

# **Artificial Neural Networks**

[2500WETANN]

**José Oramas** 



# **Modeling Sequences with Neural Networks**

José Oramas



# **Today's Lecture - Outline**

- Recap Sequence Modeling
  - → [Algorithmically] How to approach the problem
- Sequence Modeling with Recurrent Architectures
  - $\rightarrow$  RNNs, LSTMs, GRUs, etc.
- Sequence Generation
- Transformers and Attention Mechanisms



# Modeling Data Sequences [ in theory ]

#### **Some Foundations**

- Supervised Learning
- Data

$$\{x,y\}_i$$

Model

$$\hat{y} \approx f_{\theta}(x)$$

- Loss

$$L(\theta) = \sum_{i=1}^{N} l(f_{\theta}(x_i), y_i)$$

- Optimization

$$\theta^* = arg \ min_{\theta} \ L(\theta)$$

# Modeling Data Sequences [ in theory ]

#### **Some Foundations**

- Supervised Learning
- Data

$$\{x,y\}_i$$

- Model

$$\hat{y} \approx f_{\theta}(x)$$

- Loss

$$L(\theta) = \sum_{i=1}^{N} l(f_{\theta}(x_i), y_i)$$

- Optimization

$$\theta^* = arg \ min_{\theta} \ L(\theta)$$

- Modeling Sequences
- Data

$$\{x\}_i$$

- Model

$$p(x) \approx f_{\theta}(x)$$

- Loss

$$L(\theta) = \sum_{i=1}^{N} log \ p(f_{\theta}(x_i))$$

- Optimization

$$\theta^* = arg \ max_{\theta} \ L(\theta)$$

# **Modeling Sequences - classical approaches**

### **Summarizing**

+ Good / Intuitive Models

Not Scalable



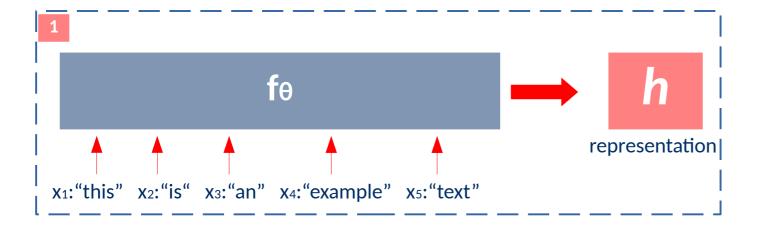
# **Modeling Data Sequences**

[ How to do it... in practice ]



# Modeling Data Sequences [in practice]

- 1. Describing/Vectorizing the Context
- Idea: Learn how to represent a sub-sequence

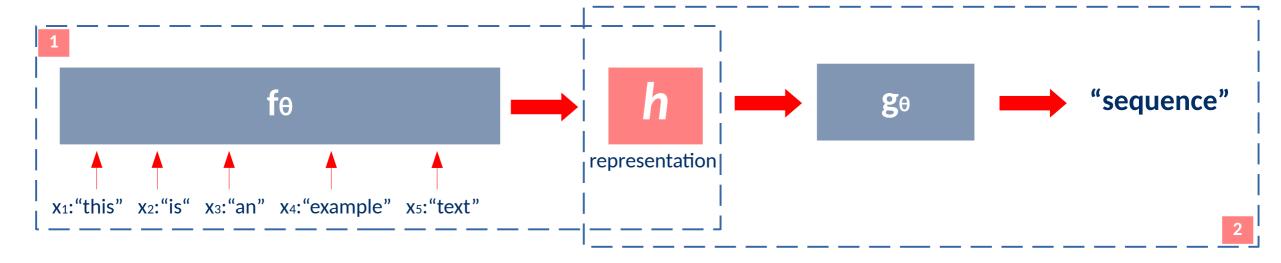




# Modeling Data Sequences [ in practice ]

### 1. Describing/Vectorizing the Context

Idea: Learn how to represent a sub-sequence



### 2. Modeling Conditional Probabilities

• Idea: Predicting the next element given the context



# Recurrent Architectures

[RNNs, LSTMs, GRUs, etc.]



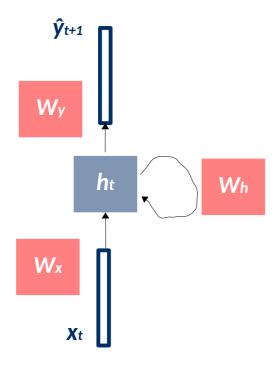
# Recurrent Neural Networks

[ The Most Popular Architecture ]



### **Provide Neural Networks with Memory**

Idea: Use a persistent state h that encodes past observations (context)



#### **Defined by three equations**

$$h_t = \tanh(W_h h_{t-1} + W_x x_t)$$

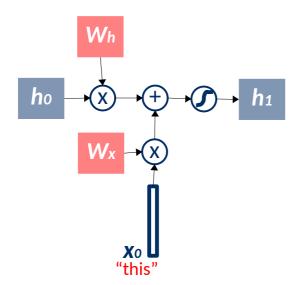
$$p(y_{t+1}) = softmax(W_y h_t)$$

$$L_{\theta}(y, \hat{y})_t = -y_t \log \hat{y}_t$$



### **Provide Neural Networks with Memory**

Idea: Use a persistent state h that encodes past observations (context)

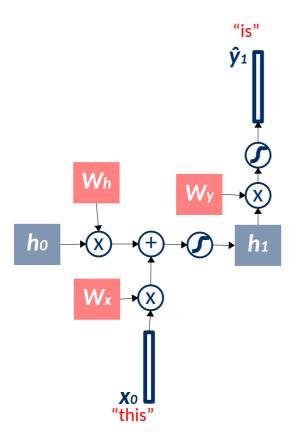


$$h_t = \tanh(W_h h_{t-1} + W_x x_t)$$



### **Provide Neural Networks with Memory**

Idea: Use a persistent state h that encodes past observations (context)



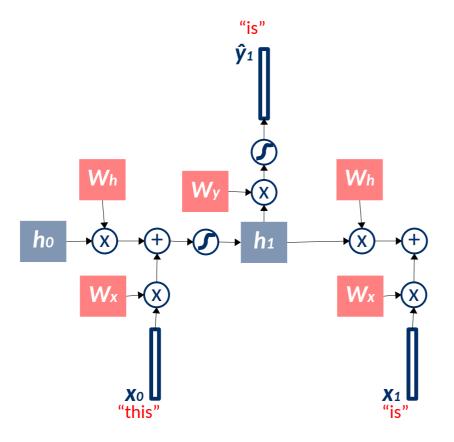
The probability of next element is obtained from the state **h** 

$$p(y_{t+1}) = softmax(W_y h_t)$$



### **Provide Neural Networks with Memory**

Idea: Use a persistent state h that encodes past observations (context)

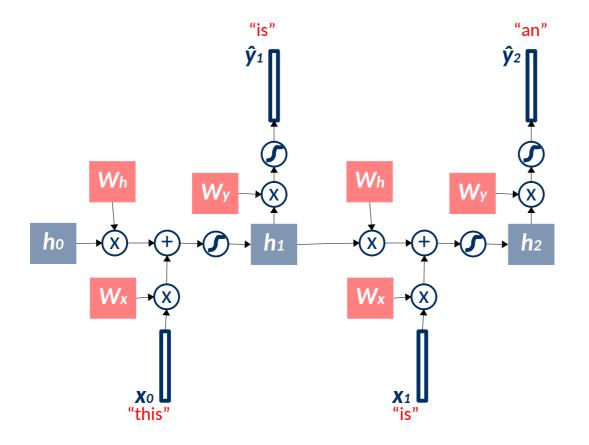


Input the next element **x**<sub>1</sub> from the sequence



### **Provide Neural Networks with Memory**

Idea: Use a persistent state h that encodes past observations (context)

