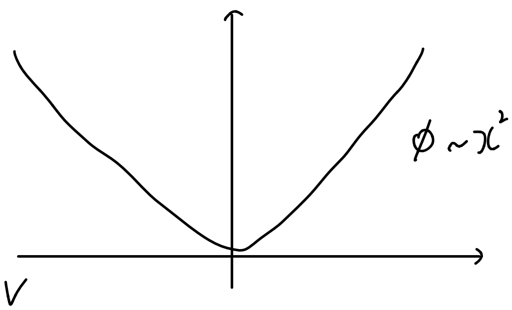
pro.8 in Ch.4

$$-u_{xx} + x^2 u = \lambda u \quad , x \in \mathbb{R}$$

$$\begin{cases} D_N^{(2)} : (3, 12) \leftarrow N \times N \\ u_{xx} \simeq D_N^{(2)} \cdot V \end{cases}$$

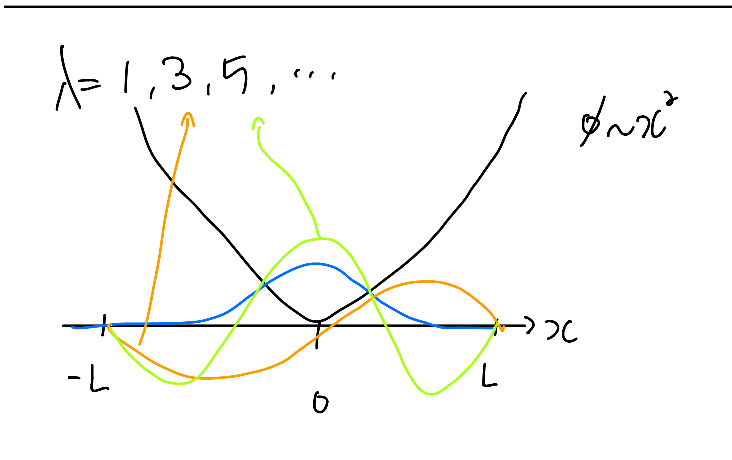
$$: (\underline{-D_N^{(2)} + S}) V = \lambda \cdot V$$

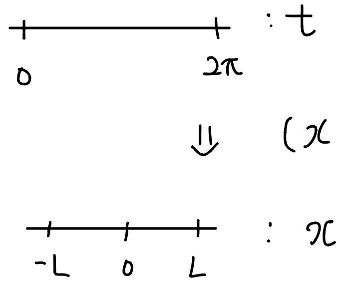
$$M \cdot V = \lambda \cdot V$$



$$S = x^2$$

$$S = \begin{pmatrix} x_1^2 & & & 0 \\ & x_2^2 & & \\ & & \ddots & \\ 0 & & & x_N^2 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{pmatrix}$$





$$\Downarrow \quad \left(x = \frac{L}{\pi} t - L\right)$$

$$\frac{d^2}{dt^2} \to D_N^{(2)} \quad (3, 12)$$

$$\frac{d^2}{dx^2} = \left(\frac{\pi}{L}\right)^2 \frac{d^2}{dt^2} \to \underbrace{\left(\frac{\pi}{L}\right)^2 \cdot D_N^{(2)}}_{D2}$$

$\hookrightarrow M \equiv -D2 + S$

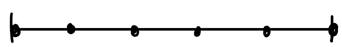
Spectral Differentiation

i) Given v_j , $p(x)$ such that $v_j = p(x_j)$

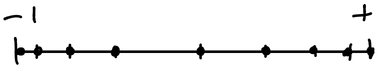
ii) $u^{(n)}(x) \simeq p^{(n)}(x)$

$p(x) \rightarrow$ 다항식 , $= a_0 x^0 + a_1 x^1 + a_2 x^2 + \cdots a_N x^N$: N^{th} order

② How to choose grid point?

 X ,  O

-1 $+1$

 x $\Rightarrow \Delta x \sim O(N^{-2})$

Point density: $\dfrac{\Delta N}{\Delta x} \sim \dfrac{N}{\pi\sqrt{1-x^2}}$, $\Delta x = \dfrac{1}{N}\sqrt{1-x^2}$

$\Delta x(x \rightarrow \pm 1) \sim N^{-1} \lim\limits_{x \rightarrow \pm 1} \sqrt{1-x}\sqrt{1+x}$

$= N^{-1}\sqrt{2}\underbrace{\sqrt{1 \pm x}}_{\sqrt{\Delta x}}$

$\Rightarrow \Delta x(x \rightarrow \pm 1) \sim O(N^{-2})$

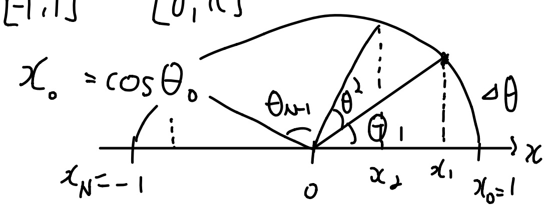
$\hookrightarrow_{x \rightarrow 0}$ $N^{-1}\sqrt{1}\sqrt{1} \sim O(N^{-1})$

Let $\pi \dfrac{\Delta N}{N} = \Delta\theta$, $\theta \in [0, \pi]$

$\hookrightarrow \dfrac{\Delta x}{\sqrt{1-x^2}} = \Delta\theta \Rightarrow \displaystyle\int_1^x \frac{dx}{\sqrt{1-x^2}} = \int_0^\theta d\theta \Rightarrow \cos^{-1} x = \theta$

$x_j = \cos\theta_j$: chebyshev points.

[-1, 1] $[0, \pi]$



$$\theta_j = \underbrace{j \cdot \frac{\pi}{N}}_{\Delta\theta}$$

$$\Delta x = \frac{2}{N}$$

$$x_j = \cos\left(j\frac{\pi}{N}\right)$$