Dynamic Knapsack Problem

KV

December 2020

1 Decision Epochs

Decisions are made at the beginning of each time period (will be weeks)

2 State Space

State space is defined by current units available for various PPEs for future periods, amount of budget already spent on PPEs, current patient waitlist, expected period demand, and number of patients already scheduled

$$\vec{s} = (\vec{be}, \vec{bu}, \vec{pw}, \vec{pe})$$

- $\vec{be} = be_{mp}$ - Expected unist for period m, and PPE p
- $\vec{bu} = bu_{mp}$ - Used units for period m, and PPE p
- $\vec{pw} = pw_{mdc}$ - Number of patients of complexity d, CPU c, on a wait list for m periods
- $\vec{pe} = pe_{dc}$ - Number of patients of complexity d, CPU c expected to arrive this period
- $\vec{ps} = ps_{tmdc}$ - Number of patients of complexity d, CPU c, scheduled to period t, who have been on the waitlist for m periods (m of 0 stands for people who have just arrived)

3 Action Sets

At the beginning of each period, decision maker must cancel appointments as necessary (if patient complexity increased and too much PPE is being used, or if expected units of PPE have changed negatively). And decision maker must also schedule patients to surgeries

$$\vec{a} = (\vec{sc}, \vec{usc})$$

- $\vec{sc} = sc_{tmdc}$ - Number of patients of difficulty d, CPU c, who have been in wait list for m periods, to schedule in period t (m of 0 stands for people who have just arrived)
- $\vec{usc} = usc_{tmdc}$ - Number of patients of difficulty d, CPU c, who have been on the waitlist for m periods, to cancel from period t

The actions must satisfy the following constraints:

- Total allocated budged must not be exceeded (U_{pdc} - usage of PPE p per patient difficulty d, CPU c)

$$\sum_{dc} s_{mdc} U_{pdc} <= (be - bu) \quad \forall m, p$$

- Cannot schedule past waitlist horizon limit
- number of people scheduled/cancelled must be consistent

4 Transition Probabilities

There are three sources of uncertainty:

1. Number of patients arriving this period - pe_{dc}
   - let's assume pea_{dc} - is the random variable that represents the number of patients arrived this period. It follows a poisson distribution.
2. Transition between patient difficulties within the wait list - pw_{mdc}
   - let's assume $pwfi_{mdc}$ represents an intermediary variable showing the number of patients of complexity d, for CPU c entering the waitlist of period m in any way.
   - let's assume $pwfo_{mdc}$ represents an intermediary variable showing the number of patients of complexity d, for CPU c leaving the waitlist of period m in any way
   - let's assume pwt_{mdc} is the random variable that represents the number of patients of priority d, CPU c, that have been waiting for m period, that have moved a more complex category. It follows binary distribution.
3. Amout of expected units of PPE resource for the next time period - bc_{1dc}
   - let's assume bed_{md} is the random variable that represents the deviation of PPE units from the expectation for the next period only. It follows some uniform distribution.
   - let's assume ben_d is the default value to be used for expected number of PPE units p per period

Assume pe_{dc} follows some poisson distribution.
Assume transition within pw_{mdc} follows some binomial distribution with a probability that patient of difficulty d in CPU c moves to a more difficult category at each state transition.
Assume deviation from expected number of PPE units follows a uniform distribution (bcd_p)
State transitions have the following constraints:

- Transition from $\vec{bc}$ to $\vec{bc'}$ - Expected Units of PPE

$$be'_{1p} = be_{2p} + bed_p \quad \forall p$$

$$be'_{m-1,p} = be_{mp} \quad \forall m \in \{3...M\}, p$$

$$be'_{Mp} = ben_d \quad \forall p$$

- Transition from $\vec{bu}$ to $\vec{bu'}$ - Used Units of PPE

$$bu'_{m-1,p} = bu_{mp} - \sum_{tdc}(sc_{t,m-1,dc}U_{pdc}) + \sum_{tdc}(usc_{t,m-1,dc}U_{pdc}) \quad \forall m \in \{2...M\}, p$$

$$bu'_{Mp} = 0 \quad \forall p$$

- Transition from $\vec{pe}$ to $\vec{pe'}$ - Expected number of patients for this month

$$pe_{dc} = pea_{dc} \quad \forall dc$$

- Transition from $\vec{pw}$ to $\vec{pw'}$ - Flow of patients between difficulties/scheduling/cancelling for waitlist

$$pw'_{1dc} = pe_{dc} - \sum_t sc_{t0dc} + \sum_t usc_{t0dc} \quad \forall dc$$

$$pw'_{m+1,dc} = pw_{mdc} - \sum_t sc_{tmdc} + \sum_t usc_{tmdc} + pwt_{m,d-1,c} - pwt_{m,d,c} \quad \forall m \in \{1...M-1\}, dc$$

$$pw'_{Mdc} = 0 \quad \forall dc$$

- Transition from $\vec{ps}$ to $\vec{ps'}$ - Flow of patients between difficulties/scheduling/cancelling for scheduled appointments

$$ps'_{t-1,m+1,dc} = ps_{tmdc} - \sum_t sc_{tmdc} + \sum_t usc_{tmdc} + inflow - outflow \quad \forall t \in 2...T, m \in \{0...M-1\}, dc$$

$$ps'_{Tmdc} = 0 \quad \forall mdc$$

5 Costs

Cost will come from two source:

- cost of having patients wait (cw)
  - this cost will apply to 2 things: number of patients remaining on the waitlist, and number of patients scheduled in advance
- cancellation cost (cc)

$$c(\vec{s},\vec{a}) = \sum_{dc} cw_m pw_{mdc} + \sum_{tdc} cw_m ps_{tmdc} + \sum_{tmdc} cc * usc_{tmdc}$$

Where cw_m (wait cost) is computed as follows (where val is an arbitrary number that would allow us to describe cost growth well):

$$cc_m = val^m \quad \forall m \in \{0, ..., M-1\}$$

$$cc_M = val^m * 100$$

The reason there is a very high cost at the last month is because patients are essentially removed if they are not scheduled and they are in the last wait month (in order to be able to have a limit on m while modeling). So theoretically, the model should schedule in such a way that people never reach that point.