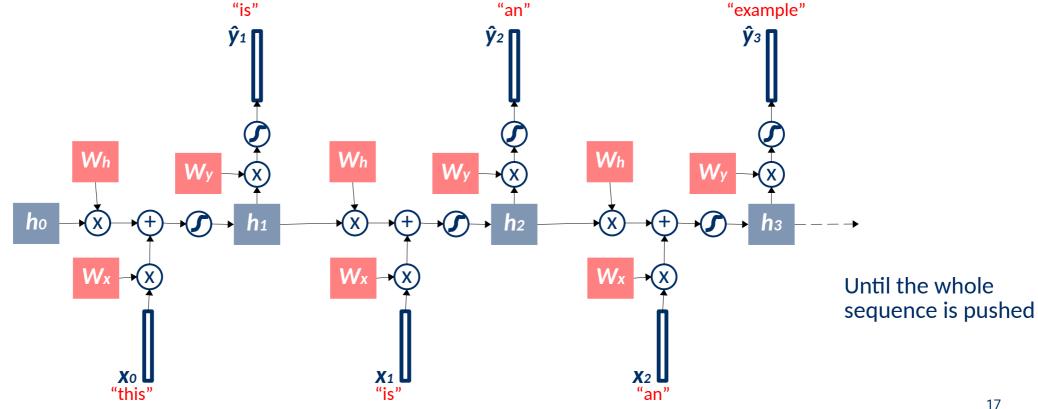


Keep going



### **Provide Neural Networks with Memory**

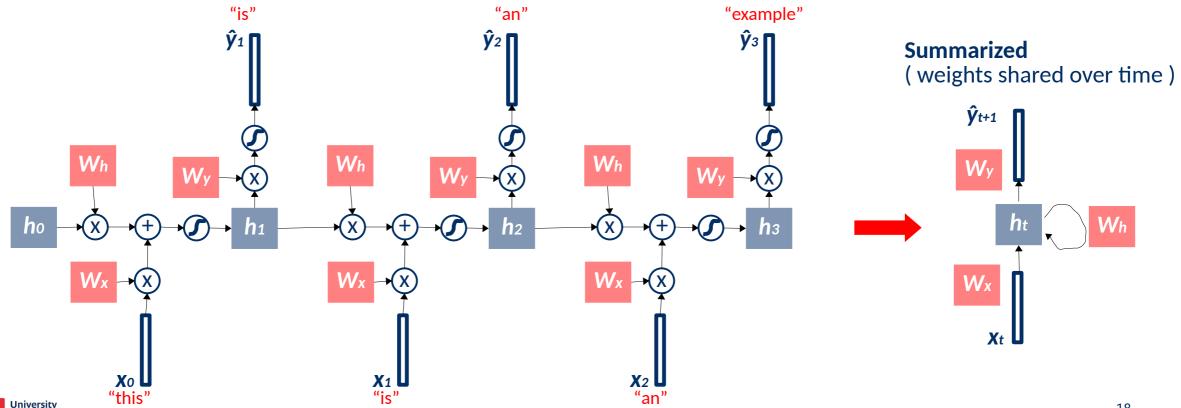
**Idea:** Use a persistent state **h** that encodes past observations (context)





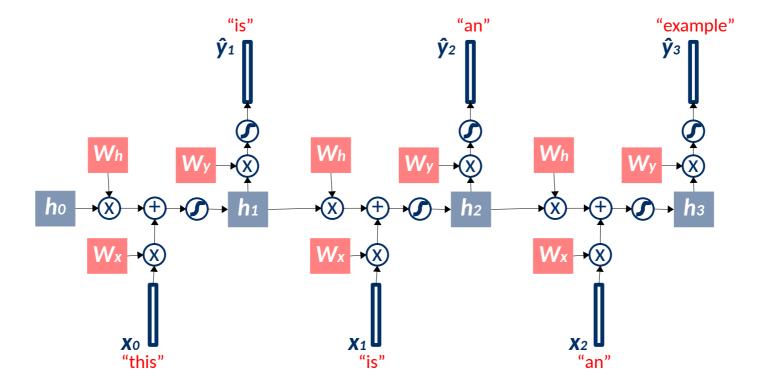
### **Provide Neural Networks with Memory**

**Idea:** Use a persistent state **h** that encodes past observations (context)



### **Learning | Training**

- Idea: Formulate the next-word prediction as a classification problem
- Number of classes = vocabulary\_size → Use the cross-entropy loss.





### **Learning | Training**

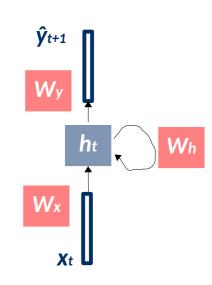
- Idea: Formulate the next-word prediction as a classification problem
- Number of classes = vocabulary\_size → Use the cross-entropy loss.

#### Given a sequence of **T** elements:

For one element 
$$t \to L_{\theta}(y, \hat{y})_t = -y_t \log \hat{y}_t$$

For the sequence 
$$\rightarrow L_{\theta}(y,\hat{y}) = -\sum_{t=1}^{T} y_t \log \hat{y}_t$$

Trainable parameters 
$$\rightarrow \theta = \{W_x, W_h, W_y\}$$





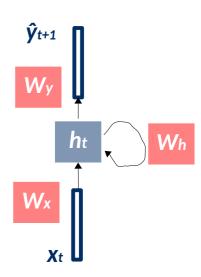
### Back-Prop. - Differentiation wrt. the parameters $\theta = \{W_x, W_h, W_y\}$

$$h_t = \tanh(W_h h_{t-1} + W_x x_t)$$

$$p(y_{t+1}) = softmax(W_y h_t)$$

$$L_{\theta}(y, \hat{y})_t = -y_t \log \hat{y}_t$$

$$\frac{\partial L_{\theta,t}}{\partial W_y} = \frac{\partial L_{\theta,t}}{\partial \hat{y}_t} \frac{\partial \hat{y}_t}{\partial W_y}$$
$$= (y_t - \hat{y}_t)h_t$$





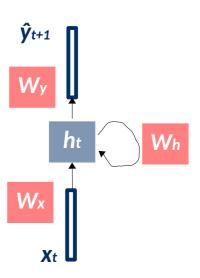
# Back-Prop. - Differentiation wrt. the parameters $\theta = \{W_x, W_h, W_y\}$

$$h_t = \tanh(W_h h_{t-1} + W_x x_t)$$

$$p(y_{t+1}) = softmax(W_y h_t)$$

$$L_{\theta}(y, \hat{y})_t = -y_t \log \hat{y}_t$$

$$\frac{\partial L_{\theta,t}}{\partial W_h} = \frac{\partial L_{\theta,t}}{\partial \hat{y}_t} \frac{\partial \hat{y}_t}{\partial h_t} \frac{\partial h_t}{\partial W_h}$$





