Math For Economists
Problem Set 1

September 18, 2023

1. Show that a set is convex if and only if its intersection with any line is convex.
2. Find a solution of the following problem ($A \in S^n_{++}$)

$$\begin{aligned} &\min_x c^\intercal x \\ &\text{s.t.} \quad x^\intercal A x \leq 1 \end{aligned}$$

   How does a solution of the problem is related to a solution of

$$\begin{aligned} &\min_x c^\intercal x \\ &\text{s.t.} \quad x^\intercal A x = 1 \end{aligned}$$

   Prove your results.

3. A firm uses three inputs (such as materials, unskilled and skilled labor force) to produce output. If it uses inputs in quantities K, L, H the resulting quantity of output is ($A > 0, a > 1, \alpha \in (0,1)$)

$$F(K,L,H) = AK^{1-\alpha}\left(L^\rho + aH^\rho\right)^{\alpha/\rho}$$

   Your task is the following

   (a) Establish values of ρ for which the production function is concave.

   (b) Formulate a cost minimization problem for the firm and write down first order conditions.

   (c) Find elasticity of substitution between capital and unskilled labor, capital and skilled labor and unskilled and skilled labor. Try to provide economic intuition.

   (d) Solve the problem analytically and present cost as a function of output and factor prices.

   (e) Let $A = 1, a = 2, \alpha = 1/2, \rho = 1/2$, quantity produced and factor prices are $q = 2, P_K = 100, P_L = 200, P_H = 300$. Find optimal cost and factors. Check your results for the factors using Envelope Theorem.

4. Solve the following consumer problem

$$\begin{aligned} &\max_{x_1,x_2} \sqrt{x_1} + x_2 \\ &p_1 x_1 + p_2 x_2 \leq S \\ &x_1 \leq 1 \\ &x_2 \leq 2 \end{aligned}$$

   where $p_1 > 0, p_2 > 0$ and $S > p_1 + p_2$ are parameters