Unobservable Subspace

- **Unobservable subspace: null-space of** $\mathcal{O}_k = \mathcal{N}(\mathcal{O}_k)$
- It is basically the space (i.e., set of states $x \in \mathcal{X}$ that you cannot estimate or observer
- Notice that if $x(0) \in Null(\mathcal{O}_k)$, and $u(k) = 0$, then the output is going to zero from $[0, k-1]$
- Notice that input $u(k)$ does not impact the ability to determine $x(0)$
- The unobservable subspace $\mathcal{N}(\mathcal{O}_k)$ is A-invariant: if $z \in \mathcal{N}(\mathcal{O}_k)$, then $Az \in \mathcal{N}(\mathcal{O}_k)$

Unobservable Space

The null spaces $Null(\mathcal{O}_k) = \mathcal{N}(\mathcal{O}_k)$ satisfy

$$\mathcal{N}(\mathcal{O}_0) \supseteq \mathcal{N}(\mathcal{O}_1) \supseteq \cdots \supseteq \mathcal{N}(\mathcal{O}_n) = \mathcal{N}(\mathcal{O}_{n+1}) = \cdots$$

This means that the more output measurements you have, the smaller the unobservable subspace.
It also implies that you cannot get more information if you go above $k > n$. You can prove this by C-H theorem ($A^n = \sum_{i=0}^{n-1} \alpha_i A^i$)