STATISTICAL QUALITY CONTROL

Second Project

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Title: Statistical Quality Control in Solar Panel Production Process

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Abstract

This report is about the application of statistical process control methods and more specifically about the construction of several control charts as regards the manufacturing process of solar panels in a company. More specifically, we are interested on monitoring the length consistency of the solar panels that the company produce. We will try to detect any assignable shift in the production process by using different type of control charts such as X-bar, S, CUSUM, and EWMA. So, through the simulation of production data, the report will illustrate how random variation within the process can be distinguished from a significant shift which will set our process out of control. This shift can be happened because of changes in the production process. The findings of this report highlight the sensitivity and timeliness of each control chart in detecting small and large shifts in the process. In this way, the report gives insight to the company for monitoring and improving the ongoing process of manufacturing of the solar panels.

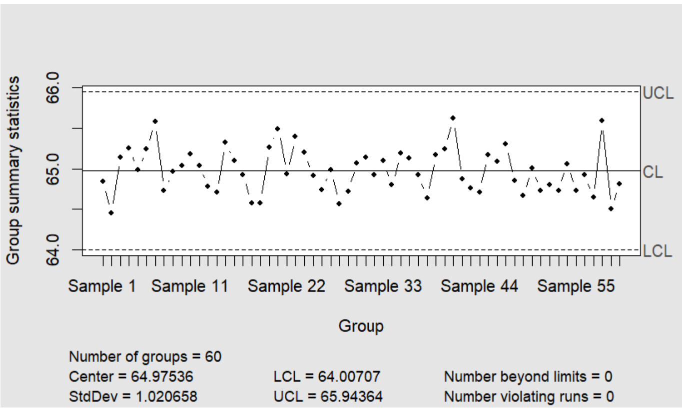
Introduction

So, let's suppose that we work in a company that produces solar panels where the average length of each panel is about 65 cm. The daily production amounts to 2000 pieces. To monitor the quality of the production process, the company draws 60 samples of 10 solar panels each. So as we understand the sample size n is 10. With these samples, we will measure the quantity of every product in order to detect if the procedure is under control or not. We will monitor the process and more specifically the length of the solar panels produced with the X-bar, S, CUSUM, and EWM control charts. Also, we will evaluate the performance of these charts in both small and large shifts in the process mean, as well as changes in process variability. Finally, we will compare these charts and we will result in which of them is better and in which case.

X Bar Chart

The very first thing to do was to construct a X Bar Chart. We denote that every sampling has sample size 10 and our data follow the Normal distribution with mean 65 cm and standard deviation 1 cm. This type of control chart is a tool that we use regularly in Statistical Quality Control to help us study and analyze the process average and monitor the between-sample variability. The corresponding chart for our experiment is the one bellow:

Figure 1: Phase 1 X Bar Control Chart

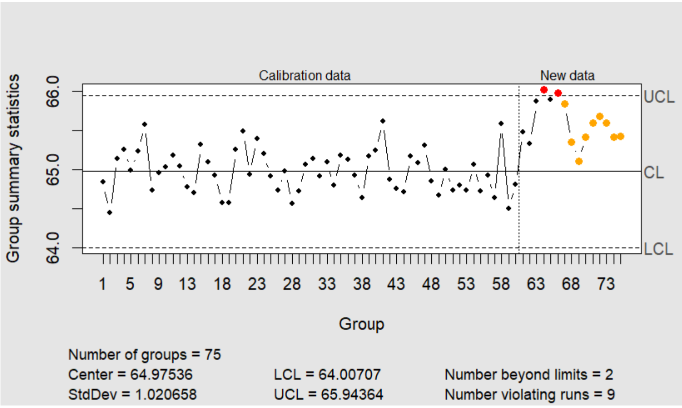


As we can see from the graph above, all the samples are within the control limits (LCL=

64.007 and UCL= 65.943), which verify that the process is under control. The center line, representing the average of all sample means is about 64.97 cm and that is the average length of the solar panels across all batches. Also, as we can see there are 60 samples (batches) represented on the X-axis. Finally, we do not observe any specific patterns on the plot, which could be a sign of systematicity in the process.

