Report

1 Problem Statement

Consider a Maximum Concurrent Flow problem.

$$\max_{\lambda, x \succeq 0} \lambda,$$
$$\text{s.t.} \quad \sum_{p \in \mathcal{P}} x_p \delta_{ep} \leq f_e \quad \forall e \in E,$$
$$\sum_{p \in \mathcal{P}_k} x_p = \lambda d_k \quad k = \overline{1, r}$$

Now we start formulating a dual problem. We define $\mathcal{D}_\lambda = \{x \ : \ \sum_{p \in \mathcal{P}_k} x_p = \lambda d_k, \quad k = \overline{1, r}\}$.

$$\max_{\lambda \geq 0, x \succeq 0} \min_{y \succeq 0} [\lambda + \sum_{e \in E} y_e (f_e - \sum_{p \in \mathcal{P}} x_p \delta_{ep})] =$$
$$= \min_{y \succeq 0} \max_{\lambda \geq 0, x \succeq 0, x \in \mathcal{D}_\lambda} [\lambda + \sum_{e \in E} y_e (f_e - \sum_{p \in \mathcal{P}} x_p \delta_{ep})] =$$
$$= \min_{y \succeq 0} \max_{\lambda \geq 0, x \succeq 0, x \in \mathcal{D}_\lambda} [\lambda + \sum_{e \in E} y_e f_e - \sum_{p \in \mathcal{P}} \sum_{e \in E} \delta_{ep} x_p y_e] =$$
$$= \min_{y \succeq 0} \max_{\lambda \geq 0, x \succeq 0, x \in \mathcal{D}_\lambda} [\lambda + \sum_{e \in E} y_e f_e - \sum_{k=1}^{r} \sum_{p \in \mathcal{P}_k} x_p \sum_{e \in E} \delta_{ep} y_e] =$$
$$= [\text{solve the inner problem analytically. } \ell_k(y) \text{shortest path corresponding to k-th commodity}] =$$
$$= \min_{y \succeq 0} \max_{\lambda \geq 0} [\sum_{e \in E} y_e f_e + \lambda (1 - \sum_{k=1}^{r} d_k \ell_k(y))].$$

Therefore, the dual problem takes the followign form:

$$\min_{y \succeq 0} \sum_{e \in E} y_e f_e,$$
$$\text{s.t.} \quad \sum_{k=1}^{r} d_k \ell_k(y) \geq 1.$$

2 Optimization

We exploit a subgradient method derived at the lecture. More precisely, $y_{k+1} = y_k - \eta \frac{\partial g_k}{\|\partial g_k\| \sqrt{k}}$. Also note that the subgradient of $\ell_k(y)$ is calculated as if the optimal path does not change when we change y.

3 Metrics

Spectral Gap. Given a symmetric adjacency matrix W, where $W_{ij} = f_{ij}$. Also given a degree matirix $D = \mathrm{diag}(d_1, \ldots, d_n)$, where $d_i = \sum_j W_{ij}$. We calculate the spectum of the matrix $D - W$ defined as $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_n$. Finally, we define a spectral gap as $\lambda_1 - \lambda_2$.

Effective number of neightbours. Without going into details, for each $\gamma \in [0, 1)$ we define an effective number of neightbours as

$$\mathrm{ENN}_\gamma = \frac{\frac{1}{1-\gamma}}{\sum_{i=1}^{n} \frac{\lambda_i^2}{1 - \gamma \lambda_i^2}},$$

where λ_i are eigenvalues of W.

