Dimos Kompitselidis (AEM 9985)

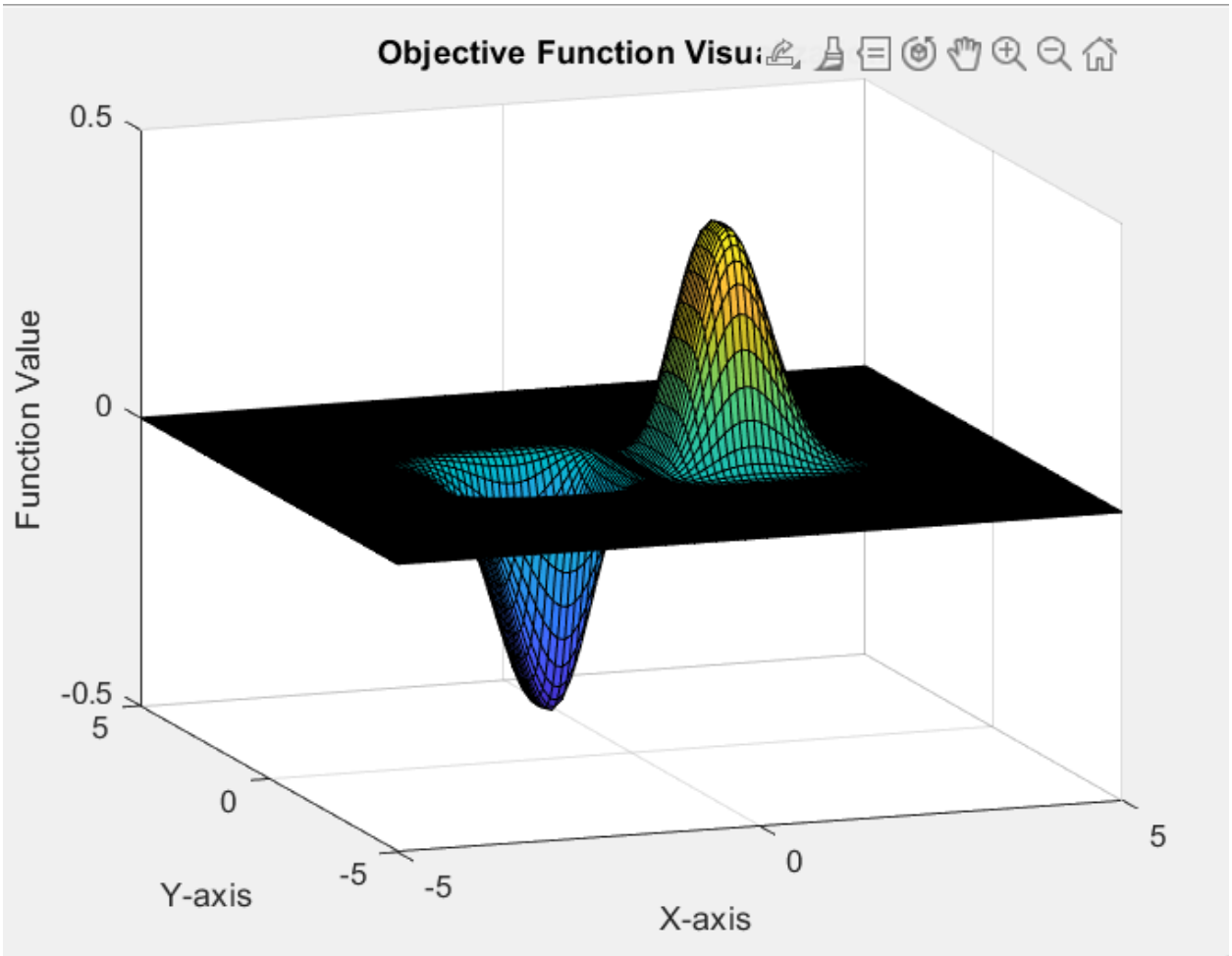
Optimization Techniques

Assignment 2: Unconstrained Minimization of Multivariable Functions

Task 1: Function Visualization

Plot the function for a comprehensive understanding of its shape and behavior.

Here is the 3d plot I made for x in [-4,4] and y in [-4,4]:



The reason for giving values only in these intervals (from -5 to +5) is because the rest of the graph has values close to zero.

