

Maple Exercise 1: Calculus Review

1 Functions and Polynomials

1. Let $f(x) = x^7 - 9x^6 + 29x^5 - 59x^4 + 95x^3 - 71x^2 - 8x + 4$.
 - a) Factor $f(x)$ over the real numbers.
 - b) Factor $f(x)$ over the complex numbers.
2. Let $f(x) = e^{\sin(x)}$.
 - a) Define $f(x)$ as a function.
 - b) Evaluate the values of f at $x \in \{-5, -4, -3, \dots, 5\}$.
 - c) Plot $f(x)$ on $x = [-5, 5]$.
3.
 - a) Plot the function $y = x^3$ for $-1 \leq x \leq 1$.
 - b) Now plot the functions $y = x, x^2, \dots, x^6$ together.
 - c) Describe the pattern in words. What do you think that the graph of x^{100} looks like? Use Maple to check your answer.

2 Limit

4. Let $f(x) = \frac{\sin(x)}{x}$.
 - a) Evaluate the values of f at $x \in \{1, 0.1, 0.01, \dots, 0.000001\}$.
 - b) Find $\lim_{x \rightarrow 0} f(x)$.
5. Let $x_n = \frac{a^{n+1} + b^{n+1}}{a^n + b^n}$ and $y_n = \frac{a^n b + a b^n}{a^n + b^n}$, where $a = 10$ and $b = 20$.
 - a) Find the values of x_n and y_n for $n = 1, 2, 3, \dots, 50$.
 - b) What do you think are the limiting values? (No need for proof, let's just do Experimental math.).

3 Derivative

6. Find the first and second derivative of

a) $f(x) = \frac{x^2 + 1}{x - 2}$.

b) $g(x) = \frac{\cot(x) + 1}{\csc(x)}$.

4 Riemann Sum

7. Estimate the area under the graph of $f(x) = \sin(x)$ from $x = 0$ to $x = 3$ using

a) three approximating rectangles and mid-points. ie. $f(0.5) + f(1.5) + f(2.5)$.

b) six approximating rectangles and mid-points.

ie. $\frac{f(0.25)}{2} + \frac{f(0.75)}{2} + \frac{f(1.25)}{2} + \frac{f(1.75)}{2} + \frac{f(2.25)}{2} + \frac{f(2.75)}{2}$

c) twelve approximating rectangles and mid-points.

d) twenty four approximating rectangles and mid-points.

e) Compare your results to the actual area, $\int_0^3 \sin(x) dx$.

5 Integration

8. Evaluate the following integrals:

a) $\int_0^2 (1 + x^2)^3 dx$.

b) $\int_4^9 (\sqrt{x} + \frac{1}{\sqrt{x}})^2 dx$.

c) $\int \frac{\arcsin(x)}{\sqrt{1-x^2}} dx$.

d) $\int_0^5 \frac{1}{\sqrt{x^2-1}} dx$.

6 Parametric Curve

9. Look up help `plot` to find the command to plot the parametric curve:

$$x = \sin(t + \sin(t)), y = \cos(t + \cos(t)), -4\pi \leq t \leq 4\pi.$$

7 Sequences and Series

10. List the first 10 terms of the sequence, $a_n = \frac{n^3}{n!}$.

11. a) Find $\sum_{n=1}^k \frac{1}{n(n+1)}$ for $k=10, 100, 1000$ and 10000 .

What do you think about the value of the infinite sum: $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$.

b) Find $\frac{1}{\pi^2} \sum_{n=1}^k \frac{1}{n^2}$ for $k=10, 100, 1000$ and 10000 .

What do you think about the value of the infinite sum: $\frac{1}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2}$. Answer in term of fraction.

Note: You might curious about other infinite sums like $\sum_{n=1}^{\infty} \frac{1}{n^4}, \sum_{n=1}^{\infty} \frac{1}{n^6}, \dots$

These are options for you to explore!

12. Use command `taylor` to find the first ten terms of the Taylor series for $f(x) = \cos(x^2)$ centered at $x = \pi$.

8 Vector and Matrix

13. Let $A = \langle a, b, c \rangle$ and $B = \langle x, y, z \rangle$.

Add package `VectorCalculus` to perform the operation $A \cdot B$ and $A \times B$.

14. Come up with your own 3-by-3 matrix A , use package `LinearAlgebra` to find $A^2, A^{-1}, \det(A)$, eigenvalues and eigenvectors of A .

9 Surfaces in Three dimension

15. Use the command `plot3d` to sketch the following surfaces:

a) $x^2 + y^2 + z^2 = 1$.

b) $x^2 + y^2 = z^2$.

c) $x^2 + y^2 = z$.

d) $x^2 + y^2 - z^2 = 1$.

e) $x^2 - y^2 = z$.