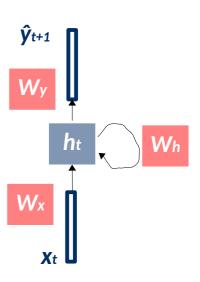
# Back-Prop. - Differentiation wrt. the parameters $\theta = \{W_x, W_h, W_y\}$

$$h_t = \tanh(W_h h_{t-1} + W_x x_t)$$

$$p(y_{t+1}) = softmax(W_y h_t)$$

$$L_{\theta}(y, \hat{y})_t = -y_t \log \hat{y}_t$$

$$\frac{\partial L_{\theta,t}}{\partial W_h} = \frac{\partial L_{\theta,t}}{\partial \hat{y}_t} \frac{\partial \hat{y}_t}{\partial h_t} \frac{\partial h_t}{\partial W_h}$$
$$\frac{\partial h_t}{\partial W_h} = \frac{\partial h_t}{\partial W_h} + \frac{\partial h_t}{\partial h_{t-1}} \frac{\partial h_{t-1}}{\partial W_h}$$





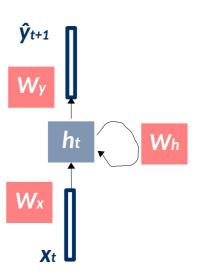
# Back-Prop. - Differentiation wrt. the parameters $\theta = \{W_x, W_h, W_y\}$

$$h_t = \tanh(W_h h_{t-1} + W_x x_t)$$

$$p(y_{t+1}) = softmax(W_y h_t)$$

$$L_{\theta}(y, \hat{y})_t = -y_t \log \hat{y}_t$$

$$\begin{split} \frac{\partial L_{\theta,t}}{\partial W_h} &= \frac{\partial L_{\theta,t}}{\partial \hat{y}_t} \frac{\partial \hat{y}_t}{\partial h_t} \frac{\partial h_t}{\partial W_h} \\ \frac{\partial h_t}{\partial W_h} &= \frac{\partial h_t}{\partial W_h} + \frac{\partial h_t}{\partial h_{t-1}} \frac{\partial h_{t-1}}{\partial W_h} \\ &= \frac{\partial h_t}{\partial W_h} + \frac{\partial h_t}{\partial h_{t-1}} \left[ \frac{\partial h_{t-1}}{\partial W_h} + \frac{\partial h_{t-1}}{\partial h_{t-2}} \frac{\partial h_{t-2}}{\partial W_h} \right] \end{split}$$



### Back-Prop. - Differentiation wrt. the parameters $\theta = \{W_x, W_h, W_y\}$

$$h_t = \tanh(W_h h_{t-1} + W_x x_t)$$

$$p(y_{t+1}) = softmax(W_y h_t)$$

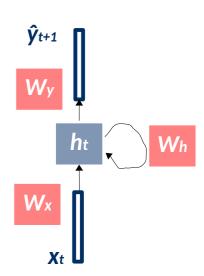
$$L_{\theta}(y, \hat{y})_t = -y_t \log \hat{y}_t$$

$$\frac{\partial L_{\theta,t}}{\partial W_h} = \frac{\partial L_{\theta,t}}{\partial \hat{y}_t} \frac{\partial \hat{y}_t}{\partial h_t} \frac{\partial h_t}{\partial W_h}$$

$$\frac{\partial h_t}{\partial W_h} = \frac{\partial h_t}{\partial W_h} + \frac{\partial h_t}{\partial h_{t-1}} \frac{\partial h_{t-1}}{\partial W_h}$$

$$= \frac{\partial h_t}{\partial W_h} + \frac{\partial h_t}{\partial h_{t-1}} \left[ \frac{\partial h_{t-1}}{\partial W_h} + \frac{\partial h_{t-1}}{\partial h_{t-2}} \frac{\partial h_{t-2}}{\partial W_h} \right]$$

$$= \sum_{k=1}^{t} \frac{\partial h_t}{\partial h_k} \frac{\partial h_k}{\partial W_h}$$





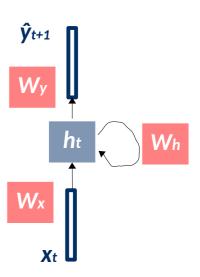
### Back-Prop. - Differentiation wrt. the parameters $\theta = \{W_x, W_h, W_y\}$

$$h_t = \tanh(W_h h_{t-1} + W_x x_t)$$

$$p(y_{t+1}) = softmax(W_y h_t)$$

$$L_{\theta}(y, \hat{y})_t = -y_t \log \hat{y}_t$$

$$\begin{split} \frac{\partial L_{\theta,t}}{\partial W_h} &= \frac{\partial L_{\theta,t}}{\partial \hat{y}_t} \frac{\partial \hat{y}_t}{\partial h_t} \frac{\partial h_t}{\partial W_h} \\ \frac{\partial h_t}{\partial W_h} &= \frac{\partial h_t}{\partial W_h} + \frac{\partial h_t}{\partial h_{t-1}} \frac{\partial h_{t-1}}{\partial W_h} & \text{Back-propagation through time} \\ &= \frac{\partial h_t}{\partial W_h} + \frac{\partial h_t}{\partial h_{t-1}} \left[ \frac{\partial h_{t-1}}{\partial W_h} + \frac{\partial h_{t-1}}{\partial h_{t-2}} \frac{\partial h_{t-2}}{\partial W_h} \right] \\ &= \sum_{k=1}^t \frac{\partial h_t}{\partial h_k} \frac{\partial h_k}{\partial W_h} \end{split}$$





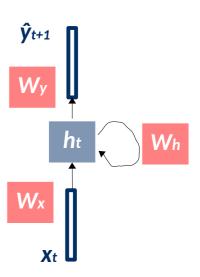
### Back-Prop. - Differentiation wrt. the parameters $\theta = \{W_x, W_h, W_y\}$

$$h_t = \tanh(W_h h_{t-1} + W_x x_t)$$

$$p(y_{t+1}) = softmax(W_y h_t)$$

$$L_{\theta}(y, \hat{y})_t = -y_t \log \hat{y}_t$$

$$\frac{\partial L_{\theta,t}}{\partial W_h} = \frac{\partial L_{\theta,t}}{\partial \hat{y}_t} \frac{\partial \hat{y}_t}{\partial h_t} \frac{\partial h_t}{\partial W_h}$$



### Back-Prop. - Differentiation wrt. the parameters $\theta = \{W_x, W_h, W_y\}$

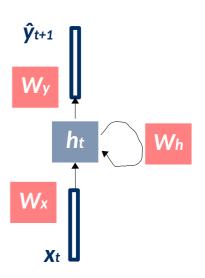
$$h_t = \tanh(W_h h_{t-1} + W_x x_t)$$

$$p(y_{t+1}) = softmax(W_y h_t)$$

$$L_{\theta}(y, \hat{y})_t = -y_t \log \hat{y}_t$$

$$\frac{\partial L_{\theta,t}}{\partial W_h} = \frac{\partial L_{\theta,t}}{\partial \hat{y}_t} \frac{\partial \hat{y}_t}{\partial h_t} \frac{\partial h_t}{\partial W_h}$$

$$= \frac{\partial L_{\theta,t}}{\partial \hat{y}_t} \frac{\partial \hat{y}_t}{\partial h_t} \sum_{k=1}^t \frac{\partial h_t}{\partial h_k} \frac{\partial h_k}{\partial W_h}$$





