Dynamic Knapsack Problem

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2 Introduction

2.1 Goals

The goal of this paper is to describe a patient surgery scheduling policy. This policy should consider the following items:

- Different surgery types
- Usage of various resources
- Consider changes in patient complexity and patient priority
- Consider randomness in various aspects
- Take into account long term impact of scheduling
- Modeling of the wait list
- It should allow for rescheduling

2.2 Outcomes

The outcomes of the model should be the following:

- The model should optimize overall system metrics
- It should provide suggested schedule times for an input

2.3 Model Description

The model's decisions will be done at the beginning of each period (in our case weekly). At the beginning of the period the model will take the current number of patients waiting, current waitlist, and current resource into account to make a decision. The model will output how many of each patient type to schedule into which period. And it will also say how many of each patient type to reschedule to which period.

3 Assumptions

1. Assume arrivals follow poisson distribution and all arrivals are at the beginning of the period
2. It is assumed that patients do not change complexities/priorities within some limit (TL).
3. We assume patient complexity/priority transitions follow binary distribution
4. It is assumed if a patient is scheduled in period one, it means they are served "immediately", regardless if in practice the appointment is at the beginning or end of the period.
5. We distinguish between two types of reschedules: good and bad reschedules
   - Good reschedules are reschedules where a patient is rescheduled to an earlier period
   - Bad reschedules are reschedules where a patient is rescheduled to a later period
6. We assume that only specific reschedules are allowed (to simplify the model, and remove redundancies)
   - Good reschedules are allowed from any period after 1 into period 1
   - Bad reschedules are only allowed from period 1 to any period after 1
7. It is assumed that there is a certain default expected number of PPE units available for all periods
   - However, in the period 1, there is some random deviation from the expected number of units
   - This random deviation follows some uniform distribution
8. We allow some violation of PPE units, in rare cases (with high cost), in order to accomodate changes due to variability.
   - If we have violation of PPE units - the extra capacity comes from external source

4 Data

4.1 Sets

- $\vec{m}$ - period defining how long a patient has been waiting so far. 0 represents patients who have arrived. $m \in \{0...M\}$
- $\vec{t}$ - denotes time of various things - scheduled into, waitlist period, rescheduled from. $t \in \{1...T\}$
- $\vec{d}$ - denotes complexity of a patient. $d \in \{1...D\}$
- $\vec{k}$ - denotes priority of a patient. $d \in \{1...K\}$
- $\vec{c}$ - denotes surgery type of a patient. $c \in \{1...C\}$
- $\vec{p}$ - denotes resource type. $p \in \{1...P\}$

4.2 Input Data

- p_co_p - set of resources that carries over between periods
- p_nco_p - set of resources that doesn't carry over between periods
- p_ue_p - amount of resources expected each period
- p_ued_p - Deviation from p_ue. Uniform, defined by lower and upper value. The mean must be zero
- U_{pdc} - Usage of resource p, of complexity d, surgery type c
- pea_{dkc} - Patient Expected Arrival
- TL_c - Time Limit after which patient can transition in complexity/priority
- ptp_d_{dc} - Probability to transition to higher complexity
- ptp_p_{kc} - Probability to transition to higher priority

4.3 Model Parameters

- cw_k - defines cost of waiting, by priority k
- cc_k - additional cost of rescheduling, by priority k
- α - discount factor to be used in the model

5 MDP Model

5.1 Decision Epochs

Decisions are made at the beginning of each time period. There are 3 relevant time intervals to look at:

- Pre-decision state (S)
  - Pre-decision state defines the initial state on which a decision is required
  - This information drives decision making
- Post-decision state ($\hat{S}$)
  - Post-decision state is the time when the decision has been executed, but no new info came in
  - This state defines the immediate cost of an action
- Post-transition state (S')
  - This is the state when new information has come in (transition randomness). Defines next pre decision state
  - It is primarily used to generate expectation for the ADP

