

Knapsack Dynamic Problem

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1 Decision Epochs

Decisions are made at the beginning of each month

2 State Space

State space is defined by current budget for various PPEs for future month, amount of budget already spent on PPEs, current patient waitlist, and expected monthly demand (?)

$$\vec{s} = (\vec{bc}, \vec{bu}, \vec{pw}, \vec{pe})$$

- $\vec{bc} = bc_{mp}$ - Expected budget for month m , and PPE p
- $\vec{bu} = bu_{mp}$ - Used budget for month m , and PPE p
- $\vec{pw} = pw_{mdc}$ - Number of patients of difficulty d , CPU c , on a wait list for m months
- $\vec{pe} = pe_{dc}$ - Number of patients of difficulty d , CPU c expected to arrive this month (I don't like having it here, because the way it is in the modelled right now, it is assumed this information is essentially know in advance, while in real world this would be realized over the transition)

3 Action Sets

At the beginning of each month, decision maker must decide how to use the PPE budget, ie how many patients of each group to assign to this month.

$$\vec{a} = (\vec{s})$$

- $\vec{s} = s_{tmdc}$ - Number of patients of difficulty d , CPU c , who have been in wait list for m month, to schedule in month t

The actions must satisfy the following constraints:

- Total allocated budget must not be exceeded (U_{pdc} - usage of PPE p per patient difficulty d , CPU c)

$$\sum_{dc} s_{mdc} U_{pdc} \leq (bc - bu) \quad \forall m, p$$

4 Transition Probabilities

There are three sources of uncertainty:

1. Number of patients arriving this month - pe_{dc}
2. Transition between patient difficulties within the wait list - pw_{mdc}

3. Amount of resources used during the operation - U_{pdc}

Assume pe_{dc} follows some poisson distribution.

Assume transition within pw_{mdc} follows some binomial distribution with a probability that patient of difficulty d in CPU c moves to a more difficult category at each state transition.

These two sources of uncertainty only deal with pe_{dc} and pw_{mdc} part of the states and must follow the following constraints

- Total number of patients on the wait-list must be consistent over time

$$\sum_d pw'_{m+1,dc} = \sum_d pw_{mdc} - \sum_{td} s_{tmdc} \quad \forall m, c$$

- Patients can only get more difficult over time

$$\sum_{di=d}^D pw'_{m+1,di,c} \geq \sum_{di=d}^D pw_{mdc} - \sum_{t,d=di}^D s_{tmdc} \quad \forall m, d, c$$

- Unscheduled expected arrivals are moved into the wait list

$$pw'_{m=1,dc} = pe_{dc} - \sum_t s_{t,m=0,dc} \quad \forall dc$$

5 Costs

I want to try a very simple cost for now, because I think it might be enough.

Cost will come only from one source: the number of people unscheduled after action is performed:

$$c(\vec{s}, \vec{a}) = \sum_{dc} c_m pw_{mdc}$$

Where c_m is computed as follows (where val is an arbitrary number that would allow us to describe cost growth well):

$$c_m = val^m \quad \forall m \in \{0, \dots, M-1\}$$

$$c_M = val^m * 100$$

The reason there is a very high cost at the last month is because patients are essentially removed if they are unscheduled and they are in the last wait month (in order to be able to have a limit on m while modeling). So theoretically, the model should schedule in such a way that people never reach that point.