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 COIS 3050
 Assignment 1

- 1) Give a recursive definition for the language WEIRD which is defined over the alphabet $\Sigma = \{a, b\}$ and contains words that have an odd number of b's and an even number of a's.**

Rule 1: $b \in \text{WEIRD}$

Rule 2: if $W \in \text{WEIRD}$, then $aaW, aWa, Waa, bbW, bWb, Wbb \in \text{WEIRD}$

Rule 3: No other words belong to WEIRD

- 2) Consider the following recursive definition of 4-PERMUTATION:**

Rule 1: 1234 is a 4-PERMUTATION.

Rule 2: If wxyz is a 4-PERMUTATION, then so are zxyw and yxzw.

Rule 3: no other words belong to 4-PERMUTATION.

Give all the words in the language 4-PERMUTATION.

Rule 1: $1234 \in 4\text{-PERMUTATION}$

Rule 2: Use 1234, $w = 1, x = 2, y = 3, z = 4$.

Then: $4231, 3241 \in 4\text{-PERMUTATION}$

Rule 2: Use 4231, $w = 4, x = 2, y = 3, z = 1$.

Then: ~~1234~~, $3214 \in 4\text{-PERMUTATION}$

Rule 2: Use 3241, $w = 3, x = 2, y = 4, z = 1$.

Then: $1243, 4213 \in 4\text{-PERMUTATION}$

Rule 2: Use 3214, $w = 3, x = 2, y = 1, z = 4$.

Then: ~~4213~~, $1243 \in 4\text{-PERMUTATION}$

Rule 2: Use 1243, $w = 1, x = 2, y = 4, z = 3$.

Then: ~~3241~~, $4231 \in 4\text{-PERMUTATION}$

Rule 2: Use 4213, $w = 4, x = 2, y = 1, z = 3$.

Then: ~~3214~~, $1234 \in 4\text{-PERMUTATION}$

Rule 3: No more words can be created by applying rule 1, rule 2, or rule 3.

Therefore: $1234, 1243, 3214, 3241, 4213, 4231 \in 4\text{-PERMUTATION}$

3) Construct a regular expression defining each of the following languages over the alphabet {a, b}.

(a) $L = \{aab, ba, bb, baab\}$;

$(aab + ba + bb + baab)$

- Word 1 or Word 2 or Word3 or Word 4

(b) The language of all strings containing exactly two b's.

$a^* b a^* b a^*$

- Any amount of a's, exactly two b's, any amount of a's

(c) The language of all strings containing at least one a and at least one b.

$b^* (a^* a) (b^* b) a^*$

- Any amount of b's, any amount of a's, at least one a, any amount of b's, at least one b, any amount of a's

(d) The language of all strings that do not end with ba.

$(a+b)^* b$

- Any amount of a or b, always end with b so that word does not end with ba

(e) The language of all strings that do not containing the substring bb.

$a^* a^* + b a^*$

- Any amount of a's, any amount of a's or a b, any amount of a's

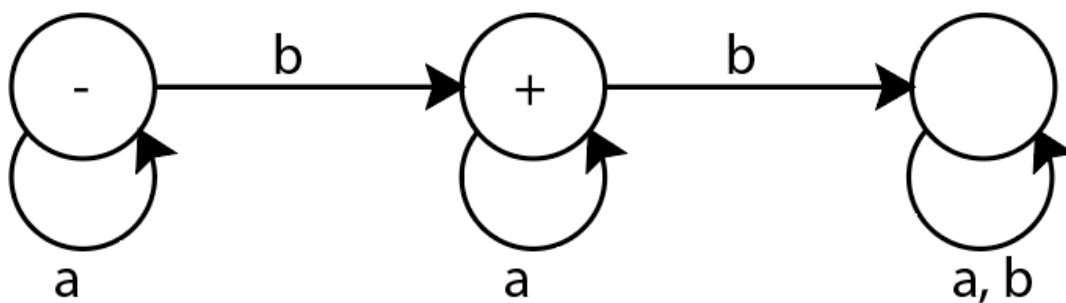
(f) The language of all strings in which every b is followed immediately by aa.

$a^* baa^*$

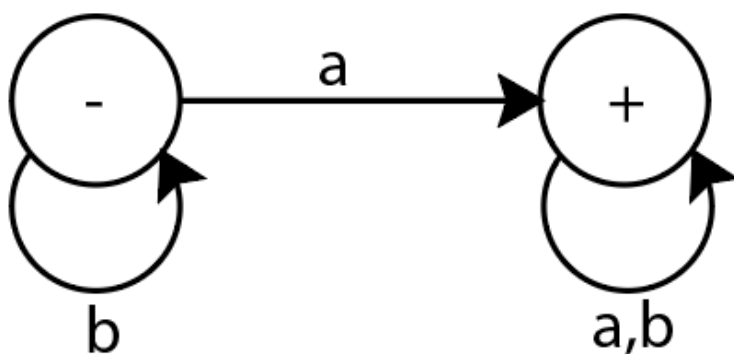
- Any amount of a's, any amount of b's that are followed immediately by aa

4) For the alphabet $S = \{a, b\}$, construct an FA that accepts the following languages.

