# Detailed Documentation

## Konrad Krämer

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### Documentation

The package ast2ast translates a R function into a C++ function and returns an external pointer (XPtr) to this function. The scope of ast2ast is to generate functions which can be used during solving ode-systems (derivative function or jacobian function) or during optimization. More generally the translated function can be used in fields where it is necessary to evaluate a function very often. Especially if the function is evaluated by C++ the generated external pointer is very sufficient.

First of all the supported objects and functions listed below are explained in detail. Here the arguments which have to be passed to the functions are described and it is explained what the function returns. Furthermore, for each function a small example is given showing how to use it. Moreover, it is explained how the function differ from R equivalents. If other differences were detected please report it.

The last section of the documentation describes how the external pointers produces by ast2ast can be used in packages. This information is intended for package authors who want to use ast2ast in order to enable a simple user interface as it not necessary anymore for the user to write C++. This paragraph explains how the examples in this vignette are executed. First of all a Rcpp function is created, which executes the output of the function translate. If you want to know how this functions work in detail you can go to Information for Package authors. In the examples only the R code is shown in order to show how to write the code. Sometimes the result of the executed code is also presented.

#### Supported objects:

- vectors (containing numbers)
- matrices (containing numbers)

## Supported functions:

- assignment: = and <-
- allocation: vector and matrix
- information about objects: length and dim
- Basic operations: +, -, \*, /
- Indices: [] and at

- mathematical functions: sin, asin, sinh, cos, acos, cosh, tan, atan, tanh, log, ^ and exp
- concatenate objects: c
- control flow: for, if, else if, else
- comparison: ==, !=, >, <, >= and <=
- printing: print
- returning objects: return

### **Objects**

There exists two containers which can be used in ast2ast functions. Both containers can only hold the numeric type of R (is equivalent to double). The first container are vectors and the second one are matrices. \*\*It is possible to declare a variable of a scalar numeric data type. This is done by adding \*\_db\* (e.g. varname\_db) to the end of the variable. Each time \*\_db\* is found the variable is declared as a scalar numeric data type. In this case the object cannot change its type!\*\*

#### Nice features:

- The variables can change the type within a function. This is normally not possible when using C++.
- The index of vectors and matrices starts at 1 as in R.
- The index has to be in the boundaries of the vector or matrix. Even though this is different from the behaviour in R it is a nice feature. If you access a element outside the boundaries of a vector in R NA is returned. This is in 99% of the cases not what the user wants.
- The memory of the matrices is columnwise arranged as in R.
- In R arguments passed to a function are always copied. In ast2ast functions it is possible to pass only the memory address of a object (called a reference). To do this you have to set the reference parameter of the translate function to TRUE. If you pass a function by reference you can modify the object without returning it (see Example 1). In the Rcpp function the variable x is printed before and after the call of the function fetr. Notably, if no return is used in the R code translated by ast2ast nothing is returnd (in R the last object is returned in this case). You see that x is 10 before the call of the function and it is 1 after the call of the function. But the function does not return anything. Thus, the object x is modified in the function without copying it.

### Example 1

```
f <- function(variable) {
  variable <- 1
}
library(ast2ast)
fetr <- translate(f)
x <- 10
output <- byref(fetr, x)

## x before call of function:
## 10
## x after call of function:
## 1
output</pre>
```

## ## NULL

#### Caveats:

- Sometimes large overhead of the containers
  - Variables which are scalars are represented as vectors of length 1. This is also how R handels scalar variables. As in C++ scalar variables are not defined as vectors the speed of the translated R function can be substantially lower compared to a native C++ function.

#### Variable declaration

In **Example 2** the various ways of declaring variables are presented. To assign a value to a variable you can use <- or =. As already mentioned only numeric values are possible. If you want to assign a vector you can use either the c or vector function. The c function works in the same way as R and can handle any combinations of scalars, vectors or matrices. The function vector differs in two ways from the R equivalent. First of all you cannot use terms such as vector(length = size) as this is not possible in C++. In contrast you just write vector(size). The R function rep is not available in ast2ast but it is possible to write vector(value, size) which in R would be written as rep(value, size). A third way to use the vector function is to pass another vector and the size to it vector(other\_vector, size). The matrix function works in the same way as the vector function with the difference that instead of the size two arguments were needed the number of rows and the number of columns.

