Stress-testing interconnected portfolios in the South African banking sector

Tina Koziol

***Abstract*—The paper conducts stress-tests on South African banks by calibrating a model of price-mediated contagion. Using longitudinal data of balance sheet positions of the largest 10 banks from 2010 to 2020, two types of shocks are studied: one to a non-marketable asset of the largest retail bank and the other to a marketable asset held by all banks. Overall, the paper finds that second-order feedback effects from bank' de-leveraging are muted and that the concentrated structure of the banking system has a positive effect on shock absorption. However, a gradual trend towards more similar asset portfolios in the past 10 years has increased the exposure to the price-mediated contagion channel.**

Keywords: Stress-testing, portfolio similarity, fire-sale externalities

I. Introduction

AMPLIFICATION and feedback effects that compound losses in financial networks are at the centre of attention to understand systemic risk. Price-mediated contagion becomes particularly potent when banks hold similar portfolios as price shocks amplify relative to common balance sheet asset holdings. This paper investigates this type of contagion channel and conducts stress-tests on South African banks across two types of shocks; one to a non-marketable asset of an individual bank and the other to a marketable asset held by all banks. I rank individual banks according to their contribution to systemic risk arising from this contagion channel.

The paper is particularly concerned with the following questions: 1) How can we quantify price spillover amplification in the South African banking system? Are they relevant for systemic risk? 2) Which Bank contributes more to this amplification process? 3) Has the exposure to this type of systemic risk changed over time and 4) what is the role of portfolio similarity?

The source code and replication files for this paper can be found https://github.com/blackrhinoabm and https://github.com/t1nak/ba900. All errors are my own.

University of Cape Town

The main results are the following: the amplification of losses in second and third order de-leveraging rounds is largely contained when the initial shock hits the portfolios of unsecured loans to the largest retail bank. This is because the bank and the liquidity reserves are large enough to absorb the shock, while the structure of the financial system characterised by large, few banks has a positive absorptive effect on financial system stability. In the case of a shock to the marketable asset held by all banks (SA Government bonds), banks' de-leveraging behaviour becomes more pronounced, however, they do not exceed the magnitude of the initial impact. Again, the concentrated structure of the banking system has a positive effect on shock absorption.

When examining each bank's contribution to spillover loses, their systemic relevance is fairly stable over time. The top 4 banks each contribute between 20 and 27% of exposure to price-mediated contagion, while similarly moving "closer together" in terms of asset size, leverage and portfolio composition. However, this development has lead to a gradual increase in exposure to this type of contagion channel for the overall banking sector. As banks become more similar in their balance sheet set-up, the price-mediated contagion channel becomes more potent. While still being at low levels, the aggregate sector vulnerability to this type of contagion has doubled between 2010 and 2020.

Last, but not least, the paper finds that leverage, while being an important factor for spillover losses in general, has decreased in the South African banking sector in the last 10 years, therefore mitigating the risk of indirect contagion.

What is price-mediated contagion?

This paper studies shock scenarios to quantify systemic losses arising from the price-mediated contagion channel, also known as fire-sale externalities. Fire-sales occur in situations where financial institutions experience sudden constraints, e.g. a large liquidity requirement, which lead to forced liquidation of assets [1]. When a bank faces a liquidity crisis and is forced to sell-off assets in a short amount of time to meet counterparties' claims, it accepts prices that can be substantially below market value, so-called fire-sale prices. The discount on the market value is higher, the more illiquid the asset. Fire-sale externalities pose a threat to the financial system because they amplify price shocks across assets and institutions and thus, may lead to liquidation spirals (see e.g. [2], [3]). This type of contagion becomes particularly potent when banks hold similar portfolios as price shocks amplify relative to common balance sheet asset holding patterns [4].

Measuring similarity: What are common asset holdings of South African banks and how do they affect financial stability? The similarity between two banks m and n can be measured as the Euclidean distance (ED) between them in K-dimensional space as in [5]:

$$\text{Distance}_{m,n,t} = \sqrt{\sum_{k=1}^{K} (w_{m,k,t} - w_{n,k,t})^2}$$

where $w_{m,k,t}$ is the portfolio weight invested in asset class k by bank m. Figure 3 shows the pair-wise euclidean distance between the top 10 banks as of February 2020. The five largest banks (A to E) are much closer in portfolio composition than the rest (G to J).

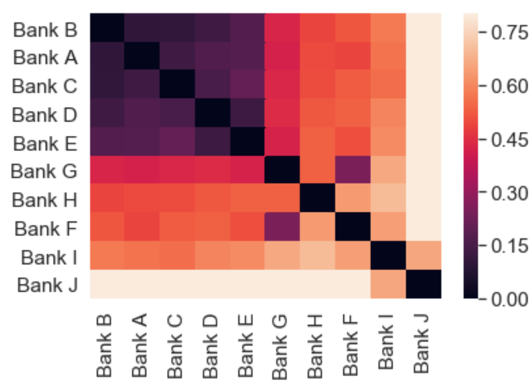


Fig. 1. Pair-wise euclidean distance between top 10 banks as of February 2020. The closer (darker) the value to 0, the more similar the portfolios. Source: SARB's BA 900 forms, aggregated to 27 asset classes.

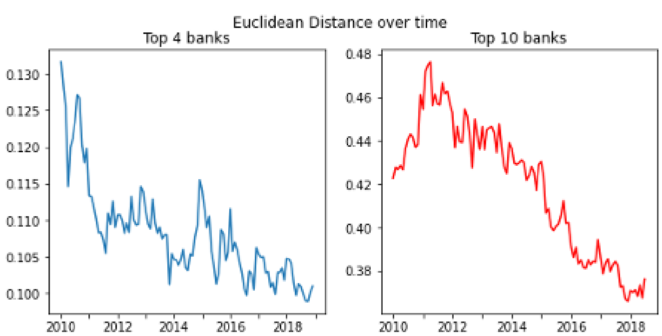


Fig. 2. Average euclidean distance, 2010 to 2020.