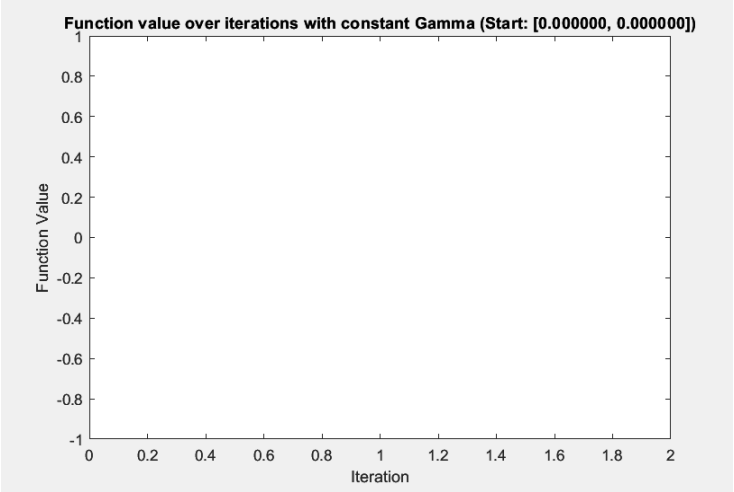
Task 2: Steepest Descent Method

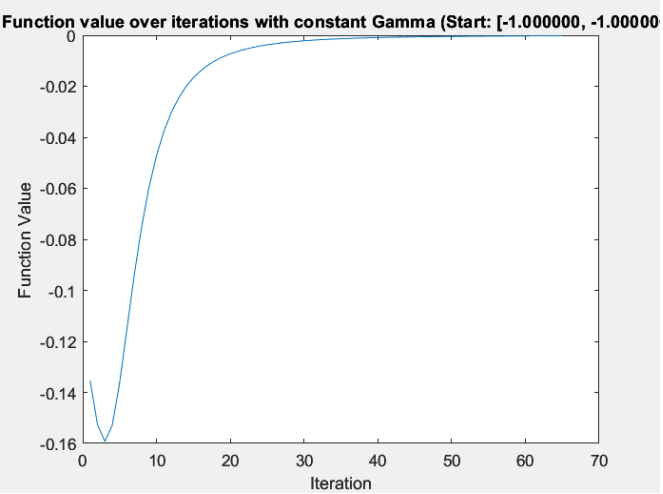
With Constant Gamma = 0.1 :

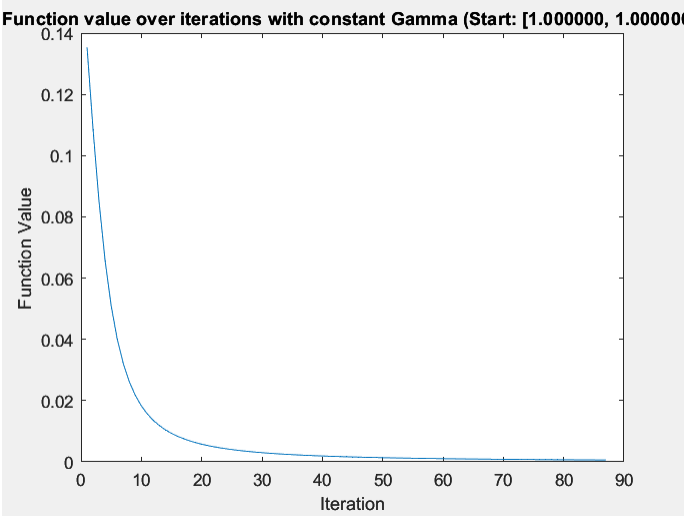
And the starting points (0,0), (1,-1), (-1,1):

Minimazation of f With Constant Gamma
Initial Point: (0.000000, 0.000000)
The lowest point found (0.000000, 0.000000) with 0.000000 value and 0 steps
Initial Point: (-1.000000, -1.000000)
The lowest point found (-0.063627, -0.681864) with -0.000207 value and 64 steps
Initial Point: (1.000000, 1.000000)
The lowest point found (1.061856, 1.593294) with 0.000616 value and 86 steps

The graph of the function value throughout the interactions:







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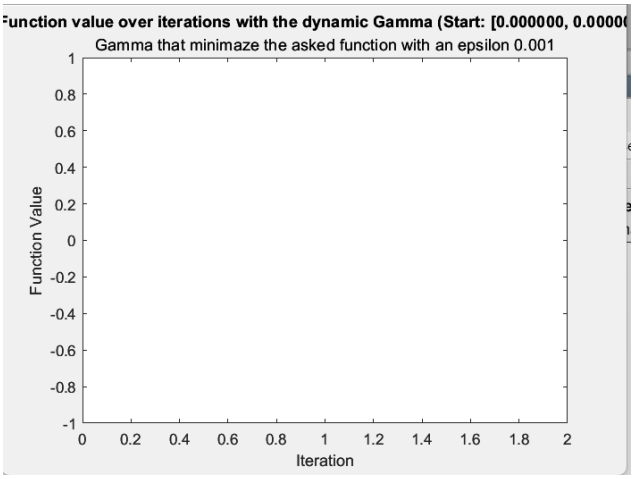
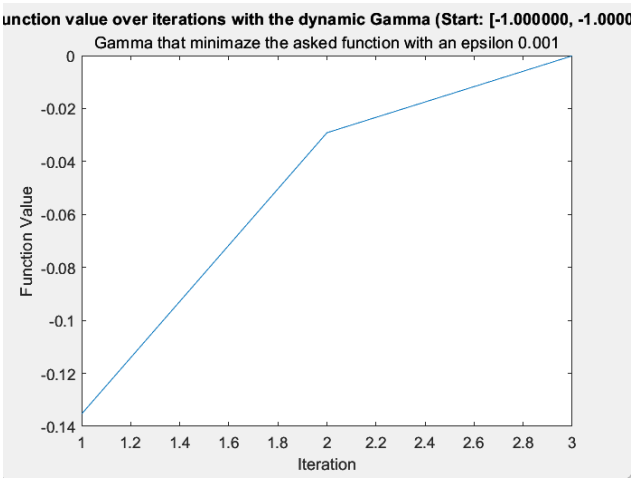
With Gamma calculated for the minimization of the asked function:

