frac{d}{dt} \frac{\partial T}{\partial \dot{\theta}} - \frac{\partial T}{\partial \theta} + \frac{\partial U}{\partial \theta} = 0$$

the dynamic differential equation of swinging degree of freedom θ is derived as equation bellow, which is the equation (1) in section II of the article.

$$\ddot{\theta} + \frac{1}{l_p} \ddot{y} \cos(\phi + \theta) + \ddot{\phi} \left[\left(1 - \frac{h_p}{l_p} \right) \cos\theta + 1 \right] - \dot{\phi}^2 \left(1 - \frac{h_p}{l_p} \right) \sin\theta + \frac{g}{l_p} \sin(\phi + \theta) + c_d \dot{\theta} = 0 \tag{9}$$