The figures show that South African banks have become more similar in terms of asset composition over the past 10 years, potentially aggravating the systemic risk arising from overlapping portfolios in the sector [6].

Literature

Greenwood et al (2015) [3] were one of the first authors to calibrate an indirect price-mediated contagion model to empirical data. Their framework uses a constant holding structure and fixed leverage ratio to study the effect of a debt haircut for European sovereign bonds on capital losses in the European banking system. Duarte et al 2013 [7] apply Greenwood et al.'s model to a panel data analysis of US broker-dealer banks to investigate the effect of price declines in assets financed by repurchase-agreements. They find that a one percent decline in the price of all assets financed with repos leads to losses owing to fire-sale spillovers accumulating to eight percent of total equity. Greenwood et al (2015)'s framework is also the basis of Cont and Schaanning 2017's [8] recent stress-test analysis of the European banking sector. They extend the original framework by introducing asymmetric liquidation behaviour and a concave price impact function which depends on assets' market depth and selling volumes. They perform a stress-test on the European banking sector and show that the quantification of systemic losses based on those kind of indirect fire-sale contagion effects yields substantially different results than traditional stress-test methods.

II. MODEL

To quantify price spillover effects from different shock scenarios, the paper employs a computational stress-test simulation model. Computational models are useful to conduct policy-relevant research because they can be studied by incorporating more realistic assumption and behaviour. Adding layers of complexity to mathematical models comes with the caveat that these models are very difficult to solve analytically and hence, need to be studied by simulation.

The framework of the model is similar to Greenwood et al (2015) [3] but is extended by incorporating a cash liquidity buffer and allowing for changing portfolio weights. The purpose of the model is to describe sequential rounds of price spillovers and bank de-leveraging following an initial external shock. It's important to define banks' balance sheets and portfolio weights m_k for each asset class before proceeding with the shock implementation.

Assume a set of n banks $B = \{1, ..., n\}$ and k asset classes $K = \{1, ..., k\}$, with $K = \{C, LB, TB\}$. We define a subset of asset classes Cash $C = \{k^c\}$, trading book assets $TB = \{1, ..., k^{tb}\}$ and loan book assets $LB = \{1, ..., k^{lb}\}$. Each individual bank b_i has total assets a_i with portfolio weight w_k on asset k such that $\sum_k w_k = 1$. On the liability side, bank i has debt d_i and equity capital e_i, resulting in leverage $l_i = d_i/e_i$.

Balance sheet

Asset	Liabilities
Cash w_{k^c} a_i	Equity e_i
Loan Book $w_{k^{lb}}$ a_i	Debt d_i
Trading Book $w_{k^{tb}}$ a_i	

Algorithm and parameters

In addition to the definition of banks' balance sheets, it's important to formulate assumptions that guide the simulation. A full description can be found in annex A. In short, when banks are exposed to an initial shock, they move away from their target leverage position. They respond by scaling down their asset side by either depleting their liquidity reserves or liquidating assets. If this happens on a large scale, cumulative banks' sales lead to a price effect which in turn induces a second round (and third and fourth order etc) price shock. It's those second-degree price spillover that are at the heart of the fire-sale externality channel. The price effect depends on the illiquidity parameter ρ_k, which determines the magnitude of feedback effects. In the simulation, I choose ρ_k, in the same neighborhood as in

Greenwood et al. (2015)[1]. Furthermore, there are two important vulnerability indicators that are derived from the framework: 1) *Aggregate Vulnerability*, i.e. the percentage of aggregate banking system equity that are wiped out by **only** spillover effects, and 2) *systemicness*, i.e. each bank's contribution to this Aggregate Vulnerability [3].

III. Data and simulation study

I use balance sheet data for the largest ten banks from the BA900 forms of the South African Reserve Bank and simulate general shock scenarios. The aim is to quantify systemic losses arising from the fire-sale contagion channel, as well as individual banks' contribution to overall fragility of the financial system conditional on the shock. Banks' portfolios consist of 27 asset classes which are aggregated from the BA 900 forms. A key characteristic of the banking sector is its high concentration of assets among few retail banks, i.e. the four largest banks account for approx. 80% of total assets in the sector. Figure depicts the relationship between banks' size and leverage ratios in 2015 and 2020.

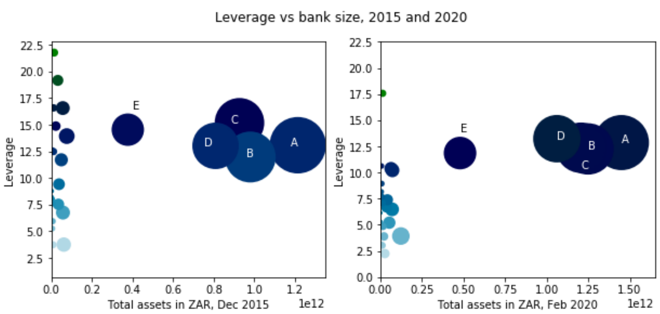


Fig. 3. Leverage and total asset of top10 banks in December 2015 and February 2020. Bubble size represents market share in terms of assets.

Stress-test scenarios

This section describes the stress-test scenarios conducted to identify determinants of banking sector fragility to price-mediated contagion. The shock scenarios are hypothetical and chosen to be artificially large to maximise stress-testing exposure.

Scenario 1 - loan portfolio of individual bank

The largest bank in the banking system is *Bank A* with approx. R1.45 tn total assets for February 2020. As the unsecured lending category is the part of the loan book that is most exposed to defaults, we study knock-on effects from a hypothetical devaluation shock of -10%, -20% and -50%. The number of periods post-shock is chosen large enough so the system reaches a steady state.

