Seminar in Logic

Non-Monotonic Reasoning

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Introduction

Monotonic Reasoning

Monotonicity (Intuition)

Additional information can not invalidate previous conclusions.

Global Monotonicity

A logic is called globally monotonic if for A and B being theories, such that $A \subseteq B$ it follows that $Th(A) \subseteq Th(B)$, with $Th(S) := \{p \mid S \vDash p\}$ (or syntactically $Th(S) = \{p \mid S \vdash p\}$)

For example: $\Gamma \vDash \varphi$ implies $\Gamma \cup \{\psi\} \vDash \varphi$ or *weak-l* in Sequent Calculus.

Local Monotonicity

A logic is called locally monotonic, if it allows for "Strengthening the Antecedent".

For example: In propositional logic it is a tautology $(A \to C) \to (A \wedge B \to C)$.

Bochman 2005; McDermott and Doyle 1980; McCarthy 1981s

Motivating Problems: Tweety Problem

Stage 1: Naive Model

$$\mathcal{T}_1 := \{\forall x\ \mathit{Person}(x) \rightarrow \mathit{Innocent}(x), \mathit{Person}(\mathit{Tweety})\}$$

Stage 2: Expansion 1

$$\begin{aligned}\mathcal{T}_2 := \mathcal{T}_1 \cup \{&\mathit{Murderer}(\mathit{Tweety}),\\ &\forall x\ \mathit{Murderer}(x) \rightarrow \mathit{Person}(x),\\ &\forall x\ \mathit{Murderer}(x) \rightarrow \neg \mathit{Innocent}(x)\}\end{aligned}$$

Stage 3: Naive Solution

$$\begin{aligned}\mathcal{T}_3 := \mathcal{T}_2 &\setminus \{\forall x\ \mathit{Person}(x) \rightarrow \mathit{Innocent}(x)\}\\ &\cup \{\forall x\ \mathit{Person}(x) \wedge \neg \mathit{Murderer}(x) \rightarrow \mathit{Innocent}(x)\}\end{aligned}$$

Motivating Problems: Tweety Problem 2

Stage 4: Expansion 2

$$\mathcal{T}_4 := \mathcal{T}_3 \cup \{\mathit{Person}(\mathit{Polly})\}$$

But now

$$\mathcal{T}_4 \not\models \mathit{Innocent}(\mathit{Polly})$$
$$\mathcal{T}_4 \cup \{\neg \mathit{Murderer}(\mathit{Polly})\} \models \mathit{Innocent}(\mathit{Polly})$$

Hence,

Innocent when proven to be not a murderer?

Problems:

- listing all exceptions (not yet classified crimes)
- expansion may lead to inconsistency
- checking all exceptions (absence of complete information)

Reiter 1980; McCarthy 1981

Motivating Problems: Frame Problem

Temporal Projection Problem

- Frame / Persistence Problem → "What does not change?"
- Ramification Problem → "What changes implicitly?"
- Qualification Problem → "When is an action possible?"

Ginsberg and Smith 1987; Bochman 2007; Strasser and Antonelli 2018; Shanahan 2016; Ginsberg and Smith 1987

Why Non-Monotonic Reasoning?

What is desired?

Capture statements such as:

- Normally X holds
- Typically X is the case
- Assume X as a default

In order to

- modelling normality and abnormality
- modelling a reasoned use of assumptions
- distinguish between information
- allow for safe reasoning in a dynamic environment

Strasser and Antonelli 2018; Bochman 2007

Approaches to Non-Monotonic Reasoning

General Approaches to Non-Monotonic Reasoning

Preferential Non-Monotonic Reasoning

Type: locally non-monotonic (possibly globally monotonic)

Core Concept: Cumulative / Preferential Models

Formalisms: Circumscription, Closed World Assumption, Conditional Logic

(Strongly connected to Meta-Theoretic Approach)

Explanatory Non-Monotonic Reasoning

Type: globally non-monotonic (possibly locally monotonic)

Core Concept: Explanatory Closure / Stable Extensions

Formalisms: Default Logic, Modal Non-Monotonic Logic

Other methods of characterisation

logic formulas vs. inference rules

credulous vs. sceptical inference

cumulative vs. not-cumulative

Bochman 2005; Bochman 2007; Brewka, Dix, and Konolige 1997

Preferential Non-Monotonic Reasoning: Core Concept

Cumulative Models

$\langle S, I, \prec \rangle$ where

- S states
- $I : S \to \wp(\mathcal{U})$ with $\mathcal{U}$ set of all interpretations
- $\prec$ is a strict partial order on S satisfying the smoothness condition.