Now, suppose that we have a small shift at the mean by adding 0.7 cm (that means 0.7 standard deviations) and we want to examine if the X Bar-Chart will identify the shift. The chart below represents the X Bar Control Chart including this shift :

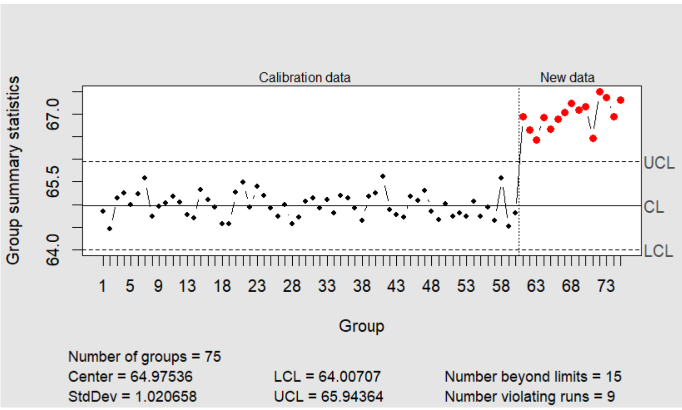
Figure 2: X Bar Control Chart with the small shift in the mean from 65 to 65.7



So, as we see in the above chart, the X Bar control chart identified this shift but not with great success, as there are only 2 red points beyond the upper control limit (UCL), which indicates that the process went out of control at the sample 64. Also, there are 9 violating runs which may suggest a non-random pattern within the data, something which also confirms the shift of the process. In this plot, there are 75 samples which are on the X-axis, and each sample as we have said corresponds to a subgroup of solar panel length measurements. Here, we have data before and after the change of the process. Finally, the new x bar chart made the shift visible and this shift indicates an increase in the mean of the solar panel length.

Now suppose that instead of having a shift of only 0.7 σ we have a big shift in the process and that is 2σ (2cm). We want to examine again how well will the X Bar-Chart identify this bigger change. The chart below represents the X Bar Control Chart for this bigger shift.

Figure 3: X Bar Control Chart with big shift in the mean from 65 to 67

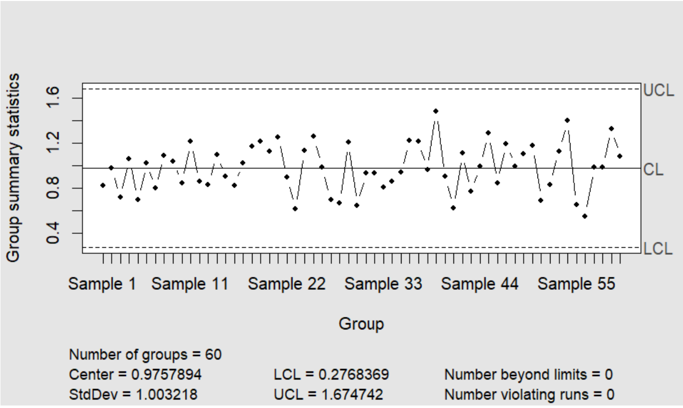


From this graph, we understand that the x bar control chart fully identified the shift in the process and this makes sense because we know that the x bar control chart is much more effective at detecting large shifts rather than small ones. Here, the shift is much clearer and it is a shift upwards from the original process mean. Now, we have 15 points out of the control limits so the process is significantly out of control. All these 15 points indicate that the mean of our process has changed and it is much higher than before. Also, in the chart we have 9 violating runs, which suggest that there is a pattern in the process. This is evidenced by consecutive points on one side of the center line, which signals a loss of control in the process.