The lower panel in Figure 4 shows the evolution of total assets in the banking sector. In all three cases, systemic losses peak in the first iteration post-shock and level off in subsequent periods. These losses can be attributed to Bank A's direct exposure to defaults in the unsecured lending segment, as well as to de-leveraging effects on the part of other banks. The

[1]Greenwood et al. (2015) use an illiquidity parameter of 10^{-13}, which means that a selling volume of USD 10 bn leads to a price drop of 0,1%, or 10 basis points. The estimate is empirically found in studies from the European bond market (see e.g. Duffie 2010, Newman 2003)

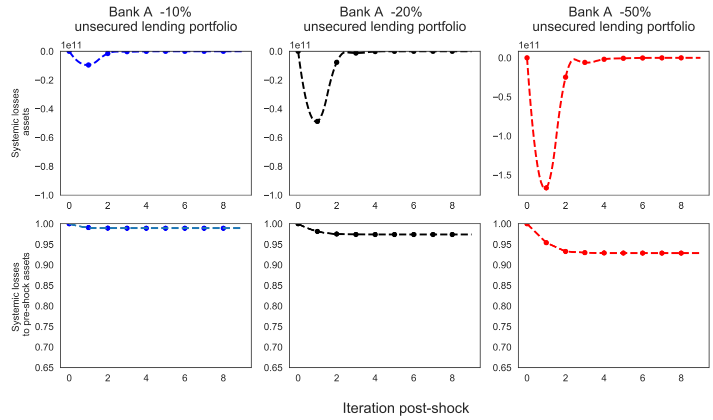


Fig. 4. Systemic asset losses over multiple rounds following a shock on Bank A's household unsecured lending portfolio. Left: Initial de-valuation shock is 10%. Middle: 20% de-valuation Right: 50% de-valuation. Lower charts show the effect of on total assets in the banking system (1 is 100% pre-shock assets). Source: Author's simulation based on SARB BA 900 forms' balance sheet data for February 2020.

hypothetical cumulative effect on total assets as a share of pre-shock assets in the banking system is shown in the lower charts. A 10% shock reduces pre-shock banking system assets by 1%, a 20% shock by 3% and a 50% shock by 7%. Thus, from the perspective of a fire-sale contagion channel, defaulting unsecured loans on the part of Bank A have an insignificant effect on the stability in the South African banking sector overall. This finding can be explained by inspecting banks' liquidity reserves and individual selling behaviour.

The heat map in Figure 5 shows the occurrence of fire-sales by bank. The rows show which bank engages in asset sales at which post-shock period. The darkest color displays asset sales in the order of R1 bn and shades reach a lighter color every R200 m. For a hypothetical de-valuation shock of -10% (left chart) on Bank A's unsecured lending portfolio, only Bank A is forced to de-leverage by selling assets. All other banks display no occurrence of de-leveraging because the second-round price effect is small enough to be absorbed by banks' liquidity buffers. In a hypothetical larger shock scenario of -50% devaluation on Bank A's unsecured lending portfolio, fire-sale externalities become more pronounced and cause asset-sales across a number of financial institutions, i.e. Bank B, Bank E and Bank G who sell assets on the market because they no longer can use liquidity reserves to pay back debt contracts necessary to de-leverage their balance sheet size. One should note that large retail banks Bank C and Bank D are unaffected even in the large shock scenario. This is the primary reason why systemic risk is muted overall.

Scenarios 2 - Marketable asset of all banks

In an alternative scenario, I shock the price of a marketable asset held by all banks, i.e. South African Government bonds held in the trading book. I conduct the hypothetical scenario that the price of a basket of Government bonds drops by -10% and -30% respectively. One should note that the -30% price shock is extremely unlikely and only chosen to maximise the stress-test envelope (the largest price drop for the 10-year SA Government bond in the last 20 years was -23% on 28 January 2004, see Figure 6). Most banks are exposed to the initial shock as supposed to only Bank A in the previous scenario. To shed more light on this, the heat map in Figure 7 displays fire-sales for each bank for the 10% and 30% shock on SA

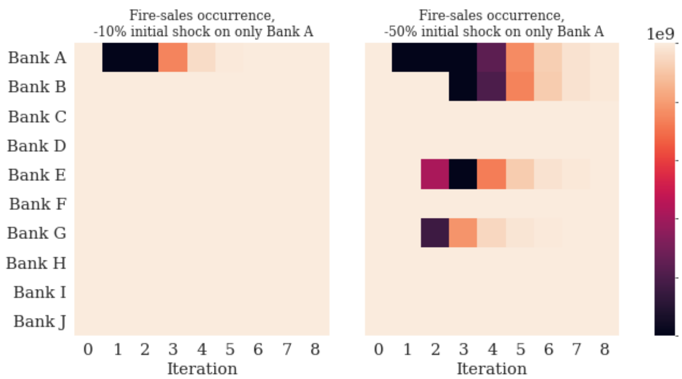


Fig. 5. Asset sales per bank over multiple rounds. Initial impact is a 10% (left chart) and a 50% (right chart) de-valuation shock on Bank A. Color shades range from 0 to R1 bn. Source: Author's simulation based on SARB BA 900 forms' balance sheet data for February 2020.

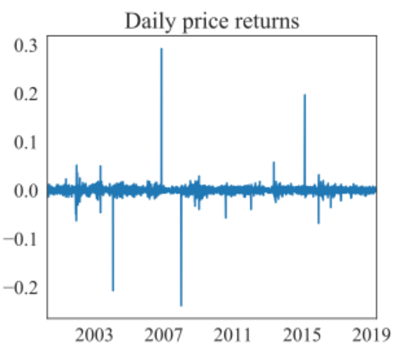


Fig. 6. R186 10-year RSA Government bond daily price returns

government bonds. Feedback price effects are caused mainly by Bank A to E. Bank H does not experience any stress in the small shock scenario, but contributes to systemic losses given a -30% shock. Interestingly, Bank F and Bank I do not liquidate any of their assets even in the large shock scenario, which can be attributed to two reasons. Firstly, they have very little asset holdings in SA Government bonds overall and, thus, no direct exposure to the initial shock. Second, the feedback price effects that occur in subsequent iterations can be absorbed by their liquidity buffers.