5.2 State Space

5.2.1 Description

State is defined by carried over resources, deviation from expected resources, patient waitlist/demand, and patients already scheduled

$$\vec{S} = (\vec{ul}, \vec{pw}, \vec{ps})$$

- $\vec{ul} = ul_p$ - Units left over from previous period for resource p plus deviation for that unit for period 1.
- $\vec{pw} = pw_{mdkc}$ - Patients of complexity d, priority k, surgery type c, on a wait list for m periods (m of 0 - just arrived)
- $\vec{ps} = ps_{tmdkc}$ - Patients of complexity d, priority k, surgery type c, scheduled to period t, waiting for m periods

5.3 Action Sets

5.3.1 Description

Actions consist of rescheduling currently scheduled patients, and scheduling patients on wait list. There are also some goal and auxiliary variables

$$\vec{A} = (\vec{sc}, r\vec{sc}, \text{auxiliary variables})$$

- $\vec{sc} = sc_{tmdkc}$ - Patients of complexity d, priority k, surgery type c, waiting for m, to schedule in period t
- $r\vec{sc} = rsc_{tt'mdkc}$ -Patients of complexity d, priority k, surgery type c, waiting for m, reschedule from t to t'

5.3.2 Auxiliary Variables

- $\vec{uv} = uv_{tp}$ - goal variable, violation on number of resources used for period t, of resource p
- uvb_{tp} - binary variable to enforce uv variable without objective function
- $\hat{ul}_{p \in p\_co}$ - post-decision unit leftover at period 1
- $ulb_{p \in p\_co}$ - binary variable to enforce ul variable without objective function
- $\hat{uu}_{tp}$ - post-decision units used
- $\hat{pw}_{mdkc}$ - post-decision patients waiting
- $\hat{ps}_{tmdkc}$ - post-decision patients scheduled
- pwt_d_{mdkc} - patients waiting transitioned in complexity
- pwt_k_{mdkc} - patients waiting transitioned in priority
- pst_d_{tmdkc} - patients scheduled transitioned in complexity
- pst_k_{tmdkc} - patients scheduled transitioned in priority

5.3.3 Auxiliary Variable Definition

$$\begin{aligned}
\hat{uu}_{tp} &= \sum_{mdkc} U_{pdc} \hat{ps}_{tmdkc} && \forall tp && (1)\\
\hat{pw}_{mdkc} &= pw_{mdkc} - \sum_{t} sc_{tmdkc} && \forall mdkc && (2)\\
\hat{ps}_{tmdkc} &= ps_{tmdkc} + sc_{tmdkc} + \sum_{t} rsc_{tt'mdkc} - \sum_{t'} rsc_{tt'mdkc} && \forall tmdkc && (3)\\
pwt_d_{mdkc} &= ptp_dc * \hat{pw}_{mdkc} && \forall mdkc && (4)\\
pwt_k_{mdkc} &= ptp_kc * (\hat{pw}_{mdkc} + pwt_d_{md-1kc} - pwt_d_{mdkc}) && \forall mdkc && (5)\\
pst_d_{tmdkc} &= ptp_dc * \hat{ps}_{tmdkc} && \forall tmdkc && (6)\\
pst_k_{tmdkc} &= ptp_kc * (\hat{ps}_{tmdkc} + pst_d_{tmd-1kc} - pst_d_{tmdkc}) && \forall tmdkc && (7)
\end{aligned}$$

