$u_{xx} = e^{u} = f(x)$, $-1 < x < 1$, $u(\pm 1) = 0$

$u_{xx} = D_N^2 v = f$ | $a_0x_0 + a_1x_1 + a_2x_2 = f_0$

$\hookrightarrow L \cdot v = f$ | $b_0x_0 + b_1x_1 + b_2x_2 = f_1$

| $c_0x_0 + c_1x_1 + c_2x_2 = f_2$

$$\Rightarrow \begin{pmatrix} a_0 & a_1 & a_2 \\ b_0 & b_1 & b_2 \\ c_0 & c_1 & c_2 \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} f_0 \\ f_1 \\ f_2 \end{pmatrix}$$

$$\begin{pmatrix} x_0 \\ x_1 \\ x_2 \end{pmatrix} \hookrightarrow \begin{pmatrix} v_0 \\ v_1 \\ v_2 \end{pmatrix}$$

If) $u(+1) = \alpha = v_0$

$u(-1) = \beta = v_N$

$\hookrightarrow$ Dirrichlet 조건

$1 \cdot v_0 + 0 \cdot v_1 + 0 \cdot v_2 = \alpha$

$b_0 \cdot v_0 + \cdots + \cdots = f_1$

$0 \cdot v_0 + \cdots + 1 \cdot v_2 = \beta$

$$\hookrightarrow \begin{pmatrix} 1 & 0 & 0 \\ b_0 & b_1 & b_2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} v_0 \\ v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} \alpha \\ f_1 \\ \beta \end{pmatrix}$$

$$\hookrightarrow \begin{pmatrix} 1 & 0 & 0 \\ D_{10}^2 & D_{11}^2 & D_{12}^2 \\ 0 & 0 & 1 \end{pmatrix}$$

노이만 조건 $v_0(+1) = \alpha$, $w_N = u_x(-1) = \beta$

$1 \cdot v_0 + 0 \cdot v_1 + 0 \cdot v_2 = \alpha$

$\vdots$

$D_{20}^1 v_0 + D_{21}^1 v_1 + D_{22}^1 v_2 = \beta$

$u_x \simeq D_N^1 v$

$\hookrightarrow u(-1) = \beta$

$$\hookrightarrow \begin{pmatrix} 1 & 0 & 0 \\ D_{10}^2 & D_{11}^2 & D_{12}^2 \\ D_{20}^1 & D_{21}^1 & D_{22}^1 \end{pmatrix} \begin{pmatrix} v_0 \\ v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} \alpha \\ f_1 \\ \beta \end{pmatrix}$$

$u_{xx} + u_{yy} = f(x,y) \quad u=0$ in bound

$u(x,y)$, $N_x = 1$, $N_y = 2$

① $v = \begin{pmatrix} v_{00} & v_{01} & v_{02} \\ v_{10} & v_{11} & v_{12} \end{pmatrix}$ (y →, x ↓) $\Rightarrow (v_{00}\ v_{01}\ v_{02}\ v_{10}\ v_{11}\ v_{12})$

Flattening

$f \Rightarrow (f_{00}\ f_{01}\ \cdots\ )$

② $L = D^{(2)}_{N,x} \otimes I_{N,y} + I_{N,x} \otimes D^{(2)}_{N,y}$

바운드 값이 0

③ $\tilde{L} = \tilde{D}^2_{N,x} \otimes \tilde{I}_{N,y} + \tilde{I}_{N,x} \otimes \tilde{D}^2_{N,y}$: Laplace (7.4)

$\Rightarrow \tilde{L}\tilde{v} = \tilde{f}$

↳ $D^2_{N,x}, D^2_{N,y} = \begin{pmatrix} 1 & 0 & 0 & \cdots \\ \cdots & 0 & 0 & 1 \end{pmatrix}$

$f = \begin{pmatrix} 0 & 0 & \cdots & 0 \\ 0 & & & 0 \\ \vdots & & & \vdots \\ 0 & 0 & \cdots & 0\ 0 \end{pmatrix}$

