STATISTICAL QUALITY CONTROL

First Project

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Abstract

This report presents a comprehensive overview of different control charts which are useful in monitoring processes with multinomial data. Firstly, we explain how the control charts are designed, we introduced their principles and we note the importance of them in statistical quality control. After that, we refer to the traditional control charts (like p and c charts) and their limitations in such processes due to their inherent design for continuous or binary data. The need for designing and constructing more complex control charts to monitor processes with multinomial data is now obvious. So in this the report we also investigate more specialized control charts like those based on the Chi-square statistic and Cumulative Sum (CUSUM) control charts and we highlight their properties and capabilities. Also, we explore the Multiattribute Control Charts (MACCs) and Likelihood Ratio-based Multiattribute Control Charts (LR-MACCs) which represent a significant advancement in monitoring processes with multiple correlated attributes and can be really useful also in processes with multinomial data. Also, further exploration is dedicated to Generalized Likelihood Ratio (GLR) control charts, which utilize statistical tests to detect subtle process shifts, offering an alternative for processes exhibiting more complex patterns. Additionally, we explore the concept of Fuzzy Multinomial Control Charts which is a novel approach that is extremely useful when the classification of a product/service is not so straight forward. Finally, we refer to Multivariate Attribute Control Charts utilizing the Mahalanobis distance, which is a technique that stands out in the context of multinomial data, as this approach considers all the categories together and because is more sensitive to shifts in quality as it considers the overall distribution of defects across categories, not just the rate within each one.

Introduction

One of the most useful tool in Statistics Process Control is control charts. Control charts have become one of the most commonly used tools for monitoring process variations in multiple environments, like manufacturing industry or health industry. Using control charts we are able to detect a shift in the process and identify abnormal conditions. In this way, if for example we are talking about a production process, someone can diagnose easily and fast enough a problem in the process, reduce process and product variation, reduce the loses and also improve the quality of the product and the financial performance of the company. All the control charts are constructed by gathering and using data plenty of samples of data from the process that we want to monitor, then compute a sample statistic and plot it on control charts. In general, control chart design requires specification of sample size, control limit width, and sampling frequency. In control charts we have 2 phases: in phase 1 we establish and estimate the control chart's parameters and we calculate the initial control limits based on our observed data. Control limits are derived from natural process variability, or the natural tolerance limits of a process. Once I have constructed the control chart phase 2 begins, in which, we plot new data points on the control chart to monitor the process's current performance against the established control limits. Here I want to be able to identify signals and patterns that indicate a shift in the process, such as points beyond the control limits. I do not want to see any patterns (e.g. many points below the center line) . The nice thing to see is randomness as regards the samples. If I see something out of control I implement some necessary actions to bring the process back in control.

In order to understand better what is a control chart Shewhart propose a general model for constructing control charts to identify the occurrence of assignable causes as the following : suppose that we have a sample test statistic which measures some quality of interest in a process. The mean of this sample statistic is μ and the standard deviation of it is σ. In a control chart there are : the **center line** (CL), the **upper control limit** (UCL), and the **lower control limit** (LCL) . These are defined like this: $UCL = \mu + \kappa*\sigma$, $CL = \mu$ and $LCL = \mu - \kappa*\sigma$, where k is the "distance" of the control limits from the center line and it is expressed in standard deviation units. The usual choice for k is 3 which means that the Type I error probability a,that is the probability of rejecting Ho when Ho (my process is in controll) is actually true, is 0.0027. However, control limits can be designed by specifying the Type I error level for the test. This type of control limits are called probability limits. Also, there are the warning limits which are basically 2 sigma limits. Generally, there are 2 types of control charts: first the Variables Control Charts which are used for characteristics that can be measured on a continuous scale and are useful when the quality characteristic can be quantified and precise measures are possibl and second the Attributes Control Charts which are used for characteristics that are counted and can be categorized, such as the number of defects or defectives. Some of the most known variables control charts are the X-bar which is used for controlling the process average and basically it monitors the between-sample variability, the R(xmax-xmin) control charts which measure the within-sample variability and S chart where both are used for controlling the process variance. For the x bar chart, the logic is to choose as small a sample size is consistent with magnitude of process shift one is trying to detect. All of these control charts are designed for continuous data that often assume a normal distribution. R chart is relatively insensitive to changes in process standard deviations and is more sensitive to departures from normality than x bar chart, while S chart is better for larger samples. We can construct X bar and R charts using probability limits. These control charts are better in detecting big changes and not so good in finding the small shifts in the process.We can see an example of a Shewhart control chart in the picture below.