Contribution to spillovers by bank

The question arises as to which bank is most *systemic* and which bank is most *vulnerable*. From [3]'s framework, two stress indicators can be computed for each bank in the shock scenario: a) **systemicness**, i.e. a bank's contribution to banking sector spillover losses and b) **indirect vulnerability**, i.e. the share of the bank's equity lost 'indirectly' through other banks' de-leveraging. Bank size enters the systemicness indicator, but not the indirect vulnerability indicator, which is driven by leverage and shock exposure to the bank's assets. For example, a smaller bank can be vulnerable but not systemic. Table I shows each bank's contribution to total banking sector **spillover losses** for December 2015 and February 2020. The relevance of the top 4 banks to systemic risk is fairly stable over time (Figure 8), with Bank C overtaking Bank D in 2010. While Bank A is still the most systemic, contributing approx. 26% to total banking sector equity losses arising from price spillovers, the top 4 banks are moving closer together in terms of their role in

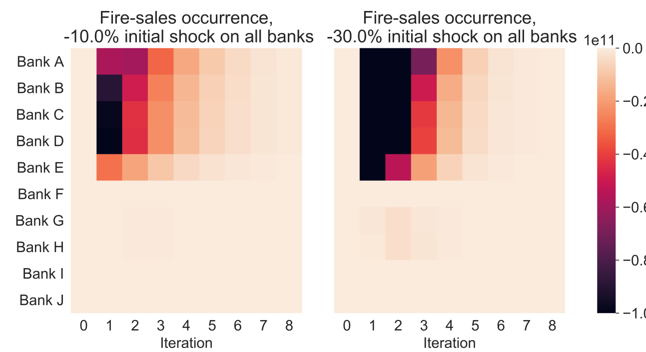


Fig. 7. Asset sales per bank post-shock. All banks holding SA Government bonds in their investment book are affected by a 10% (left chart) and 30% (right chart) price shock. Color shades range from 0 to R100 bn. Source: Author's simulation based on SARBBA 900 forms' balance sheet data for February 2020.

	Systemicness		
	Dec-15	Rank	Feb-20
Bank A	28%	1	26%
Bank B	23%	2	24%
Bank C	22%	3	20%
Bank D	18%	4	19%
Bank E	7%	5	9%
Rest	2%	6	2%

TABLE I

BANKS' 'SYSTEMICNESS' IS THEIR CONTRIBUTION TO AGGREGATE BANKING SECTOR VULNERABILITIES IN THE 30% SHOCK SCENARIO ON SA GOVERNMENT BONDS FOR DECEMBER 2015 AND FEBRUARY 2020. BANK A IS STILL THE MOST SYSTEMIC, CONTRIBUTING APPROX. 26% TO TOTAL BANKING SECTOR EQUITY LOSSES ARISING FROM PRICE SPILLOVERS.

facilitating price-mediated contagion. Interestingly, while Bank A is the largest and most systemic bank in the stress-test for February 2020, Bank B is the most vulnerable to the given shock scenarios (Table III).

	Vulnerability to spillover (IV)			
	Dec-15	Rank	Feb-20	Rank
Bank B	-9.2	1	-10.2	1
Bank A	-7.7	2	-9.7	2
Bank D	-7.5	3	-10.2	3
Bank E	-7.4	4	-9.7	4
Bank C	-6.5	5	-8.2	5
Rest	-3.2	5	-3.1	6

TABLE II

BANKS INDIRECT VULNERABILITY, I.E. THEIR SHARE OF EQUITY LOST DUE TO PRICE-MEDIATED CONTAGION DURING THE TWO STRESS-TESTS (30% SHOCK ON SA GOVERNMENT BONDS) FOR DECEMBER 2015 AND FEBRUARY 2020.

Sensitivity analysis

When modelling feedback price effects, it's also important to inspect the sensitivity of results to parameter variation.

1) Leverage

Leverage plays a crucial role in determining losses from price-mediated contagion. To investigate this further, a wide range of simulations is carried out where the shock size and leverage parameters are gradually increased. The results can be seen in Figure 12 in the appendix which shows how total banking sector assets evolve (y-axis) following an initial price shock to SA government bonds (line graphs) and varying degrees of banking sector leverage from 1 to 2.5 times the current levels (x-axis). One should note that the price shocks are artificially

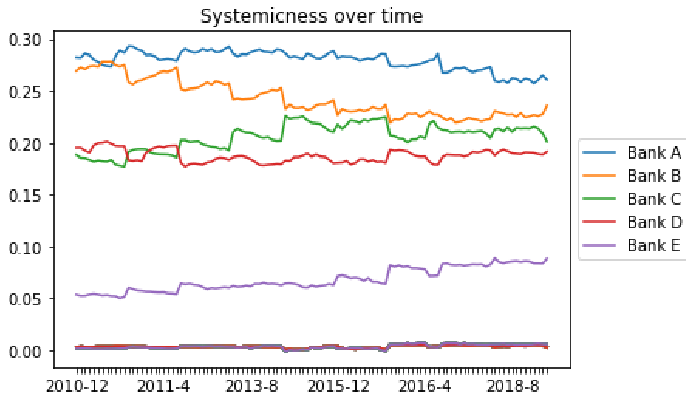


Fig. 8. Banks systemic relevance, i.e. their contribution to spillover losses in stress-test scenario 2, over time.

high for demonstration purposes. It becomes apparent that the spillover risk to banking sector asset losses is not linear, but increases exponentially with higher leverage ratios. Considering price shocks from 10% to 30%, a system with 1.5 times of current leverage levels is very exposed.