- Define Resource Violation Variable

$$
\begin{aligned}
uv_{tp} &= max\{0, \hat{uu}_{1p} - p\_ue_{1p} - ul_p\} && \forall p && (8)\\
uv_{tp} &= max\{0, \hat{uu}_{tp} - p\_ue_{tp}\} && \forall t \in \{2..T\}p && (9)\\
& && && (10)\\
uv_{tp} &\geq 0 && \forall tp && (11)\\
uv_{tp} &\leq M(uvb_{tp}) && \forall tp && (12)\\
uv_{1p} &\geq \hat{uu}_{1p} - p\_ue_{1p} - ul_p && \forall p && (13)\\
uv_{1p} &\leq (\hat{uu}_{1p} - p\_ue_{1p} - ul_p) + M(1 - uvb_{tp}) && \forall p && (14)\\
uv_{tp} &\geq \hat{uu}_{tp} - p\_ue_{tp} && \forall t \in \{2..T\}p && (15)\\
uv_{tp} &\leq (\hat{uu}_{tp} - p\_ue_{tp}) + M(1 - uvb_{tp}) && \forall t \in \{2..T\}p && (16)\\
& && && (17)
\end{aligned}
$$

- Define Units Left Over Variable

$$
\begin{aligned}
\hat{ul}_p &= max\{0, p\_ue_{1p} + ul_p - \hat{uu}_{1p}\} && \forall p \in \{p_co\} && (18)\\
& && && (19)\\
\hat{ul}_p &\geq 0 && \forall p \in p\_co && (20)\\
\hat{ul}_p &\leq M(ulb_p) && \forall p \in p\_co && (21)\\
\hat{ul}_p &\geq p\_ue_{1p} + ul_p - \hat{uu}_{1p} && \forall p \in p\_co && (22)\\
\hat{ul}_p &\leq (p\_ue_{1p} + ul_p - \hat{uu}_{1p}) + M(1 - ulb_p) && \forall p \in p\_co && (23)
\end{aligned}
$$

5.3.4 State-Action Constraints

- Resource Usage Constraint

$$
\begin{aligned}
\hat{uu}_{1p} &\leq p\_ue_{1p} + ul_p + uv_{1p} && \forall p && (24)\\
\hat{uu}_{tp} &\leq p\_ue_{tp} + uv_{tp} && \forall t \in \{2..T\}p && (25)
\end{aligned}
$$

- Custom bounds on when reschedules are allowed

$$
\begin{aligned}
rsc_{tt'mdkc} &= 0 && \forall t \in \{2...T\}, t' \in \{2...T\}mdkc && (26)\\
rsc_{tt'mdkc} &= 0 && \forall tt'mdkc, \text{where } t = t' = 1 && (27)
\end{aligned}
$$

- Bounds on Schedules/Reschedules

$$
\begin{aligned}
\sum_{t'} rsc_{tt'mdkc} &\leq ps_{tmdkc} && \forall tmdkc && (28)\\
\sum_{t} sc_{tmdkc} &\leq pw_{mdkc} && \forall mdkc && (29)
\end{aligned}
$$

- Bounds on states

$$
\begin{aligned}
ul_p &\leq p\_ue_p && \forall p && (30)\\
pw_{mdkc} &\leq pea_{dkc} * 2 && \forall mdkc && (31)\\
ps_{tmdkc} &\leq pea_{dkc} && \forall tmdkc && (32)
\end{aligned}
$$

5.4 Transition

In this section we will have the following:

- *p_ued* will represent the random variable from *p_ued* distribution
- *pea* will represent the random variable from *pea* distribution.
- *pwt* and *pst*, will represent number transitioned in complexity/priority on waitlist and currently scheduled list.

1. Transition from $\vec{ul}$ to $\vec{ul'}$ - Resource Carry Over

$$ul'_p = \hat{ul}_p + p\_ued_p \quad \forall p \in \{p\_co\} \tag{33}$$
$$ul'_p = p\_ued_p \quad \forall p \in \{p\_nco\} \tag{34}$$