## Example 2

```
f <- function() {</pre>
  a <- 1
  a_db <- 3.14
  b = 2
  c \leftarrow c(1, 2, 3)
  d = vector(2)
  e \leftarrow vector(3.14, 4)
  f <- vector(c, 3)
  g <- matrix(2, 2)</pre>
  h <- matrix(6, 2, 2)
  i <- matrix(e, 2, 2)
  print("a")
  print(a)
  print(a_db)
  print()
  print("b")
  print(b)
  print()
  print("c")
  print(c)
  print()
  print("d")
  print(d)
  print()
  print("e")
  print(e)
  print()
  print("f")
  print(f)
  print()
  print("g")
  print(g)
  print()
  print("h")
  print(h)
  print()
  print("i")
  print(i)
  print()
```

```
}
library(ast2ast)
fetr <- translate(f)</pre>
vardec(fetr)
## a
## 1
## 3.14
##
## b
## 2
##
## c
## 1
## 2
## 3
##
## d
## 2
## 2
##
## e
## 3.14
## 3.14
## 3.14
## 3.14
##
## f
## 1
## 1
## 1
##
## g
## 4.6423e-310 1.27246e+232
## 0
        2.05586
##
## h
## 6
        6
        6
## 6
##
## i
## 3.14 3.14
## 3.14 3.14
```

# Basic arithmetics

As usual in R it is possible to use basic arithmetic operations on scalars, vectors and matrices (**Example 3**).

## Example 3

```
f <- function() {
a <- 2
b <- 3</pre>
```

```
print("scalar operations")
print(a + b)
print(a - b)
print(a / b)
print(a * b)
print()
print("vector & scalar operations")
a \leftarrow c(1, 2, 3)
b <- 4
print(a + b)
print(b - a)
print()
print("vector & vector operations (same length)")
a <- 6:8
b <- 1:3
print(a / b)
a <- 1:6
b <- 1:3
print(a / b)
print("vector & vector operations (different length)")
print("longer object length is a multiple of shorter object length")
a <- 1:6
b <- 1:3
print(a / b)
print("longer object length is not a multiple of shorter object length")
a <- 1:5
b <- 1:3
print(a / b) # different to R no warning
print()
print("matrix & scalar operations")
a <- 3
b <- matrix(3, 2, 2)
print(a*b)
print(b + 4)
print()
print("matrix & vector operations")
a <- 5:6
b <- matrix(3, 2, 2)
print(b - a)
print(a / b)
print()
print("matrix & matrix operations")
a <- matrix(3, 2, 2)
```

```
b <- matrix(4, 2, 1) # difference to R!
print(a + b)
print()
print("mixed operations")
a <- 1
b <- 2:5
c <- matrix(50, 2, 2)</pre>
d <- a + b - c/2
print(d)
}
library(ast2ast)
fetr <- translate(f)</pre>
call_fct(fetr)
## scalar operations
## 5
## -1
## 0.666667
## 6
##
## vector & scalar operations
## 5
## 6
## 7
## 3
## 2
## 1
## vector & vector operations (same length)
## 6
## 3.5
## 2.66667
## 1
## 1
## 1
## 4
## 2.5
## vector & vector operations (different length)
## longer object length is a multiple of shorter object length
## 1
## 1
## 1
## 4
## 2.5
## longer object length is not a multiple of shorter object length
## 1
## 1
## 1
## 4
```

```
## 2.5
##
## matrix & scalar operations
## 9
## 9
        9
## 7
        7
## 7
##
## matrix & vector operations
## -2
        -2
## -3
        -3
## 1.66667
            1.66667
## 2
##
## matrix & matrix operations
## 7
## 7
        7
##
## mixed operations
## -22 -20
## -21 -19
```

## Subsetting

If you want to subset a vector or a matrix object you can use either [] or the at function. The [] is slower then at but more powerful (**Example 4**).

The following objects can be passed to // when using a vector or matrix:

- nothing
- numeric scalar
- logical
- vector
- matrix
- result of comparison
- caveat:
- it is not possible to pass the results of calculations!

In case of a matrix it is possible to pass one of the above objects to access specific rows or columns respectively ([rows, cols]). In contrast to at only a scalar can be passed. Thus, only a single element is accessed by this function! However, this function works faster. The result of at cannot be subsetted further.

