Reservation Planning for Elective Surgery Under Uncertain Demand for Emergency Surgery

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A Typical Day in an Operating Room

- At the beginning of the day, there are n requests for elective surgery.
- The scheduler decides how many of them, m, to admit today.
- There is a random, unknown duration of emergency surgeries throughout the day.
- These emergency cases take priority over elective cases.
- Overloading operating room capacity leads to high costs of overtime pay and/or transporting patients to nearby hospitals.

The Problem

- How do you effectively schedule elective surgeries in an operating room?
- While the number of elective surgery requests is known, the duration of each surgery is unknown.
- The number and duration of emergency surgeries is unknown.
- Hospitals make revenue from elective surgeries.
- Hospitals lose money by exceeding operating room capacity and having to pay overtime.
- We also consider a penalty to delaying elective surgeries.

Motivation

- A cutoff policy is a policy in which every elective surgery is admitted, up to a cutoff number M. After M elective surgeries are admitted, no others are admitted that day.
- M is based on hospital capacity and expected duration of emergency cases.
- Most hospitals use a cutoff policy.
- We want to see if there is a better policy for admitting elective surgery patients.

Expected Revenue in One Day

- Let X_j be the number of new requests for surgery on the j^{th} day.
- Let n be the total number of current requests for surgery, some of which are new and some of which are past requests which have not been fulfilled.
- Let m be the number of elective surgeries to admit today.
- Let π be the expected revenue from each elective surgery.

- Present day expected revenue: πm

Expected Overtime Costs in One Day

- Let Z_i be the unknown duration of the i^{th} elective surgery of the day ($i \in \{1, 2, ..., m\}$).
- Let $S_m = \sum_{i=1}^{m} Z_i$ be the total duration of elective surgeries today.
- Let Y be the random duration of emergency surgeries today.
- Let T be the duration the operating room can be used before overtime pay begins. This value is known.
- Let c be the penalty, per unit time, of exceeding the day's capacity, T.

- Present day expected overtime cost: $c\mathbb{E}[(Y + S_m - T)^+]$

Expected Delay Penalty in One Day

- The waiting patient and/or society often incurs a cost when individuals are fully or partially unable to function normally.
- Let p be the average daily penalty for the postponement of each elective case by one day.

- Present day expected penalty from postponement: $p(n-m)$

Present day expected profit function

- Out of a pool of n possible elective surgeries, m are performed
- Expected Profit = Elective Surgery Revenue - Expected Overtime Costs - Postponement Penalties

$$g(m,n) = \pi m - c\mathbb{E}[(Y + S_m - T)^+] - p(n-m)$$

Concavity of Present Day Profit

Theorem: The function $g(m, n)$ is jointly concave in m and n for every $T \geq 0$. Furthermore, if $\pi + p < c\mathbb{E}[Z_1]$, then $g(m, n)$ is bounded from above.

Interpretation:

- We expect the cost of overtime to be greater than the sum of the profit of the extra surgery and the savings of not postponing the surgery.
- There is a maximum on how much profit can be made in this case. The maximum profit is dependent on the operating room capacity, T.

Discounting Future Profit

- Let $f_i(n)$ be the maximal expected discounted profit with i days remaining if there are n outstanding elective cases at the beginning of that day.
- Dynamic programming recursion for determining today's allotment:

$$f_i(n) = \max_{m \leq n}(g(m,n) + \alpha \mathbb{E}[f_{i-1}(n-m+X)]),$$

  where $g(m,n)$ is the present day expected profit, α is the daily discount factor ($0 < \alpha < 1$), and X is the number of new arrivals the next day
- $f_0(n) = 0 \ \forall \ n$
- We will apply the backward recursion an "infinite" number of times to model the reality of an ongoing operating room.

Concavity of $f_i(n)$

Theorem: $f_i(n)$ is concave in $n \; \forall \; i$. If $p = 0$, then $f_i(n)$ is also increasing in n.

Interpretation:

- Like our present-day profit examined in the previous theorem, our discounted future profit is also concave in n.
- If there is no penalty to postponing surgeries, then our profit cannot be diminished by having more possible surgeries to schedule.

Discounted Profit of an Ongoing Operating Room

Theorem: If $\pi + p < c\mathbb{E}[Z_1]$, then there exists a concave function f such that $\lim_{i\to\infty} f_i = f$, and which satisfies

$$f(n) = \max_{m \le n} (g(m,n) + \alpha\mathbb{E}[f(n-m+X)])$$

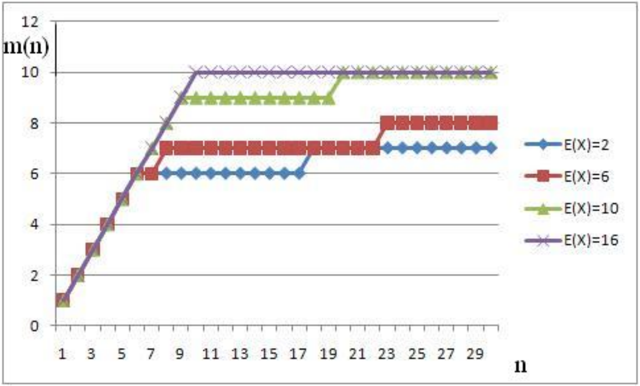
Interpretation: The use of most operating rooms is expected to continue indefinitely. So, without an ending date in sight, this theorem tells us that there is a concave discounted profit function to use to find the optimal policy.

