Project For Mathematic Software, T1-2016

This project worths 22% of your course grade (7% preparation and 15% presentation). You may choose one of the suggested projects below or come up with your own problem. On Friday, December 2 you will make a 10-20 minutes presentation using beamer program. Please feel free to ask me for help.

1 Monty Hall

a) Proof rigorously the probability of winning when use the switching strategy.

b) Simulate the probability of winning with switching strategy if there are $n+m$ doors with n cars and m goats.

c) Simulate the probability of winning with switching strategy if there are $n+m$ doors with n cars and m goats. This time the host reveals k goat door where $0 \leq k \leq m-1$.

d) What is the chance to win a car if the host reveals the goat door and offers an option to switch the door with probability p?

2 Coupon Collector

a) Proof rigorously that $T(n) = nH(n)$.

b) Let $U(n)$ be average number of pictures to buy to complete Two albums with n motif each. Is there anything you could say about the rate of growth of $U(n)$?

3 Integer Sequences

Write a computer program to find at least three new sequences that are not yet on Integer Sequences website. Then submit them to claim the ownership.

4 Formulas for Restricted Walk

Use examples given in the class, i.e. positive walk or Lower-triangle walk, to come up with other restricted walks in 1d and 2d. Conjecture the formulas and try to prove them rigorously.

5 Dreidel

The rule:
The k players take turns spinnig. They start out with NUTS nuts each, and immediately donate each one nut to the pot. If the dreidel landed on *Gimel* the turner gets to take the whole (Ganze) pot, (and then everyone donates one nut to the pot). If it landed on *Hay* , the player gets the (smaller) Half (Halbe). If it landed on *Nun*, then the player gets Nothing (Nix), and if it landed on *Shin*, the player pays (Shalem) one nut to the pot.

a) For 2 players ($k = 2$), find an average number of spins that starts with each player having $NUTS$ nuts, $0 \leq NUTS \leq 25$, and terminates as soon as one of the players runs out of nuts.
Example, for $NUTS = 2$, the average number of spins is 2.4. For $NUTS = 3$, the average number of spins is 8.329934935.

b) From part a) predicts the rate of growth of average number of spins.

6 Knight Tour

a) Find the number of Knight tour on 4×4 board.

b) Find the number of Knight tour on $3 \times n$ board. Check the sequence with Sloane's integer sequence website. Is there a closed form formula for it?

c) Find the number of Knight tour on $4 \times n$ board. Check the sequence with Sloane's integer sequence website. Is there a closed form formula for it?

d) Try part a)-c) again. This time the board is wrapped around from north-south and east-west.

7 Non-attacking Queens

a) Write the program to find the formula of the number of ways to place n non-attacking queens on an $n \times n$ board. Is there a formula as a function of n?

b) Find, $R(n)$, the maximum number of non-attacking Rook that can be placed on an $n \times n$ board.

c) Do similarly with the previous part with King, Knight and Bishop replacing the Rook. Find the formulas in term of n if possible.

8 Finite Values of Erdós's Conjecture

a) Let $A \subset \{1, 2, 3, \dots\}$ and the function $v(A) = \sum_{a \in A} \frac{1}{a}$. Find the maximum value of $v(A)$ from any set A which does not contain a 3-term arithmetic progressions.

Example If $A = \{1, 2, 4, 8, \dots\}$, A does not contain 3-term arithmetic progressions (why?) and $v(A) = \sum_{n=0}^{\infty} \frac{1}{2^n} = 2$. Can you do better?

b) Similarly, find the maximum value of $v(A)$ from any set A which does not contain a 4-term arithmetic progressions.

9 The Pancake Problem

The simplest pancake sorting algorithm requires at most $2n - 3$ flips. The minimum number of flips required to sort any stack of n pancakes has been shown to be no more than $\frac{18}{11}n$.

a) Given a positive integer n, write the program to find the minimum number of flips of any stack of n pancakes.

b) Given a stack of n pancakes, come up with your own sorting algorithm that takes less than $2n - 3$ flips.

Projects that have been picked and unpicked

1. Monty Hall (Bright)
2. Coupon Collector (Sam)
3. Integer Sequences
4. Formulas for Restricted Walk (Mind)
5. Dreidel
6. Knight Tour (Eksuree)
7. Non-attacking Queens
8. Finite Values of Erdós's Conjecture
9. The Pancake Problem (Marvin)