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import numpy as np
from scipy.sparse import coo_matrix as coo
from matplotlib import pyplot as plt

N_vec = np.power(2, range(3,12 +1))
plt.figure(figsize=(16,8))

for N in N_vec:

    h = 2 * np.pi / N
    x = - np.pi + np.matrix(range(1,N + 1)).T * h

    u_array = []
    for i in np.array(x.T):
        u_val = np.power(np.e,np.sin(i)**2)
        u_array.extend(u_val)

    u = np.matrix(u_array).T
    uprime = np.matrix(2*np.array(np.sin(x))*np.array(np.cos(x)) * np.array(u))

    e = np.ones(N)
    row = np.array(range(0,N))
    col = np.array(range(1, N))
    col = np.append(col,0)
    data = 2/3 * e
    D = coo((data, (row,col)), shape=(N,N)).toarray()
    col_2 = np.array(range(2,N))
    col_2 = np.append(col_2, [0,1])
    data_2 = -1/12 * e
    D_2 = coo((data_2, (row,col_2)), shape=(N,N)).toarray()

    matrix = D + D_2
    matrix = 1/h * (matrix - matrix.T)

    error_matrix = matrix * u - uprime
    error = error_matrix.sum(axis=1).max()

    plt.scatter(N,error,color='black')

plt.plot(N_vec,1/ N_vec ** 4,color='black',linestyle = '--')
plt.ylim(10**(-15), 10**0)
plt.xlim(10**0,10**4)
plt.xscale('log')
plt.yscale('log')
plt.grid(which='both',linestyle=':')
plt.text(105, 5*10**(-8), r '$N^{-4}$', fontsize=30)
plt.xlabel('N',fontsize=20)
plt.ylabel('Error',fontsize=20)
plt.title('Convergence of 4th-order finite differences',fontsize = 15)

plt.show()

```



