Maple Class 4

1 Lecture

1. **1729** 1729 is the smallest positive integers that can be written as a sum of two cube in more than one way.

$$1729 = 12^3 + 1^3 = 10^3 + 9^3$$

We will investigate this kind of numbers with different powers.

Name of procedure: `Ramanujan`
Input: Power k, maximum number N
Output: Set of all the numbers less than N that can be written in the form $a^k + b^k$ in more than one way.
Example: Input: `Ramanujan`$(3, 10000)$; Output: {`1729,4104`}

2. **Guess Polynomial**
Patterns are everywhere in Mathematics. Given a sequence, we will guess the polynomial that fits the sequence.

Name of procedure: `GuessPol`
Input: The list of sequence L, starting number a, and variable name n.
Output: polynomial expression of the given sequence.
Example: Input: `GuessPol`$([1, 3, 6, 10, 15, 21, 28, 36, 45, 55], 1, n)$; Output: $\frac{n(n+1)}{2}$

3. **Monty Hall Problem**
The Monty Hall problem is a brain teaser, in the form of a probability puzzle, loosely based on the American television game show *Let's Make a Deal* and named after its original host, *Monty Hall*.

Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No.2?" Is it to your advantage to switch your choice?

It is obvious that if you don't switch the probability to win a car is $\frac{1}{3}$. In this problem, we will investigate the probability to win a car if you switch the door.

Program
Name of function: `SwitchMonty`

Input: the door's number p
Output: 1 if he wins the car and 0 otherwise.
Example: Input: `SwitchMonty(2);` Output: `0`

Name of procedure: `ProbWin`
Input: door's number, p and number of time to do simulation N
Output: The winning probability
Example: Input: `ProbWin(1, 100);` Output: `0.61`

2 Homework: Turn in both your maple-code and maple-worksheet

1. **Almost Famous**

   According to Fermat's Last Theorem, the equation

   $$x^3 + y^3 - z^3 = 0$$

   has no positive integer solution (x, y, z). However $x^3 + y^3 - z^3 = 1$ does have positive integer solutions. Write the program to find them.

   Name of procedure: `Fermat`
   Input: N, the range of x, y
   Output: The set of list $[x, y, z]$, $1 < y \leq x \leq N$ satisfies the equation.
   Example: Input: `Fermat(20);` Output: {`[10, 9, 12], [94, 64, 103]`}

2. **Prime Arithmetic Progressions**

   During my talk last week, Prof. Prezmo asked me to find 13-term arithmetic progressions of primes starting at 13, i.e. the sequence of primes in the form

   $$\{13, 13 + d, 13 + 2d, \ldots, 13 + 12d\}$$

   Let's explore it!

   Name of procedure: `PrimeAP`
   Input: the prime number p, length k
   Output: set of k-term arithmetic progressions starting at p
   Example: Input: `PrimeAP(5,5);` Output: {`5,11,17,23,29`}
   Input: `PrimeAP(7,7);` Output: {`7,157,307,457,607,757,907`}
   Input: `PrimeAP(11,7);` Output: {`11,123491,246971,370451,493931,617411,740891`}

3. **Generalized Monty Hall**

   Modified the program we did to find the probability of winning when there are 5 doors with 2 cars and 3 goats.