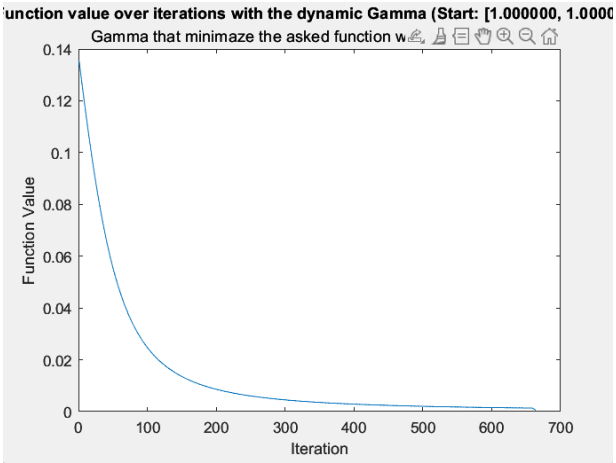
And the starting points (0,0), (1,-1), (-1,1):

Minimazation of f with dynamic gamma
Initial Point: (0.000000, 0.000000)
The lowest point found (0.000000, 0.000000) with 0.000000 value and 0 steps
Initial Point: (-1.000000, -1.000000)
The lowest point found (-0.046977, -0.447417) with -0.000099 value and 2 steps
Initial Point: (1.000000, 1.000000)
The lowest point found (1.059383, 1.600461) with 0.000547 value and 663 steps

The graph of the function value throughout the interactions:





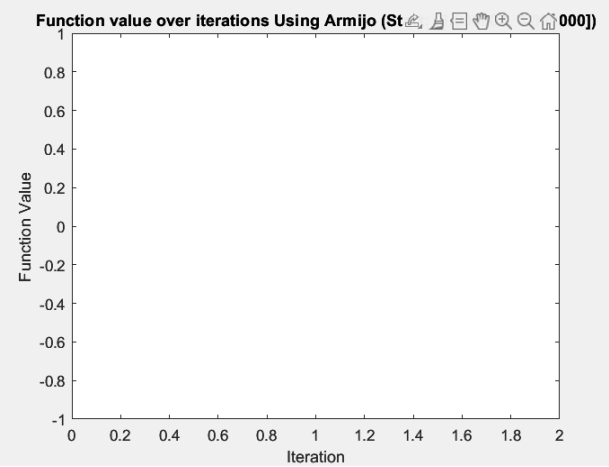


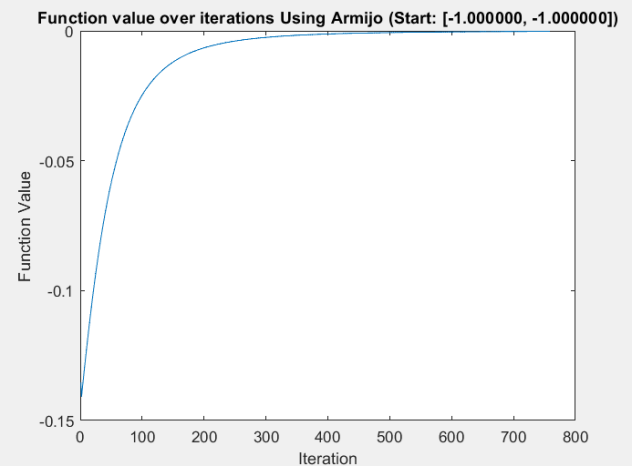
With Gamma calculated based on the Armijo rule:

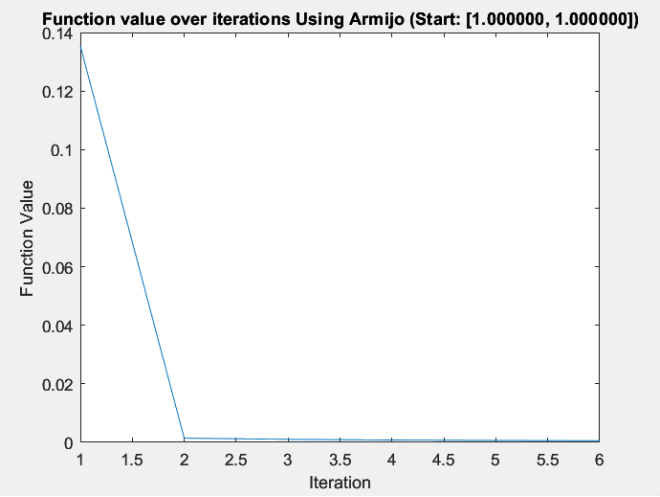
And the starting points (0,0), (1,-1), (-1,1):