S Chart

Next, we will construct a S control chart and we will apply a different scenario where we change the standard deviation. The S control chart is used in order to monitor the variance of the process. The S control chart for our experiment (without any shift) is the chart below:

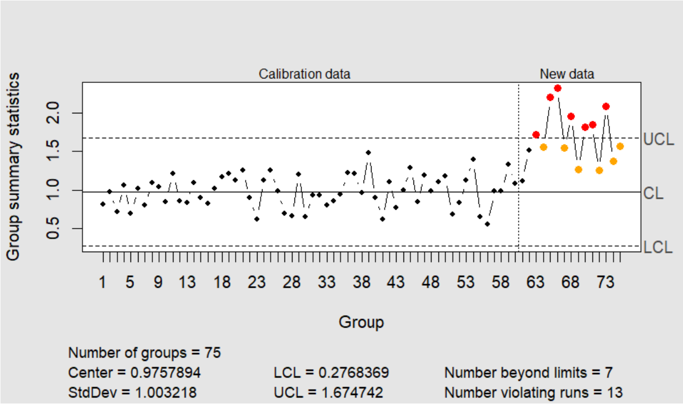
Figure 4: S Control Chart



The above S control chart has a central line equal to 0.97 which is the average of the sample standard deviations and also indicates the average variability of the solar panel lengths within each group of samples. Also we observe that all the points are within the control limits which are: the UCL is around 1.67, and the LCL is about 0.28. Also this chart proves that the process is under control.

Now, we performed a small change to the standard deviation and we set it equal to 1.5. The corresponding S chart is the one below:

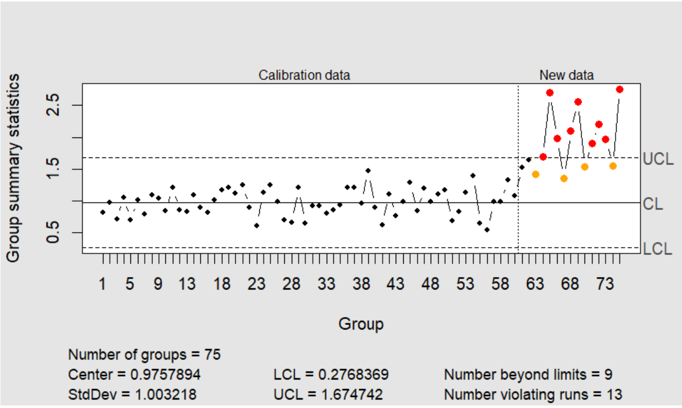
Figure 5: S Control Chart with small change on σ from 1 to 1.5



Initially, the calibration data shows that the process variability is stable with sample standard deviations fluctuating around the central line, which represents the average standard deviation across all groups. But, with new data arrived, as we can see from the plot there are 7 points beyond the control limits, something which suggests that there is excessive variability in the process and now our process is out of statistical control. Also there are 13 violating runs, and of course it's clear from the above S control chart that there are sequences of consecutive points like those above the Upper Control Limit. Those points show that there is a pattern in the process and confirm that the process variation has changed and is no longer stable. In conclusion, the above S chart effectively shows that there has been a small shift in process variability (σ), denoted by the increase in the sample standard deviations above the central line.

Another thing that we did was to perform a bigger change to the standard deviation and set it equal to 2 this time, so the sd doubled, from 1 now is 2. The corresponding S chart is the one below:

Figure 6: S Control Chart with big change on σ from 1 to 2



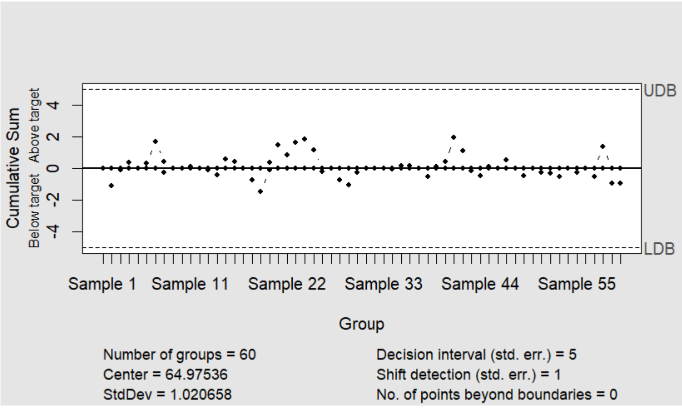
Now we see that the samples out of the control limits are even more, there are 9 this time. This indicates that our process is now significantly out of control. Again we count 13 violating runs which is a strong indicator of non-random patterns. In conclusion, S control chart is effective not only at demonstrating a small increase but also a big increase in process variability.