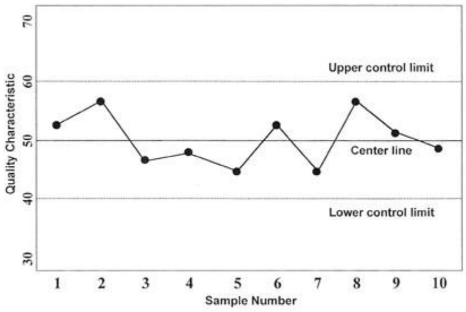

Figure 1: Control Chart example

In the above plot, all the points are within the control limits fact that indicates the process is in control and that we do not need to take any action. If we had at least one point out of the control limits, we should begin the investigation and the corrective action to find the cause of this.

In statistical quality control, shifts are divided into chance shifts and assignable shifts. Chance shift means that the shift is just a random variation which is caused under the natural condition, is inevitable, and its influence is small so that we do not care. In general, we say that a process which is operating with only chance causes of variation is in statistical control. Now, the changes that we care are the assignable ones should not occur in the regular production process, because when a shift like this happens it causes a malfunction into the system and it means that our process is out of control. If an assignable change has occurred then the sample statistic will plot a sample outside the control limits that we have already constructed by the above way. From the time that we will observe an assignable shift it has to begin a procedure so as the problem will be corrected and the process will return to the in control state. Vining and Reynolds, said that in the process of constructing a control chart, someone has to consider three factors: 1) the false alarm rate of the chart, 2) the ability of the chart to detect an out-of-control process, and 3) the sampling costs of the chart. We need a balance between these three factors. Of course, effective use of control charts requires periodic review and revision of control limits and centre lines.

Traditional Control Charts

There are plenty of extremely well known attribute control charts which monitor processes with only two categories. Some of those charts are the Fraction nonconforming chart (p chart) , Defective chart (np chart) which are used for binomially distributed processes, the control chart for the number of defects or non-conformities (c chart) and the Defects per unit chart (u chart) which are used for Poisson distributed processes. Np charts monitor the number of defects while p charts the percentage of them. In p chart the sample size n can be either constant or non-constant. We have to note here that NP chart cannot be designed if n is variable and that's why in this case we construct u control charts. All of those 4 control charts are known as Shewhart attribute control charts. P and c charts are one-dimension or univariate charts. This means that these 2 types of control charts track a single quality characteristic and assess the overall quality of a product based only on this single attribute. As a result, these control charts may not capture complex scenarios where multiple interrelated quality attributes affect the overall quality of a product or a service. Traditional control charts primarily monitor the mean or the standard deviation of a process to detect shifts. Also, usually this type of control charts use normal or other relevant distributions to define the control limits for the data. For more information about p ,u, np and c control charts someone can also see the book of Douglas C. Montgomery (Montgomery).

However, today there are many situations where the quality of a product can and have to be assessed by several correlated attributes at the same time and because of the modern technology like sensors and powerful computers someone can monitor all these attributes quite easily. So, in some cases we have more than 2 categories which characterize the situation of process control and the quality control environments in which we deal with multi-attributes and multinomial data are plenty.

So, as we see instead of just classifying items into just 2 categories, conforming and non-conforming, products can be classified into many classes. Products are classified either as "conforming" or "nonconforming", depending upon whether or not they meet specification. This type of classification exists mostly in processes where the quality characteristics cannot be easily measured on a numerical scale. Quality characteristics of this type are called attributes. Generally, when we have only two classes, the process can be considered as a Bernoulli process. But, products can also be categorized to many categories such as extremely bad, bad, ordinary, good, really good or into 1%, 2%, 5%, 10% and greater than 10%. In all these examples, we deal with multinomial processes with a number of categories (for example here 5) with parameter vector p = (p1, p2, p3, p4,p5), where p1 , p2 , p3, p4 and p5 are the probabilities that an outcome from the process falls into each of the five categories, respectively. In practice, usually if items are classified into T classes we use T-1 of them and the remaining class can be determined from these T-1. Multinomial processes lead to the construction of multinomial control charts which enable businesses, health departments and other industries to identify not just the occurrence of shifts but also the distribution of output quality across a spectrum of classifications. So, there is a need of construction of multinomial control charts which are based on the multinomial distribution, which is suitable for categorical data like the examples we said above (bad, good etc.) and keep track of the proportions of each category over time, focusing on changes in the distribution pattern among the different categories. Of course, all this analysis of the multinomial control charts may be more complex than the traditional ones because we need to consider multiple categories at the same time. Also, we need to note that control limits in multinomial control charts are determined based on the multinomial distribution properties, which consider the probability of observations falling into each category.

