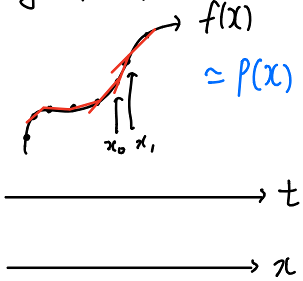
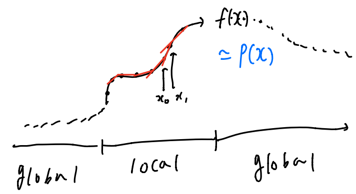
$y = f'(x)$



① $y \simeq \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}$: finite difference method

② $y_0 \simeq P'(x_0)$: finite element method (local)

③ $y \simeq P(x)$: spectral method (global)



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① system of linear eq.
② Determinent
③ EigenValues & Eigen vectors
④ linear transformation

① system of linear eq.

$ax = b$, $x = \dfrac{b}{a} = a^{-1} b$: if $a \neq 0$

• if $a = 0$

$0 \cdot x = b$ $\begin{cases} b = 0 \\ b \neq 0 \end{cases}$

• $ax = 0$

Homogeneous Eq.
✓ $x = 0$: Trivial solution
✓ $a = 0$, $x \neq 0$: Non-trivial

$\begin{cases} a_1 x + b_1 y = C_1 \\ a_2 x + b_2 y = C_2 \end{cases}$, $\begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}$

$A \cdot r = b$

$Ar = b \Rightarrow r = A^{-1} b$,

$\therefore A^{-1} = \dfrac{1}{|A|} C^T \longrightarrow C_{ij}$ = cofactor of A

$Ar = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$. Homogeneous

✓ $x = 0, y = 0$: Trivial solution

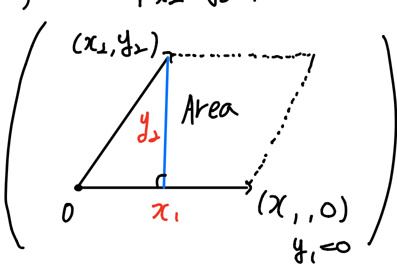
✓ $|A| = 0$: Non-trivial

↳ $\begin{matrix} a_1 x + b_1 y = 0 \\ a_2 x + b_2 y = 0 \end{matrix}$: Same Eq.

---

② Determinent (Geometrical Meaning)

$M = \begin{pmatrix} x_1 & y_1 \\ x_2 & y_2 \end{pmatrix}$, $|M| = \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} = x_1 y_2 - x_2 y_1 = x_1 y_1$



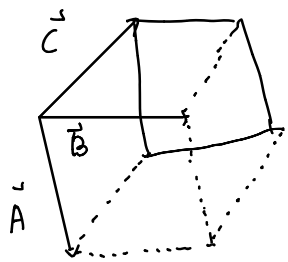
if $|M| = 0$. No Area

✓ Parallel

$$M = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix}, \quad \vec{A} = (a_1, a_2, a_3),\ \vec{B} = (b_1, b_2, b_3),\ \vec{C} = (c_1, c_2, c_3)$$

$$|M| = (a_1 b_2 c_3 + a_2 b_3 c_1 + a_3 b_1 c_2) - (a_3 b_2 c_1 + a_2 b_1 c_3 + a_1 b_3 c_2)$$

$= \vec{A} \cdot (\vec{B} \times \vec{C})$: Scalar triple product



: Volume. if $|M| = 0$. 3 Vectors → Same Plane
→ line

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③ linear transformation

$Mr = R$ ↳ linear transformation

- M : linear operator
- f : operation

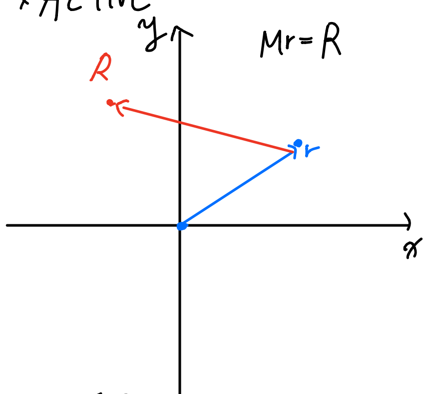
Linearity $f(x)$

i) $f(x+y) = f(x) + f(y) \rightarrow M(r_1 + r_2) = Mr_1 + Mr_2$

ii) $f(cx) = cf(x) \rightarrow M(cr) = cMr$

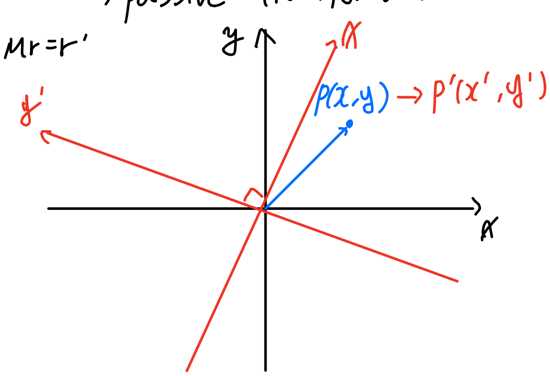
× Active transformation

$Mr = R$



× passive transformation

$Mr = r'$

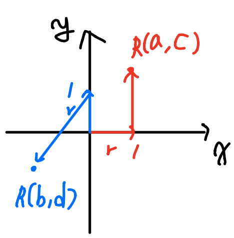


✓ Active

$$M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$M\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} a \\ c \end{pmatrix}$$

$$M\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} b \\ d \end{pmatrix}$$

