Observability for CT LTI Systems — 2

- We can write the previous equation as:

$$\begin{bmatrix} y(t_0) \\ \dot{y}(t_0) \\ \ddot{y}(t_0) \\ \vdots \\ y^{(n-1)}(t_0) \end{bmatrix} Y(t_0) = \underbrace{\begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}}_{=\mathcal{O}\in\mathbb{R}^{np\times n}} x(t_0) \Rightarrow$$

$$x(t_0) = \mathcal{O}^\dagger Y(t_0) = (\mathcal{O}^\top \mathcal{O})^{-1} \mathcal{O} Y(t_0)$$

- Hence, the initial conditions can be determined if the observability matrix is full column rank
- This condition is identical to the DT case where we also wanted to obtain $x(k=0)$ from a set of output measurements
- The difference here is that we had to obtain derivatives of the output at t_0
- Can you rederive the equations if $u(t) \neq 0$? It won't make an impact on whether a solution exists, but it'll change $x(t_0)$