$\vec{V}$

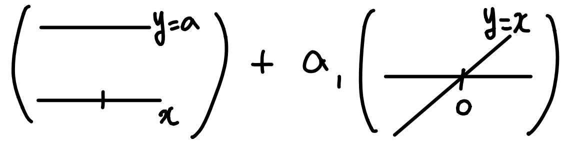
$$= V_x(\hat{x}) + V_y(\hat{y}) + \cdots = V_i \hat{e}_i$$

$$= \vec{V}\cdot\hat{x} + \vec{V}\cdot\hat{y} + \cdots = \vec{V}\cdot\hat{e}_i$$

「Power function」

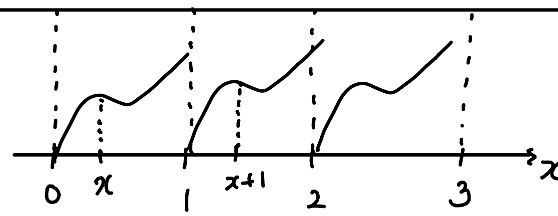
$$f(x) = a_0x^0 + a_1x^1 + a_2x^2 + a_3x^3 + \cdots = \sum_{n=0}^{\infty} a_n x^n$$

$$= a_0\left(\quad\right) + a_1\left(\quad\right) + \cdots \qquad (x = [-\infty, \infty])$$



$$a_0 = f(0),\ a_1 = f'(0),\ a_2 = \frac{1}{2}f''(0)\cdots \qquad a_n = \frac{1}{n!}\left.\frac{d^nf}{dx^n}\right|_{x=0}$$

「Fourier Series」

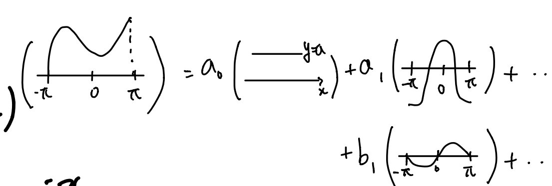


$$f(x) = f(x + nP) \quad \rightarrow \text{Period}$$

$$= a_0 + \sum_{n=0}^{\infty}\left(a_n\cos\frac{2\pi n}{P}x + b_n\sin\frac{2\pi n}{P}x\right) \quad \left[\cos\frac{2\pi n}{P}x = \cos\frac{2\pi n}{P}(x+mP) = \cos\frac{2\pi n}{P}\cdot x + \cos\frac{2\pi n}{\cancel{P}}\cdot m\cancel{P}\right]$$

$P = 2\pi$

$$= a_0 + \sum_{n=0}^{\infty}(a_n\cos nx + b_n\sin nx)$$



$$\llcorner\ e^{ix} = \cos x + i\sin x$$

$$= \sum_{n=-\infty}^{\infty} C_n e^{inx}$$

$$f(x) = \sum_{k=-\infty}^{\infty} C_k e^{ikx}$$

$p = 2\pi$

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} e^{ikx}e^{-ilx}\,dx \quad 0 \ (\frac{\theta}{0}

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} e^{i(k-l)x}\,dx \begin{cases} i)\ k=l \rightarrow 1. \\ ii)\ k\neq l \rightarrow \frac{1}{2\pi}\frac{1}{i(k-l)}e^{i(k-l)x}\Big|_{-\pi}^{\pi}=0 \\ \quad \hookrightarrow p=2\pi \end{cases}$$

$\downarrow$

기함수, 우함수

$$\Rightarrow \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{i(k-k')x}\,dx = \delta_{kk'}$$

$$\rightarrow \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\cdot e^{-ikx}\,dx = \frac{1}{2\pi}\int_{-\pi}^{\pi}\sum_{k'=-\infty}^{\infty} C_{k'}\, e^{ik'x}\, e^{ikx}\,dx$$

$$= \sum_{k'=-\infty}^{\infty} C_{k'}\,\delta_{kk'} = C_k$$

$$\therefore C_k = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)e^{-ikx}\,dx$$

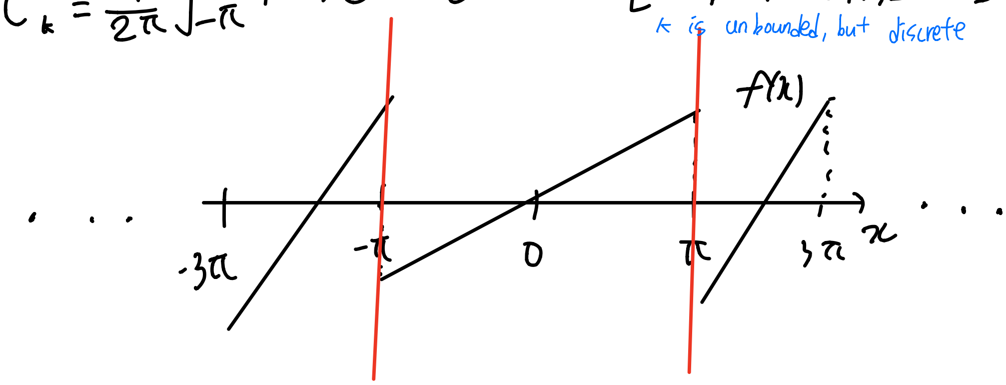
$$\begin{cases} f(x) = \sum_{k=-\infty}^{\infty} c(k)e^{ikx} & \rightarrow x=[-\infty,\infty] = (-\pi,\pi) \\ C_k = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)e^{-ikx}\,dx & \rightarrow k=[\cdots,-2,-1,0,1,2\cdots] \end{cases}$$

