Introduction to Observability

- **Observability:** The ability to observer what's happening inside your system (i.e., to know system states $x(t)$)
- Observability: In order to see what is going on inside the system under observation (i.e., output $y(t)$), **the system must be observable**. Observation: output $y(t)$
- Given this dynamical system:

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k), \quad x(0) = x_0, \\ y(k) &= Cx(k) + Du(k), \\ \text{or } \dot{x}(t) &= Ax(t) + Bu(t), \quad x(0) = x_0, \\ y(t) &= Cx(t) + Du(t) \end{aligned}$$

  a natural question arises: can we learn anything about $x(t)$ given $y(t)$ and $u(t)$ for a specific time t?
- Clearly, if we know $x(0)$ and $u(t)$ for all t, we can determine $x(t)$ via

$$x(t) = e^{A(t-t_0)}x(t_0) + \int_{t_0}^{t} e^{A(t-\tau)}Bu(\tau)d\tau$$

- However, if $x(0)$ if unknown, can you obtain $x(t)$ via only $y(t), u(t)$?