

Project For Mathematic Software, T1-2016

This project worths 22% of your course grade (7% preparation and 15% presentation). You may choose one of the suggested projects below or come up with your own problem. On Friday, December 2 you will make a 10-20 minutes presentation using beamer program. Please feel free to ask me for help.

1 Monty Hall

- a) Proof rigorously the probability of winning when use the switching strategy.
- b) Simulate the probability of winning with switching strategy if there are $n + m$ doors with n cars and m goats.
- c) Simulate the probability of winning with switching strategy if there are $n + m$ doors with n cars and m goats. This time the host reveals k goat door where $0 \leq k \leq m - 1$.
- d) What is the chance to win a car if the host reveals the goat door and offers an option to switch the door with probability p ?

2 Coupon Collector

- a) Proof rigorously that $T(n) = nH(n)$.
- b) Let $U(n)$ be number of pictures to buy to complete Two Album with n motif each. Is there anything you could say about the rate of growth of $U(n)$?

3 Knight Tour

- a) Find the number of Knight tour on 4×4 board.
- b) Find the number of Knight tour on $3 \times n$ board. Check the sequence with Sloane's integer sequence website. Is there a closed form formula for it?
- c) Find the number of Knight tour on $4 \times n$ board. Check the sequence with Sloane's integer sequence website. Is there a closed form formula for it?
- d) Try part a)-c) again. This time the board is wrapped around from north-south and east-west.

4 Finite Values of Erdős's Conjecture

- a) Let $A \subset \{1, 2, 3, \dots\}$ and the function $v(A) = \sum_{a \in A} \frac{1}{a}$. Find the maximum value of $v(A)$ from any set A which does not contain a 3-term arithmetic progressions.

Example If $A = \{1, 2, 4, 8, \dots\}$, A does not contain 3-term arithmetic progressions (why?) and $v(A) = \sum_{n=0}^{\infty} \frac{1}{2^n} = 2$. Can you do better?

- b) Similarly, find the maximum value of $v(A)$ from any set A which does not contain a 4-term arithmetic progressions.

5 The Pancake Problem