The Handy Dandy Book of Algebraic Tricks

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1 Addition and subtraction

$$a_1 + a_2 \ \cdots \ + a_n = n \cdot a \qquad a + (-b) = a - b \qquad a - (-b) = a + b$$

2 Exponents and roots

$$a^n \cdot a^m = a^{(n+m)} \qquad \frac{a^n}{a^m} = a^{(n-m)} \qquad a^{-n} = \frac{1}{a^n}$$

$$(a^n)^m = a^{(n \cdot m)} \qquad (a \cdot b)^n = a^n \cdot b^n \qquad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$\sqrt[q]{a} = a^{\frac{1}{q}} \qquad \sqrt[q]{a^p} = a^{\frac{p}{q}} \qquad \sqrt[q]{a \cdot b} = \sqrt[q]{a} \cdot \sqrt[q]{b}$$

$$\sqrt[q]{\frac{a}{b}} = \frac{\sqrt[q]{a}}{\sqrt[q]{b}} \qquad a^0 = 1 \qquad a^1 = a$$

$$(a \pm b)^2 = a^2 + b^2 \pm 2ab \qquad (-1)^n = \begin{cases} 1 & \text{if } n \text{ is even} \\ -1 & \text{if } n \text{ is odd} \end{cases} \qquad \frac{1}{\sqrt{a}} = \frac{\sqrt{a}}{a}$$

$$(a + b)(a - b) = a^2 - b^2 \qquad a \cdot \sqrt{a} = a^{\frac{3}{2}}$$

3 Fractions

$$\frac{a}{b} \pm \frac{c}{d} = \frac{a \cdot d \pm c \cdot b}{b \cdot d} \qquad \frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d} \qquad c \cdot \frac{a}{b} = \frac{c \cdot a}{b}$$

$$\frac{a}{c} \pm \frac{b}{c} = \frac{a \pm b}{c} \qquad \frac{a}{b} = \frac{\frac{a}{c}}{\frac{b}{c}} \qquad \frac{a}{b} = \frac{a \cdot c}{b \cdot c}$$

$$a = \frac{a}{1} \qquad \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} \qquad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \cdot \frac{d}{c} \qquad \frac{\frac{a}{b}}{c} = \frac{a}{b \cdot c} \qquad \frac{a}{\frac{b}{c}} = \frac{a \cdot c}{b}$$

$$-\frac{a}{b} = \frac{-a}{b} = \frac{a}{-b} \qquad \frac{-a}{-b} = \frac{a}{b} \qquad \frac{1}{a} = a^{-1}$$

$$\frac{a}{a} = 1 \qquad \frac{a}{c} \cdot \frac{c}{b} = \frac{a}{b} \qquad \frac{a}{b} \cdot b = a$$

$$a \pm \frac{b}{c} = \frac{a \cdot c \pm b}{c} \qquad \frac{b}{c} - a = \frac{b - a \cdot c}{c}$$

4 Logarithms

These rules apply to all positive numbers a and c and all $n \neq 0$. b and d denote the logarithm base and is above 1.

$$\log_b(a \cdot c) = \log_b(a) + \log_b(c) \qquad \log_b\left(\frac{a}{c}\right) = \log_b(a) - \log_b(c)$$

$$\log_b\left(a^n\right) = n \cdot \log_b(a) \qquad \log_b\left(\sqrt[n]{a}\right) = \frac{\log_b(a)}{n}$$

$$b^{\log_b(a)} = a \qquad \log_b\left(b^a\right) = a$$

$$\log_b(b) = 1 \qquad \log_b(1) = 0$$

$$\log_b(a) = \frac{\log_d(a)}{\log_d(b)}$$

5 Multiplication

$$a \cdot b \pm a \cdot c = a(b \pm c) \qquad a_1 \cdot a_2 \ \cdots \ \cdot a_n = a^n \qquad n \cdot a = a_1 + a_2 \ \cdots \ + a_n$$

$$a \cdot 1 = a \qquad a \cdot 0 = 0 \qquad -a \cdot (-b) = a \cdot b$$

$$-a \cdot b = a \cdot (-b)$$

6 Differentials

x is a variable, k, a and b are constants, and f and g are functions.

Function	**Derivative**
x	1
x^n	nx^{n-1}
$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$
$\frac{k}{x}$	$\frac{-k}{x^2}$

$ax + b$	a
$\sin$	$\cos$
$\cos$	$-\sin$
$\tan x$	$1 + \tan^2 x \quad \text{or} \quad \frac{1}{\cos^2 x}$
$k \cdot f$	$k \cdot f'$
$f \pm g$	$f' \pm g'$
$f \cdot g$	$f' \cdot g + f \cdot g'$
$\frac{f}{g}$	$\frac{f' \cdot g - f \cdot g'}{g^2}$

7 Trigonometry

$$\sin(\alpha + \beta) = \cos(\alpha) \cdot \sin(\beta) + \sin(\alpha) \cdot \cos(\beta) \qquad \cos(\alpha + \beta) = \cos(\alpha) \cdot \cos(\beta) + \sin(\alpha) \cdot \sin(\beta)$$

$$\sin(2\alpha) = 2 \cdot \cos(\alpha) \cdot \sin(\alpha) \qquad \cos(2\alpha) = \cos^2(\alpha) - \sin^2(\alpha)$$