Controllability-Observability Duality, Minimality

Duality

The CT LTI system with state-space matrices $(\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D})$ is called the **dual** of another CT LTI system with state-space matrices (A, B, C, D) if

$$\tilde{A} = A^\top, \quad \tilde{B} = C^\top, \quad \tilde{C} = B^\top, \quad \tilde{D} = D^\top.$$

Controllability-Observability Duality

CT system (A, B, C, D) is observable (controllable) if and only if its dual system $(\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D})$ is controllable (observable).

Minimality

A system (A, B, C, D) is called minimal if and only if it is both controllable and observable.