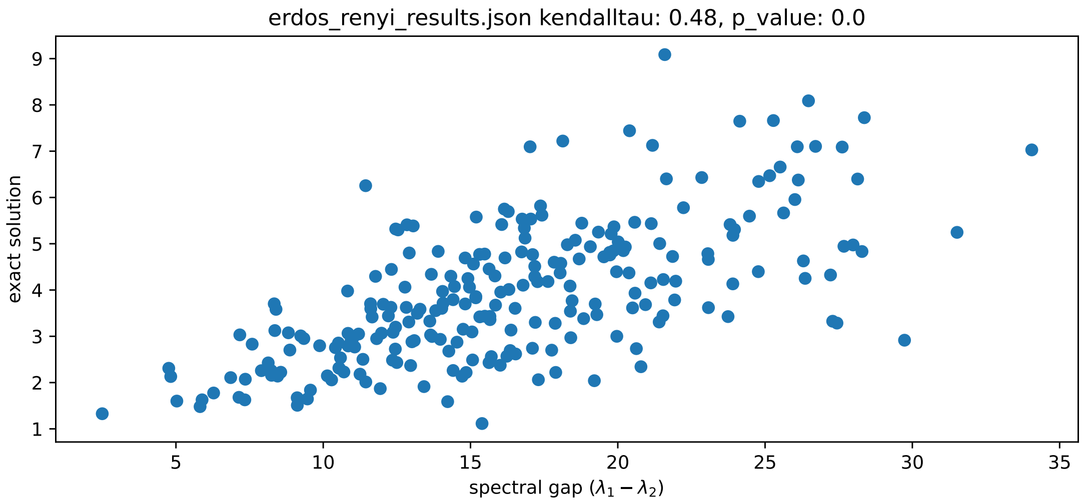
4 Experimental protocol

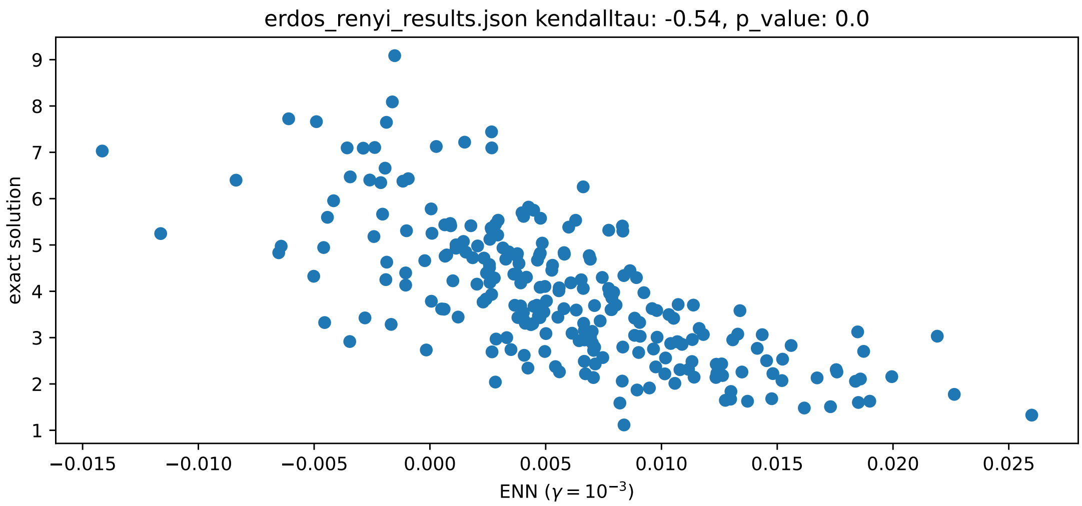
The goal of the computational experiment is to investigate the correlation of the metrics with the exact solution of MCFP.

Datasets. We consider a set of diverse datasets: Transport Flows, Survivable fixed telecommunication Network Design dataset, synthetic data (Erdos-Renyi topologies with lognormal traffic), and Topology Zoo dataset with uniform traffic.