CUSUM Control Chart

The CUSUM control chart (Cumulative Sum) is another useful tool that we use in Statistical Quality Control in order to detect the small shifts in the mean that other charts like the Shewhart ones could not detect so easily. CUSUM control charts plot the cumulative sums of the deviations of the sample values from a target value and in this way they determine if a process has shifted off this target. So, in cumulative sum control charts, sums are accumulated, but an observation is accumulated only if it differs from the goal value by more than K units. Parameter K is named as a reference value.
So we constructed a CUSUM control chart in which the decision interval was set at 5 standard errors and the shift detection is set at 1 standard error.

For our experiment the CUSUM control chart is the following one:

Figure 7: CUSUM Control Chart

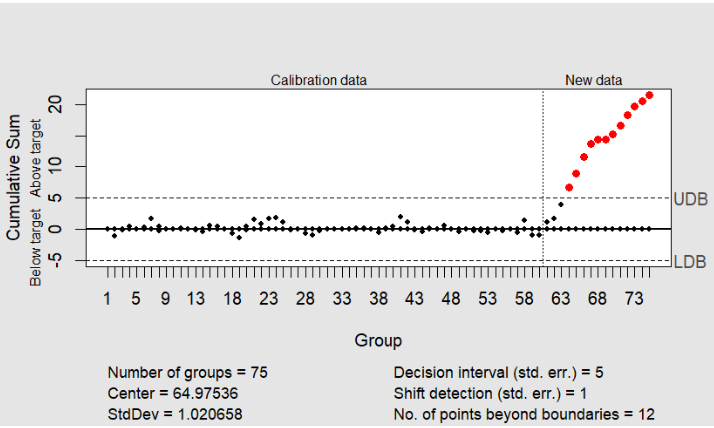


As we can see from the above graph, there are no points beyond the control limits neither above the Upper Decision Boundary (UDB) nor below the Lower Decision Boundary (LDB), so the process appears to be in statistical control. Also, there are no signs of a significant shift in the process mean based on the cumulative sum of deviations, which implies that the process is stable over time.

Now, suppose we have the same small shift at the mean as before of 0.7 cm (that means 0.7 standard deviations) and we want to examine if the CUSUM Control Chart will detect this shift and if it is more effective in identifying such small shifts than the X bar control chart. In general, we know that the Cusums are better in detecting small shifts.

The chart below represents the CUSUM Control Chart including this shift :

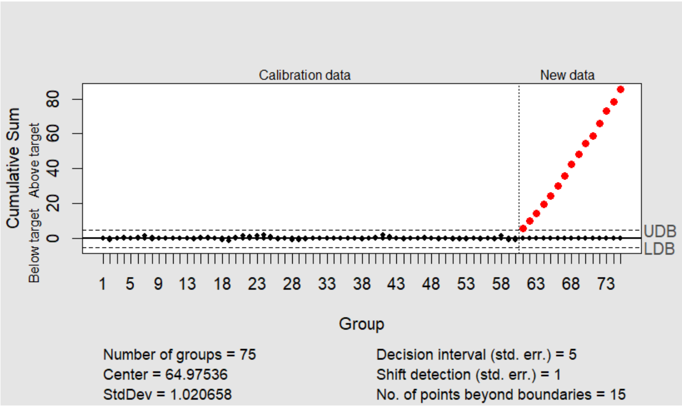
Figure 8: CUSUM Control Chart with the small shift in the mean from 65 to 65.7



In the above chart, we see a clear upward trend with 12 samples beyond the upper decision boundary (UDB). This indicates that a shift in the process mean has occurred, so the CUSUM control chart indeed did identify the shift fast and effectively.
Also, if we see again the X bar chart and this CUSUM control chart we can see that this one is more sensitive to the small shift than the X-bar and so it is better in detecting such small changes in the process. This sensitivity is evident in the way that the cumulative sum begins trending upward as soon as the small shift occurs. Also we can see the superiority of CUSUM. from the fact that the X-bar chart shows only 2 samples out of the control limits, while the CUSUM shows 12 samples. In addition, the CUSUM chart provides an earlier and clearer indication of the shift. This is critical in a quality control setting where early detection of process changes can prevent further deviation from the target. Finally, we can easily see that the trend in the CUSUM chart is more pronounced and clear, providing a strong visual indication of the shift.