# Example 4

```
f <- function() {

print("pass nothing")
a <- 1:8

print(a)
a[] <- 100

print(a)
print()</pre>
```

```
print("pass logical")
a <- 1:8
print(a)
a[TRUE] <- 100
print(a)
print()
print("pass scalar")
a <- 1:8
print(a)
a[1] <- 100
print(a)
print()
print("pass vector")
a <- 1:8
b <- 2:5
print(a)
a[b] <- 100
print(a)
print()
print("pass result of ==")
a <- 1:8
a[a < 5] <- 100
print(a)
print()
print("pass result of !=")
a <- 1:8
b \leftarrow c(1, 2, 3, 0, 0, 0, 0, 8)
a[a != b] <- 100
print(a)
print()
print("pass result of <=")</pre>
a <- 1:8
b <- c(1, 2, 3, 0, 0, 0, 0, 8)
a[a \le b] < 100
print(a)
print()
print("pass result of >=")
a <- 1:8
b \leftarrow c(1, 2, 3, 0, 0, 0, 0, 9)
a[a >= b] <- 100
print(a)
print()
```

```
print("pass result of >")
a <- 1:8
b \leftarrow c(0, 2, 3, 0, 0, 0, 0, 9)
a[a > b] <- 100
print(a)
print()
print("pass result of <")</pre>
a <- 1:8
b \leftarrow c(0, 2, 3, 0, 0, 0, 0, 9)
a[a < b] <- 100
print(a)
print()
print("pass scalar, scalar")
a <- matrix(3, 4, 4)
a[1, 1] \leftarrow 100
print(a)
print()
print("pass vector, vector")
a <- matrix(3, 4, 4)
b < -c(1, 3)
c < -c(2, 4)
a[b, c] \leftarrow 100
print(a)
print()
print("pass ==, >=")
a <- matrix(1:16, 4, 4)
b <- 1:4
c \leftarrow c(1, 8, 3, 8)
a[b == c, b >= c] <- 100
print(a)
print()
print("at")
a <- 1:16
at(a, 2) <- 100
print(a)
print()
print("at")
a <- matrix(1:16, 4, 4)
at(a, 1, 4) <- 100
print(a)
print()
```

```
}
library(ast2ast)
fetr <- translate(f)</pre>
call_fct(fetr)
## pass nothing
## 1
## 2
## 3
## 4
## 5
## 6
## 7
## 8
## 100
## 100
## 100
## 100
## 100
## 100
## 100
## 100
##
## pass logical
## 1
## 2
## 3
## 4
## 5
## 6
## 7
## 8
## 100
## 100
## 100
## 100
## 100
## 100
## 100
## 100
##
## pass scalar
## 1
## 2
## 3
## 4
## 5
## 6
## 7
## 8
```

## 100 ## 2 ## 3

```
## 4
## 5
## 6
## 7
## 8
##
## pass vector
## 1
## 2
## 3
## 4
## 5
## 6
## 7
## 8
## 1
## 100
## 100
## 100
## 100
## 6
## 7
## 8
##
## pass result of ==
## 100
## 100
## 100
## 100
## 5
## 6
## 7
## 8
##
## pass result of !=
## 1
## 2
## 3
## 100
## 100
## 100
## 100
## 8
##
## pass result of <=
## 100
## 100
## 100
## 4
## 5
## 6
## 7
## 100
##
```

```
## pass result of >=
## 100
## 100
## 100
## 100
## 100
## 100
## 100
## 8
##
## pass result of >
## 100
## 2
## 3
## 100
## 100
## 100
## 100
## 8
##
## pass result of <
## 1
## 2
## 3
## 4
## 5
## 6
## 7
## 100
##
## pass scalar, scalar
## 100 3
          3
              3
## 3
       3
           3
              3
## 3
       3
          3 3
## 3
       3
          3
              3
##
## pass vector, vector
## 3
       100 3
               100
## 3
       3 3
               3
## 3
       100 3
               100
## 3
       3
           3
               3
##
## pass ==, >=
## 100 5
          100 13
## 2
       6
           10 14
## 100 7
           100 15
## 4
           12 16
##
## at
## 1
## 100
## 3
## 4
## 5
```

```
## 6
## 7
## 8
## 9
## 10
## 11
## 12
## 13
## 14
## 15
## 16
##
## at
## 1
            9
                 100
## 2
        6
            10 14
        7
## 3
            11
                15
## 4
            12 16
```

## Helper functions

There exists three helper function. The *length* function returns the number of elements of a vector or matrix. The *dim* function returns the number of rows and columns of a matrix. The : function can be used to create a range of numbers. See **Example 5** in order to see how the functions work.