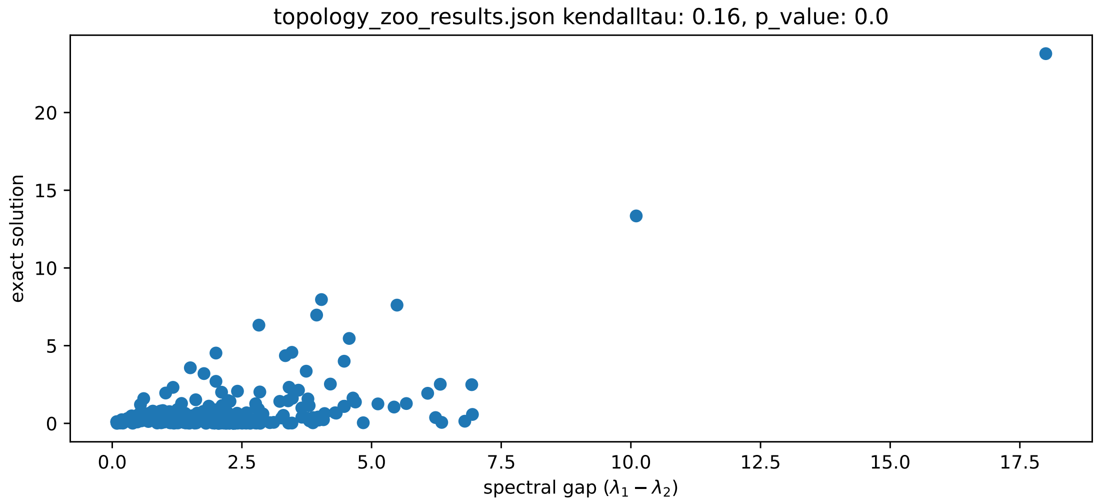
Results

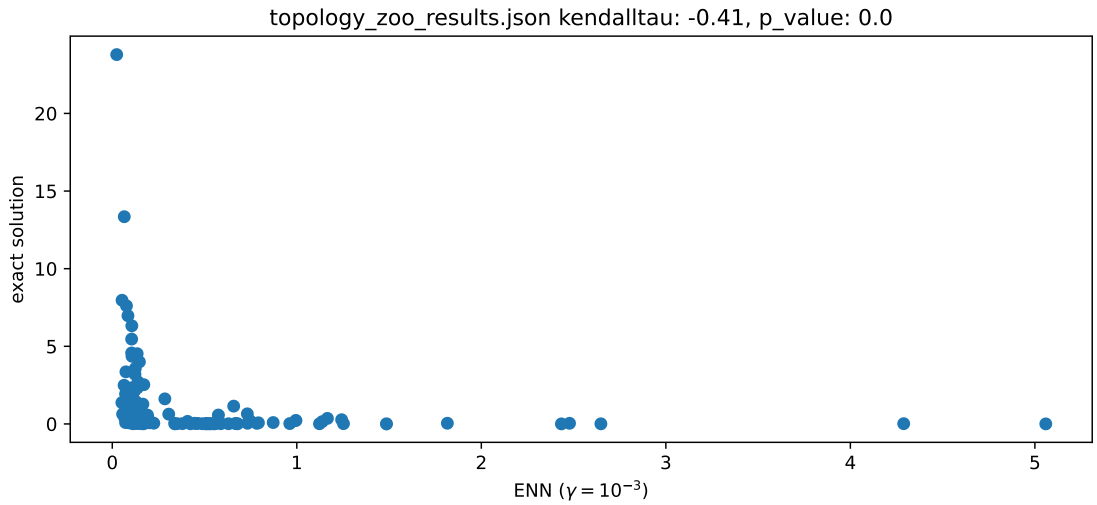
Synthetic data



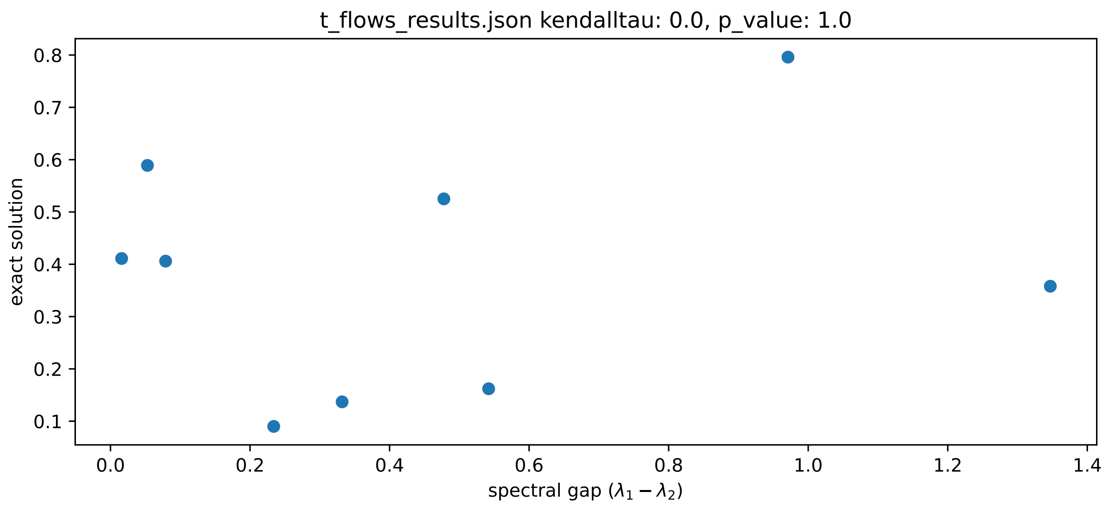


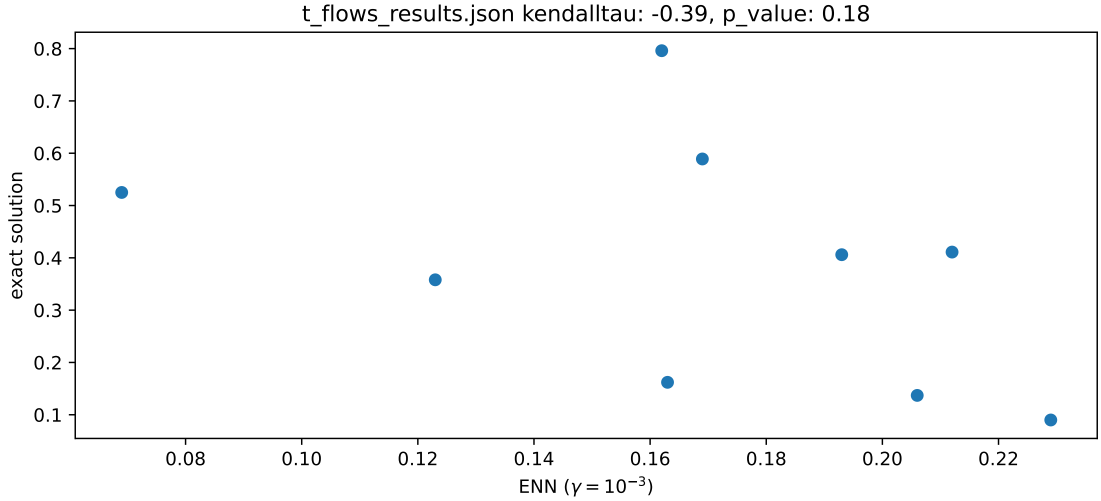
Topology Zoo



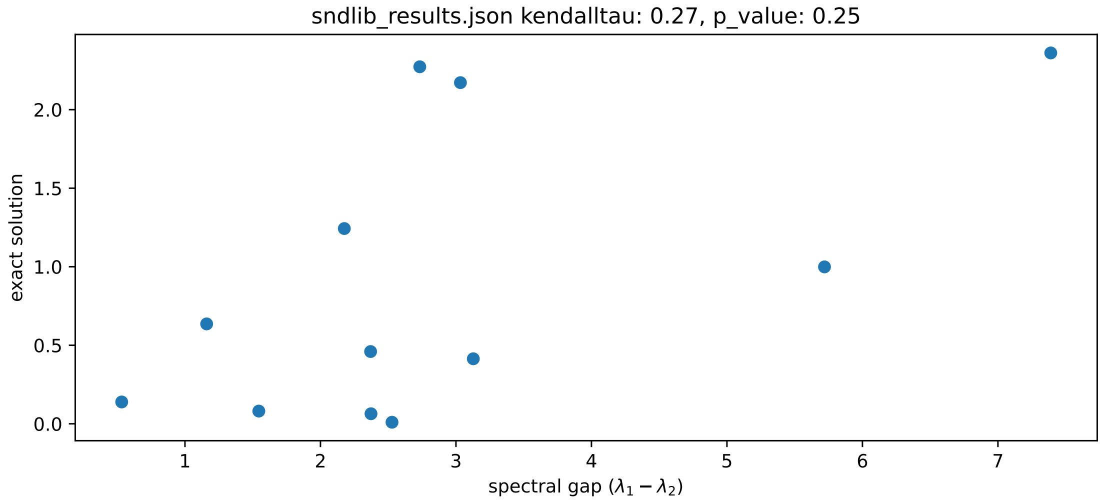


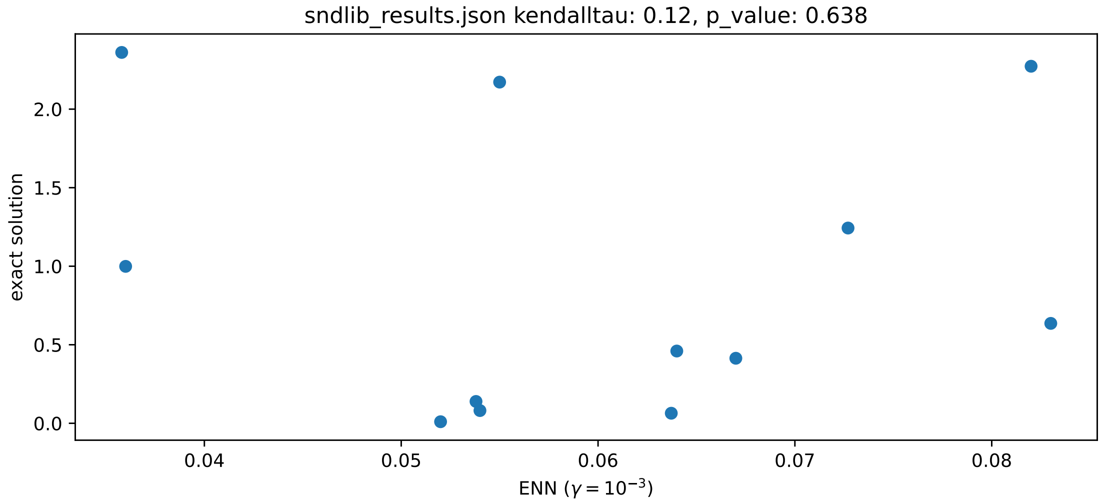
Transport Flows





SNDlib





It could be clearly seen that effective number of neighbours and spectral gap are informative features for the synthetic data and topology zoo.

5 List of contributions

Yakovlev Konstantin: subgradient method implementation, experiments on Transport Flows, SNDlib, synthetic data.
Igor Kochetkov: debugging the subgradient method, experiment on Topology Zoo dataset.