The multinomial control charts can be extremely helpful and valuable for plenty of industries. For example, if we are talking about the quality of a product and this quality is multidimensional, means that may depend on a lot of factors, different levels of quality have different implications for customer satisfaction, cost, and safety, so we need to know the size and the direction in each component and we can do this by using this type of control charts. Some general cases in which multinomial control charts are really helpful are the following:

- Manufacturing Industry: In manufacturing industry products can have multiple types of defects (minor, major, and critical), so multinomial control charts can monitor the proportion of units falling into each defect category over time.

- Service Industry : In service industries, where customer feedback or service quality may be categorized into several categories or levels (e.g., excellent, good, poor, extremely poor). Multinomial control charts can track changes in the distribution of these ratings over time to assess the service.

- Healthcare: In healthcare, treatment responses may be classified into several categories (e.g., significantly improved, improved, same, deteriorated). Here, multinomial control charts can help in monitoring the distribution of these outcomes to ensure the effectiveness of the treatment.

Control Charts based on Pearson's chi-square

Until today, most of the control charts which are used in order to monitor multinomial processes rely on the Pearson's chi-square statistic where items are grouped into samples of size n and then we plot the proportion of items with Xi= 1 for each sample. Basically, is the use of independent samples for monitoring changes in parameters of a multinomial process with a Pearson X^2 goodness-of-fit test, which means that the X^2 statistic will detect deviations from the expected distribution of the multiple categories. The in-control probabilities corresponding to the k categories are either assumed to be known or they are estimated from a preliminary sample. If the in-control probabilities for the categories are known to be p1,p2,…pk, then the control chart statistic is simply Pearson's goodness-of-fit statistic where n is the sample size for the sampling period and ni is the number of items in the sample falling into category i, with i = 1, 2 , ... , k. But, we need to have a large sample size n e.g. Marucci (Marcucci, 1985) and Nelson (Nelson, 1987) so the test statistic to follow the asymptotic chi-square distribution with k-l degrees of freedom when the process is in-control. As we know, the asymptotic chi-square distribution of Pearson's chi-square statistic is specifically known for an infinite sample size. So, to understand it better, we calculate X^2 values and then they we plot them on a control chart over time. Control limits are determined based on the X^2 distribution, taking into account the degrees of freedom and the desired level of statistical significance (usually a=0.05). If the X^2 statistic exceeds the upper or under control limit, this indicates a significant deviation from the expected distribution of charts and signals a change in the quality target parameters meaning that the process is out of control now but we cannot distinguish between increasing or decreasing levels of quality. So, we cannot say whether the defect proportion is increasing or not. A difference from traditional charts which focus on the mean or the standard deviation of a process is that those control charts specifically monitor the distribution of categorical outcomes across multiple categories and the fact that they are particularly useful for identifying shifts in the process that affect the distribution among multiple categories. However, these methods require only a nominal measurement scale, which is not always the case since classifications are often made with some ordinal relationship and although we may use the X^2 statistic also when we deal with ordinal data it is almost sure that some information will be lost in this case. For more information about control charts based on Pearson Chi Squared someone can check the paper by Nelson L.S. (Nelson, 1987).

Now, when the sample size is small, it is not appropriate to adopt the asymptotic chi-square distribution of Pearson's chi-square statistic to construct the multinomial-proportion control chart. The reason behind is that the average run length ARL of the asymptotic control charts (is the average number of samples until I have one out-of-control sample) may significantly deviate from the pre-specified ARL. So, as a result we may over- or under-adjust our multinomial process. Generally, when our process is in control a long ARLo is preferred, while in the case that our process is out of control a short ARL1 is preferred. So, as we see the ARL is a measure to see the efficiency of a control chart.

MACC and LR-MACC

Multiattribute Control Charts (MACCs) are extremely useful in monitoring and managing quality in multivariate attribute processes that deal with multinomial data. In general, MACCs have the ability to simultaneous monitor multiple quality attributes and in this way they have an advantage over traditional univariate control charts. However, one of the limitations of MACCs is that they focus on detecting process deterioration, without the ability to identify improvements or specific shifts in the quality distribution across multiple categories. MACCs are particularly crucial in cases where quality attributes are correlated and can effectively identify shifts in the process that might not be apparent when examining each attribute individually.

