Abstract: Statistical Games

Playful approach to statistics

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1 Brief introduction

It might appeal to some.

Abraham Wald [20]

The first and main part of this work has a mathematical character. It explores and analyses a few simple two-player non-cooperative games (for the explicit definition, see 2.1, or 3.2), in which many concepts of probability theory and statistics naturally emerge. In these games – termed Fisher and Bayesian games – an adversarial player can choose from a set of possible scenarios, while the other player can collect data, based on which, she has to make a guess or bet on the scenarios. Besides the mathematical exploration of games, in which concepts from Frequentist and Bayesian statistics can be identified in equilibrium, in section 5, a generalized betting game, termed "Statistical game" is introduced (for the definition, see 5.1). Unification of Bayesian and Fisher games is possible by interpreting these as general Statistical games, differing only in the agent's relative risk aversion. From a mathematical point of view, the emergent structures and nontrivial limit behaviour of such games, along with their equilibrium solutions, seem worthy of detailed examination. This work can be viewed as the beginning of the mathematical investigation and exploration of statistical games.

Later, in section 6, these games are proposed to be models or analogues for statistical inference itself. This suggestion is more philosophical – and in some sense more radical. The proposition aims to ground statistical and probabilistic concepts with non-cooperative games, instead of devices of chance or subjective degree of belief. This can be viewed as a different framework for the *interpretation* of probability and related statistical procedures.

A secondary contribution of this work comes from a non-exhaustive but broad review of the diverse scientific literature, spread both in time and context. The main philosophical and some technical concepts promoted in this work have been present in the literature in a fragmented form, often referred to as minimax regret criterion [14, 17]. (The ideas of Wald [24], Savage [20], Good [6], Kelly [11], Kashyap [10, 9] already contained the fundamental concepts from which the framework could have been constructed.) The scope of the topic – decision making, statistics, and probabilistic inference – is enormous and highly interdisciplinary. An incomplete list of related fields includes: Economics [13], Philosophy [7, 2], Statistics [4], Computer science [23, 11], Mathematics [12], Physics [18], Biology [3], Machine learning [15, 1, 19, 21] et cetera[1]. The collection of related ideas – which do not necessarily refer to each other – and presenting these concepts in a unified, coherent framework will hopefully inspire further research and stimulate interdisciplinary collaboration.

Hopefully, the presented simple but clear toy models can serve as a solid foundation for future research and development, and – together with the listed future directions in section 7 – build a compelling case for a more general and coherent framework for decision making in the face of uncertainty.

[1] Finance [8, 16], Control theory [22], Operations Research [25, 5]...

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