Maple Exercise 2: Still Command-line in Maple

1 Sequence and Series

1. Find the first ten terms of the following sequences:

   a) $a_n = n^2$

   b) $a_n = 5 \cdot 7^n - 3 \cdot 4^n$

   c) $a_n = sn + t$

2. Calculate the first ten coefficients of the power series of the functions:

   a) $\dfrac{1}{\cos(x)}$

   b) $\arcsin(x)$

   c) $(1+x)^{-m}$

2 Linear Algebra

3. Use command `solve` to solve the system of linear equations:

$$x_1 + 5x_3 - x_4 = 7$$
$$2x_1 - x_2 + 6x_3 = 8.$$

4. (Experimental Math.) Consider the following permutation matrix

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

   Find the sequence $A, A^2, A^3, ..., A^{10}$. Do you notice any pattern (or relation)?

5. Give an example of a three 2-by-2 matrix A, B, C such that

$$(AB)C \neq C(BA).$$

3   Binomail Sum

6. Try `add` function to evaluate the following sum

   a) $\displaystyle\sum_{i=0}^{n} 2^i$ for $n = 0, 1, ..., 10$.

   b) $\displaystyle\sum_{i=0}^{n} i \cdot 2^i$ for $n = 0, 1, ..., 10$.

   c) $\displaystyle\sum_{i=0}^{n} i^2 \cdot 2^i$ for $n = 0, 1, ..., 10$.

7. Use Maple to find the closed form formula for $\displaystyle\sum_{k=0}^{n} k \binom{2n+1}{2k+1}$. Also check your answer with specific n for $1 \le n \le 10$.

8. Use command `sum` to find formulas of the following sums,

   a) $\displaystyle\sum_{i=0}^{n} i \cdot 2^i$

   b) $\displaystyle\sum_{i=0}^{n} i^2 \cdot 2^i$

   c) $\displaystyle\sum_{i=0}^{n} \frac{i^4 \cdot 4^i}{\binom{2i}{i}}$

   d) $\displaystyle\sum_{i=0}^{n} \frac{\binom{2i}{i}^2}{(i+1)4^{2i}}$

4   Number Theory

9. List the $(2k+1)^{th}$ prime where $k = 0, 1, 2, 3, ..., 15$.

10. In the old time, Fermat made a bold conjecture that $2^{2^n} + 1$ is a prime for all integer $n \ge 0$. Try to prove or disprove his conjecture.