On the other hand, Likelihood Ratio-based Multiattribute Control Charts (LR – MACCs) use the likelihood ratio test within a multinomial framework to detect any change in the process. It does not matter if it is a good or a bad change. This method is particularly effective for multinomial data, where quality outcomes can be divided into multiple categories. LR-MACC can also locate the specific attributes responsible for the shifts.

For more information someone can see the review of Ms Topalidou and Mr. Stelios Psarakis (Psarakis & Topalidou, 2009).

CUSUM Control Charts

Now, in the cases where observations become available individually, someone should use a chart based on individual observations (n=1) rather than wait until a group of

$n > 1$ observations have been formed. A case like this is when there is automated inspection and measurement technology and every unit manufactured is analyzed and there is no basis for rational subgrouping. The rational subgroup concept means that subgroups or samples should be selected so that if assignable causes are present, chance for differences between subgroups will be maximized, while chance for difference due to assignable causes within a subgroup will be minimized. Another situation now can be when the data become available really slow, and we cannot wait until sample sizes of $n>1$ to be accumulated. As you may understand the long interval between observations may cause problems with rational subgrouping. For all these cases and many more where the sample size used for process monitoring is $n=1$, that is samples consist only of 1 unit, CUSUM (Cumulative Sum) control charts can be very effective. CUSUM control charts plot the cumulative sums of the deviations of the sample values from a target value and in this way they determine if a process has shifted off this target. So, in cumulative sum control charts, sums are accumulated, but an observation is accumulated only if it differs from the goal value by more than K units. Parameter K is named as a reference value. Ryan (Ryan, Wells, & Woodall, 2011) proposed the CUSUM (cumulative sum control chart) which as we said is a control chart really helpful for monitoring a multinomial process using individuals observations instead of groups of them. The multinomial CUSUM chart assumes that the direction and the size of the out-of-control shift in the parameter vector p can be prespecified. This means that it relies on pre-specified out-of-control multinomial proportions. CUSUM control charts work well whatever the magnitude of process shifts but the analysis of average run length (ARL) shows that they work better for small and medium shifts, so they could be an effective tool for quickly identifying the change point in a process. So CUSUM control charts work by cumulatively summing deviations of individual process measurements from the target value and in this way they manage to detect the shift in the process. In the case of multinomial data, CUSUM control charts would focus on monitoring the cumulative sum of deviations in the probabilities of each category from their expected values. This involves tracking changes in the distribution of categorical outcomes over time. However implementing CUSUM charts for multinomial data is a bit complex.

So, the construction of a CUSUM control chart, requires the knowledge of the probability distribution of the quality characteristics and this is not the case in real life. These control charts as we said are better than Shewhart control charts in detecting small changes, because Shewhart ones are primarily designed to detect relatively large shifts in the process and they are not so able in efficiently detecting small, gradual changes or trends in the process. In contract, CUSUM combine information from several samples and in this way are more effective. But on the other hand, when it comes to large, sudden shifts in the process, there is no clear option between these two and usually we have to use a combination of them.

Of course the shift in the process could be in any direction and it might not be prespecified. In these situations, control charts like CUSUM which can only detect changes in specific directions are not suitable and we have to use control charts of another type. Ryan (Ryan, Wells, & Woodall, 2011) suggested using multiple one-sided Bernoulli CUSUM charts, with T Bernoulli CUSUM charts when there are T categories, for cases where the direction of the shift is unknown. Of course, as anyone can understand, constructing all these multiple control charts may not be so easy. In addition, it has been observed that the time to detect the change may be longer than the in-control expected time until a false alarm. So this option for sure is not the optimal one.

GLR Control Charts

In the search of finding control charts for monitoring multinomial processes when the size and the direction of the shift is unknown, we look at GLR control charts. GLR charts have the advantage that the size of the shift is actually estimated in the process of calculating the GLR statistic, so GLR charts can detect a wide range of shifts fast enough and without using a tuning parameter. Also, in the literature we see that when the size of the shift is unknown control charts based on the GLR algorithm seem to be better than Shewhart- and CUSUM-type charts. GLR charts for multinomial processes are called MGLR charts and are used for monitoring p based on a likelihood ratio statistic when the shift direction is unspecified and individual observations are obtained. GLR control charts are designed in a way to be sensitive to changes in multiple parameters simultaneously. The advantage of GLR control charts is that the procedure is very robust to the time that the assignable cause occurs but they have a drawback which is that they are very computationally intensive. But, with the increasing capabilities of modern computers we may see an increase in the use of GLR-based process monitoring procedures.

