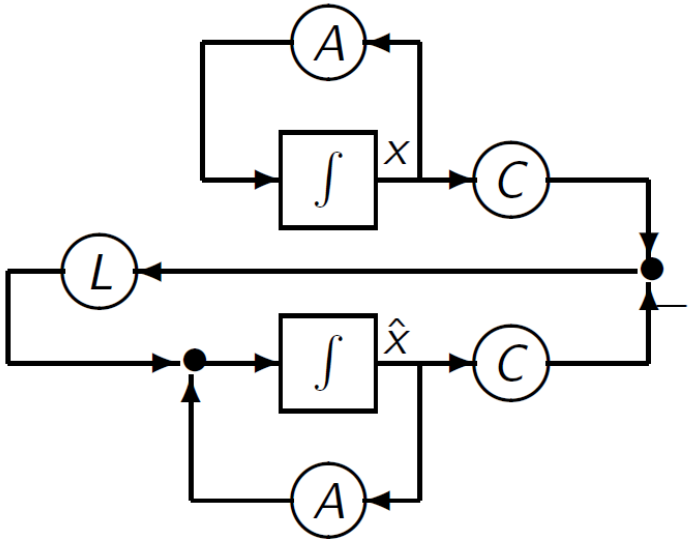
Observer Design

Original system with unknown $x(0)$:

$$\dot{x} = Ax,$$
$$y = Cx$$



Simulator with linear feedback:

$$\dot{\hat{x}} = A\hat{x} + L(y - \hat{y}), \quad \hat{x}(0) = 0$$
$$\hat{y} = C\hat{x}$$

- Objective here is to estimate (in real-time) the state of the actual system $x(t)$ given that ICs $x(0)$ are not known
- To do that, we design an observer—dynamic state estimator (DSE)
- Define dynamic estimation error: $e(t) = x(t) - \hat{x}(t)$
- Error dynamics:

$$\dot{e}(t) = \dot{x}(t) - \dot{\hat{x}}(t) = (A - LC)(x(t) - \hat{x}(t)) = (A - LC)e(t)$$

- Hence, $e(t) \to 0$, as $t \to \infty$ if $\texttt{eig}(A - LC) < 0$
- **Objective:** design observer/estimator gain L such that $\texttt{eig}(A - LC) < 0$ or at a certain location