2) Illiquidity

The fire-sale externalities measured in the stress-tests highly depend on the illiquidity parameter ρ_k used to determine feedback price declines as a function of selling volumes. Figure 9 shows the cumulative effect on total equity in the banking sector given a 10%, 30%, 50%, 70% and 90% shock on SA government bond prices and conditional on the illiquidity parameter. For example, a 50% shock (blue line) at 4×10^{-14} leads to cumulative equity losses of 41%. However, the same shock leads to 100% banking system equity losses for a parameter exceeding 3×10^{-13}.
As the chart shows, there is a critical value for the illiquidity parameter at which the slope for cumulative losses increases sharply across all shock scenarios, i.e. 1×10^{-13}. If the price effect for bond price shocks exceeds this threshold value, the South African banking system becomes highly unstable to this type of shock. This can be further examined in the lower panel of Figure 9, where shock size, illiquiidty parameter and system equity losses are displayed in a 3D chart.

How does portfolio similarity affect systemic risk to price-mediated contagion?

Finally I address the initial question of whether higher portfolio similarity leads to increased risk to price spillover as predicted by the literature [4]–[6]. A useful metric to determine overall fragility to spillover losses is *Aggregate Vulnerability (AV)* [3], i.e. the share of banking sector equity that is wiped out by second round feedback effects. Figure 10 shows that the AV for a 30% shock scenario on SA Government bonds is very low and ranges between 4% and 8% of banking system equity between 2010 and 2020. However, while still at subdued levels, the aggregate vulnerability doubled over this time period. What drove this development? One can rule out higher leverage ratios as factor because the average leverage ratio of the top 10 banks decreased over the time period (see Figure13 in the appendix). One may suspect overlapping and interconnnected portfolios as driving forces behind this trend. The scatter plot in Figure 11 shows the strong negative correlation between portfolio distance as measured by the average euclidean distance between

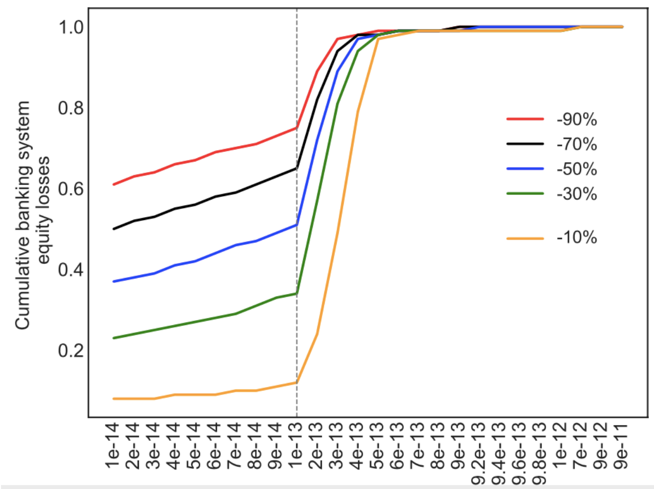
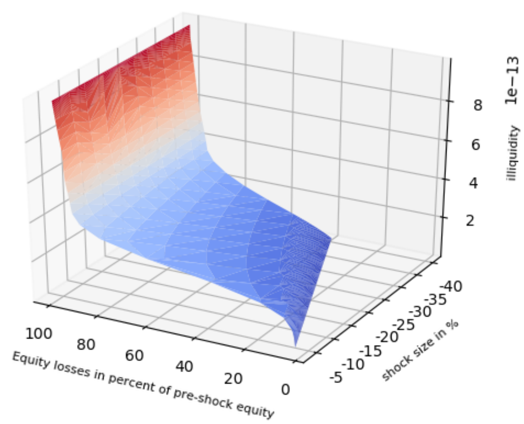


Fig. 9. *Upper*: 2D chart of cumulative equity losses in the banking system (y-axis), illiquidity parameter (x-axis) and shocks to SA Government bonds (line graphs). The critical threshold value for banking sector fragility is 10^{-13}, i.e. a selling volume of ZAR 10 bn leads to a price drop of 0,1%, or 10 basis points. Lower: 3D chart.



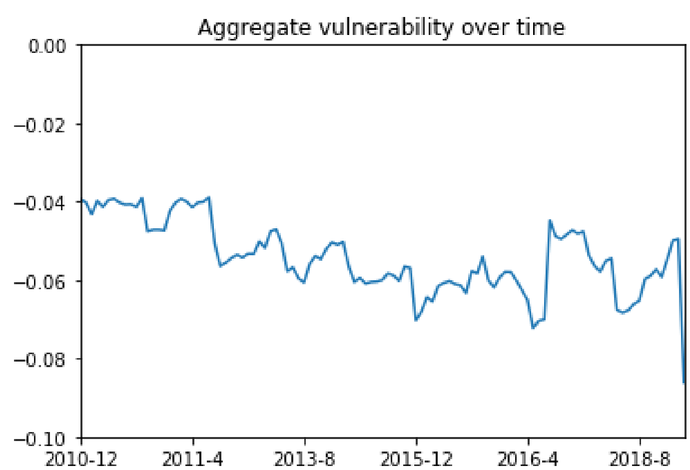


Fig. 10. Aggregate banking sector vulnerability from 2010 to 2020. Y-axis has the share of banking sector equity wiped out by spillover losses, e.g. 4% in December 2012.

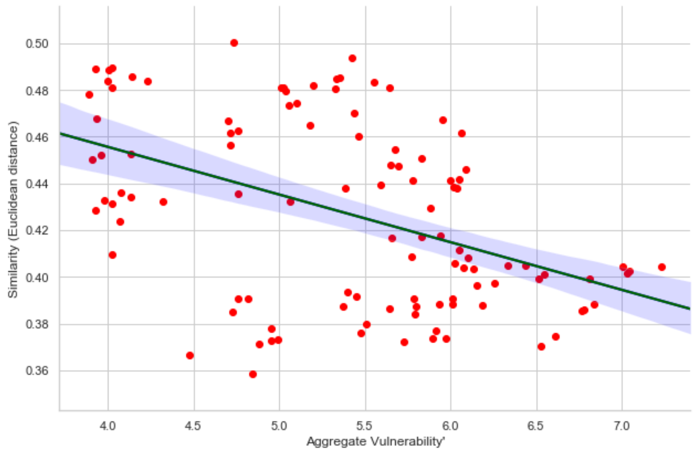


Fig. 11. Scatter plot of average portfolio similarity of top 10 banks as measured by the Euclidean distance and aggregate banking sector vulnerability to spillover losses.