Preferential Models

$\langle S, I, \prec \rangle$ cumulative where $|I(s)| = 1$

Model Preference Logics

Let $W := \langle S, I, \prec \rangle$ be a preferential model, such that

- S is a subset of all interpretations, i.e. $S \subseteq \mathcal{U}$
- I is the identity
- $\prec$ is well-founded

Kraus, Lehmann, and Magidor 1990; Brewka, Dix, and Konolige 1997

Explanatory Non-Monotonic Reasoning: Core Concept

Explanation Closure (Intuition)

Any fact holding in a model should be explained by the rules of the domain.

Stable Extensions (Intuition)

A *Stable Extension* is explanatory closed set of formulas representing one possible set of consistent beliefs.

Commonly:

- multiple stable extensions exist
- defined by a fixed-point operation

  For operator T, S is a fixed-point iff $T(S) = S$

Bochman 2005; Bochman 2007; Brewka, Dix, and Konolige 1997

Specific Non-Monotonic Formalisms

Circumscription

Let S FOL-sentence containing $P(x_1, \dots, x_n)$, short $P(\overline{x})$. Let $S(\Phi)$ replaces P with Φ.

Then the schema

$$\forall \Phi \, ((S(\Phi) \wedge \forall \overline{x} \, (\Phi(\overline{x}) \rightarrow P(\overline{x}))) \rightarrow \forall \overline{x} \, (P(\overline{x}) \rightarrow \Phi(\overline{x})))$$

is called the circumscription of P.

Model Theoretic Notion

Let $\langle S, I, \prec \rangle$ be a model preference logic, s.t.

$$s_1 \prec s_2 \textit{ iff } I(s_1) \models P(\overline{x}) \textit{ implies } I(s_2) \models P(\overline{x}) \textit{ and } \\ \textit{not } I(s_2) \models P(\overline{x}) \textit{ implies } I(s_1) \models P(\overline{x})$$

with P being circumscribed.

Note: Expresses normality based on *Abnormality Theory*

McCarthy 1981; Brewka, Dix, and Konolige 1997

Circumscription: Example 1

Let $\mathcal{I}_x := (D_{\mathcal{I}_x}, I_{\mathcal{I}_x})$ s.t.

$$I_{\mathcal{I}_0}(Guilty) := \{\}$$
$$I_{\mathcal{I}_1}(Guilty) := \{\delta\}$$
$$I_{\mathcal{I}_2}(Guilty) := \{\delta, \sigma, \eta\}$$
$$I_{\mathcal{I}_3}(Guilty) := \{\delta, \sigma, \eta, \gamma\}$$

with $D_{\mathcal{I}_x} := \{\delta, \sigma, \eta, \gamma\}$ for all x. The following preference can be established

$$\mathcal{I}_0 \prec \mathcal{I}_1 \prec \mathcal{I}_2 \prec \mathcal{I}_3$$

Given

$$S = Guilty(A) \wedge Guilty(B) \wedge Guilty(C)$$

$\mathcal{I}_1$ is chosen.

Brewka, Dix, and Konolige 1997

Circumscription: Example 2

Recall

$$\forall \Phi \, (S(\Phi) \wedge \forall \overline{x} \, (\Phi(\overline{x}) \rightarrow P(\overline{x}))) \rightarrow \forall \overline{x} \, (P(\overline{x}) \rightarrow \Phi(\overline{x}))$$

Given

$$S = Guilty(A) \wedge Guilty(B) \wedge Guilty(C)$$
$$\Phi(x) = (x = A \vee x = B \vee x = C)$$

$$\Phi(A) \wedge \Phi(B) \wedge \Phi(C) \wedge \forall \overline{x} \, (\Phi(\overline{x}) \rightarrow Guilty(\overline{x}))) \rightarrow \forall \overline{x} \, (Guilty(\overline{x}) \rightarrow \Phi(\overline{x}))$$

$$(A = A \vee A = B \vee A = C) \wedge (B = A \vee B = B \vee B = C) \wedge (C = A \vee C = B \vee C = C)$$
$$\wedge \, \forall \overline{x} \, ((\overline{x} = A \vee \overline{x} = B \vee \overline{x} = C) \rightarrow Guilty(\overline{x})))$$
$$\rightarrow \forall \overline{x} \, (Guilty(\overline{x}) \rightarrow (\overline{x} = A \vee \overline{x} = B \vee \overline{x} = C))$$

