Observability — 2

$$\begin{bmatrix} y(0) \\ y(1) \\ \vdots \\ y(k-1) \end{bmatrix} = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{k-1} \end{bmatrix} x(0) + \begin{bmatrix} D & 0 & \dots & 0 \\ CB & D & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ CA^{k-2}B & \dots & CB & 0 \end{bmatrix} \begin{bmatrix} u(0) \\ u(1) \\ \vdots \\ u(k-1) \end{bmatrix}$$

$$Y(k-1) = \mathcal{O}_k x(0) + \mathcal{T}_k U(k-1) \implies \mathcal{O}_k x(0) = Y(k-1) - \mathcal{T}_k U(k-1)$$

- Since $\mathcal{O}_k, \mathcal{T}_k, Y(k-1), U(k-1)$ are all known quantities, then we can find a unique $x(0)$ iff $\mathcal{O}_k$ is full rank

Observability Definition

DTLTI system **is observable at time** k if the initial state $x(0)$ can be uniquely determined from any given

$$u(0), \dots, u(k-1), y(0), \dots, y(k-1).$$