the top 10 banks and aggregate sector vulnerability. Note that banks are the more similar, the lower the distance between their portfolios. Hence, we have this inverse correlation between distance and vulnerability. To quantify this relationship further, I perform a pooled OLS regression of the log of Aggregate Vulnerability on the log of portfolio similarity (euclidean distance). Table III shows a highly statistically significant β coefficient of -0.8, i.e. a 1% decrease in the average euclidean distance leads to an increase in aggregate sector vulnerability of 0.8%. Hence, the hypothesis that higher similarity of portfolios in the South African banking sectors leads to higher exposure to price-mediated contagion could be confirmed. No suspicious patterns in residuals were detected in the post-regression analysis (see Figure15 and Figure14 in appendix B)

VARIABLES	(1) log Aggregate Vulnerability
log similarity	-0.806***
	(0.149)
Constant	-3.622***
	(0.128)
Observations	108
R-squared	0.196

Robust standard errors in parentheses
*** p<0.01

TABLE III
OLS ESTIMATION. REGRESSING THE LOG OF AGGREGATE BANKING SECTOR VULNERABILITY ON THE LOG OF PORTFOLIO SIMILARITY.

IV. Conclusion

This paper presents stress-tests to the South African banking sector across two scenarios, a shock to the largest bank's loan portfolio and a shock to a marketable asset held by all banks, i.e. SA government bonds. Overall, the simulations demonstrate that second-order feedback effects from banks' deleveraging are muted. In the first scenario, asset sales are not large enough to trigger de-stabilising liquidation cascades. The main reason for this is that knock-on price effects can be absorbed by liquidity buffers of most other banks. One could argue that the characteristic of the South African banking system to be highly concentrated amongst Bank A - Bank D has a positive absorptive effect on financial system stability in terms of the fire-sale contagion channel. In the second stress-test simulation, most banks are involved in de-leveraging from the initial impact, but Bank A contributes most to the spillover losses due to its connectedness in the system and the magnitude of its down-sizing. Given a 30% shock to SA Government bonds held in banks' trading book, second round equity losses amount to approx. 8% of pre-shock levels. This exposure is twice as large in 2020 as it was in 2010. Furthermore, the stress-tests confirm that banks' contribution to price spillover from contagion through common asset holdings is higher, the higher their leverage, their total assets (size), their connectedness (i.e. they own illiquid and large assets that are also held by other banks) and the larger the initial shock they are exposed. However, amplification can be substantially reduced by enlarging liquidity buffers.

Policy Implications

To mitigate the risk of price-mediated contagion in the banking system, the findings point to two crisis intervention instruments. First, the provision of emergency liquidity is crucial during a crisis to reduce the likelihood of banks' asset liquidation. The stess-test demonstrates the importance of liquidity buffers to dampen banks' de-leveraging spirals through fire-sales. Second, the results suggest that regulators put maximum leverage requirements on hold during times of stress. Maximum leverage is a regulatory instrument that prevents high risk-taking behaviour ex-ante. In times of stress, however, this regulation has the potential to aggravate the situation by incentivising de-leveraging through asset liquidation. To lessen these amplificatio effects, banks should be allowed to have larger than normal leverage ratios temporarily until systemic risk subsides.

APPENDIX A
PRICE-MEDIATED CONTAGION MODEL - EXTENSION OF GREENDWOOD ET AL'S 2015 FRAMEWORK

This sections describes the spillover model in detail.

Algorithm: Assume an initial exogenous shock hits the banking system, triggering the following process:[2]

1) *Direct exposure*: In time t, every bank holding the shocked assets incurs direct losses which can be quantified by

$$a_{i,t}\sum_{k} w_{i,k,t} f_{k,t} \text{ for bank } b_i \tag{1}$$

where $f_{k,t} \in$ [-1, 0] is the devaluation shock on asset k The bank can be hit with shocks on multiple asset classes, which is why the product of the portfolio weight and the shock value per asset class is summed up before multiplying by total assets $a_{i,t}$. This impact on bank's assets reduces equity on the liability side, which leads to an increase in the bank's leverage ratio. An important assumption of the model is *leverage targeting*, i.e. banks maintain a constant leverage ratio over time. This assumption is backed by [9], who provide some empirical evidence that large financial institutions maintain fairly stable levels of leverage in the medium term. The change in the binding leverage ratio[3] will prompt banks to become active in the market.

2) *Liquidity buffer*: [3] assume that banks immediately pay off debt to return to their initial leverage ratio l_i in response to the direct losses. A convenient modelling feature that follows from their assumption is that portfolio weights of the k assets are held constant, i.e. banks sell assets in the manner that keeps their portfolio composition the same throughout the de-leveraging phase. However, it is more realistic to assume that banks first use their liquidity buffer to pay off their debt before liquidating assets. Thus, portfolio weights are allowed to fluctuate in our model. The critical value determining the shortfall $\Gamma_{i,t}$ that bank i needs to cover is given by

$$\Gamma_{i,t}: \quad d_{i,t} - \big(\, l_i \ \max\{e_{i,t} - a_{i,t}\sum_{k} w_{i,k,t} f_{k,t}; 0\}\big)^4 \tag{2}$$

$$\text{with } \Gamma_{i,t} \in [0, d_{i,t}] \quad \text{and}$$
$$\Gamma_{i,t} > 0 \quad \text{if} \quad f_{k,t} < 0$$
$$\Gamma_{i,t} = 0 \quad \text{if} \quad f_{k,t} = 0$$

The intuition behind equation 2 is as follows. If the direct exposure is 0 because the shock is 0%, the shortfall bank i needs to cover is also 0. This is because in the absence of a shock on balance sheets, the composition of the liability side does not change, i.e. equity does not change and the difference between the previous period's debt and next period's debt is also 0. If the shock is negative, the shortfall will be larger than 0 with its maximum at the previous period's level of debt.[5]