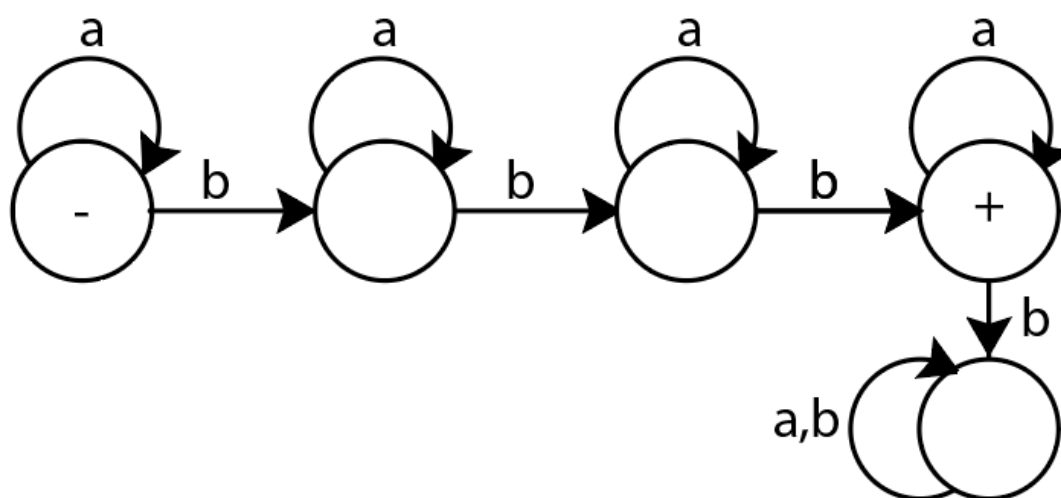
(a) $L = \{\text{all strings with exactly one } b\}$.



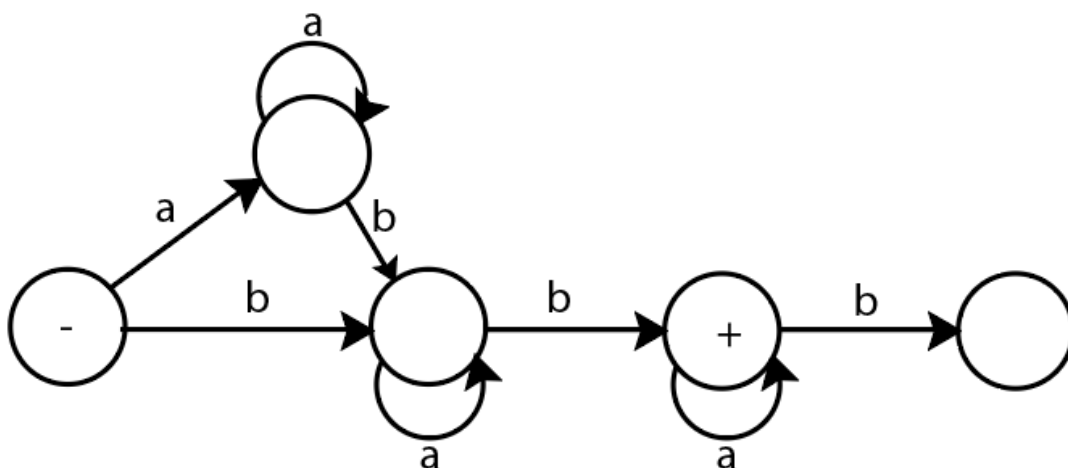
(b) $L = \{\text{all strings with at least one } a\}$.



