The thrust allocation library in this example is used for control of water-surface omnidirectional robotic platform called Mallard. The actuators used in the project are Blue robotics T-100 thrusters that are placed in "+" shaped configuration, see Figure 1.

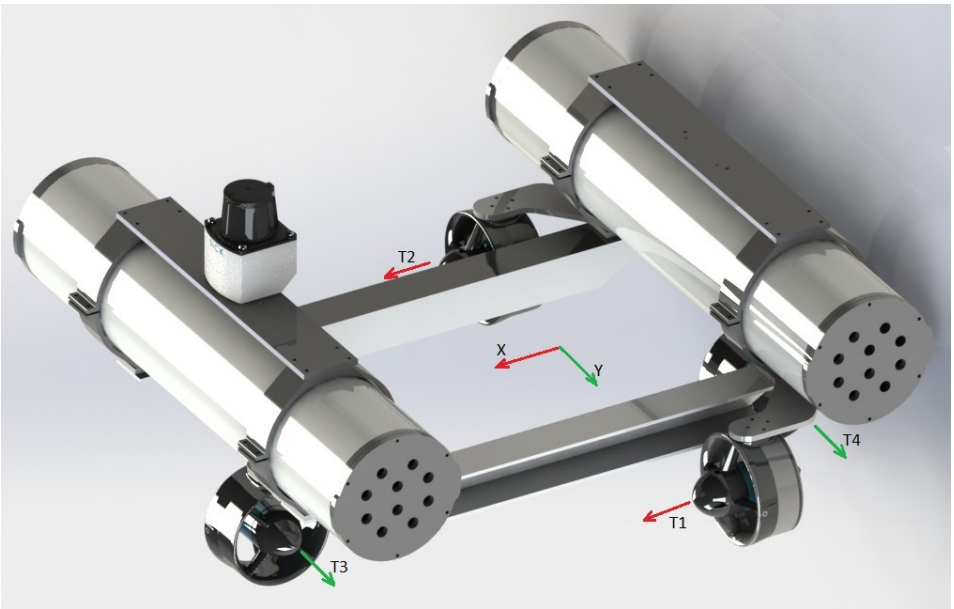

Figure 1: Mallard and its thruster configuration. Arrows show forward movement direction produced by each thruster (T_1, T_2, T_3 and T_4).

Each thruster is rigidly attached to Mallard's chassis and can generate forward and backward forces. To turn, each propeller's thrust is multiplied by its moment arm L_x and L_y, see Figure 2.

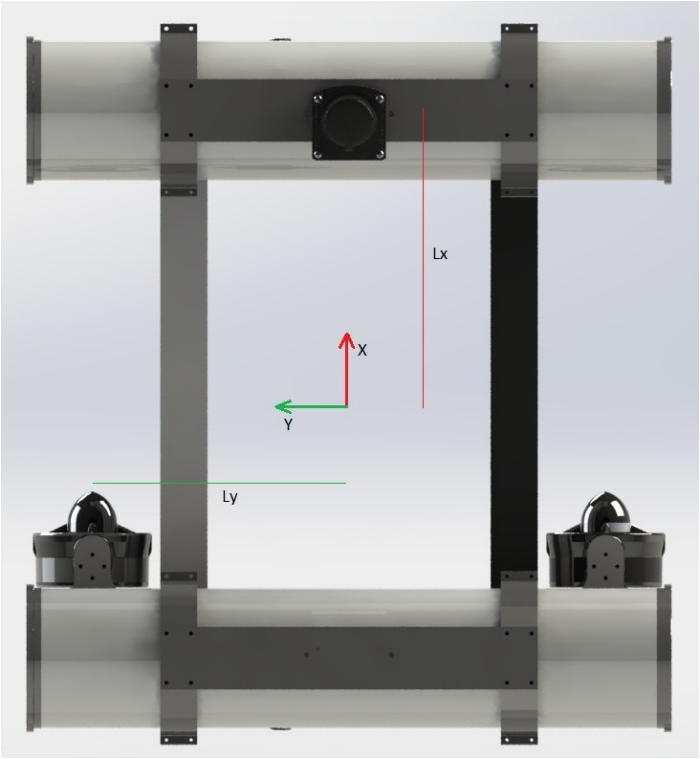

Figure 2: Top view. Moment arms L_x and L_y that are measured from the centre of Mallard, i.e. half the distance between thrusters.

To move Mallard in x-y plane the force and moment vector $\boldsymbol{\tau} = [U\ V\ R]^T$ is generated, where velocity commands U, V are in x, y direction and R is rotation about z-axis (perpendicular to x-y plane). Moreover, the $\boldsymbol{\tau}$ is defined in body (Mallard's) reference frame. The control input $\boldsymbol{u} = [T_1\ T_2\ T_3\ T_4]^T$ are forces generated by each thruster and $\boldsymbol{B}$ is control allocation matrix defining thruster configuration.

$$\boldsymbol{\tau} = \boldsymbol{B} * \boldsymbol{u} \tag{1}$$

All this can be explicitly written as:

$$\begin{bmatrix} U \\ V \\ R \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ L_y & -L_y & L_x & -L_x \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix} \tag{2}$$

To get desired control input $\boldsymbol{u}$ one need to take an inverse of matrix $\boldsymbol{B}$, however $\boldsymbol{B}$ is not a square matrix, therefore the Moore-Penrose pseudoinverse $\boldsymbol{B}^{+}$ needs to be taken:

$$\boldsymbol{u} = \boldsymbol{B}^{+} * \boldsymbol{\tau} \tag{3}$$

which is the same as:

$$\begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix} = \begin{bmatrix} 0.5 & 0 & a \\ 0.5 & 0 & -a \\ 0 & 0.5 & b \\ 0 & 0.5 & -b \end{bmatrix} \begin{bmatrix} U \\ V \\ R \end{bmatrix} \tag{4}$$

where a and b depend on chosen values for moment arms L_x and L_y. The matrix $\boldsymbol{B}^{+}$ can be found using pinv() function in Matlab. For simple code example see attached matlab file.