# Back-Prop. - Differentiation wrt. the parameters $\theta = \{W_x, W_h, W_y\}$

$$h_t = \tanh(W_h h_{t-1} + W_x x_t)$$

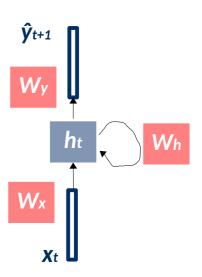
$$p(y_{t+1}) = softmax(W_y h_t)$$

$$L_{\theta}(y, \hat{y})_t = -y_t \log \hat{y}_t$$

$$\frac{\partial L_{\theta,t}}{\partial W_h} = \frac{\partial L_{\theta,t}}{\partial \hat{y}_t} \frac{\partial \hat{y}_t}{\partial h_t} \frac{\partial h_t}{\partial W_h}$$

$$= \frac{\partial L_{\theta,t}}{\partial \hat{y}_t} \frac{\partial \hat{y}_t}{\partial h_t} \sum_{k=1}^t \frac{\partial h_t}{\partial h_k} \frac{\partial h_k}{\partial W_h}$$

$$= \sum_{k=1}^t \frac{\partial L_{\theta,t}}{\partial \hat{y}_t} \frac{\partial \hat{y}_t}{\partial h_t} \frac{\partial \hat{y}_t}{\partial h_t} \frac{\partial h_t}{\partial h_k} \frac{\partial h_k}{\partial W_h}$$





### **Summarizing**

- + Good at modeling sequences of variable length
- + Trainable via Back-Prop. → differentiable

Suffer from vanishing gradients for long sequences



# Long Short-Term Memory Networks

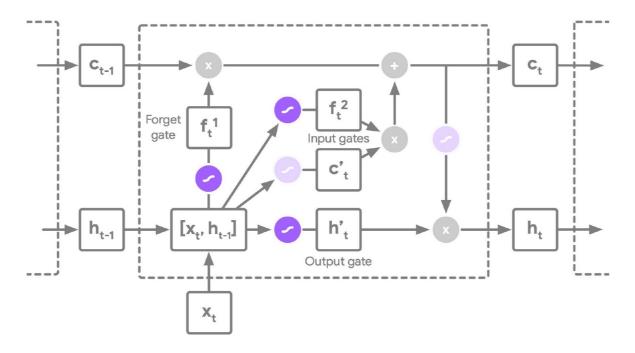
[ RNNs with Extra Memory ]



[ Hochreiter & Jurgen , 1997 ]

### Provide the capability of choosing what to remember/forget

Idea: Provide special gates to control the flow of "memories"

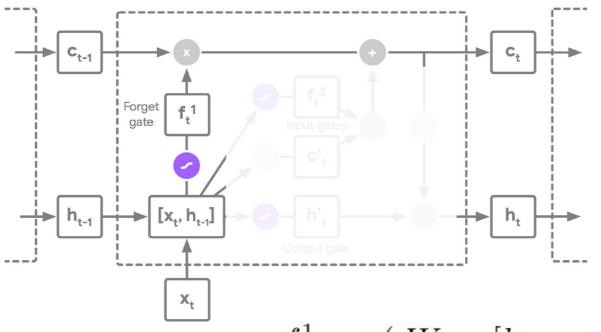




[ Hochreiter & Jurgen, 1997]

### Provide the capability of choosing what to remember/forget

• f1: Forget Gate



regulate what information to keep/ignore

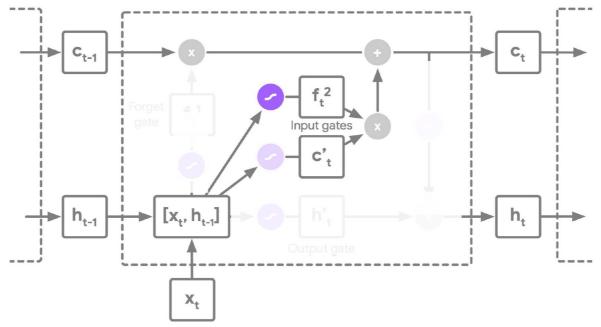
$$f_t^1 = \sigma(W_{f^1} \cdot [h_{t-1}, x_t] + b_{f^1})$$



[ Hochreiter & Jurgen, 1997]

### Provide the capability of choosing what to remember/forget

• f<sub>2</sub>: Input Gate



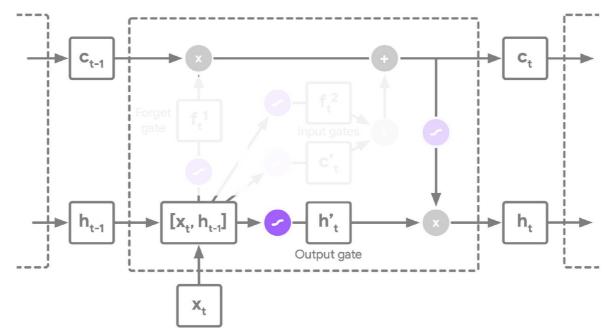
Decides what information to update

$$= \sigma(W_{f^2} \cdot [h_{t-1}, x_t] + b_{f^2}) \odot \tanh(W_{c'}[h_{t-1}, x_t] + b_{c'})$$

[ Hochreiter & Jurgen, 1997]

### Provide the capability of choosing what to remember/forget

h't: Output Gate



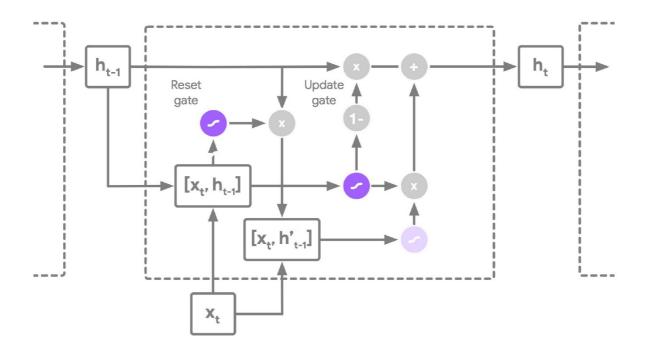
$$= \sigma(W_{h'_t} \cdot [h_{t-1}, x_t] + b_{h'_t}) \odot \tanh(c_t)$$



# Gated Recurrent Units (GRUs) [ Cho et al., 2014 ]

### **A Simplifed LSTM Network**

• Idea: Provide special gates to control the flow of "memories"





#### LSTMs and GRUs

### **Summarizing**

- + Good at modeling sequences of variable length
- + Trainable via Back-Prop. → differentiable

+ Capable of handling long sequences

(robust to vanishing/exploding gradients)



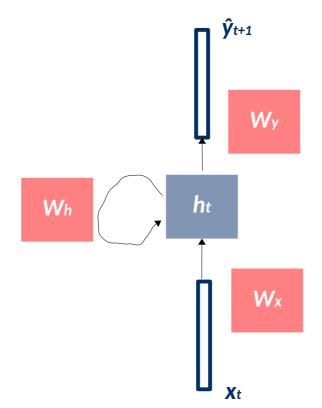
# Predictions from Sequences



# **Predictions from Sequences**

### **Training Classifiers/Regressors from Sequences**

- 1) Attach a related head (classification, regression, etc.) to the persistent state
- 2) Measure the loss wrt. the prediction task