McCarthy 1981

Circumscription: Example 3

$$S = Guilty(A) \land Guilty(B) \land Guilty(C) \land Guilty(D)$$
$$\Phi(x) = (x = A \lor x = B \lor x = C)$$

$$\begin{aligned}
&(A = A \lor A = B \lor A = C) \land (B = A \lor B = B \lor B = C) \\
&\land (C = A \lor C = B \lor C = C) \land (D = A \lor D = B \lor D = C) \\
&\land \forall \overline{x}\, ((\overline{x} = A \lor \overline{x} = B \lor \overline{x} = C) \rightarrow Guilty(\overline{x}))) \\
&\rightarrow \forall \overline{x}\, (Guilty(\overline{x}) \rightarrow (\overline{x} = A \lor \overline{x} = B \lor \overline{x} = C))
\end{aligned}$$

McCarthy 1981

Default Logic

Default Logic

A default δ has the form

$$\frac{\varphi : \psi_1, \ldots, \psi_n}{\chi}$$

with $\varphi, \chi, \psi_1, \ldots, \psi_n$ being closed propositional formulas for $n > 0$.

Default Theory

$\Delta = (D, W)$ is a default theory. With W a set of predicate formulas and D a set of defaults. For any $S \subseteq \mathcal{L}$, let $\Gamma(S)$ be the smallest set satisfying

D1: $W \subseteq \Gamma(S)$

D2: $Th_{\mathcal{L}}(\Gamma(S)) = \Gamma(S)$

D3: if $(\varphi : \psi_1, \ldots, \psi_n / \chi) \in D$ and $\varphi \in \Gamma(S)$ and $\neg\psi_1, \ldots, \neg\psi_n \notin S$ then $\chi \in \Gamma(S)$.

Antoniou and Wang 2007; Reiter 1980

Default Logic: Example

Given $\Delta := (W, D)$

$$\begin{aligned} W &:= \{Murderer(Tweety), Person(Polly), \\ &\qquad Murderer(x) \to \neg Innocent(x), Murderer(x) \to Person(x)\} \\ D &:= \{Person(x) : Innocent(x)/Innocent(x)\} \end{aligned}$$

Possible sets:

$E_1 := W \cup \{Innocent(Polly), Person(Tweety), \neg Innocent(Tweety)\}$

$E_2 := W \cup \{Innocent(Polly), Person(Tweety), \neg Innocent(Tweety), Innocent(Tweety)\}$

E_1 is a stable extension, E_2 is not.

Antoniou and Wang 2007

Default Logic: Example - Credulous and Sceptical Reasoing

Given $\Delta := (W, D)$

$$W := \{Murderer(Tweety), Person(Polly), Murderer(x) \rightarrow Person(x)\}$$
$$D := \{Person(x) : Innocent(x)/Innocent(x),$$
$$Murderer(x) : \neg Innocent(x)/\neg Innocent(x)\}$$

Possible sets:

$$E_1 := W \cup \{Innocent(Polly), Person(Tweety), Innocent(Tweety)\}$$
$$E_2 := W \cup \{Innocent(Polly), Person(Tweety), \neg Innocent(Tweety)\}\}$$

E_1 and E_2 are stable extensions.

Strasser and Antonelli 2018; Brewka, Dix, and Konolige 1997

Applications

Areas of legal reasoning

- reasoning with laws - axiomatic view
- reasoning about laws - interpretation of legal rules
- reasoning about facts - burden of proof
- reasoning about interactions - dynamic disputes between agents
- reasoning about legal action - legality of future actions

Argumentation Theory

central to law

abstract argumentation theory

- non-monotonic
- notion of argument, attack and defeat
- semantics for Default Logic or logic programs

Prakken and Sartor 2015; Prakken 2017; Bochman 2007

Non-Monotonic Reasoning in Law

Rules with exceptions

Exceptions and Context

- higher force
- self-defence

Rules with conflicting conclusions

dynamic hierarchies of preference

- specificity, authority and recency

Possible application: preference default logic

Prakken and Sartor 2015; Prakken 2017; Antoniou and Wang 2007

Other Applications of Non-Monotonic Reasoning

Computer Science

- Truth Maintenance Systems
- Normal Logic Programs
- Database Theory

Cognitive Sciences

- closer approximation of human reasoning
- possible bridge between symbolic and connectionist approaches