Minimazation of f with Armijo
Initial Point: (0.000000, 0.000000)
The lowest point found (0.000000, 0.000000) with 0.000000 value and 0 steps
Initial Point: (-1.000000, -1.000000)
The lowest point found (-0.062919, -0.642777) with -0.000209 value and 758 steps
Initial Point: (1.000000, 1.000000)
The lowest point found (1.135587, 1.601707) with 0.000559 value and 5 steps

The graph of the function value throughout the interactions:







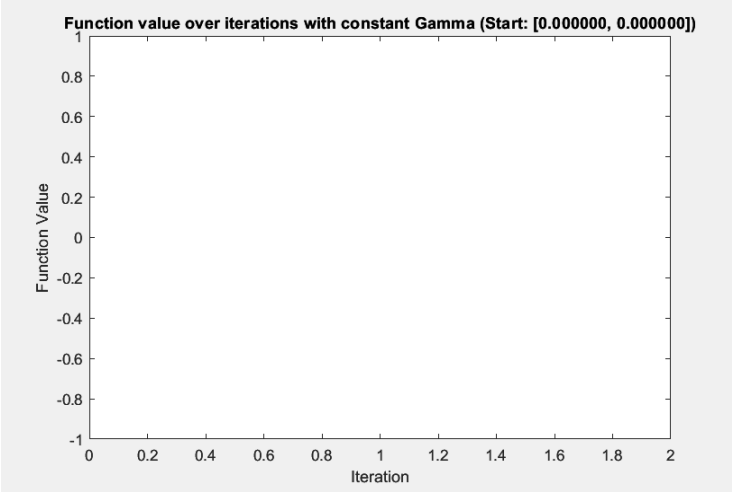
Task 3: Newton's Method

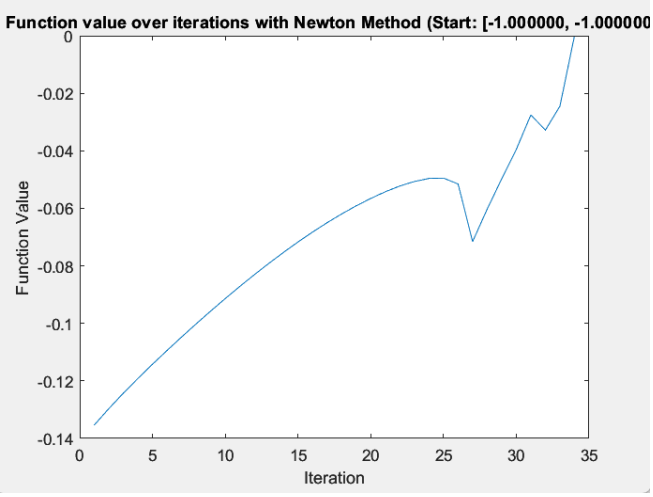
With Constant Gamma = 0.1 :

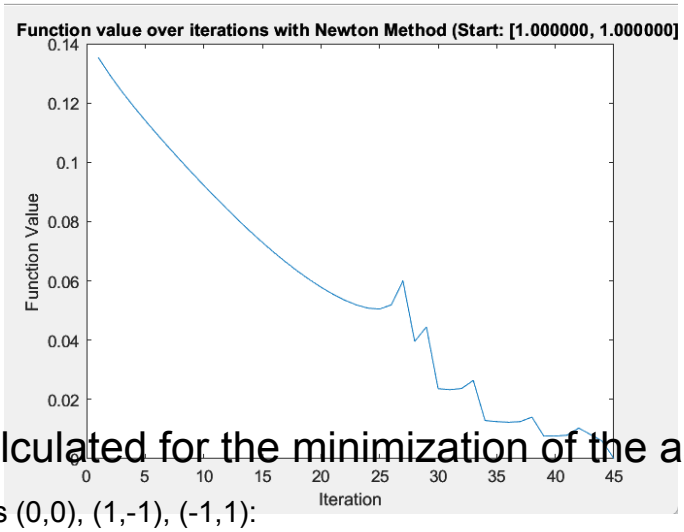
And the starting points (0,0), (1,-1), (-1,1):

Minimization of f With Newton Method
Initial Point: (0.000000, 0.000000)
The lowest point found (0.000000, 0.000000) with 0.000000 value and 0 steps
Initial Point: (-1.000000, -1.000000)
The lowest point found (-0.132836, -1.444892) with -0.000029 value and 33 steps
Initial Point: (1.000000, 1.000000)
The lowest point found (0.091336, 1.356932) with 0.000025 value and 44 steps

The graph of the function value throughout the interactions:







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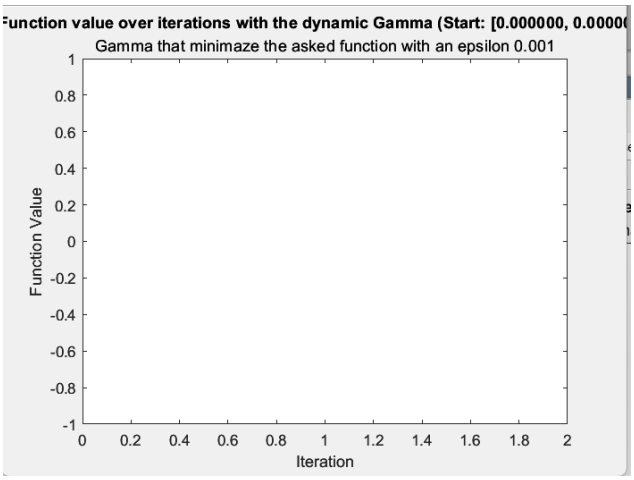
For Gamma calculated for the minimization of the asked function:

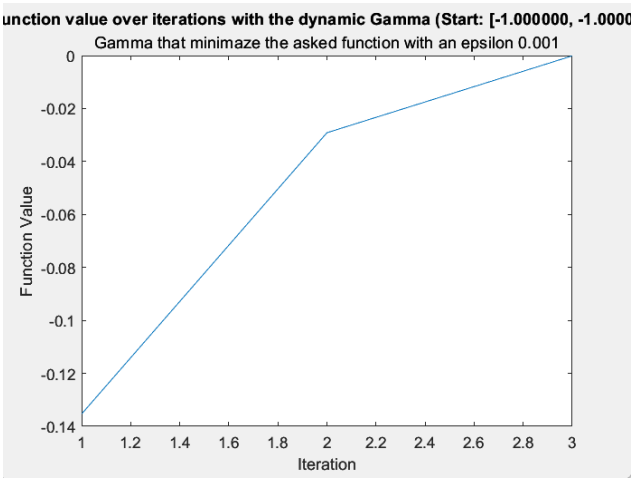
And the starting points (0,0), (1,-1), (-1,1):

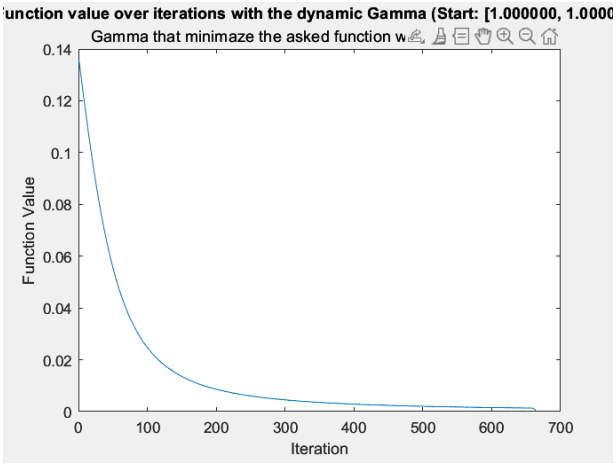
Minimazation of f with dynamic gamma
Initial Point: (0.000000, 0.000000)

The lowest point found (0.000000, 0.000000) with 0.000000 value and 0 steps
Initial Point: (-1.000000, -1.000000)
The lowest point found (0.662395, -5.601251) with 0.000000 value and 5 steps
Initial Point: (1.000000, 1.000000)
The lowest point found (0.033106, 0.269901) with 0.000036 value and 841 steps

The graph of the function value throughout the interactions:





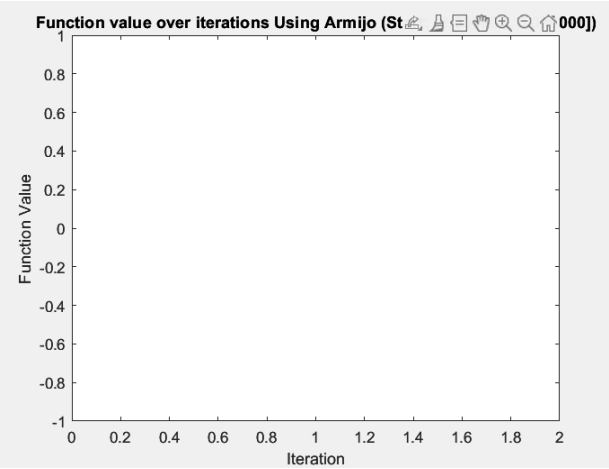


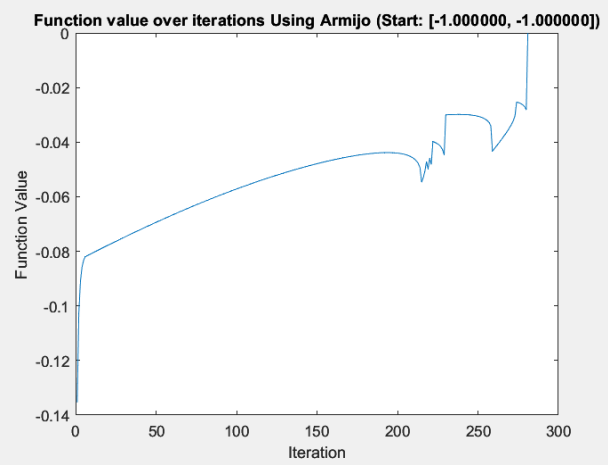
With Gamma calculated based on the Armijo rule:

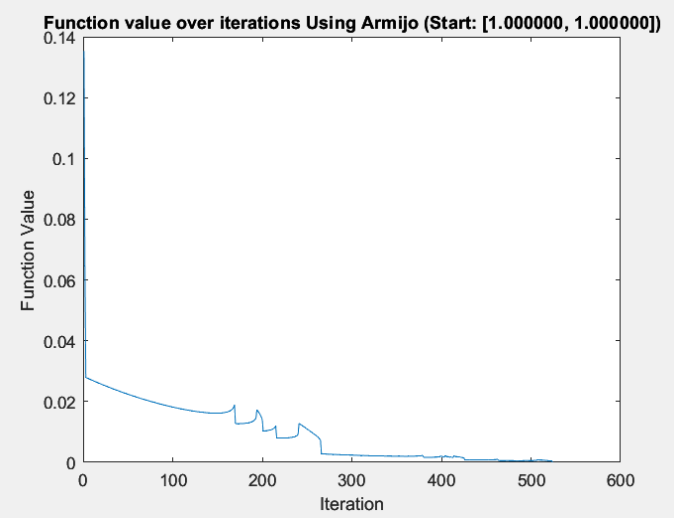
And the starting points (0,0), (1,-1), (-1,1):

Minimazation of f with Armijo
Initial Point: (0.000000, 0.000000)
The lowest point found (0.000000, 0.000000) with 0.000000 value and 0 steps
Initial Point: (-1.000000, -1.000000)
The lowest point found (-5.047357, 16.145891) with -0.000000 value and 280 steps
Initial Point: (1.000000, 1.000000)
The lowest point found (0.011035, 1.816858) with 0.000000 value and 523 steps

The graph of the function value throughout the interactions:







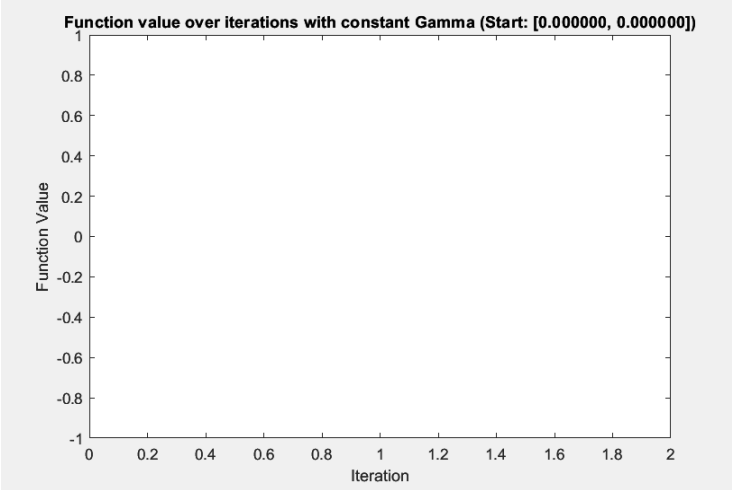
Task 4: Levenberg-Marquardt Method

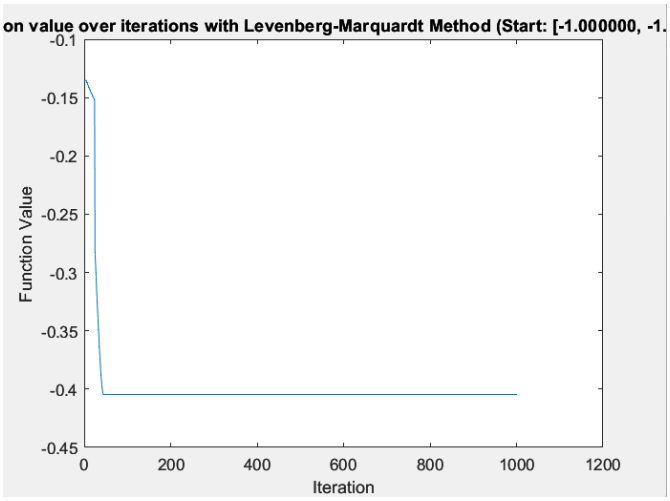
With Constant Gamma = 0.1 :

And the starting points (0,0), (1,-1), (-1,1):

Minimization of f With Levenberg-Marquardt Method
Initial Point: (0.000000, 0.000000)
The lowest point found (0.000000, 0.000000) with 0.000000 value and 0 steps
Initial Point: (-1.000000, -1.000000)
The lowest point found (-1.211775, 0.334727) with -0.404666 value and 1000 steps
Initial Point: (1.000000, 1.000000)
The lowest point found (0.001981, 1.253728) with 0.000000 value and 145 steps