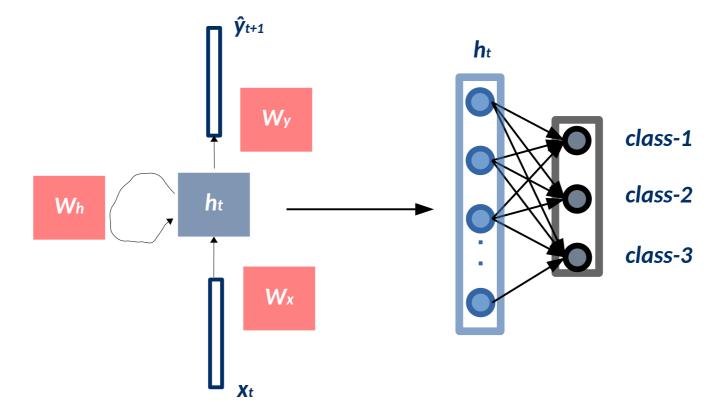




# **Predictions from Sequences**

### **Training Classifiers/Regressors from Sequences**

- 1) Attach a related head (classification, regression, etc.) to the persistent state
- 2) Measure the loss wrt. the prediction task





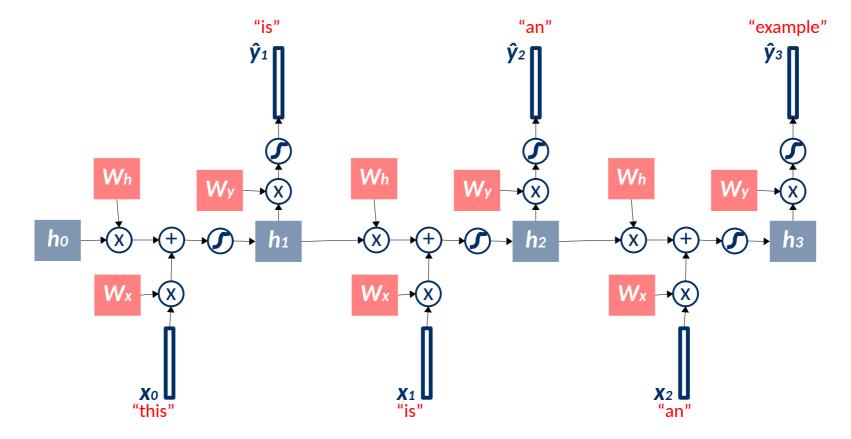
# **Generating Sequences**



# **Generating Sequences**

### Sample the next best element from the predicted distribution

• Idea: Use the predicted element  $\hat{y}_t$  as input in the next iteration

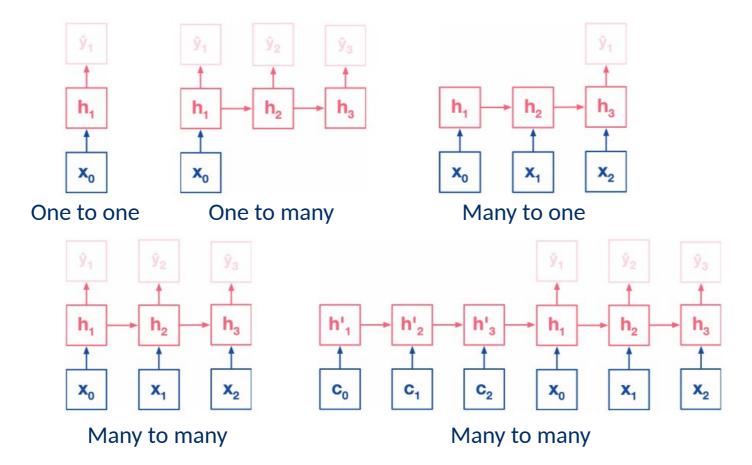




# **Generating Sequences**

### Several options are possible - beyond text sequences

Idea: Different ways to define inputs, context and outputs



#### **Some Applications**

- Language Translation
- Speech-to-Text
- Contextual Search
- Image Captioning

# Break

[ Let's meet again in 15 mins. ]



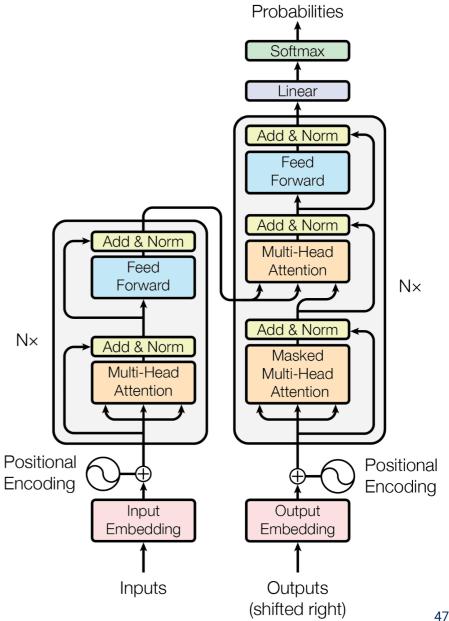
# **Transformers**



# **Transformers** [Vaswani et al., 2017]

#### Some specs

- Removed Recurrence components
- Based solely on the attention mechanism
- Originally addressed translation tasks [English-German | English-French]



Output



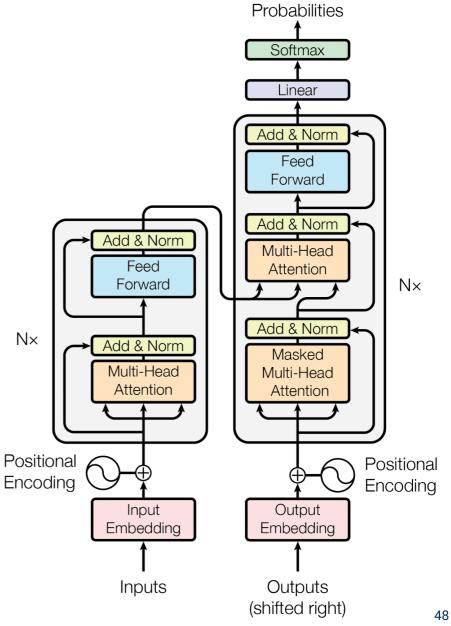
## **Transformers** [Vaswani et al., 2017]

#### Some specs

- Removed Recurrence components
- Based solely on the attention mechanism
- Originally addressed translation tasks [English-German | English-French]



Scary-looking yes, difficult not Let's follow Thomas' presentation



Output



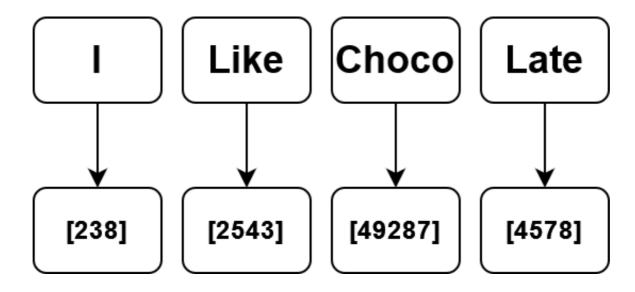
Lets consider the following sequence

# "I like chocolate"



### Tokenization: defining granular unit of processing

- Break the input into smaller units (tokens)
- Different levels of codification possible (character, word, etc.)