$u_{xx} + u_{yy} + k^2 u = f(x,y)$, k = constant

$\tilde{L} = \tilde{D}^2_{N,x} \otimes \tilde{I}_{N,y} + \tilde{I}_{N,x} \otimes \tilde{D}^2_{N,y} + k^2 \underline{\tilde{I}_{N,x} \otimes \tilde{I}_{N,y}}$

$1 \cdot u = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} u \Rightarrow k \cdot u = \begin{pmatrix} k & & 0 \\ & k & \\ 0 & & k \end{pmatrix} \cdot u$ / $k^2 \begin{pmatrix} 0 & \cdots & 0 \\ & \boxed{1 \ \ddots \ 1} & \\ 0 & \cdots & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & \cdots & 0 \\ & \boxed{1 \ \ddots \ 1} & \\ 0 & \cdots & 0 \end{pmatrix}$

$\tilde{I}_{N,x}$ $\quad \tilde{I}_{N,y}$

✓과제를 수행할 때 Book은 $\tilde{A}$만으로 그림을 도출했는데 Full-Matrix로 작성 해보기.

「Program 21」 : Mathieu E.q $\boxed{\text{periodic}}$

$$-u_{xx} + 2q\cdot\cos(2x)\cdot u = \lambda u$$

if) $q=0$

↳ simple $-u_{xx} = \lambda u$ (7.3)

$$\underbrace{-D_N^{(2)} v + 2q \begin{pmatrix} \cos 2x_0 & & \\ & \ddots & \\ & & \cos 2x_N \end{pmatrix}}_{L} v = \lambda v$$

(3.12)

「Program 22」 : Airy E.q.

$$u_{xx} = \lambda x u, \quad -1 < x < 1, \quad u(\pm 1) = 0$$

<일반적인 고유벡터>

$$Av = \lambda B v$$

$$\tilde{L}\tilde{v} = \lambda \tilde{B}\tilde{v} \quad \rightarrow \quad B = \begin{pmatrix} x_0 & & & & \\ & x_1 & & & \\ & & \ddots & & \\ & & & x_{N-1} & \\ & & & & x_N \end{pmatrix}$$

$\tilde{B}$

↳ $(B^{-1}A)v = \lambda v$

「Program 23」 Laplace E.q.

$$-u_{xx} - u_{yy} + f(x,y)u = \lambda u, \quad -1 < x, y < 1, \quad u=0$$

$$-\tilde{L}\tilde{v} = \lambda\tilde{v} \Rightarrow \tilde{L} = \tilde{D}^2_{N,x} \otimes \tilde{I}_{N,y} + \tilde{I}_{N,x} \otimes \tilde{D}^2_{N,y}$$

$u(x,y) \sim \sin(k_x(x+1))\sin(k_y(1+y))$

$k_x^2 = \frac{\pi^2}{4} i^2$ ⇒ 허수 i 아님.

$k_y^2 = \frac{\pi^2}{4} j^2$

$\lambda = \frac{\pi^2}{4}(i^2 + j^2)$, $i, j = 1, 2, 3 \cdots$

$u_{rr} \simeq 2^2 D_N^2$

$u_r \simeq 2\, D_N^1$

「Excise 9.5」: Ressel E.4.

$\frac{1}{r}(r \cdot u_r)_r - \frac{1}{r^2} m^2 u = -\omega^2 u$, $0 < r < 1$, $u_r(0) = u(1) = 0$

$\frac{d}{dr}$

↳ $r^2 u_{rr} + r u_r - m^2 u = -\omega^2 r^2 u$

↳ $Lv = \lambda B v$

$-1 < x < 1$

$r = (x+1)\frac{1}{2}$ 이용,

$\frac{d}{dr} = \frac{dx}{dr} \cdot \frac{d}{dx} = 2\frac{d}{dx}$

$$\begin{pmatrix} 1 & 0 & \cdots & 0 \\ & & & \\ D_{N0}^1 & D_{N1}^1 & \cdots & D_{NN}^1 \end{pmatrix} \begin{pmatrix} v_0 \\ \vdots \\ w_N \end{pmatrix}$$

$$B = \begin{pmatrix} r_0^2 & & & & \\ & r_1^2 & & & \\ & & \ddots & & \\ & & & r_{N-1}^2 & \\ & & & & r_N^2 \end{pmatrix}$$

0

0