

THE HANDY DANDY BOOK OF ALGEBRAIC TRICKS

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1 Addition and subtraction

$$a_1 + a_2 + \cdots + a_n = n \cdot a$$

$$a + (-b) = a - b$$

$$a - (-b) = a + b$$

2 Exponents and roots

$$a^n \cdot a^m = a^{(n+m)}$$

$$\frac{a^n}{a^m} = a^{(n-m)}$$

$$a^{-n} = \frac{1}{a^n}$$

$$(a^n)^m = a^{(n \cdot m)}$$

$$(a \cdot b)^n = a^n \cdot b^n$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$\sqrt[q]{a} = a^{\frac{1}{q}}$$

$$\sqrt[q]{a^p} = a^{\frac{p}{q}}$$

$$\sqrt[q]{a \cdot b} = \sqrt[q]{a} \cdot \sqrt[q]{b}$$

$$\sqrt[q]{\frac{a}{b}} = \frac{\sqrt[q]{a}}{\sqrt[q]{b}}$$

$$a^0 = 1$$

$$a^1 = a$$

$$(a \pm b)^2 = a^2 + b^2 \pm 2ab$$

$$(-1)^n = \begin{cases} 1 & \text{if } n \text{ is even} \\ -1 & \text{if } n \text{ is odd} \end{cases}$$

$$\frac{1}{\sqrt{a}} = \frac{\sqrt{a}}{a}$$

$$(a + b)(a - b) = a^2 - b^2$$

$$a \cdot \sqrt{a} = a^{\frac{3}{2}}$$

3 Fractions

$$\frac{a}{b} \pm \frac{c}{d} = \frac{a \cdot d \pm c \cdot b}{b \cdot d}$$

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$$

$$c \cdot \frac{a}{b} = \frac{c \cdot a}{b}$$

$$\frac{a}{c} \pm \frac{b}{c} = \frac{a \pm b}{c}$$

$$\frac{a}{b} = \frac{\frac{a}{c}}{\frac{b}{c}}$$

$$\frac{a}{b} = \frac{a \cdot c}{b \cdot c}$$

$$a = \frac{a}{1}$$

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \cdot \frac{d}{c}$$

$$\frac{\frac{a}{b}}{c} = \frac{a}{b \cdot c}$$

$$\frac{a}{\frac{b}{c}} = \frac{a \cdot c}{b}$$

$$-\frac{a}{b} = \frac{-a}{b} = \frac{a}{-b}$$

$$\frac{-a}{-b} = \frac{a}{b}$$

$$\frac{1}{a} = a^{-1}$$

$$\frac{a}{a} = 1$$

$$\frac{a}{c} \cdot \frac{c}{b} = \frac{a}{b}$$

$$\frac{a}{b} \cdot b = a$$

$$a \pm \frac{b}{c} = \frac{a \cdot c \pm b}{c}$$

$$\frac{b}{c} - a = \frac{b - a \cdot c}{c}$$

4 Logarithms

These rules apply to all positive numbers a and b and all $n \neq 0$. These logarithms are base 10 (\log_{10}).

$$\log(a \cdot b) = \log(a) + \log(b)$$

$$\log\left(\frac{a}{b}\right) = \log(a) - \log(b)$$

$$\log(a^n) = n \cdot \log(a)$$

$$\log(\sqrt[n]{a}) = \frac{\log(a)}{n}$$

$$10^{\log(a)} = a$$

$$\log(1) = 0$$

5 Multiplication

$$a \cdot b \pm a \cdot c = a(b \pm c)$$

$$a_1 \cdot a_2 \cdot \cdots \cdot a_n = a^n$$

$$n \cdot a = a_1 + a_2 + \cdots + a_n$$

$$a \cdot 1 = a$$

$$a \cdot 0 = 0$$

$$-a \cdot (-b) = a \cdot b$$

$$-a \cdot b = a \cdot (-b)$$

6 Differentials

x is a variable, k , a and b are constants, and f and g are functions.

Function	Derivative
x	1
x^n	nx^{n-1}
\sqrt{x}	$\frac{1}{2\sqrt{x}}$
$\frac{k}{x}$	$\frac{-k}{x^2}$
$ax + b$	a
\sin	\cos
\cos	$-\sin$

$\tan x$	$1 + \tan^2 x \quad \text{or} \quad \frac{1}{\cos^2 x}$
$k \cdot f$	$k \cdot f'$
$f \pm g$	$f' \pm g'$
$f \cdot g$	$f' \cdot g + f \cdot g'$
$\frac{f}{g}$	$\frac{f' \cdot g - f \cdot g'}{g^2}$

7 Trigonometry

$$\sin(\alpha + \beta) = \cos(\alpha) \cdot \sin(\beta) + \sin(\alpha) \cdot \cos(\beta)$$

$$\cos(\alpha + \beta) = \cos(\alpha) \cdot \cos(\beta) - \sin(\alpha) \cdot \sin(\beta)$$

$$\sin(2\alpha) = 2 \cdot \cos(\alpha) \cdot \sin(\alpha)$$

$$\cos(2\alpha) = \cos^2(\alpha) - \sin^2(\alpha)$$