Quantifying Observability

Observability Test

For a system with n states and p outputs, the test for observability is that

matrix $\mathcal{O} = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix} \in \mathbb{R}^{np \times n}$ has full column rank (i.e., $\text{rank}(\mathcal{C}) = n$).

The test is equivalent for DTLTI and CTLTI systems

Theorem

The following statements are equivalent:

1. $\mathcal{O}$ is full rank, system is observable
2. PBH Test: for any $\lambda \in \mathbb{C}$, $\text{rank} \begin{bmatrix} \lambda I - A \\ C \end{bmatrix} = n$
3. Eigenvector Test: for any right evector of A, $Cv_i \neq 0$
4. The following matrices are nonsingular

$$\sum_{i=0}^{n-1} (A^\top)^i C^\top C A^i \quad \text{(DTLIT)} \quad \& \quad \int_0^t e^{A^\top \tau} C^\top C e^{A\tau} \, d\tau \quad \text{(CTLTI)}$$