The graph of the function value throughout the interactions:





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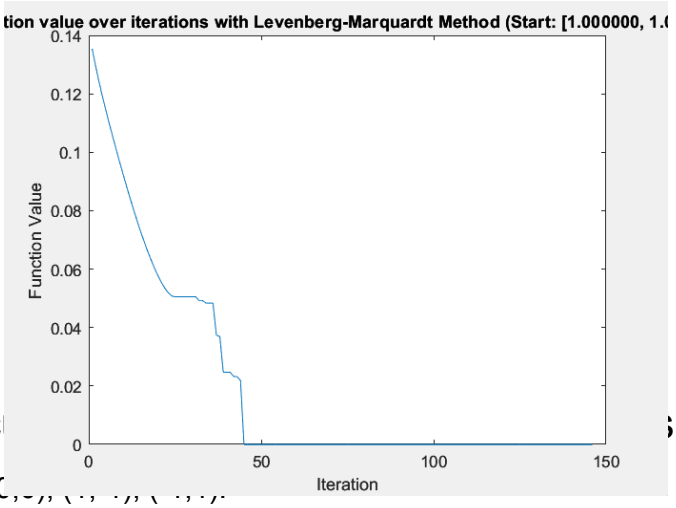


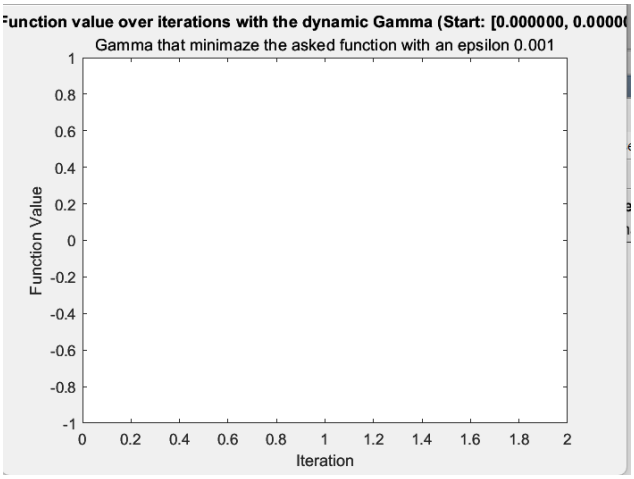
For Gamma calc sked function:

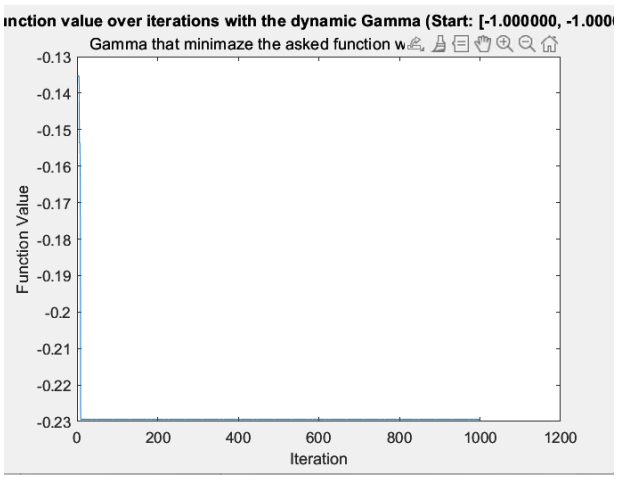
And the starting points (0,0), (1,-1), (-1,1):

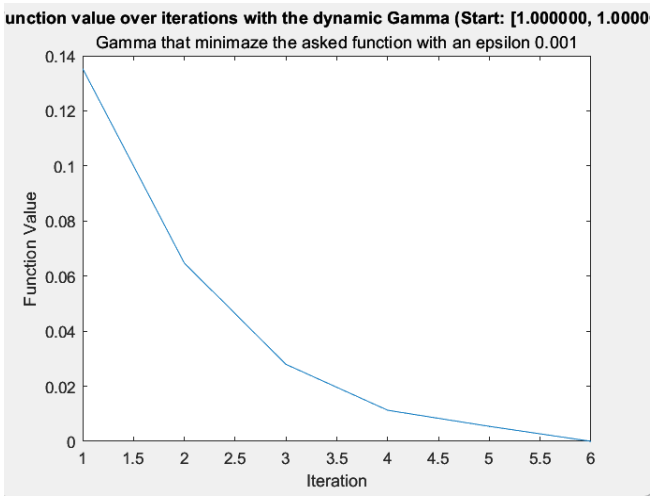
Minimazation of f with dynamic gamma

Initial Point: (0.000000, 0.000000)
The lowest point found (0.000000, 0.000000) with 0.000000 value and 0 steps
Initial Point: (-1.000000, -1.000000)
The lowest point found (-0.795796, -0.625930) with -0.229460 value and 1000 steps
Initial Point: (1.000000, 1.000000)
The lowest point found (-9.892995, 63.242106) with -0.000000 value and 5 steps