2. Transition from $\vec{pw}$ to $\vec{pw'}$ - Flow of patients on waitlist

$$pw'_{0dkc} = pea_{dkc} \quad \forall dc \tag{35}$$
$$pw'_{mdkc} = \hat{pw}_{m-1,dkc} \quad \forall m \in \{1...(TL_c - 1)\}dkc \tag{36}$$
$$pw'_{mdkc} = \hat{pw}_{m-1,dkc} + \overbrace{pwt\_d_{m-1,d-1,c} - pwt\_d_d}^{\text{change in complexities}} + \overbrace{pwt\_k_{k-1} - pwt\_k_k}^{\text{change in priorities}} \quad \forall m \in \{TL_c...M-1\}dkc \tag{37}$$
$$pw'_{Mdc} = \sum_{M-1}^{M} \left(\hat{pw}_{mdc} + \overbrace{pwt\_d_{md-1,c} - pwt\_d_d}^{\text{change in complexities}} + \overbrace{pwt\_k_{k-1} - pwt\_k_k}^{\text{change in priorities}}\right) \quad \forall dc \tag{38}$$

3. Transition from $\vec{ps}$ to $\vec{ps'}$ - Flow of patiensts scheduled

$$ps'_{t0dkc} = 0 \quad \forall tdkc \tag{39}$$
$$ps'_{Tmdkc} = 0 \quad \forall mdkc \tag{40}$$
$$ps'_{tmdkc} = \hat{ps}_{t+1,m-1,dkc} \quad \forall t \neq Tm \in \{1...TL-1\}dkc \tag{41}$$
$$ps'_{tmdkc} = \hat{ps}_{t+1,m-1,dkc} + \overbrace{pst\_d_{t+1,m-1,d-1,c} - pst\_d_d}^{\text{change in complexities}} + \overbrace{pst\_k_{k-1} - pst\_k_k}^{\text{change in priorities}} \quad \forall t \neq Tm \in \{TL_c...M-1\}dkc \tag{42}$$
$$ps'_{tMdkc} = \sum_{M-1}^{M} \left(\hat{ps}_{t+1mdc} + \overbrace{pst\_d_{t+1,md-1,c} - pst\_d_d}^{\text{change in complexities}} + \overbrace{pst\_k_{k-1} - pst\_k_k}^{\text{change in priorities}}\right) \quad \forall t \neq Tdkc \tag{43}$$

5.5 Costs

- $cs_{tk} = \sum_t (\alpha^t * cw_k)$

$$\begin{aligned} C = c(\vec{pw}, \vec{ps}, \vec{rsc}, \vec{uv}) = & \overbrace{\sum_{mdkc} cw_k(\hat{pw}_{mdkc})}^{\text{cost of waiting}} + \overbrace{\sum_{tmdkc} cs_{tk}(sc_{tmdkc})}^{\text{Prefer earlier appointments}} \\ & + \overbrace{\sum_{\substack{tt`mdkc \\ t'>t}} (cs_{t'-t,k} + cc) * (rsc_{tt'mdc})}^{\text{Bad Reschedule}} - \overbrace{\sum_{\substack{tt`mdkc \\ t'<t}} (cs_{t-t',k} - cc) * (rsc_{tt'mdc})}^{\text{Good Reschedule}} + cv \sum_{tp} uv_{tp} \end{aligned} \tag{44}$$

6 LP Model

6.1 Full LP

Given a full MDP model, the equivalent LP would look as follows:

$$\max_{\vec{v}} \sum \alpha(\vec{S}) v(\vec{S}) \tag{45}$$

subject to

$$c(\vec{S}, \vec{A}) + \gamma \sum_{\vec{p}} p * v(\vec{S'}) \geq v(\vec{S}) \quad \forall \vec{S}\vec{A} \tag{46}$$

And all auxiliary constraints in "Auxiliary Variable Definition"

7 Aproximate Dynamic Programming Model

7.1 Expectation ADP Model

Let's convert it into ADP. We do that by changing $v(\vec{S})$ to an approximation as follows:

$$v(\vec{ul}, \vec{pw}, \vec{ps}) = \beta^0 + \sum_p \beta_p^{ul} ul_p + \sum_{mdkc} \beta_{mdkc}^{pw} pw_{mdkc} + \sum_{tmdkc} \beta_{tmdkc}^{ps} ps_{tmdkc} \tag{47}$$

