OBC Example

Design an OBC (i.e., $u(t) = -K\hat{x}(t)$) for the following SISO system

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t), \quad y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t)$$

1. Before doing anything, check whether system is cont. (or stab.) and obs. (or det.): **system is cont. AND obs.**
2. First, design a stabilizing state feedback control, i.e., find K s.t.

$$eig(A-BK) < 0, A-BK = \begin{bmatrix} 0 & 1 \\ -k_1 & -k_2 \end{bmatrix} \Rightarrow K = \begin{bmatrix} 4 & 2 \end{bmatrix} \text{ does the job}$$

3. Second, design a stabilizing observer (estimator), i.e., find L s.t.

$$eig(A-LC) < 0, A-LC = \begin{bmatrix} -l_1 & 1 \\ -l_2 & 0 \end{bmatrix} \Rightarrow L = \begin{bmatrix} 10 & 100 \end{bmatrix}^\top \text{ does the job}$$

4. Finally, overall system design:

$$\begin{aligned} u(t) &= -K\hat{x}(t) = -4\hat{x}_1(t) - 2\hat{x}_2(t) \\ \dot{\hat{x}}_1(t) &= \hat{x}_2(t) + 10(y(t) - \hat{x}_1(t)) \\ \dot{\hat{x}}_2(t) &= u(t) + 100(y(t) - \hat{x}_1(t)) \end{aligned}$$