Maple Class 5

1 Lecture

1. **Recursive sequences!**
   (a) Factorial; $a_n := na_{n-1}$ where $a_0 = 1$.
   (b) Fibonacci numbers; $F_n := F_{n-1} + F_{n-2}$ where $F_0 = 1$ and $F_1 = 1$.
   (c) Number of Walk on the line with backward (-1) and forward $(+1)$ steps from the origin after n steps.
   (d) Returning walk: Number of walks with forward and backward steps from the origin and back to the origin after n steps.

2. **Two-dimensional recursive sequences!**
   (a) **Binomial**, $\binom{n}{k}$ $, n \geq 0:$ using definition

   $$\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$$

   where $\binom{n}{0} = \begin{cases} 1 & \text{if} \quad n = 0 \\ 0 & \text{otherwise.} \end{cases}$

   (b) **Lattice walk**
   $W(m,n)$ be the number of ways to walk from $(0,0)$ to (m,n) where each step is up or right.

   (c) **Lower-triangle walk (Catalan numbers)**
   $C(m,n)$ be the number of ways to walk from $(0,0)$ to (m,n) where each step is up or right and for each step $(x,y), x \geq y$.

3. **The Coupon Collector**

   Children buy photos of soccer stars for their albums, but they buy them in little non-transparent envelops, so they don't know which photo they will get.
   If there are n different photos, what is the expected number of pictures a kid has to buy until he or she gets every motif at least once?

   Program
   Name of procedure: `Coupon`

Input: Number of motif, n.
Output: The number of pictures to buy to complete their album.
Example: Input: `Coupon(5);` Output: `7`

2 Homework: Turn in both your maple-code and maple-worksheet

1. **Lucas numbers**

   Write the program to compute Lucas number which defines as follows

   $$L_n := L_{n-1} + L_{n-2}$$

   where $L_0 = 2$ and $L_1 = 1$.

   Name of procedure: `Lucas`
   Input: The non-negative number n
   Output: $L(n)$
   Example: Input: `Lucas(35);` Output: `20633239`

2. **Non-negative Walk:**

   Let $PW(n)$ be the number of walks with forward and backward step from the origin after n steps such that always stay in the non-negative axis.

   Find $PW(n)$ for $n = 1 \ldots 15$.

3. **The Coupon Collector**

   Let $T(n)$ be the average number of pictures to buy to complete the album of n motif.

   a) Observe the rate of growth of $T(n)$.
   Predict the ratio $\dfrac{T(n)}{nH(n)}$ as $n \to \infty$ where the harmonic series, $H(n) = 1 + \dfrac{1}{2} + \cdots + \dfrac{1}{n}$.

   b) The rate of growth of $H(n)$:

   $$H(n) \sim \log(n) + C \quad \text{as} \quad n \to \infty.$$

   Use Maple to predict the constant C.

   c) Write the program to simulate $U(n)$ the number of pictures to buy to complete **TWO** Album with n motif each.