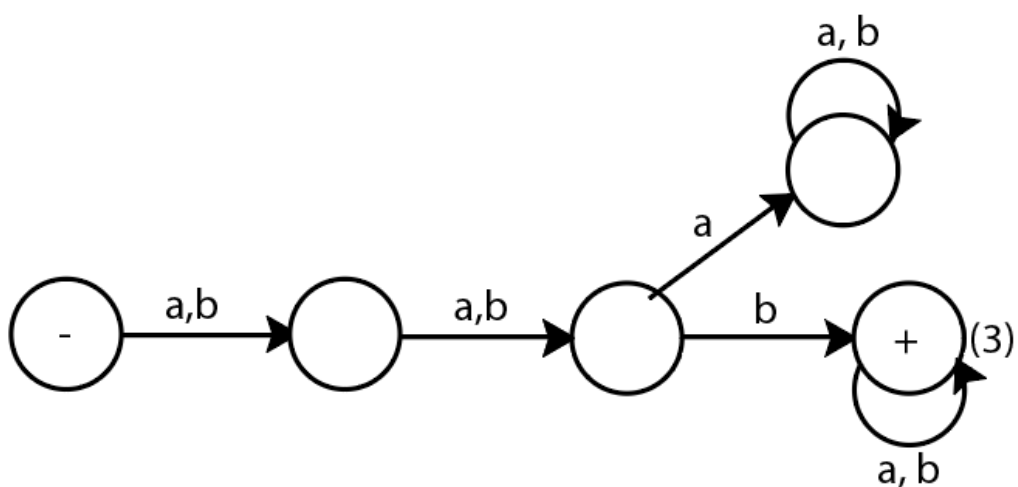
(c) $L = \{\text{all strings with no more than three } b\text{'s}\}$.



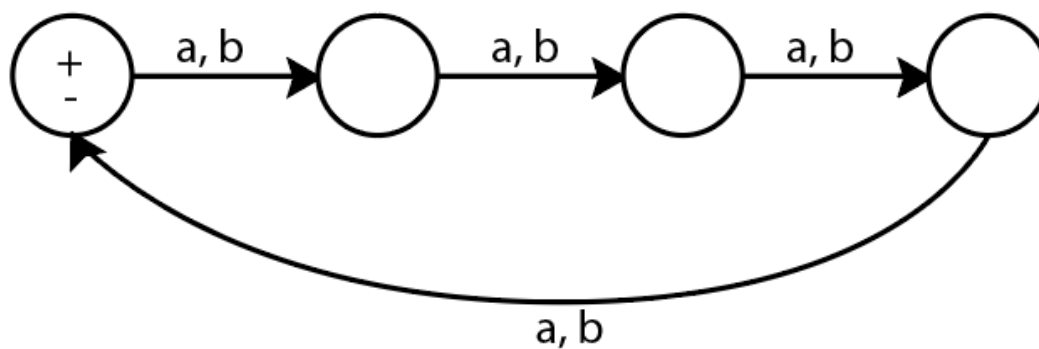
(d) $L = \{\text{all strings with at least one } a \text{ and exactly two } b\text{'s}\}$



(e) $L = \{\text{all strings with } b \text{ as the third letter}\}$

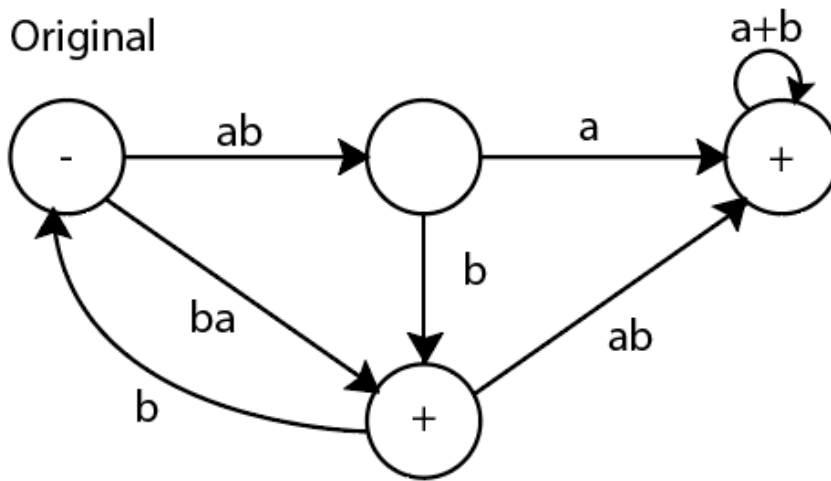


(f) $L = \{w, |w| \bmod 4 = 0\}$ // the cardinality of the word is a multiple of 4

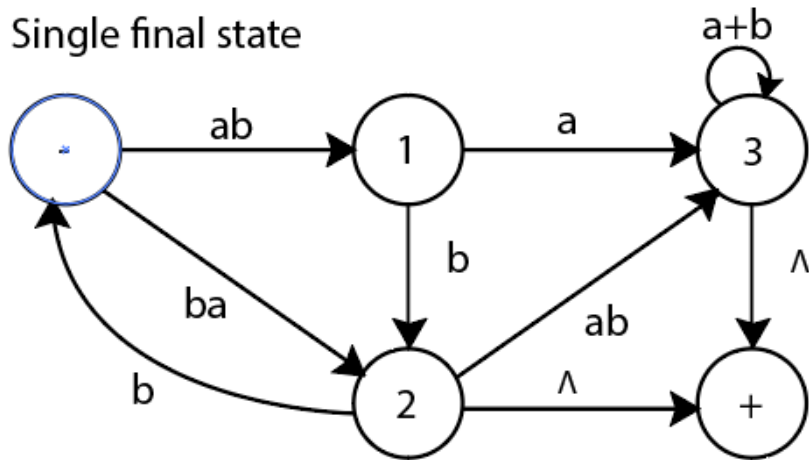


- 5) Using the algorithm provided by Kleene's Theorem, convert the following TG into a Regular Expression (show your work).