Next we need to convert a part of equation 46 into expectation.
(To make it easier later, we will format is as $v(\vec{S}) - E[v(\vec{S}')]$)

- $E[p\_ued_p] = 0$
- $E[pwt_{mdkc}] = ptp\_d_{dc} * \hat{pw}_{mdkc}$
- $E[pst_{tmdkc}] = ptp\_k_{kc} * \hat{ps}_{tmdkc}$

$$\beta^0 : \qquad (1-\gamma)\beta^0 \tag{48}$$

$$\beta_p^{ul} : \qquad \sum_{p \in \text{p\_co}} \beta_p^{ul} \Big(ul_p - \gamma(\hat{ul}_p) \Big) + \sum_{p \in \text{p\_nco}} \beta_p^{ul} (ul_p) \tag{49}$$

$$\begin{aligned}
\beta_{mdkc}^{pw} : \qquad & \sum_{dkc} \beta_{0dkc}^{pw} \Big(pw_{0dkc} - \gamma(pea_{dkc}) \Big) + \sum_{m=1dkc}^{TL_c - 1} \beta_{mdkc}^{pw} \Big(pw_{mdkc} - \gamma(\hat{pw}_{m-1dkc}) \Big) + \\
& \sum_{m=TL_cdkc}^{M-1} \beta_{mdkc}^{pw} \Big(pw_{mdkc} - \gamma(\hat{pw}_{m-1dkc} + E[pwt_d_{m-1d-1c}] - E[pwt_d_d] + E[pwt_k_{k-1}] - E[pwt_k_k]) \Big) + \\
& \sum_{dc} \beta_{Mdkc}^{pw} \Big(pw_{Mdkc} - \gamma \sum_{M-1}^{M} (\hat{pw}_{mdkc} + E[pwt_d_{md-1c}] - E[pwt_d_d] + E[pwt_k_{k-1}] - E[pwt_k_k]) \Big)
\end{aligned} \tag{50}$$

$$\begin{aligned}
\beta_{tmdkc}^{ps} : \qquad & \sum_{tdkc} \beta_{t0dkc}^{ps} \Big(ps_{t0dkc} \Big) + \sum_{mdkc} \beta_{Tmdkc}^{ps} \Big(ps_{Tmdkc} \Big) + \sum_{t=1m=1dkc}^{T-1, TL_c - 1} \beta_{tmdkc}^{ps} \Big(ps_{tmdkc} - \gamma(\hat{ps}_{t+1m-1dkc}) \Big) + \\
& \sum_{t=1m=TL_cdkc}^{T-1, M-1} \beta_{tmdkc}^{ps} \Big(ps_{tmdkc} - \gamma(\hat{ps}_{t+1m-1dkc} + E[pst_d_{t+1m-1d-1c}] - E[pst_d_d] + E[pst_k_{k-1}] - E[pst_k_k]) \Big) + \\
& \sum_{t=1dkc}^{T-1} \beta_{tMdkc}^{ps} \Big(ps_{tMdkc} - \gamma \sum_{M-1}^{M} (\hat{ps}_{t+1mdkc} + E[pst_d_{t+1md-1c}] - E[pst_d_d] + E[pst_k_{k-1}] - E[pst_k_k]) \Big)
\end{aligned} \tag{51}$$

Using the numbers above we rearrange the original equation as follows:

$$\max_{\vec{\beta}} \Big(\sum \beta^0 + \sum \beta^{ut} E[ut] + \sum \beta^{pw} E[pw] + \sum \beta^{ps} E[ps] \Big) \tag{52}$$

Subject to:

$$v(\vec{S}) - E[v(\vec{S}')] \leq c(\vec{S}, \vec{A}) \quad \forall \vec{S}\vec{A} \tag{53}$$