### Example 5

```
f <- function() {
    a <- 1:4
    print(a)
    a <- 1.1:5.2
    print(a)

a <- 1:16
    print(length(a))

b <- matrix(1:4, 2, 2)
    print(dim(b))
}

library(ast2ast)
fetr <- translate(f)
call_fct(fetr)

## 1
## 2</pre>
```

## 2 ## 3 ## 4 ## 1.1 ## 3.1 ## 3.1 ## 5.1 ## 5.1 ## 2 ## 2

### Comparison functions

As usual in R it is possible to compare two objects.

- ==
- <=
- >=
- !=
- <
- >

### Control flow

- for loop
- if, else if, else, &&, ||

### **Printing**

- print() is a difference to R
- print("string")
- print(logical)
- print(scalar)
- print(vector) is different to R
- print(matrix)

## Math functions

• sin, asin, sinh, cos, acos, cosh, tan, atan, tanh, log, ^ and exp

## Interpolation

 $\bullet$  cmr

# Information for Package authors

- the sexp object
- VEC class

## Rcpp Interface

- NumericVector = sexp
- sexp = NumericVector
- $NumericMatrix = sexp\_mat$
- sexp\_mat = NumericMatrix

## RcppArmadillo Interface

- vec = sexp
- sexp = vec
- $mat = sexp\_mat$
- $sexp_mat = mat$

### Pointer Interface

- sexp(size, pointer, integer) -> integer = 0, 1 or 2 = copy, take ownership, borrow
- sexp(rows, cols, pointer, integer) -> integer = 0, 1 or 2 = copy, take ownership, borrow
- modify NumericVector

- modify NumericMatrix
- modify arma::vec
- modify arma::mat
- modify std::vector

### Examples

## How to use ast2ast

### R code is translated

```
add_two <- function(a) {
  return(a + 2)
}
pointer_to_f_cpp <- ast2ast::translate(add_two)</pre>
```

## Defining C++ Code which uses the translated R Code

```
// [[Rcpp::depends(ast2ast)]]
// [[Rcpp::depends(RcppArmadillo)]]
#include "etr.hpp"

// [[Rcpp::plugins("cpp17")]]

typedef sexp (*fp)(sexp a);

// [[Rcpp::export]]
void call_fct(Rcpp::XPtr<fp> inp) {
    fp f = *inp;
    sexp a = coca(1, 2, 3);
    print(a);
    a = f(a);
    print("a is now:");
    print(a);
}
```

## Calling the final function

```
#Rcpp::sourceCpp("cpp_code.cpp") # if not run in Rmarkdown
call_fct(pointer_to_f_cpp)

## 1
## 2
## 3
## a is now:
## 3
## 4
## 5
```

### r2sundials

#### r2sundials code

```
setwd("/home/konrad/Documents/OUni/programming")
#install.packages("ast2ast", type = "source", repos = NULL)
library(Rcpp)
library(ast2ast)
library(r2sundials)
## Loading required package: rmumps
library(RcppXPtrUtils)
library(microbenchmark)
# R version
ti <- seq(0, 5, length.out=101)
p \leftarrow list(a = 2)
p \leftarrow c(nu = 2, a = 1)
y0 <- 0
frhs <- function(t, y, p, psens) -p["nu"]*(y-p["a"])</pre>
res exp <- r2sundials::r2cvodes(y0, ti, frhs, param = p)</pre>
attributes(res exp) <- NULL
\#stopifnot(diff(range(1-exp(-p$a*ti) - res_exp)) < 1.e-6)
# External pointer
ptr_exp <- cppXPtr(code = '</pre>
int rhs_exp(double t, const vec &y, vec &ydot, RObject &param, NumericVector &psens) {
  double a = 1;
  double nu = 2;
  ydot[0] = -nu*(y[0] - a);
  return(CV_SUCCESS);
', depends=c("RcppArmadillo", "r2sundials", "rmumps"),
includes="using namespace arma;\n#include <r2sundials.h>", cacheDir="lib", verbose=FALSE)
pv \leftarrow c(a = 1)
res_exp2 <- r2sundials::r2cvodes(y0, ti, ptr_exp, param = pv)</pre>
attributes(res_exp2) <- NULL</pre>
```