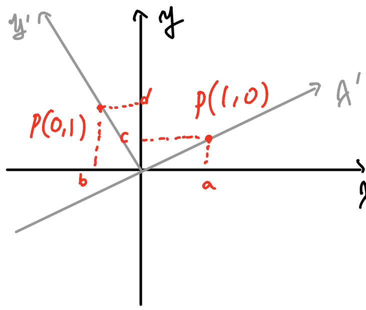


$$M\begin{pmatrix} x \\ y \end{pmatrix} = xM\begin{pmatrix} 1 \\ 0 \end{pmatrix} + yM\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$= x\begin{pmatrix} a \\ c \end{pmatrix} + y\begin{pmatrix} b \\ d \end{pmatrix}$$

✓ Passive

$M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$



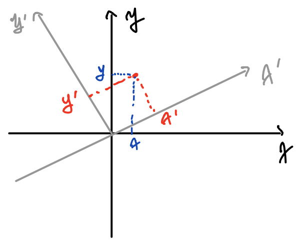
$r' = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$r = Mr' = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} a \\ c \end{pmatrix}$

$r = Mr' = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} b \\ d \end{pmatrix}$

$$\begin{pmatrix} x \\ y \end{pmatrix} = M\begin{pmatrix} x' \\ y' \end{pmatrix} = x'M\begin{pmatrix} 1 \\ 0 \end{pmatrix} + y'M\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$= x'\begin{pmatrix} a \\ c \end{pmatrix} + y'\begin{pmatrix} b \\ d \end{pmatrix}$$



$x = x'a + y'b$

$y = x'c + y'd$

✓ Othogonal trans.

$r = Mr'$

$x'^2 + y'^2 = x^2 + y^2$

✓ similarity trans.

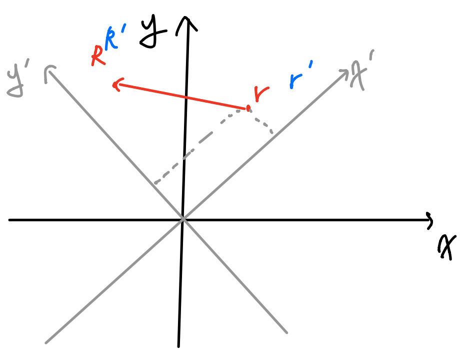
$r = \begin{pmatrix} x \\ y \end{pmatrix}$, A / $r' = \begin{pmatrix} x' \\ y' \end{pmatrix}$, B

C : Coord trans matrix

$r = Cr' \rightarrow r' = C^{-1}r \quad (CC^{-1} = I)$

A: point r → another R ⇒ $R = Ar$

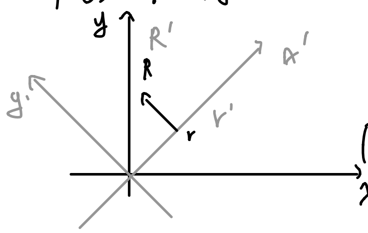
Q. $A' \leftarrow R' = A'r'$



$$R' = C^{-1}R = C^{-1}Ar = \underbrace{C^{-1}AC}_{A'}r'$$

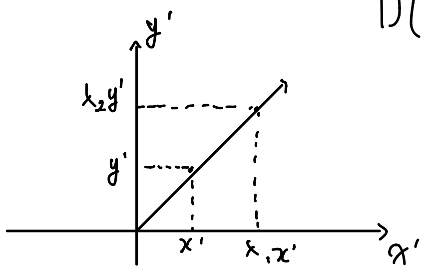
$$= A'r'$$

④ Eigen Values & Eigen vectors



$A,\ A'(C^{-1}AC)$

$D = A' = \begin{pmatrix}\lambda_1 & 0\\ 0 & \lambda_2\end{pmatrix}$: Diagonalized



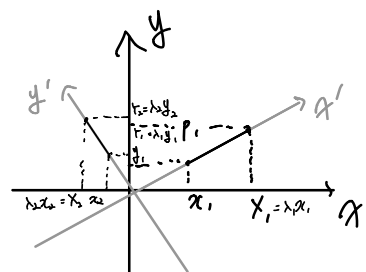
$$D\begin{pmatrix}x'\\y'\end{pmatrix} = x'D\begin{pmatrix}1\\0\end{pmatrix} + y'D\begin{pmatrix}0\\1\end{pmatrix}$$
$$= \lambda_1\begin{pmatrix}x'\\0\end{pmatrix} + \lambda_2\begin{pmatrix}0\\y'\end{pmatrix}$$

$$A = CA'C^{-1} = CDC^{-1} \longleftrightarrow D = C^{-1}AC$$

Q. what is C ?

$C = (V_1, V_2)$

↳ Eigenvectors



$Ar = \lambda r$

$(A-\lambda I)r = 0$

↳ Homogeneous

Non-tri : $|A-\lambda I| = 0$. λ (Eigen values)

↳ characteristic eq.

$$\begin{pmatrix}a-\lambda & b\\ c & d-\lambda\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$$

$$\begin{pmatrix}a-\lambda_1 & b\\ c & d-\lambda_1\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$$

↳ $y = \alpha x$

Ex. $\begin{pmatrix}X\\Y\end{pmatrix} = \underbrace{\begin{pmatrix}5 & -2\\ -2 & 2\end{pmatrix}}_{A}\begin{pmatrix}x\\y\end{pmatrix}$

① $|A-\lambda I| = 0$

$\begin{vmatrix}5-\lambda & -2\\ -2 & 2-\lambda\end{vmatrix} = 0$, $(5-\lambda)(2-\lambda) - 4 = 0$

$\lambda^2 - 7\lambda + 6 = 0$

$\therefore \lambda = 1, 6$

ii) $\lambda = 1$

$$\begin{pmatrix} 4 & -2 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0$$

$y = 2x$

$\lambda = 6$

$$\begin{pmatrix} -1 & -2 \\ -2 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0$$

$y = -\frac{1}{2}x$