Now, an alternative instead of using MGLR would be a set of two-sided Bernoulli CUSUM charts but it is shown that the MGLR chart have better overall performance over a wide range of unknown shifts. Someone can see more about this type of control charts in the paper of Lee J, Peng Y, Wang N, Reynolds Jr MR. A GLR control chart for monitoring a multinomial process (Jaeheon , Yiming , Ning , & Marion R, 2017)

Fuzzy Multinomial Control Charts

Also there is a fuzzy multinomial chart (FM -chart) for monitoring a multinomial process. Control limits of FM -chart are obtained by using the multinomial distribution. This type of control charts are extremely useful when the classification is not so straight forward meaning that for example a product cannot be easily classified into distinct, non-overlapping categories. The effectiveness and accuracy of an FM chart depend on two main components: on the degrees of membership (the representative value) and the form of the fuzzy membership function. In fuzzy logic, the degree of membership is a way to say how much probable is a specific observation to belong to a specific category. It ranges between 0 and 1, where 0 means "does not belong at all" and 1 means "fully belongs. For example, a product may be 0.8 excellent and 0.2 average, so there is a part on both categories. The membership function now is the way of how the degree of membership is calculated. This function takes into account the characteristics of an observation and then it calculates a membership degree between 0 and 1 for each observation. There are several membership functions in statistical quality control (linear, triangular, trapezoid, S, etc.). For instance when using triangular fuzzy number (TFN) and fuzzy mode transformation, the degrees of membership for the four categories could be obtained such as 0 for “excellent”, 0.25 for “good”, 0.5 for “medium” and 1 for “bad”. Also, as someone can imagine the shape of this function shows us how is the transition between different degrees of membership occurs. Fuzzy multinomial control charts in many circumstances tend to accurately reflect the real-world complexity and vagueness of classifications and as a result tend to give better results than traditional control charts like the p chart. A situation of when the FM charts work better can be when the process or product quality assessment involves vagueness or when there are multiple categories of classification with overlapping characteristics. That is because FM chart can be more sensitive to small shifts in quality levels across its multiple categories and also the FM chart minimizes the risk of misclassification that can occur in traditional methods when products or processes are forced into binary categories. Raz and Wang propose assigning fuzzy sets to linguistic terms, making it possible to deal with the inherent vagueness in such classifications and also combine these terms for each sample using fuzzy arithmetic and having as a result a fuzzy set. The idea is to plot a measure of centrality of this aggregate fuzzy set on a Shewhart-type control chart. Here we have to say that a linguistic term-variable is a variable whose values are not numbers, but words or sentences in a natural or artificial language. The concept of a linguistic variable is derived because we needed a way to describe events that cannot be easily described in a quantitative way and all this led to fuzzy logic. For more information on the studies and the use of linguistic data, the interested readers are referred to Alcala (Alcala, et al., 2003) , Bordogna and Pasi (Bordogna & Pasi, 2000) . Each value of a linguistic variable is a fuzzy set. Taleb and Limam (Taleb & Limam, 2005), developed two monitoring statistics for the case of monitoring multivariate multinomial quality characteristics. The first was based on the fuzzy theory and the second on probability theory. The first statistic, is obtained by transforming fuzzy observations into their representative values and its distribution is derived using resampling methods such as bootstrap, while the other statistic is a linear combination of dependent chi-square variables and its distribution is derived by the Satterthwaite's approximation. In fuzzy approach , each fuzzy subset is converted into representative values (degree of membership) in order to maintain the standard format of the control chart. The second approach, builds a control chart based on the probability density function existing behind the linguistic terms. Both approaches lead to the construction of control charts that can be used for multivariate processes for products that are classified to more than 2 categories. As regards to which approach may be better, the fuzzy one seems to have better results.

In practice, for example in industry, a product or process outcome might be evaluated on multiple quality characteristics like functionality etc. , where each of this can be described using linguistic variables L1, L2, ..., Lp, j=1,...p .For each linguistic variable L1,L2,… Lp there exists a set of linguistic terms *(Ljk*) that describe the possible states or evaluations of that attribute, like bad, medium etc. For example, if $L1$ is durability , the linguistic terms might be $L11$= "high", $L12$= "medium", and $L13$= "low", with $c1$=3 since there are three terms used to describe durability. As Raz and Wang proposed, each Ljk is associated with a fuzzy subset Fjk which is described by a membership function μjk and can be converted into a representative value using one of the four existing transforming methods: fuzzy mode, fuzzy median, α level fuzzy midrange and fuzzy average. For more information about fuzzy multinomial control charts someone can check also 1Amirzadeh, V., M. Mashinchi, and M. A. Yaghoobi (Amirzadeh, Mashinchi, & Yaghoobi, 2008) article.