## Wrapper code in order to call translated R function (maybe written by package authors)

```
// [[Rcpp::depends(RcppArmadillo)]]
// [[Rcpp::depends(r2sundials)]]
// [[Rcpp::depends(ast2ast)]]
// [[Rcpp::plugins("cpp17")]]
#include "etr.hpp"
#include "RcppArmadillo.h"
#include "r2sundials.h"
using namespace arma;

typedef int (*fp)(double t, const vec &y, vec &ydot, RObject &param, NumericVector &psens);
```

```
typedef void (*user_fct)(sexp& y_, sexp& ydot_);
user_fct Fct;
int rhs_exp_wrapper(double t, const vec &y, vec &ydot, RObject &param, NumericVector &psens) {
 NumericVector p(param);
  const int size = y.size();
 sexp ydot_(size, ydot.memptr(), 2);
 double* ptr = const cast<double*>(y.memptr());
 sexp y_(size, ptr, 2);
 Fct(y_, ydot_);
 return(CV_SUCCESS);
}
// [[Rcpp::export]]
Rcpp::NumericVector solve_ode(Rcpp::XPtr<user_fct> inp, NumericVector time, NumericVector y) {
 Fct = *inp;
  Rcpp::XPtr<fp> ptr = Rcpp::XPtr<fp>(new fp(&rhs_exp_wrapper));
 Rcpp::Environment pkg = Rcpp::Environment::namespace_env("r2sundials");
 Rcpp::Function solve = pkg["r2cvodes"];
 Rcpp::NumericVector output = solve(y, time, ptr, time);
 return output;
}
```

## Applying ast2ast

The speed of pure C++ is not reached. However, the ast2ast function is substantially fastern then the R code.

```
# ast2ast version
ti <- seq(0, 5, length.out=101)
p <- list(a = 2)
p <- c(nu = 2, a = 1)
y0 <- 0

ode <- function(y, ydot) {
    nu_db <- 2
    a_db <- 1
    at(ydot, 1) <- -nu_db*(at(y,1) - a_db)
}

pointer_to_ode <- ast2ast::translate(ode, reference = TRUE, verbose = FALSE)
res_exp3 <- solve_ode(pointer_to_ode, ti, y0)
attributes(res_exp3) <- NULL

head(res_exp)

## [1] 0.000000000 0.09516259 0.18126923 0.25918179 0.32968002 0.39346945
head(res_exp2)</pre>
```