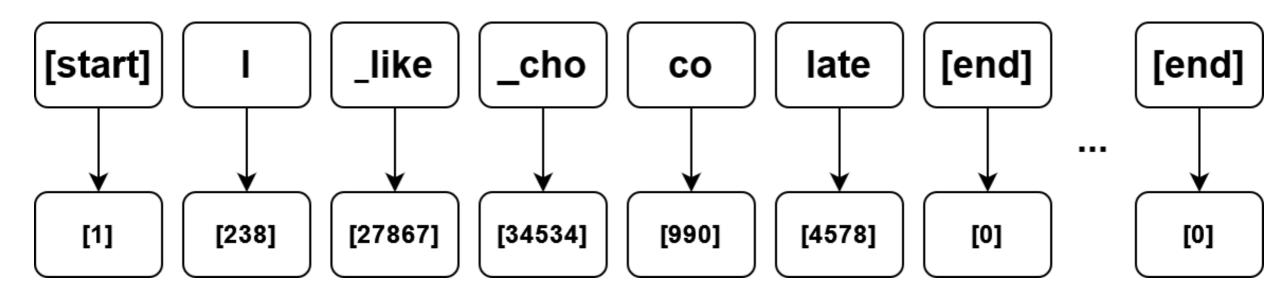




### Tokenization: defining granular unit of processing

- Break the input into smaller units (tokens)
- Different levels of codification possible (character, word, etc.)

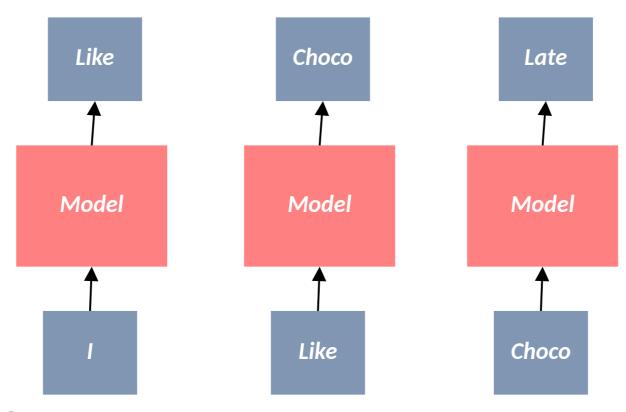
\*In reality it is not that straight-forward





[Dooms, 2024]

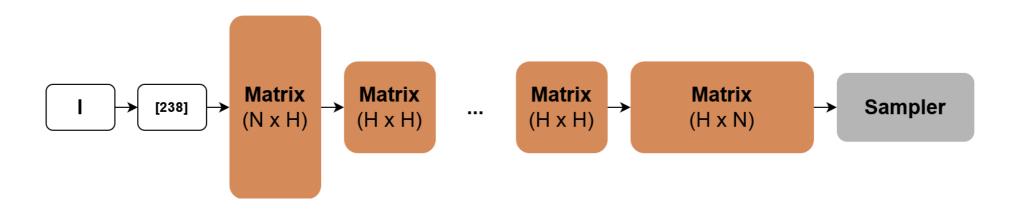
Predicting the next element (token)





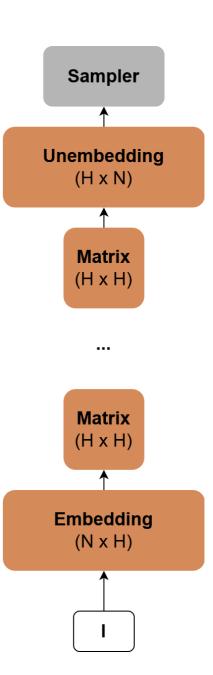
### **A Very Popular Recipe**

Moving to a lower dimensional space (determined by H)



### **A Very Popular Recipe**

Moving to a lower dimensional space (determined by H)

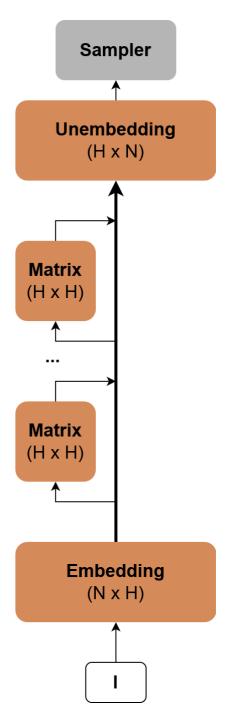




[Dooms, 2024]

### **A Very Popular Recipe**

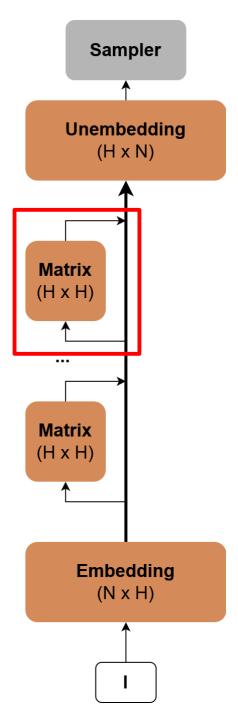
- Moving to a lower dimensional space (determined by H)
- Using residual layers (why?)





### **A Very Popular Recipe**

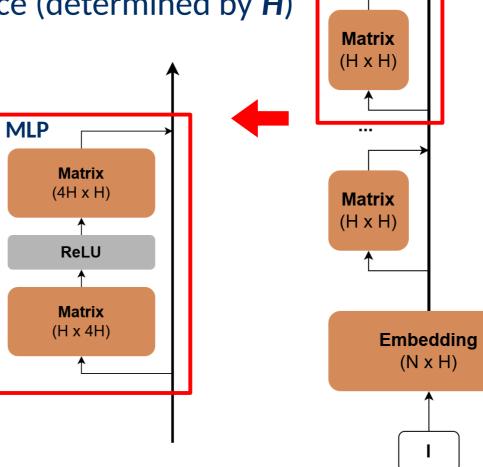
- Moving to a lower dimensional space (determined by H)
- Using residual layers (why?)





### **A Very Popular Recipe**

- Moving to a lower dimensional space (determined by H)
- Using residual layers (why?) MLP



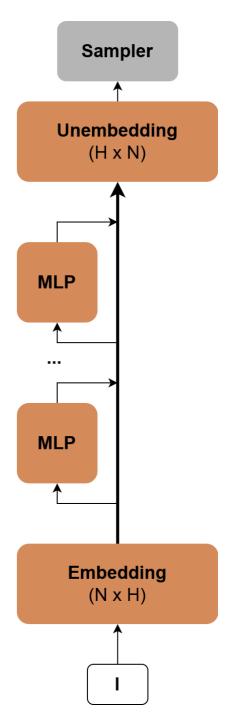
Sampler

Unembedding (H x N)



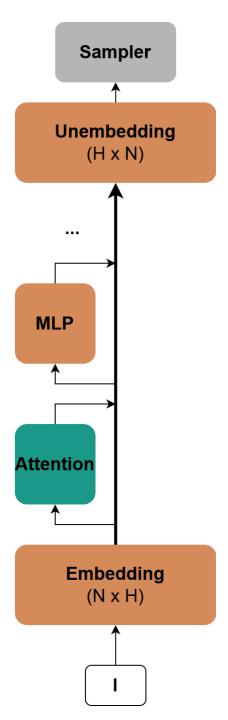
### **A Very Popular Recipe**

- Moving to a lower dimensional space (determined by H)
- Using residual layers (why?) MLP



#### **A Very Popular Recipe**

- Moving to a lower dimensional space (determined by H)
- Using residual layers (why?) MLP
- Add an [self] attention layer