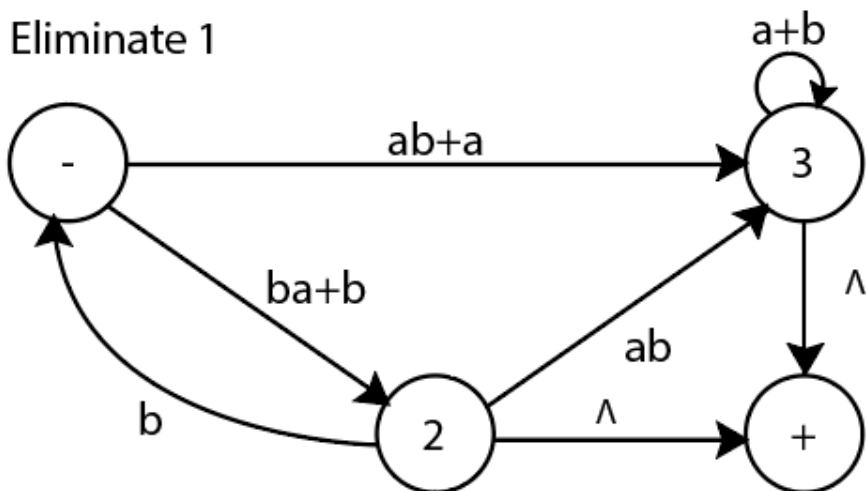
Original



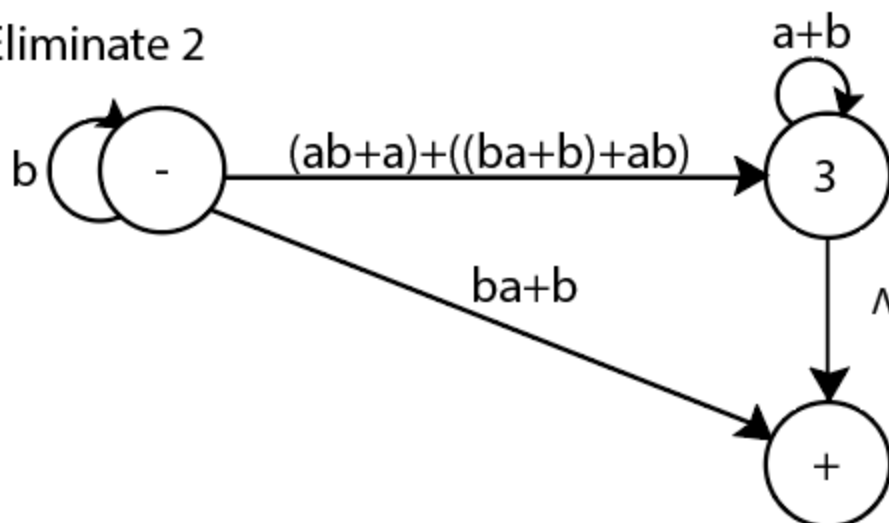
Single final state



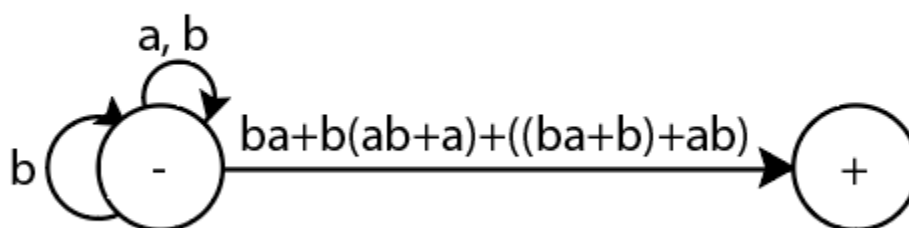
Eliminate 1



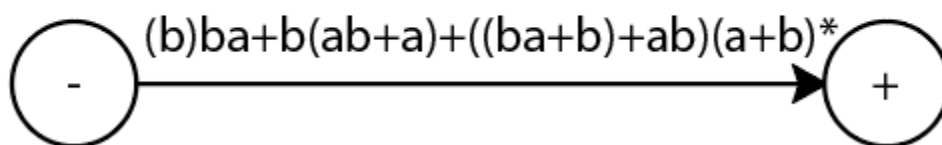
Eliminate 2



Eliminate 3



Eliminate Loops



Regular expression: $(b)ba+b(ab+a)+((ba+b)+ab)(a+b)^*$