3) *Fire-sales*: For an individual bank i, the algorithm checks two conditions that can occur in the face of a shock $f_{k,t}$ on its balance sheet. If the shock is too large and liquidity buffers are depleted, bank i starts selling assets immediately in proportion to its weights $w_{i,k,t}$[6]. In the second case, if the bank is able to absorb the shock, neither fire-sales nor spillover to other banks occur, but the balance sheet composition changes in response to transactions.
At the bank level, if the individual shortfall is larger than the bank's liquidity buffer, the total bank's de-leveraging amount is determined by the product of its leverage and its direct exposure:

$$\Omega_{i,k,t} = \begin{cases} \underbrace{\tilde{w}_{i,k,t}}_{\text{weight for asset k}} \ \underbrace{l_i}_{\text{leverage}} \ \underbrace{a_{i,t}\sum_{k} w_{i,k,t} f_{k,t}}_{\text{direct exposure}} & \text{if } \Gamma_{i,t} > \underbrace{a_{i,t} w^c_{i,k,t}}_{\text{liquidity buffer}} \\ 0 & \text{else} \end{cases} \tag{3}$$

with $\tilde{w}_{i,k,\cdot}$ being the adjusted portfolio weight for asset k after cash operations are being taken into account.
We sum up the bank-level selling volumes for asset k across all banks to get to the system-wide fire-sales for asset k:

$$\textit{Asset sales}_{\ k,t} = \sum_{i}^{n} \Omega_{i,k,t} \tag{4}$$

Note that the first term $\tilde{w}_{i,k,\cdot}$ in 3 and 4 contains the intermediate adjusted weights that follow from cash operations. We define their derivation in equation 9, however first in the law of motion is the adjustment of the liability side as described below.

How are balance sheets adjusted?
Whenever liquidity buffers are used, weights are adjusted proportionately according to the new total assets of bank i, which in turn depend on how equity and debt are affected by the direct exposure and the pay-off of debt obligations. Equity and debt in $t+1$ are defined by:

$$e_{i,t+1} = max\{e_{i,t} - a_{i,t}\sum_{k} w_{i,k,t} f_{k,t}; 0\} \tag{5}$$

$$d_{i,t+1} = max\{l_i \ \ e_{i,t+1} \ \ ; 0\} \tag{6}$$

The sum of adjusted equity and updated debt gives total assets of bank i in $t+1$ as

$$a_{i,t+1} = \max\{d_{i,t+1} + e_{i,t+1}, 0\} \tag{7}$$

On the asset side, cash is reduced by how much of the shortfall $\Gamma_{i,t}$ can be covered. In $t+1$, its value is determined by debt pay-offs transactions. The maximum amount that is payable is $\Gamma_{i,t}$, hence new cash positions in $t+1$ amount to:

$$c_{i,t+1} = \begin{cases} 0 & \text{if} \quad \Gamma_{i,t} \geq \underbrace{a_{i,t} w^c_{i,k,t}}_{\text{cash liquidity buffer}} \\ c_{i,t} - \Gamma_{i,t} & \text{else} \end{cases}$$

$$\text{with } c_t = a_{i,t} w^c_{i,k,t}$$

In the case that the cash buffer is not sufficient to de-leverage, $c_{i,t+1}$ is 0. Alternatively, the new cash position is the difference between the previous period's

[2]The description of the framework is similar to [7], pp. 5-9

[3]This constraint is not given by regulators in our simulation. For sake of simplicity we assume that banks become active as soon as they move away from initial leverage conditions. An interesting extension of the model could investigate spillover in the case of additional regulatory leverage restrictions.

[4]It is theoretically possible that equity is wiped out entirely by a very large shock; thus the max operator limits losses to 0, i.e. there is no negative equity

[5]One should note here that $f_{k,t} \in$ [-1, 0].

[6]As in [3], it is assumed that these selling volumes are accommodated in the market at the initial step at no price discount

amount and Γ_i, t.

The next step is the intermediate update of portfolio weights $\sum_k w_{i,k} = 1$. As in [3], we assume that asset weights determine how much of each asset is sold in the de-leveraging process. This assumption is a drastic simplification as selling behaviour is more complex in real markets. However, it is a necessary building block which helps to gauge the extent of overlapping portfolios in the sector, while still being reasonably simple to allow for data calibration. While in [3], weights are constant, we allow for fluctuations due to cash transactions. The update process takes place between t and $t+1$, which is why 'intermediate' adjusted weights are denoted with $\tilde{w}_{i,k,\cdot}$. Starting with cash, the intermediate portfolio weight is given by the ratio of the target positions:

$$\tilde{w}^c_{i,k,\cdot} = \frac{c_{i,t+1}}{a_{i,t+1}} \tag{8}$$

Since $\tilde{w}^c_{i,k,\cdot}$ is smaller than $w^c_{i,k,t}$ $\forall f_{k,t} < 0$, the difference needs to be accounted for so that $\sum_k w_k = 1$. For sake of simplicity, we distribute the difference proportional to the existing weights. Consider the correction factor $\tau = \frac{w^c_{i,t} - \tilde{w}^c_{i,k,\cdot}}{k-1}$, so that the remaining intermediate weights are given by

$$w_{i,k,\cdot} = w_{i,k\neq c,t} + \tau \qquad \forall f_{k,t} < 0 \tag{9}$$

To re-iterate the law of motion, the intermediate weights are used in the determination of fire-sale volumes in the de-leveraging process described in equations. Once transactions materialised overnight, the intermediate weights become the new weights for the period $t+1$.