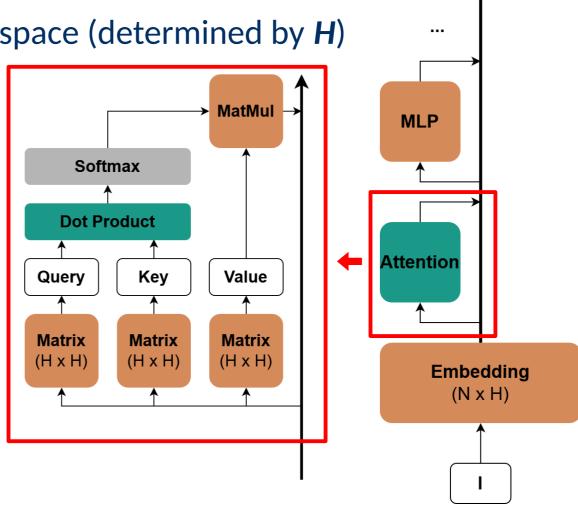




#### **A Very Popular Recipe**

Moving to a lower dimensional space (determined by H)

- Using residual layers (why?)
- Add an [self] attention layer



Sampler

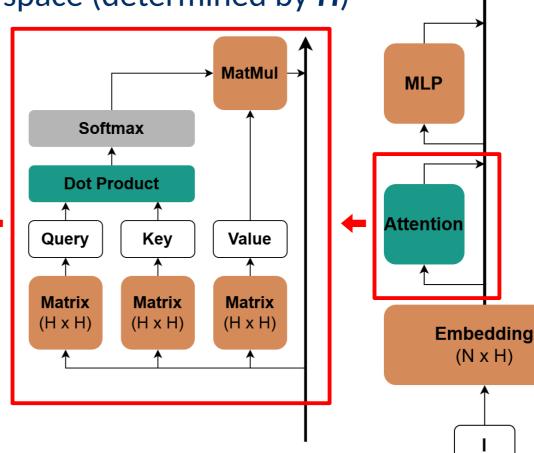
Unembedding (H x N)



### A Very Popular Recipe

- Moving to a lower dimensional space (determined by **H**)
- Using residual layers (why?)
- Add an [self] attention layer

$$\operatorname{Attention}(Q,K,V) = \operatorname{softmax}(\frac{QK^T}{\sqrt{d_k}})V$$





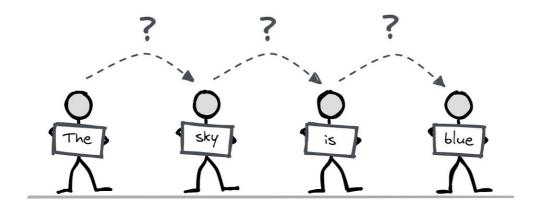
Sampler

Unembedding  $(H \times N)$ 

 $(N \times H)$ 

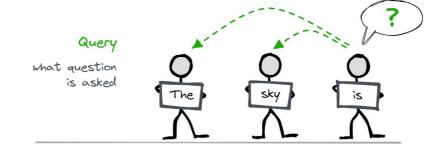
#### **Attention Mechanism - An Intuition**

Each person in the line tries to guess what word the person in front of them is holding.

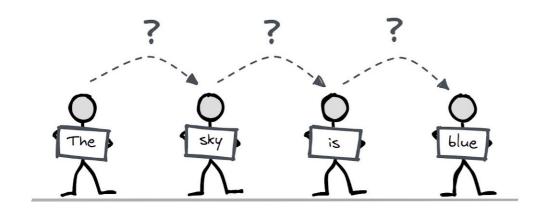




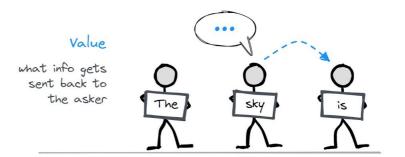
#### **Attention Mechanism - An Intuition**



Each person in the line tries to guess what word the person in front of them is holding.





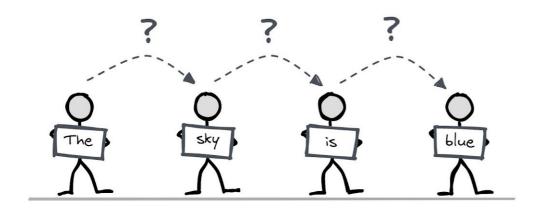


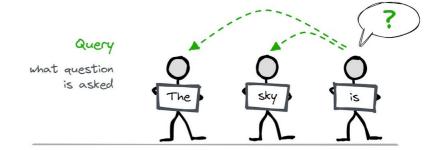


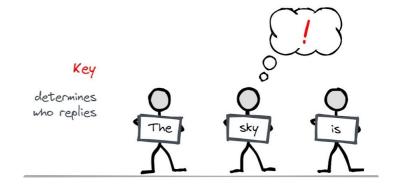
#### **Attention Mechanism - An Intuition**

- $Q \rightarrow the question$
- K  $\rightarrow$  critical element to focus on
- V → the information to be forwarded

Each person in the line tries to guess what word the person in front of them is holding.











#### **Attention Mechanism - An Intuition**

- $Q \rightarrow the question$
- K  $\rightarrow$  critical element to focus on
- V → the information to be forwarded

#### A database Analogy

	Query = 5	Values	Output
Key = 1	0	1	0
Key = 5	1	2	2
Key = 3	0	3	0
Key = 4	0	4	0
Key = 5	1	5	5

#### **Attention Mechanism - An Intuition**

- $Q \rightarrow \text{the question}$
- K  $\rightarrow$  critical element to focus on
- V → the information to be forwarded

Like convolution but with dynamic weight (query-key similarity)

#### A database Analogy

Query	=	5
-------	---	---

0.5

Key = 1

Key = 5

**Values** 

1

2

3

4

5

#### Output

0.125

2

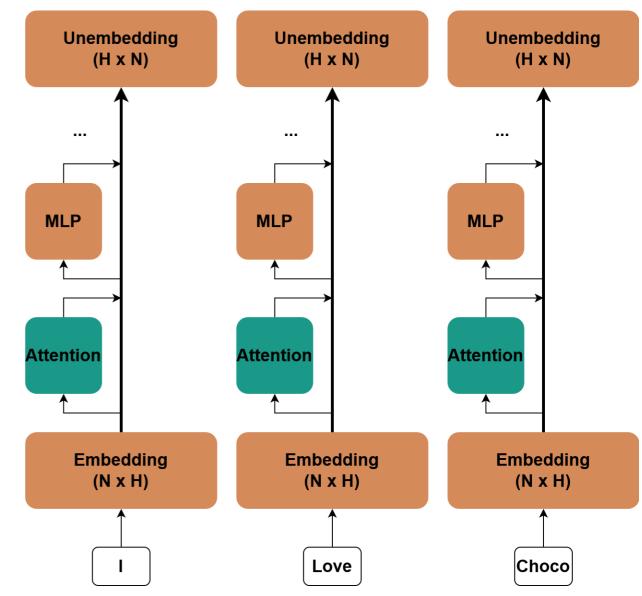
0.75

2

5

## **Strengths: Communication**

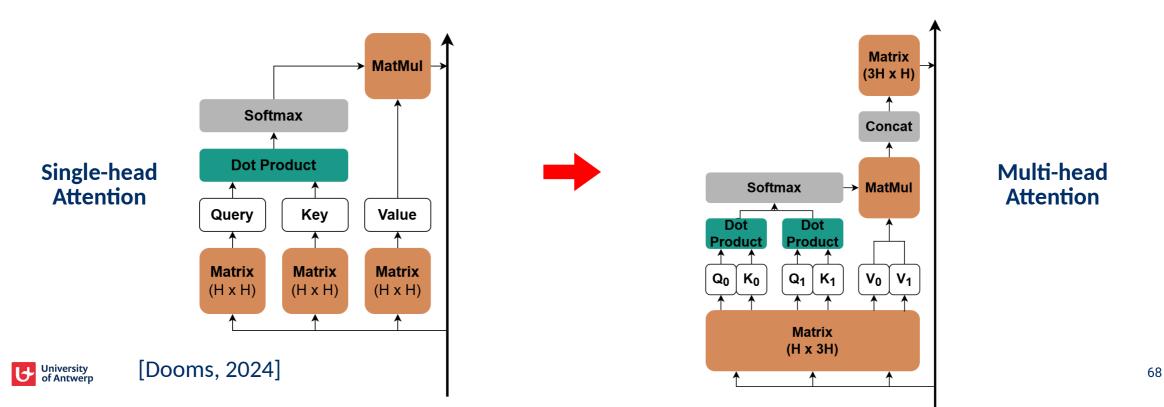
- Depth: more specialized and structured
- Through attention: across tokens