We also did a CUSUM control chart but instead of having a shift of only 0.7 σ we have a big shift in the process and that is 2σ (2cm). We want to examine again how well will the CUSUM control Chart identify this bigger change. The chart below represents the CUSUM Control Chart for this bigger shift.

Figure 9: CUSUM Control Chart with the big shift in the mean from 65 to 67



In the above chart which is about a big shift in the process mean (2cm), we can see that the chart shows a very pronounced upward trend starting around the point where the new data begin and this trend seems to be very steep and continuous. This fact combined with the 15 samples out of the control limits indicate that our process is now out of statistical control.

Both the X bar control chart and the CUSUM effectively detect the large shift in the process mean, as in both charts we see multiple samples outside of the control limits.

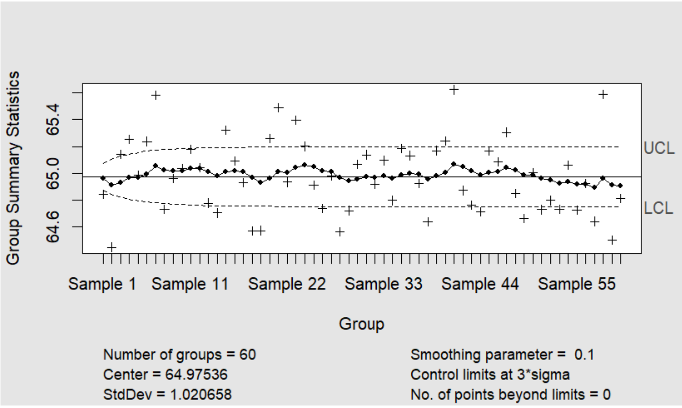
EWMA Control Chart

The EWMA (Exponentially Weighted Moving Average) control chart is used to monitor the mean of the process. It is known that EWMA control charts are extremely useful in detecting small shifts in the process. Also, they are robust when the normality assumption does not hold. So, the EWMA chart plots these weighted averages over time and applies control limits that are based on the sd and the smoothing parameter λ. EWMA chart uses the weighting smoothing parameter λ and we know that smaller values of λ are good if we want to detect small shifts in the process while bigger values of λ are for big shifts. The value of λ ranges between 0 and 1 but usually λ is between 0.05 and 0.25. A λ close to 1 gives more weight to the most recent observations and as I go back to the process I give smaller weight. A λ close to 0 gives more weight to past observations. For example if λ=0.7 this means that I give 70% weight in the last observation.

We will construct 3 EWMA control charts for our experiment and we will use λ=0.1 for all the 3 of them. The first one is about our data without any shift, the second is for the data with the small shift and the third is about the data with the big shift.

The first EWMA control chart is the following:

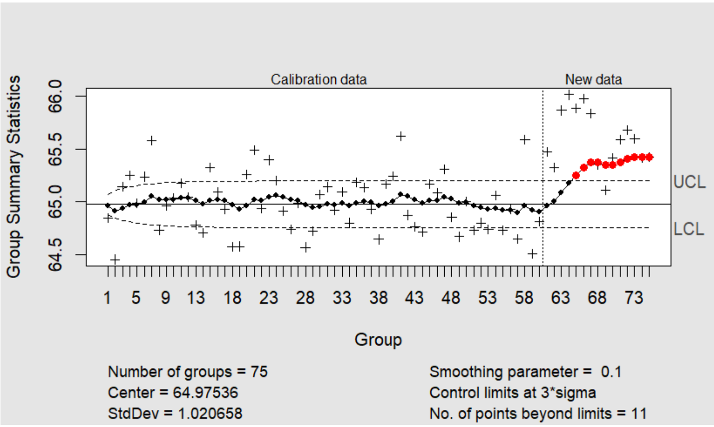
Figure 10: EWMA Control Chart



So in the above EWMA control chart we see that first of all the central line is approximately 64.97536, which corresponds to the process mean (65). Moreover, there are no samples out of the control limits (UCL and LCL) and no presence of any systematic pattern, indicating that the process is reasonably stable and in control. More specifically, we see that the control limits in this graph are LCL= 64.75322 and the UCL= 65.19749. Also, as we said also before, the smoothing parameter (λ) in this graph is 0.1, which mean that I give only 10% weight in the most recent observation and I put relatively more weight on past observations.

Now, suppose that we have a small shift at the mean by adding 0.7 cm (that means 0.7 standard deviations) and we want to examine if the EWMA Control Chart will identify the shift. The chart below represents the corresponding EWMA chart:

Figure 11:EWMA Control Chart with the small shift in the mean from 65 to 65.7



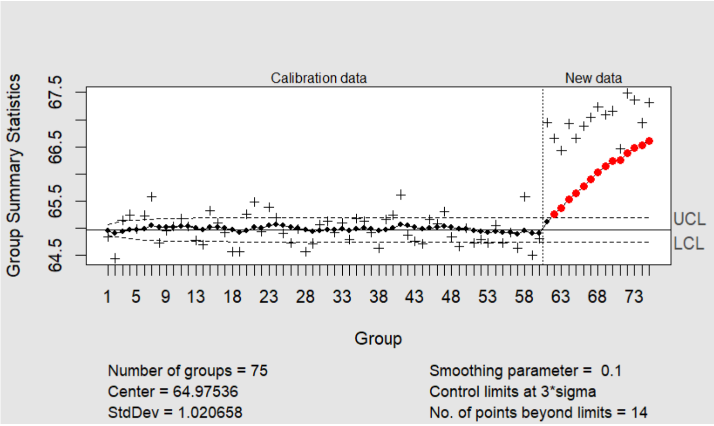
The first segment labelled "Calibration data" shows the initial process variation, with all points well within the upper and lower control limits, and the EWMA line relatively stable and close to the center line. With new data arrived, we see a clear upward trend, with 11 samples out of the control limits, something that shows us that a change has occurred in the mean of the process. So, The EWMA chart, with λ set to 0.1, appears to have effectively detected the small shift in the mean (from 65 to 65.7).

Now, if we want to compare this with the X-bar chart for detecting a small shift, the EWMA chart's ability to give more weight to recent data without ignoring the historical context gives us a clear, early detection of small shifts. So, the EWMA chart provides a more responsive indication of small shifts in the process mean.

Finally, we will examine the capability of an EWMA control chart to detect big shifts of

2σ. The corresponding EWMA chart is the one below:

Figure 12:EWMA Control Chart with the big shift from 65 to 67



In the beginning (Calibration data) the process was in control, with all points well within the control limits but when the new data arrived, where the process mean shifts to 67, we see a distinct and rapid upward trend, with 14 samples out of the control limits. This show us that our process is now out of control due to a significant shift in the mean. So, the EWMA chart, with a smoothing parameter (λ) of 0.1, has efficiently identified the big shift.

Conclusions

In conclusion, after all this analysis of the control charts we saw that each chart has its unique strengths and sensitivities in detecting shifts within the manufacturing process. The CUSUM and EWMA charts showed increased sensitivity to small shifts in the process mean, providing a clearer indication of the shift compared to the X-bar chart. In the case of large shifts in the mean, all chart types clearly indicated that the process is out-of-control by showing plenty of samples to be out of the control limits. As regards the increase in process variability (σ), the S chart was very effective in detecting the change, especially when the standard deviation doubled (the big shift in σ). All these insights indicate which chart types should the company use depending on the specific monitoring goals.