7.2 ADP Master Problem

The equation above has a reasonable amount of decision variables, but an uncountable amount of constraints. Let's use column generation to solve it. First converting to dual

$$\min_w \sum_{\vec{S}\vec{A}} w(\vec{S}\vec{A}) c(\vec{S}\vec{A}) \tag{54}$$

Subject To:

$$\beta^0: \sum_{\vec{S}\vec{A}} \vec{w}(1-\gamma) = 1 \tag{55}$$

$$\beta^{ul}: \sum_{\vec{S}\vec{A}} \vec{w}\Big(ul_p - \gamma(\hat{ul}_p)\Big) \geq E[ul_p] \qquad \forall p \in p\_co \tag{56}$$

$$\sum_{\vec{S}\vec{A}} \vec{w}(ul_p) \geq E[ul_p] \qquad \forall p \in p\_nco \tag{57}$$

$$\beta^{pw}: \sum_{\vec{S}\vec{A}} \vec{w}\Big(pw_{0dkc} - \gamma(pea_{dkc})\Big) \geq E[ps_{0dkc}] \qquad \forall dkc \tag{58}$$

$$\sum_{\vec{S}\vec{A}} \vec{w}\Big(pw_{mdkc} - \gamma(\hat{pw}_{m-1dkc})\Big) \geq E[pw_{mdkc}] \qquad \forall m \in \{1...TL_c - 1\}dkc \tag{59}$$

$$\sum_{\vec{S}\vec{A}} \vec{w}\Big(pw_{mdkc} - \gamma(\hat{pw}_{m-1dkc} + E[pwt\_d_{m-1d-1c}] - E[pwt\_d_d] + \tag{60}$$

$$+ E[pwt\_k_{k-1}] - E[pwt\_k_k])\Big) \geq E[pw_{mdkc}] \qquad \forall m \in \{TL_c...M-1\}dkc \tag{61}$$

$$\sum_{\vec{S}\vec{A}} \vec{w}\Big(pw_{Mdkc} - \gamma\sum_{M-1}^{M}(\hat{pw}_{mdkc} + E[pwt\_d_{md-1c}] - E[pwt\_d_d] + \tag{62}$$

$$+ E[pwt\_k_{k-1k}] - E[pwt\_k_k])\Big) \geq E[pw_{Mdkc}] \qquad \forall dkc \tag{63}$$

$$\beta^{ps}: \sum_{\vec{S}\vec{A}} \vec{w}\Big(ps_{t0dkc}\Big) \geq E[ps_{t0dkc}] \qquad \forall tdkc \tag{64}$$

$$\sum_{\vec{S}\vec{A}} \vec{w}\Big(ps_{Tmdkc}\Big) \geq E[ps_{Tmdkc}] \qquad \forall mdkc \tag{65}$$

$$\sum_{\vec{S}\vec{A}} \vec{w}\Big(ps_{tmdkc} - \gamma(\hat{ps}_{t+1m-1dkc})\Big) \geq E[ps_{tmdkc}] \qquad \forall \{1...T-1\}\{1...TL_c-1\}dkc \tag{66}$$

$$\sum_{\vec{S}\vec{A}} \vec{w}\Big(ps_{tmdkc} - \gamma(\hat{ps}_{t+1m-1dc} + E[pst\_d_{t+1m-1d-1c}] - E[pst\_d_d] + \tag{67}$$

$$+ E[pst\_k_{k-1}] - E[pst\_k_k])\Big) \geq E[ps_{tmdkc}] \qquad \forall \{1...T-1\}\{TL_c...M-1\}dkc \tag{68}$$

$$\sum_{\vec{S}\vec{A}} \vec{w}\Big(ps_{tMdkc} - \gamma\sum_{M-1}^{M}(\hat{ps}_{t+1mdkc} + E[pst\_d_{t+1md-1c}] - E[pst\_d_d] + \tag{69}$$