Biology

- exception hierarchies

Isaac, Szymanik, and Verbrugge 2014; Evans 2002; Bochman 2007; Antoniou and Wang 2007

References

Antoniou, Grigoris and Kewen Wang (2007). “Default Logic”. In: *The Many Valued and Nonmonotonic Turn in Logic*. Ed. by Dov M. Gabbay and John Woods. Vol. 8. Handbook of the History of Logic. North-Holland, pp. 517–555. DOI: https://doi.org/10.1016/S1874-5857(07)80011-2. URL: http://www.sciencedirect.com/science/article/pii/S1874585707800112.

Bochman, Alexander (2005). *Explanatory nonmonotonic reasoning*. Vol. 4. World scientific.

– (2007). “Nonmonotonic Reasoning”. In: *The Many Valued and Nonmonotonic Turn in Logic*. Ed. by Dov M. Gabbay and John Woods. Vol. 8. Handbook of the History of Logic. North-Holland, pp. 557–632. DOI: https://doi.org/10.1016/S1874-5857(07)80012-4. URL: http://www.sciencedirect.com/science/article/pii/S1874585707800124.

Brewka, Gerhard, Jürgen Dix, and Kurt Konolige (1997). *Nonmonotonic reasoning: an overview*. Vol. 73. CSLI publications Stanford.

Evans, Jonathan St BT (2002). “Logic and human reasoning: An assessment of the deduction paradigm.”. In: *Psychological bulletin* 128.6, p. 978.

Ginsberg, Matthew L and David E Smith (1987). “Reasoning about action I: A possible worlds approach”. In: *The frame problem in artificial intelligence*. Elsevier, pp. 233–258.

Isaac, Alistair MC, Jakub Szymanik, and Rineke Verbrugge (2014). “Logic and complexity in cognitive science”. In: *Johan van Benthem on logic and information dynamics*. Springer, pp. 787–824.

Kraus, Sarit, Daniel Lehmann, and Menachem Magidor (1990). "Nonmonotonic reasoning, preferential models and cumulative logics". In: *Artificial intelligence* 44.1-2, pp. 167–207.

McCarthy, John (1981). "Circumscription—a form of non-monotonic reasoning". In: *Readings in Artificial Intelligence*. Elsevier, pp. 466–472.

McDermott, Drew and Jon Doyle (1980). "Non-monotonic logic I". In: *Artificial intelligence* 13.1-2, pp. 41–72.

Prakken, Henry (2017). "On the problem of making autonomous vehicles conform to traffic law". In: *Artificial Intelligence and Law* 25.3, pp. 341–363. ISSN: 1572-8382. DOI: 10.1007/s10506-017-9210-0. URL: https://doi.org/10.1007/s10506-017-9210-0.

Prakken, Henry and Giovanni Sartor (2015). "Law and logic: a review from an argumentation perspective". In: *Artificial Intelligence* 227, pp. 214–245.

Reiter, Raymond (1980). "A logic for default reasoning". In: *Artificial intelligence* 13.1-2, pp. 81–132.

Shanahan, Murray (2016). "The Frame Problem". In: *The Stanford Encyclopedia of Philosophy*. Ed. by Edward N. Zalta. Spring 2016. Metaphysics Research Lab, Stanford University.

Strasser, Christian and G. Aldo Antonelli (2018). "Non-monotonic Logic". In: *The Stanford Encyclopedia of Philosophy*. Ed. by Edward N. Zalta. Summer 2018. Metaphysics Research Lab, Stanford University.

Verheij, Bart (2000). "Henry Prakken (1997). Logical Tools for Modelling Legal Argument. A Study of Defeasible Reasoning in Law". In: *Artificial Intelligence and Law* 8.1, pp. 35–65. ISSN: 1572-8382. DOI: 10.1023/A:1008318016098. URL: https://doi.org/10.1023/A:1008318016098.

Smoothness Condition

Smoothness Condition

$\langle S, l \prec \rangle$ satisfies smoothness condition if $\forall \alpha \in \mathcal{L}$ the set $\{s \in S \mid \forall m \in l(s)\ m \vDash \alpha\}$ is smooth, i.e. every state is either minimal or is in relation to a minimal state.

Kraus, Lehmann, and Magidor 1990