Observer-Based Control — 3

- Closed-loop dynamics:

$$\begin{aligned}\dot{x}(t) &= Ax(t) - BK\hat{x}(t)\\ \dot{\hat{x}}(t) &= A\hat{x}(t) + L(y(t) - \hat{y}(t)) - BK\hat{x}(t)\end{aligned}$$

- The overall system (observer + controller) can be written as follows:

$$\begin{bmatrix}\dot{x}(t)\\ \dot{\hat{x}}(t)\end{bmatrix} = \begin{bmatrix}A & -BK\\ LC & A-LC-BK\end{bmatrix}\begin{bmatrix}x(t)\\ \hat{x}(t)\end{bmatrix}$$

- Transformation: $\begin{bmatrix}x(t)\\ e(t)\end{bmatrix} = \begin{bmatrix}x(t)\\ x(t)-\hat{x}(t)\end{bmatrix} = \begin{bmatrix}I & 0\\ I & -I\end{bmatrix}\begin{bmatrix}x(t)\\ \hat{x}(t)\end{bmatrix}$
- Hence, we can write:

$$\begin{bmatrix}\dot{x}(t)\\ \dot{e}(t)\end{bmatrix} = \underbrace{\begin{bmatrix}A-BK & BK\\ 0 & A-LC\end{bmatrix}}_{A_{\mathrm{cl}}}\begin{bmatrix}x(t)\\ e(t)\end{bmatrix}$$

- If the system is controllable & observable $\Rightarrow$ $\texttt{eig}(A_{\mathrm{cl}})$ can be arbitrarily assigned by proper K and LWhat if the system is stabilizable and detectable?