System-wide de-leveraging

We now turn to the spillover effects that arise from system-wide de-leveraging. Recall from equation 4 that the amount of asset k that is sold across all banks is given by

$$\Omega_{k,t} = \sum_i \tilde{w}_{i,k,\cdot} \quad l_i \quad a_{i,t} \sum_k w_{i,k,t} f_{k,t}$$

The direct exposure of bank i is multiplied by its leverage to determine the shortfall bank i needs to cover by asset sales in case liquidity buffers are depleted. This shortfall is multiplied by asset k's portfolio weight $w_{i,k,t}$ to determine the proportional amount that bank i sells of asset k. The sales are summed up over all banks, leading to a total amount $\Omega_{k,t}$, i.e. the system-wide fire-sales of asset k following the initial shock $f_{k,t}$. The equity of bank i is reduced by direct exposure $a_{i,t} \sum_k w_{i,k,t} f_k$, while debt is paid off according to $l_i(a_{i,t} \sum_k w_{i,k,t} f_{k,t})$.

4) *Price impact*: The cumulative sales lead to a price effect $\upsilon(\rho_k, \Omega_{k,t})$ which depends on the liquidity parameter ρ_k and the selling volumes $\Omega_{k,t}$. The assumption is that an exogenous buyer steps in to accommodate the selling volumes at the fire-sold price.

5) *Spillover losses*: The price effect leads to further losses on banks' balance sheets. These are the *indirect* spillover losses arising from common asset holdings. Our analysis is particularly concerned with these kind of spillover losses as they represent the amplification mechanism in the centre of the fire-sale contagion channel. It is possible to describe total spillover losses for asset k by

$$SP_{k,t} = \sum_i (a_{i,t} \sum_k \tilde{w}_{i,k,\cdot}) \underbrace{\left[\rho_k \Omega_{k,t}\right]}_{f*} \tag{10}$$

where the expression inside the square brackets can be interpreted as second round shock f^*_k on asset k. The routine from *3.* is repeated to determine the system-wide losses $SP_{k,t}$ for asset k which result only from the second round fire-sale price-shock f^*_k. Summing up second-round sales across all asset classes gives us the system-wide spillover losses

$$\lambda_t = \sum_k SP_{k,t} \tag{11}$$

In the next step, we capture the fragility of the banking system to fire-sale spillovers by putting λ_t in relation to pre-shock banking sector equity $E_= \sum_i e_i$:

$$AV_t = \frac{\lambda_t}{E_{t-1}} \tag{12}$$

Greenwood et al 2015 [3] call this the *Aggregate Vulnerability* of the banking system to the preceding shock. It is further possible to break down AV into every bank's contribution to the overall losses in the banking system attributable to *indirect spillover losses*, i.e. $AV_t = \sum_i S_{i,t}$. To conclude, the *systemicness* of a bank depends on 4 factors and is higher, the more connected the bank is (connectedness is high when the bank owns large illiquid amounts of assets which are also held by other banks), the bigger the bank, the more leveraged the bank (l_i) and the larger the shock the bank faces.

Appendix B
Figures

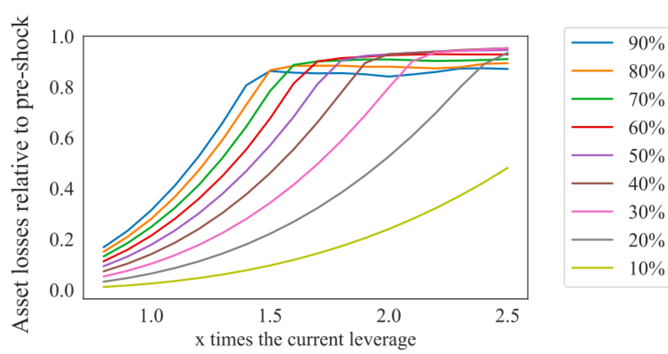


Fig. 12. The effect of price shocks to SA Government bonds (line graphs) to banking sector assets as a share of pre-shock levels when banks' leverage ratios are increased by factor 1 to 2.5.

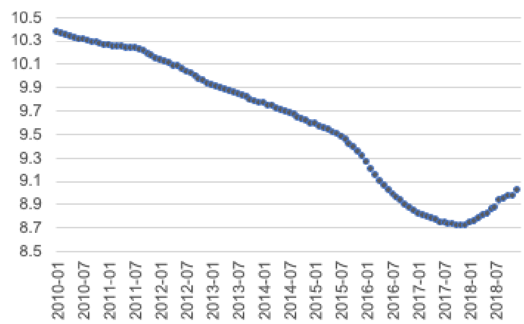


Fig. 13. Average leverage ratio for top 10 banks from January 2010 to February 2020

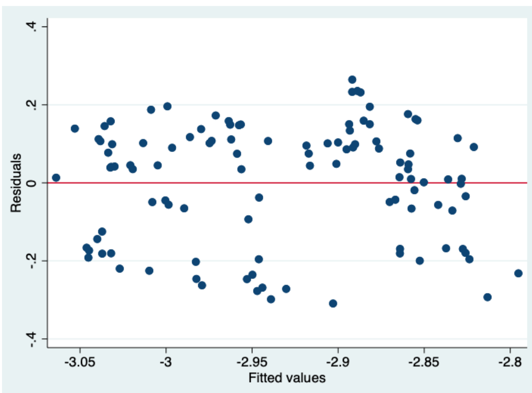


Fig. 14. Residual vs fitted plot of pooled OLS regression. Residuals do not show any meaningful patterns.

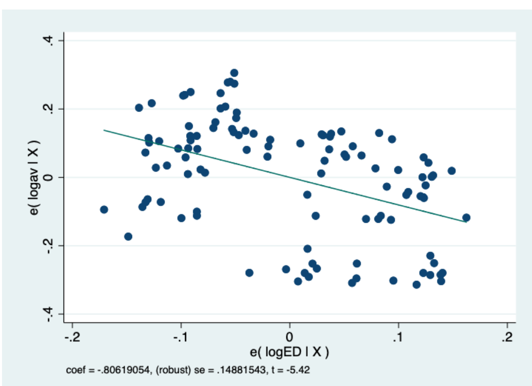


Fig. 15. Added variable plot of pooled OLS regression.

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