### Single vs. Multi-head Attention

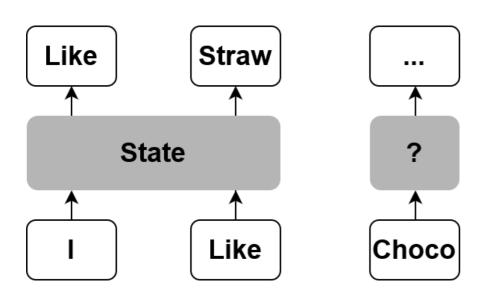
- Single: Each layer ask one question via the single QKV matrices.
- Multi: Split the QKV matrices into smaller ones → ask more/simpler questions

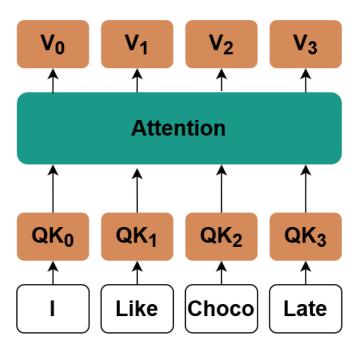


#### **RNNs VS Transfomers**

#### **Context Sources**

- Information from memory vs. tokens (error propagation)
  - → Sentence vs. Word -based



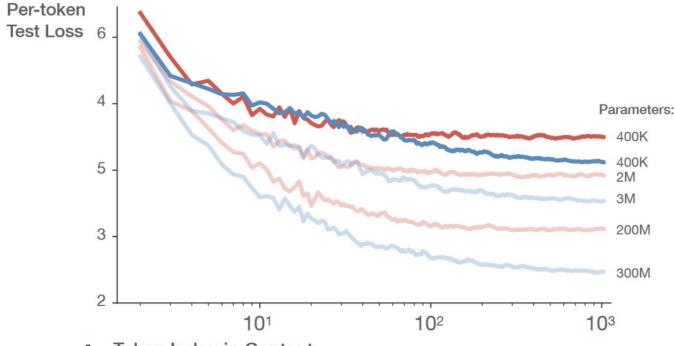




#### **RNNs VS Transfomers**

#### Performance / size trade-off

- Higher performance as # of considered tokens increases
- Better use of parameters



[Kaplan, 2020]



[Finally:D]



- Two Step-Approach

  - 1. Model Context

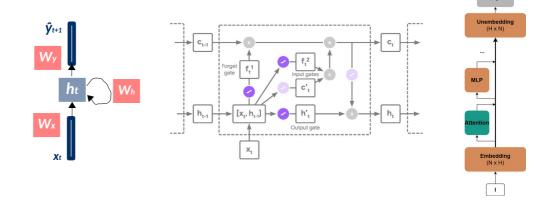
[ "this", "is", "an", "example", "text" ]



- Two Step-Approach
  - 1. Model Context

- ["this", "is", "an", "example", "text"]  $\longrightarrow$   $f_{\theta}$   $\longrightarrow$   $f_{\theta}$   $\longrightarrow$  "sequence"
- 2. Predict Next Element given Context
- Serveral architectures with different capabilities

**RNNs | LSTMs | GRUs | Transformers** 





- Two Step-Approach
  - 2. Predict Next Element given Context
  - 1. Model Context

[ "this", "is", "an", "example", "text"]

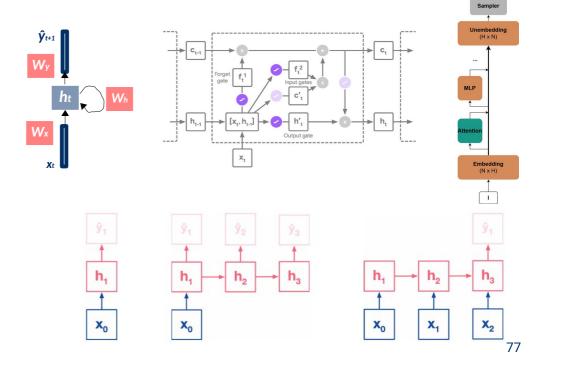
Serveral architectures with different capabilities

**RNNs | LSTMs | GRUs | Transformers** 

High Flexibility towards different problems

one-to-one | many-to-one | many-to-many ...





#### References

- K. Cho, B. van Merrienboer, Gulcehre, Caglar, D. Bahdanau; F. Bougares, H. Schwenk; Y. Bengio (2014). "Learning Phrase Representations using RNN Encoder-Decoder for Statistical Machine Translation". arXiv:1406.1078.
- Elman, J. L. (1990). Finding Structure in Time. Cognitive Science, 14(2), 179-211. https://doi.org/10.1207/s15516709cog1402\_1
- S. Hochreiter and J. Schmidhuber. 1997. **Long Short-Term Memory**. Neural Comput. 9, 8 (November 15, 1997), 1735–1780. DOI:https://doi.org/10.1162/neco.1997.9.8.1735
- The Recurrent Neural Network Theory and Implementation of the Elman Network and LSTM https://pabloinsente.github.io/the-recurrent-net
- Understanding LSTM Networks COlah's Blog
   http://colah.github.io/posts/2015-08-Understanding-LSTMs/



#### References

- K. Cho, B. van Merrienboer, Gulcehre, Caglar, D. Bahdanau; F. Bougares, H. Schwenk; Y. Bengio (2014). "Learning Phrase Representations using RNN Encoder-Decoder for Statistical Machine Translation". arXiv:1406.1078.
- Elman, J. L. (1990). Finding Structure in Time. Cognitive Science, 14(2), 179–211. https://doi.org/10.1207/s15516709cog1402\_1
- S. Hochreiter and J. Schmidhuber. 1997. Long Short-Term Memory. Neural Comput. 9, 8 (November 15, 1997), 1735–1780.
   DOI:https://doi.org/10.1162/neco.1997.9.8.1735
- J. Kaplan, S. McCandlish, T. Henighan, T. B. Brown, B. Chess, R. Child, S. Gray, A. Radford, J. Wu, D. Amodei. **Scaling Laws for Neural Language Models.** arxiv:2001.08361, 2020.
- A. Vaswani, N. Shazeer, N. Parmar, J. Uszkoreit, L. Jones, Aidan N. Gomez, L. Kaiser, I. Polosukhin. Attention is All you Need.
   NeurIPS 2017.
- The Recurrent Neural Network Theory and Implementation of the Elman Network and LSTM https://pabloinsente.github.io/the-recurrent-net
- Understanding LSTM Networks COlah's Blog
   http://colah.github.io/posts/2015-08-Understanding-LSTMs/





# **Modeling Sequences with Neural Networks**

José Oramas