## [1] 0.00000000 0.09516259 0.18126923 0.25918179 0.32968002 0.39346945

```
head(res_exp3)
## [1] 0.00000000 0.09516259 0.18126923 0.25918179 0.32968002 0.39346945
microbenchmark(r2cvodes(y0, ti, frhs, param = p), r2cvodes(y0, ti, ptr_exp, param = pv), solve_ode(poin
## Unit: microseconds
##
                                               min
                                     expr
                                                          lq
                                                                  mean
                                                                         median
##
       r2cvodes(y0, ti, frhs, param = p) 3768.491 4041.8150 5107.2214 4566.336
   r2cvodes(y0, ti, ptr_exp, param = pv) 251.067 274.6030 402.7040 368.532
##
        solve_ode(pointer_to_ode, ti, y0) 336.221 369.1185 514.8868 396.281
##
                   max neval
##
           uq
## 6137.2495 7801.331
    526.3245 717.235
##
                         100
##
     514.9035 2749.441
                         100
// [[Rcpp::depends(RcppArmadillo)]]
// [[Rcpp::depends(ast2ast)]]
// [[Rcpp::plugins("cpp17")]]
// [[Rcpp::depends(paropt)]]
#include "etr.hpp"
#include "RcppArmadillo.h"
using namespace Rcpp;
typedef int (*0S)(double &t, std::vector<double> &params, std::vector<double> &states);
int ode_system(double &t, std::vector<double> &params, std::vector<double> & states) {
  double a = params[0];
  double b = params[1];
  double c = params[2];
  double d = params[3];
  double n1 = states[0];
  double n2 = states[1];
  states[0] = n1*c*n2 - n1*d;
  states[1] = n2*a - n2*b*n1;
  return 0;
}
// [[Rcpp::export]]
Rcpp::XPtr<OS> test_optimization() {
 Rcpp::XPtr<OS> xpfun = Rcpp::XPtr<OS>(new OS(&ode_system));
  return xpfun;
}
typedef int (*paropt_fct)(double &t, std::vector<double> &params, std::vector<double> & states);
typedef void (*user_fct2)(sexp& p, sexp& y);
```

```
user_fct2 Fct_paropt;
int ode system wrapper(double &t, std::vector<double> &params, std::vector<double> & states) {
  sexp p(params.size(), params.data(), 2);
  sexp s(states.size(), states.data(), 2);
 Fct_paropt(p, s);
 return 0;
}
// [[Rcpp::export]]
Rcpp::List optimize_paropt(Rcpp::XPtr<user_fct2> inp,
                                     NumericVector time, Rcpp::DataFrame lb, Rcpp::DataFrame ub, Rcpp::D
  Fct_paropt = *inp;
  Rcpp::XPtr<paropt_fct> ptr = Rcpp::XPtr<paropt_fct>(new paropt_fct(&ode_system_wrapper));
  Rcpp::Environment pkg = Rcpp::Environment::namespace_env("paropt");
  Rcpp::Function optim = pkg["optimizer_pointer"];
 Rcpp::NumericVector abs_tols{1e-8, 1e-8};
 Rcpp::List output = optim(time, ptr, 1e-6, abs_tols, lb, ub, states,
                                     40, 1000, 0.0001, "bdf");
 return output;
}
#states
path <- system.file("examples", package = "paropt")</pre>
states <- read.table(paste(path, "/states_LV.txt", sep = ""), header = T)</pre>
# parameter
1b \leftarrow data.frame(time = 0, a = 0.8, b = 0.3, c = 0.09, d = 0.09)
ub \leftarrow data.frame(time = 0, a = 1.3, b = 0.7, c = 0.4, d = 0.7)
# Optimizing
library(paropt)
paropt
## SUNDIALS - Copyright (c) 2002-2015, Lawrence Livermore National Laboratory.
## Produced at the Lawrence Livermore National Laboratory.
## See - https://computation.llnl.gov/projects/sundials
## PSO: See - https://github.com/kthohr/optim
set.seed(1)
start time <- Sys.time()</pre>
df_cpp <- optimizer_pointer(integration_times = states$time, ode_sys = test_optimization(),</pre>
                         relative_tolerance = 1e-6, absolute_tolerances = c(1e-8, 1e-8),
                        lower = lb, upper = ub, states = states,
                        npop = 40, ngen = 1000, error = 0.0001, solvertype = "bdf")
end_time <- Sys.time()</pre>
cpp_time <- end_time - start_time</pre>
```

```
# ast2ast with at and _db
ode <- function(params, states) {</pre>
  a_db = at(params, 1)
  b_db = at(params, 2)
  c_db = at(params, 3)
  d_db = at(params, 4)
  n1_db = at(states, 1)
  n2_db = at(states, 2)
  at(states, 1) = n1_db*c_db*n2_db - n1_db*d_db;
  at(states, 2) = n2_db*a_db - n2_db*b_db*n1_db;
}
pointer_to_ode <- ast2ast::translate(ode, reference = TRUE, verbose = FALSE)</pre>
set.seed(1)
start_time <- Sys.time()</pre>
df_ast2ast <- optimize_paropt(pointer_to_ode, states$time, lb, ub, states)</pre>
end_time <- Sys.time()</pre>
a2a_time <- end_time - start_time
message("time of the pure C++ code")
## time of the pure C++ code
knitr::kable(cpp_time)
                                          Х
                                          47.77598 \text{ secs}
message()
message("parameter results of the pure C++ code")
## parameter results of the pure C++ code
knitr::kable(df_cpp[[8]])
                             1.12283
                                      0.4065349
                                                 0.0946064
                                                            0.3925324
message("time of the ast2ast code")
## time of the ast2ast code
knitr::kable(a2a_time)
```

 $\mathbf{x}$ 

 $1.100143~\mathrm{mins}$ 

message("parameter results of the ast2ast code")

## parameter results of the ast2ast code

knitr::kable(df\_ast2ast[[8]])

 $0 \quad 1.12283 \quad 0.4065349 \quad 0.0946064 \quad 0.3925324$