The graph of the function value throughout the interactions:





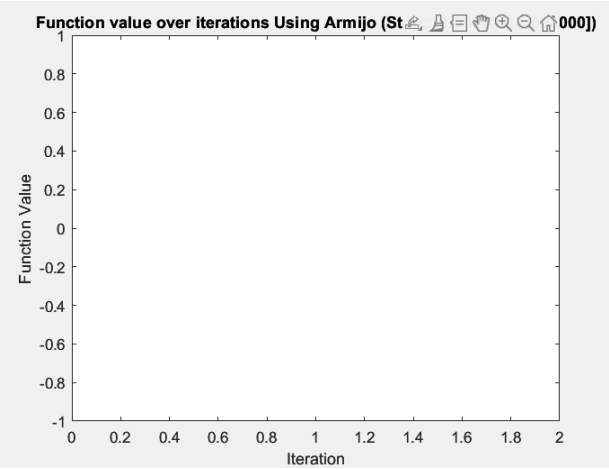


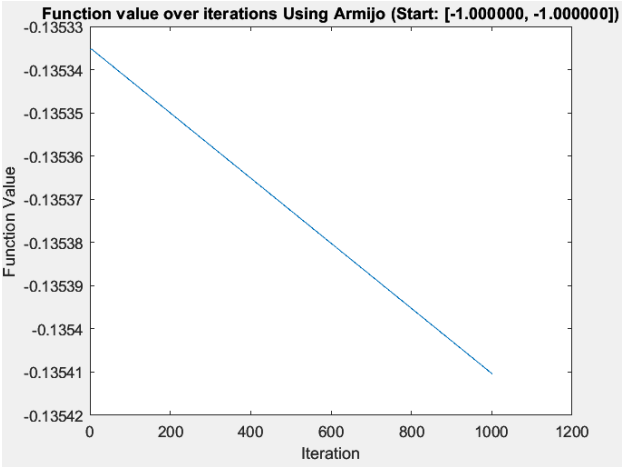
With Gamma calculated based on the Armijo rule:

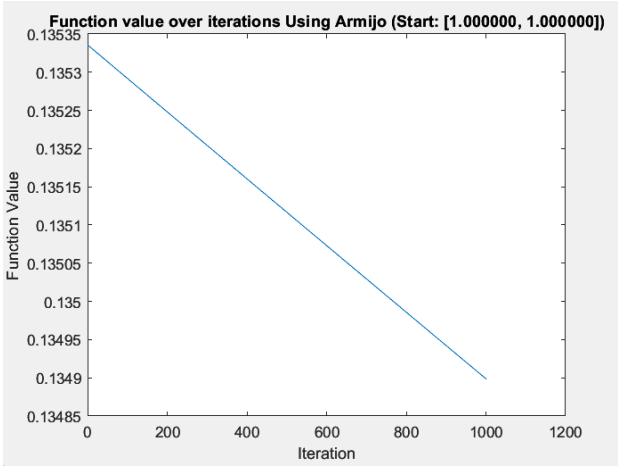
And the starting points (0,0), (1,-1), (-1,1):

Minimazation of f with Armijo
Initial Point: (0.000000, 0.000000)
The lowest point found (0.000000, 0.000000) with 0.000000 value and 0 steps
Initial Point: (-1.000000, -1.000000)
The lowest point found (-0.999813, -0.999815) with -0.135410 value and 1000 steps
Initial Point: (1.000000, 1.000000)
The lowest point found (0.997522, 1.000185) with 0.134898 value and 1000 steps

The graph of the function value throughout the interactions:







Results and Analysis

Steepest Descent Method:

Efficiency: This method demonstrated varying efficiency based on the choice of gamma (step size) and the initial starting point. With a constant gamma, the number of steps to reach the minimum was generally higher compared to dynamic gamma and the Armijo rule.
Convergence: Achieved a greater convergence when gamma wasn't constant
Local Minima: The algorithm occasionally got trapped in local minima, especially with specific starting points like (-1, -1), indicating sensitivity to initial conditions.

Newton's Method:

Efficiency: Newton's method was more efficient in terms of the number of iterations compared to the steepest descent, especially with dynamically calculated gamma.
Convergence: The convergence was rapid, achieving greater minimization of the function.
Local Minima: Unlike the steepest descent, Newton's method showed a tendency to escape local minima more effectively.

Levenberg-Marquardt Method:

Efficiency: This method showed an intermediate efficiency level, performing better than the steepest descent but not as well as Newton's method in most cases.
Convergence: It was robust in terms of convergence, handling various starting points and gamma values well. However, it sometimes required a large number of iterations to converge, especially with constant gamma, and with the armijo rule it seemed to be very slow.
Local Minima: The Levenberg-Marquardt method was effective at avoiding getting trapped in local minima.