Characterizing the Optimal Number to Admit

- Let $m(n)$ be the value of m that maximizes $f(n)$, the maximal expected discounted profit.
- $m(n)$ is non-decreasing in n
  - Numerical example:
    If you would admit 4 surgeries out of a possible 5 surgeries, then you would admit 4 or more out of a possible 6 surgeries.
- For every $n \geq 0$ and $d \geq 0$, $m(n+d) - m(n) \leq d$
  - Numerical example:
    If you would admit 5 elective surgeries out of a possible 10, then you would admit no more than 7 elective surgeries out of a possible 12.
- m can be found through Value Iteration.

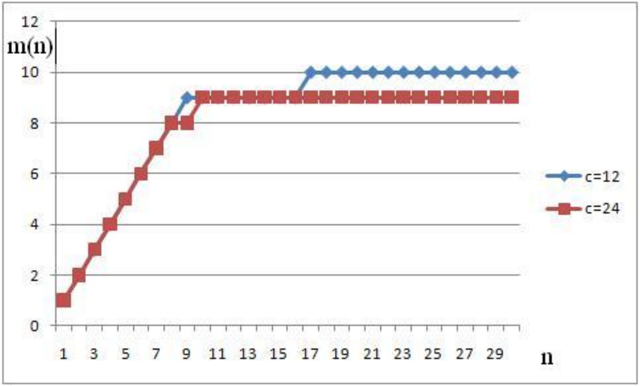
Effect of Arrival Rate

Units are minutes
$T = 960$, $Y \sim Norm(400, 640)$, $Z_i \sim Norm(60, 100)$,
$\pi = \$600$, $c = \$15$, $\alpha = .99$, $p = \$0$,
Arrivals are Poisson with rate listed



Effect of Overtime Cost

Units are minutes
$T = 960$, $Y \sim Norm(400, 640)$, $Z_i \sim Norm(60, 100)$,
$X \sim Poisson(10)$, $\pi = \$600$, $\alpha = .99$, $p = \$0$

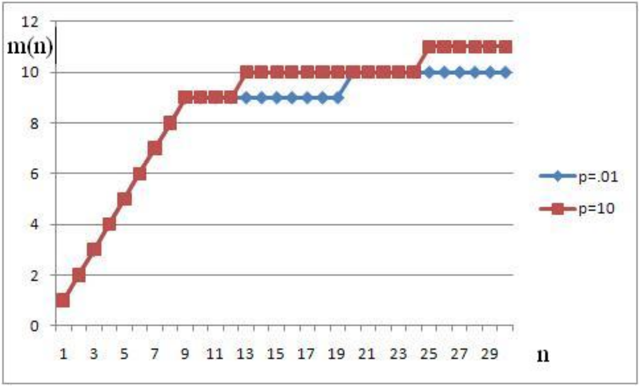


Effect of Postponement Penalty

Units are minutes

$T = 960$, $Y \sim Norm(400, 640)$, $Z_i \sim Norm(60, 100)$,

$X \sim Poisson(10)$, $\pi = \$600$, $c = \$15$, $\alpha = .99$,



Comparison to Cutoff Policy

- It is possible to re-formulate this system to find the best cutoff policy number, M.
  - Add the constraint that $m(n) = min(n, M)$, where M is the cutoff number.
  - Find the cutoff number that maximizes $f(n)$
- We will compare the optimal policy found in this model to the best cutoff policy to see if there are significant differences.

Avg Arrival Rate, $E(X) = 10$

	Cutoff # Policy		Optimal Policy		
n	m	$f(n)$	m	$f(n)$	Savings
0	0	501432.52	0	503805.96	2373.44
1	1	502032.52	1	504405.96	2373.44
2	2	502632.52	2	505005.96	2373.44
3	3	503232.52	3	505605.96	2373.44
4	4	503832.51	4	506205.94	2373.44
5	5	504432.22	5	506805.66	2373.44
6	6	505029.01	6	507402.44	2373.44
7	7	505607.15	7	507980.58	2373.44
8	8	506114.42	8	508487.85	2373.44
9	9	506457.29	9	508830.73	2373.44
10	10	506563.08	9	509134.06	2570.99
11	10	506879.56	9	509394.97	2515.41
12	10	507166.01	9	509619.49	2453.48
13	10	507430.18	9	509814.22	2384.05
14	10	507678.46	9	509985.27	2306.81
15	10	507914.90	9	510137.69	2222.79
16	10	508141.33	9	510275.43	2134.11
17	10	508358.22	9	510401.56	2043.34
18	10	508565.70	9	510518.45	1952.75
19	10	508763.98	9	510627.90	1863.92
20	10	508953.44	10	510733.68	1780.24
21	10	509134.51	10	510837.02	1702.51
22	10	509307.58	10	510935.18	1627.61

Conclusions

- It is possible to find the optimal policy for making reservations for elective surgery in the face of uncertain demand for emergency surgery.
- The optimal policy is not one of cutoff number, but the relative loss in profit from using the best cutoff number policy is small.
- Finding the optimal policy suggests the best cutoff number.