Observability — 1

DTLTI system (n states, m inputs, p outputs):

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k), \quad x(0) = x_0, && (1) \\ y(k) &= Cx(k) + Du(k), && (2) \end{aligned}$$

- **Application**: **given that** A, B, C, D**, and** $u(k), y(k)$ **are known** $\forall k = 0:1:k-1$, **can we determine** $x(0)$?
- **Solution:**

$$\begin{bmatrix} y(0) \\ y(1) \\ \vdots \\ y(k-1) \end{bmatrix} = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{k-1} \end{bmatrix} x(0) + \begin{bmatrix} D & 0 & \dots & 0 \\ CB & D & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ CA^{k-2}B & \dots & CB & 0 \end{bmatrix} \begin{bmatrix} u(0) \\ u(1) \\ \vdots \\ u(k-1) \end{bmatrix}$$