Modelo ARX :

$$y(t) + a_1 y(t-1) + \cdots + a_{n_a} y(t-n_a)$$
$$= b_1 u(t-1) + \cdots + b_{n_b} u(t-n_b) + e(t) \qquad (4.7)$$

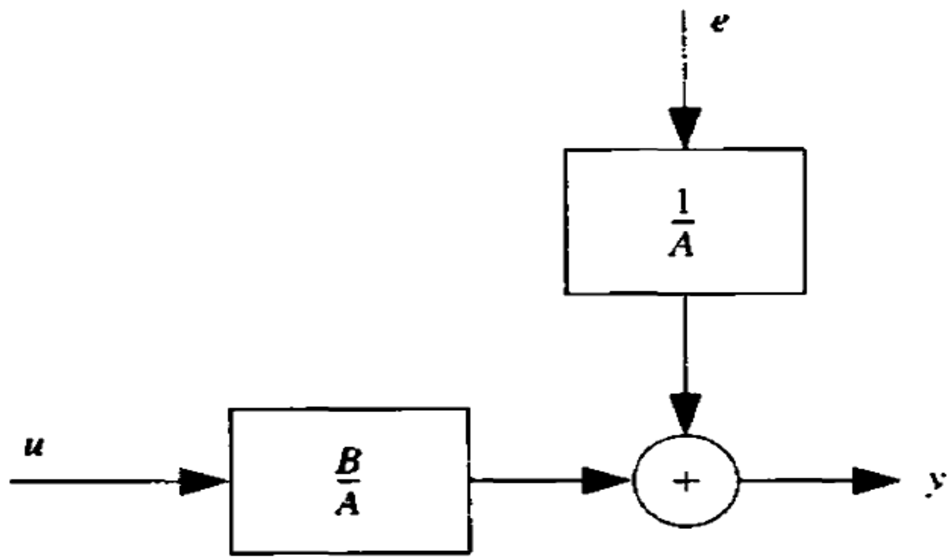


Figure 4.1 The ARX model structure.

If we introduce

$$A(q) = 1 + a_1 q^{-1} + \cdots + a_{n_a} q^{-n_a}$$

and

$$B(q) = b_1 q^{-1} + \cdots + b_{n_b} q^{-n_b}$$

we see that (4.7) corresponds to (4.4) with

$$G(q,\theta) = \frac{B(q)}{A(q)}, \qquad H(q,\theta) = \frac{1}{A(q)} \qquad (4.9)$$

Remark. It may seem annoying to use q as an argument of $A(q)$, being a polynomial in q^{-1}. The reason for this is, however, simply to be consistent with the conventional definition of the z-transform; see (2.17).

We shall also call the model (4.7) an ARX model, where AR refers to the autoregressive part $A(q)y(t)$ and X to the extra input $B(q)u(t)$ (called the exogeneous variable in econometrics). In the special case where $n_a = 0$, $y(t)$ is modeled as a finite impulse response (FIR). Such model sets are particularly common in signal-processing applications.

Para el caso ARMAX:

$$y(t) + a_1 y(t-1) + \cdots + a_{n_a} y(t-n_a) = b_1 u(t-1) + \cdots$$
$$+ b_{n_b} u(t-n_b) + e(t) + c_1 e(t-1) + \cdots + c_{n_c} e(t-n_c) \quad (4.14)$$

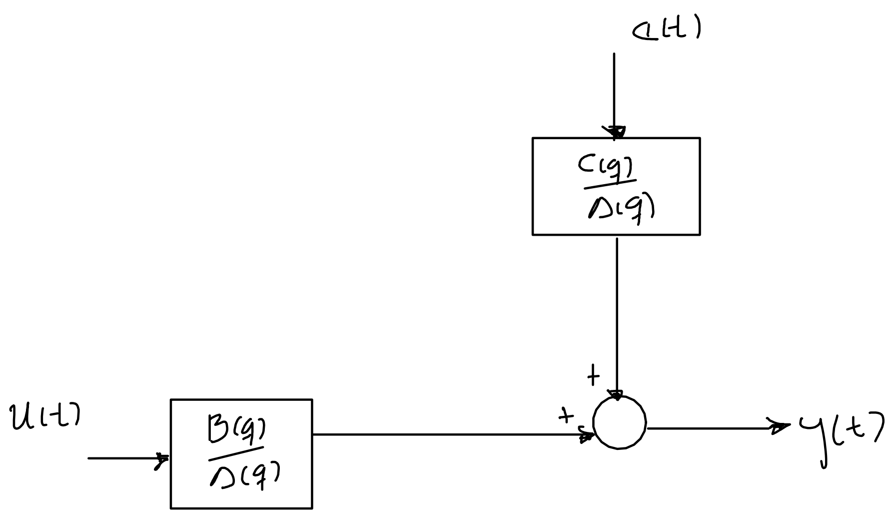
Es decir, en ARX usa *e(t)* (ruido blanco gaussiano) como error, pero en ARMAX ya no. En su lugar el error se modela como si este atrvesase antes un filtro :

$$\boxed{e(t) + c_1 e(t-1) + \cdots + c_{n_c} e(t-n_c)}$$

it can be rewritten

$$A(q)y(t) = B(q)u(t) + C(q)e(t) \quad (4.15)$$

In view of the moving average (MA) part $C(q)e(t)$, the model (4.15) will be called ARMAX. The ARMAX model has become a standard tool in control and econometrics for both system description and control design. A version with



Estructura de un modelo ARMAX