Dynamic Knapsack Problem

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1 Decision Epochs

Decisions are made at the beginning of each time period (will be weeks)

2 State Space

State space is defined by current units available for various PPEs for future periods, amount of units already used for various PPEs, current patient waitlist, expected period demand, and number of patients already scheduled

$$\vec{s} = (\vec{ue}, \vec{uu}, \vec{pw}, \vec{pe}, \vec{ps})$$

- $\vec{ue} = ue_{tp}$ - Expected unist for period t, and PPE p
- $\vec{uu} = uu_{tp}$ - Used units for period t, and PPE p
- $\vec{pw} = pw_{mdc}$ - Number of patients of complexity d, CPU c, on a wait list for m periods
- $\vec{pe} = pe_{dc}$ - Number of patients of complexity d, CPU c expected to arrive this period
- $\vec{ps} = ps_{tmdc}$ - Number of patients of complexity d, CPU c, scheduled to period t, who have been on the waitlist for m periods (m of 0 stands for people who have just arrived)

3 Action Sets

3.1 Description

At the beginning of each period, decision maker must reschedule appointments as necessary (if patient complexity increased and too much PPE is being used, or if expected units of PPE have changed negatively). And decision maker must also schedule patients to surgeries

$$\vec{a} = (\vec{sc}, r\vec{sc}, \vec{uv})$$

- $\vec{sc} = sc_{tmdc}$ - Number of patients of difficulty d, CPU c, who have been in wait list for m periods, to schedule in period t (m of 0 stands for people who have just arrived)
- $r\vec{sc} = rsc_{tt'mdc}$ - Number of patients of difficulty d, CPU c, who have been on the waitlist for m periods, to reschedule from period t to period t'
- uv_{tp} - goal variable, violation on number of resources used for period t, of PPE p

3.2 Action Constraints

- Total number of PPE units cannot be exceeded
  - (U_{pdc} - usage of PPE p per patient difficulty d, CPU c)

$$\sum_{mdc}(sc_{tmdc})U_{pdc} <= (ue_{tp} - uu_{tp}) + uv_{tp} \quad \forall tp$$

- Cannot schedule/reschedule past wait list horizon limit

$$sc_{tmdc} = 0 \quad \forall tmdc, \text{where } ((t-1)+m) > T$$

$$rsc_{tt'mdc} = 0 \quad \forall tt'mdc, \text{where } ((t'-1)+m) > T$$

- Bounds on when reschedules are allowed

$$rsc_{tt'mdc} = 0 \quad \forall t \in \{2...T\}, t' \in \{2...T\}, mdc$$

$$rsc_{tt'mdc} = 0 \quad \forall tt'mdc, \text{where } t = t'$$

- number of people scheduled/rescheduled must be consistent

$$\sum_{t'} rsc_{tt'mdc} \leq ps_{tmdc} \quad \forall tmdc$$

$$\sum_{t} sc_{t0dc} \leq pe_{dc} \quad \forall dc$$

$$\sum_{t} sc_{tmdc} \leq pw_{mdc} \quad \forall m \in \{1...M\}, dc$$

4 Transition Probabilities

4.1 Uncertainty Sources

1. Number of patients arriving this period - pe_{dc}
   - let's assume pea_{dc} - is the random variable that represents the number of patients arrived this period. It follows a poisson distribution.
2. Transition between patient difficulties within the wait list - pw_{mdc}
   - let's assume pwt_{mdc} is the random variable that represents the number of patients of priority d, CPU c, that have been waiting for m period, that have moved a more complex category. It follows binary distribution.
3. Transition between patient difficulties within the scheduled list - ps_{tmdc}
   - let's assume pst_{tmdc} is the random variable that represents the number of patients of priority d, CPU c, that have been waiting for m period, that have been scheduled into period t, that have moved a more complex category. It follows binary distribution.
4. Amout of expected units of PPE resource for the next time period - bc_{1dc}
   - let's assume ued_{md} is the random variable that represents the deviation of PPE units from the expectation for the next period only. It follows some uniform distribution.
   - let's assume uen_d is the default value to be used for expected number of PPE units p per period

4.2 Transition Constraints

1. Transition from $\vec{bc}$ to $\vec{bc}'$ - Expected Units of PPE

$$ue'_{1p} = ue_{2p} + ued_p \quad \forall p$$
$$ue'_{m-1,p} = ue_{mp} \quad \forall m \in \{3...M\}, p$$
$$ue'_{Mp} = uen_d \quad \forall p$$

2. Transition from $\vec{uu}$ to $\vec{uu}'$ - Used Units of PPE

$$uu'_{t-1,p} = uu_{tp} + \sum_{mdc}(sc_{t-1,mdc}U_{pdc}) - \sum_{t'mdc}(rsc_{tt'mdc}U_{pdc}) + \sum_{tmdc}(rsc_{tt'mdc}U_{pdc}) \quad \forall t \in \{2...T\}, p$$
$$uu'_{Tp} = 0 \quad \forall p$$

3. Transition from $\vec{pe}$ to $\vec{pe}'$ - Expected number of patients for this month

$$pe_{dc} = pea_{dc} \quad \forall dc$$

4. Transition from $\vec{pw}$ to $\vec{pw}'$ - Flow of patients between difficulties/scheduling/cancelling for waitlist

$$pw'_{1dc} = pe_{dc} - \sum_t sc_{t0dc} \quad \forall dc$$
$$pw'_{m+1,dc} = pw_{mdc} - \sum_t sc_{tmdc} + pwt_{m,d-1,c} - pwt_{mdc} \quad \forall m \in \{1...M-2\}, dc$$
$$pw'_{Mdc} = \sum_{M-1}^{M} pw_{mdc} - \sum_{t,M-1}^{M} sc_{tmdc} + \sum_{M-1}^{M} pwt_{m,d-1,c} - \sum_{M-1}^{M} pwt_{mdc} \quad \forall dc$$

5. Transition from $\vec{ps}$ to $\vec{ps}'$ - Flow of patients between difficulties/scheduling/cancelling for scheduled appointments

$$ps'_{t-1,m+1,dc} = ps_{tmdc} + \sum_t sc_{tmdc} - \sum_{t'mdc}(rsc_{tt'mdc}U_{pdc}) + \sum_{tmdc}(rsc_{tt'mdc}U_{pdc})$$
$$+pst_{tm,d-1,c} - pst_{tmdc} \quad \forall t \in \{2...T\}, m \in \{0...M-2\}, dc$$
$$ps'_{t-1,Mdc} = \sum_{M-1}^{M} ps_{mdc} - \sum_{t,M-1}^{M} sc_{tmdc} - \sum_{t',M-1,dc}^{M}(rsc_{tt'mdc}U_{pdc}) + \sum_{t,M-1,dc}^{M}(rsc_{tt'mdc}U_{pdc})$$
$$+\sum_{M-1}^{M} pst_{tm,d-1,c} - \sum_{M-1}^{M} pst_{tm,d,c} \quad \forall t \in \{2...T\}, dc$$
$$ps'_{Tmdc} = 0 \quad \forall mdc$$

5 Costs

Cost will come from two source:

- cost of waiting (cw) (comes from 2 things)
- cost of canceling (cc)
- goal variable (to eliminate constraint violation, but still allow it if necessary)

$$c(\vec{s}, \vec{a}) = \sum_{mdc} cw_m(pw_{mdc} - \sum_t sc_{tmdc}) + cc \sum_{tt`mdc} rsc_{tt'mdc} + M(uv_{tp})$$

cw_m is computed as follows (val is arbitrary number that describes cost growth):

$$cw_m = val^m \quad \forall m$$

cc is some arbitrary value