Multivariate Attribute Control Charts using Mahalanobis distance

In multinomial distribution, another case is of grouped data is the one resulting from the classification of products into several categories of non-conformities when monitoring the proportion non-conforming of all the categories. By categories of non-conforming we mean for example for a product that comes off the production line we can have different types of defects: paint errors, missing pieces, all these are categories of non-conformities. Non-conformities as we have said also above, are deviations from the specified standards or expectations for a product, for example from the standard deviation or the mean of a quality of the product. Mukhopadhyay (Mukhopadhyay, 2008) used the Mahalanobis distance within a multinomial distribution framework to develop a multivariate attribute control chart. Attribute control charts monitor the defects. Unlike the traditional approach which is to construct multiple p-charts that monitor each category in each one, basically one for each category of defect, this approach considers all the categories together. In p charts we test several hypotheses of equality of proportion defective independently (H0 : pi = p), which leads to a false type I error while with the new approach we have a simultaneous test of pi = p. By testing the equality of proportions across multiple categories simultaneously (pi = p), rather than independently, it avoids the cumulative inflation of Type I error, which is denoted as a and is the false alarm that the process has been shifted out of control when the process is actually still in control and operating as expected. In other words, Type I error is the probability of rejecting the Null Hypothesis Ho that my process is in control while Ho is actually true. Also this method is more sensitive to shifts in quality because it considers the overall distribution of defects across categories, not just the rate within each one. Being more sensitive basically means that it has bigger ability to detect smaller or more complex changes in the process that might not significantly alter the defect rate within any single category but do alter the overall pattern of defects. In this method what we do is to compare the vector of proportion defective at a specific point of time with the vector of average proportion defective, where the Mahalanobis distance (D2) plays a significant role. The resulting D2 control chart controls simultaneously proportion defectives falling in various categories of defects in a single chart. This approach enhances the ability of people working in statistical quality control departments to monitor and improve process performance comprehensively and effectively. For more information about Multivariate Attribute Control Charts using Mahalanobis distance see the article of Psarakis and Topalidou. (Psarakis & Topalidou, 2009).

Conclusions and Future Research

In conclusion, the exploration of control charts for multinomial data within this report shows the multiple and different control charts and their importance in the field of statistical process control. From the traditional control charts to the need of more advanced and complex control charts in order to be able to monitor processes with multinomial data, the report notes the importance of these tools in ensuring the quality of a process. More specifically, we saw how the control charts and their control limits are designed. Also, we saw some cases of univariate charts and their limitations when dealing with multinomial data, control charts based on Pearson X^2 which monitor changes in parameters of a multinomial process with a Pearson X^2 goodness-of-fit test. We explored MACCs which simultaneous monitor multiple quality attributes and LR-MACCs, we described CUSUM control charts which we saw that they are better than Shewhart control charts in detecting small changes. In addition, we explored the Fuzzy Multinomial control charts that are extremely useful when there are multiple categories of classification with overlapping characteristics. All the above methodologies represent offer improved sensitivity and specificity for detecting shifts in process distributions characteristic of multinomial outcomes. Finally, in this report we note several control charts which are and will be even more useful because today processes are becoming increasingly complex and the demand for quality intensifies and for sure the industries need and will need to accurately monitor and analyze multinomial and multiattribute data.

Now as regards the future research that can be done, for example for the fuzzy multinomial control charts some subjects are: how many linguistic terms should be defined? and second, how should the degrees of membership of linguistic terms be constructed? Also, future research could explore the development of models that combine fuzzy logic with other statistical or machine learning approaches to create more robust and adaptable control charts. In addition, the exact relationship between the degree of fuzziness and sensitivity of control charts is also yet to be investigated in the future.

Moreover, further research can be about the integration of machine learning algorithms with control charts. With the use of these algorithms we may be able to predict shifts in process parameters and adjusting control limits in real-time. More specifically, a case is the use of neural networks in attributes monitoring which has not been investigated a lot. Artificial intelligence algorithms could also aid in automating the interpretation of control chart signals, identifying potential causes of the assignable shifts without manual intervention, and even automating recommending of some corrective actions in order the process to be again in control.

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