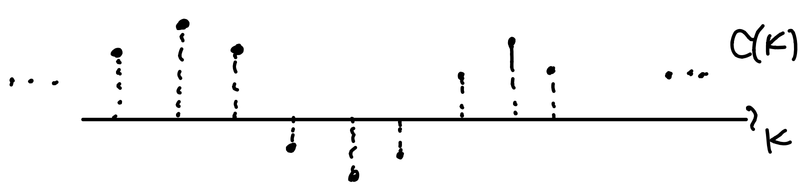
$\boxed{p=2\pi}$

x is bounded, but continuos

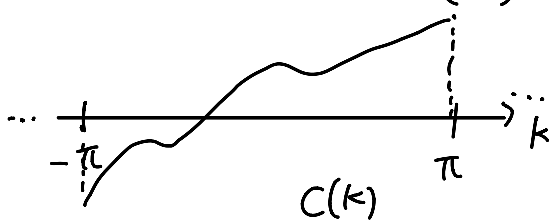
k is unbounded, but discrete

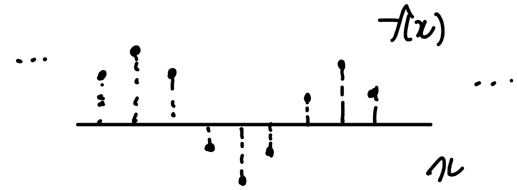
부분을 본다.





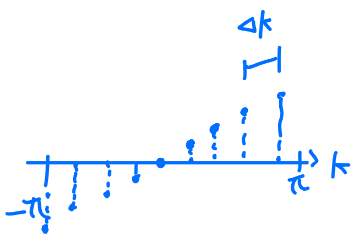
Q. Can we switch (x, k) and (C, f)?

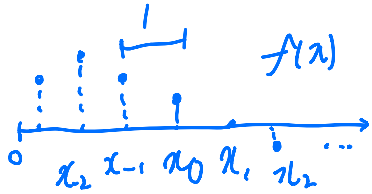




$$C(k) = \sum_{x_j=-\infty}^{\infty} f(x_j)\, e^{-ikx_j}$$

$$f(x) = \frac{1}{2\pi}\int_{-\pi}^{\pi} C(k)\, e^{ikx}\, dk$$





Total longth : L

$\Delta k \cdot L = 2\pi$

↓

$\Delta k = \frac{2\pi}{L}$

$[-\infty, \infty]$를 가지면 수는 연속.

↓

범위를 줄이면 Δk가 증가한다.

$$f(x_j) = \frac{\Delta k}{2\pi} \sum_{x_j=1}^{\cancel{L}\,N} C(k_i)\, e^{ik_i x_j} = \frac{1}{\cancel{L}\,N} \sum_{i=1}^{\cancel{L}\,N} C(k_i)\, e^{ik_i x_j}$$

$$C(k_i) = \sum_{j=1}^{\cancel{L}\,N} f(x_j)\, e^{-ik_i x_j}$$

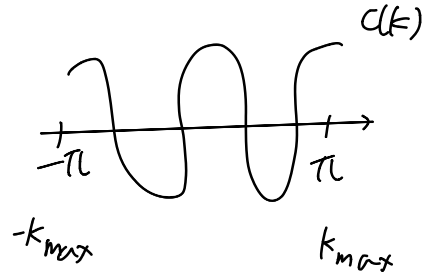
FFT

⇒ 전체적인 형태는 이 형태고 조금의 변화는 있을 수 있음.

Python Documention 확인.

「Fourier Transformation : Intergral Transformation」

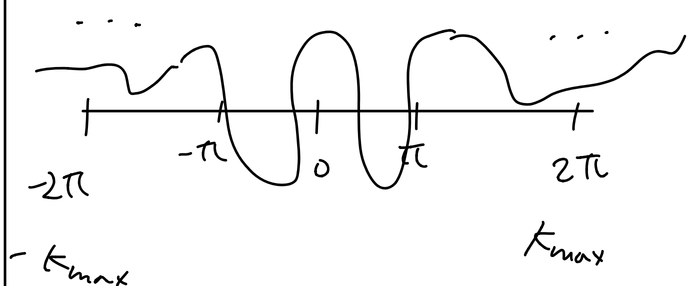
interval = 1.



point density ↑ (2배) ⇒ interval = $\frac{1}{2}$

→ extend k-space

$$\boxed{\begin{aligned} &\Delta x \cdot 2k_{max} = 2\pi \\ &\rightarrow k_{max} = \frac{\pi}{\Delta x} \\ &\Delta x \rightarrow 0 \ , \ k_{max} = \infty \end{aligned}}$$



$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} g(k) e^{ikx} dk \longleftrightarrow g(k) = \int_{-\infty}^{\infty} f(x) e^{-ikx} dx$$

$$f(x_j) = \frac{1}{N}\sum_{j=1}^{N} C(k_j) e^{ik_i x_j} \ , \ C(k_i) = \sum_{j=1}^{N} f(x_j) e^{-ik_i x_j}$$

* Check FFT, IFFT