$$+ E[pst\_k_{k-1}] - E[pst\_k_k])\Big) \geq E[ps_{tMdkc}] \qquad \forall \{1...T-1\}dkc \tag{70}$$

$$\text{State Action Bounds:} \qquad w \geq 0 \qquad \forall w \tag{71}$$

7.3 ADP Pricing Problem

$$\min_{(\vec{ue},\vec{uu},\vec{pw},\vec{pe},\vec{ps})\in S,(\vec{sc},\vec{rsc})\in A} c(\vec{pw},\vec{sc},\vec{rsc},\vec{uv}) - \Big(v(\vec{S}) - E[v(\vec{S}')]\Big) \tag{72}$$

Subject to: constraints in sections "State-Action Constraints" and "Auxiliary Variable Definition"

8 Solution Explanation

8.1 Algorithm for solving

To get $\vec{\beta}$ values, which will be used to generate an action follow steps below:

1. Perform a monte-carlo simulation (following some arbitrary policy) to get $E[ul], E[pw]E[pw]$
2. Create an initial feasible set of state-action pairs - $\vec{w}$
3. Solve model in section "ADP Master Problem" where each state-action pairs in $\vec{w}$ corresponds to a variable and parameters for all the constraints for a specific action.

4. Solve model in section "ADP Pricing Problem", where duals from problem in step 3 correspond to $\vec{\beta}$ values.
   - If objective function is less than 0, add solution as a single state-action pair to $\vec{w}$ and go to step 3
   - If objective function is greater than 0, continue to next step
5. Duals from problem in step 3 correspond to final $\vec{\beta}$ values

8.2 Generating an Action

Once $\vec{\beta}$ values have been approximated, you may use the model below to generate a recommended action for a specific state.

$$\min_{\vec{A}} c(\vec{S}, \vec{A}) + \gamma v(S')$$

$$\begin{aligned}
v(S') = \Bigg(& \beta^0 + \Big(\sum_{p \in p_co} \beta_p^{ul}(\hat{ul}_p)\Big) + \Big(\sum_{dc} \beta_{0dkc}^{pw} pea_{dkc}\Big) + \Big(\sum_{m=1,dkc}^{TL_c-1} \beta_{mdkc}^{pw}(\hat{pw}_{m-1dkc}\Big) + \\
& \Big(\sum_{m=TL_cdkc}^{M-1} \beta_{mdkc}^{pw}(\hat{pw}_{m-1dkc} + E[pwt_d_{m-1d-1c}] - E[pwt_d_d] + E[pwt_k_{k-1}] - E[pwt_k_k])\Big) + \\
& \Big(\sum_{dkc} \beta_{Mdkc}^{pw} \sum_{M-1}^{M} (\hat{pw}_{mdkc} + E[pwt_d_{md-1c}] - E[pwt_d_d] + E[pwt_k_{k-1}] - E[pwt_k_k])\Big) + \\
& \Big(\sum_{t=1m=1dkc}^{T-1,TL_c-1} \beta_{tmdkc}^{ps}(\hat{ps}_{t+1m-1dkc})\Big) + \\
& \Big(\sum_{t=1m=TL_cdkc}^{T-1M-1} \beta_{tmdkc}^{ps}(\hat{ps}_{t+1m-1dkc} + E[pst_d_{t+1m-1d-1c}] - E[pst_d_d] + E[pst_k_{k-1}] - E[pst_k_k])\Big) + \\
& \Big(\sum_{t=1dkc}^{T-1} \beta_{tmdkc}^{ps} \sum_{M-1}^{M} (\hat{ps}_{t+1mdkc} + E[pst_d_{t+1md-1c}] - E[pst_d_d] + E[pst_k_{k-1}] - E[pst_k_k])\Big)\Bigg)
\end{aligned} \tag{73}$$

Subject to: constraints in section "State-Action Constraints"